EHF - 88 - 24

H. SCHÖNAUER

Painting into the Booster

INJECTION & PAINTING

H. Schönauer

CONTRIBUTORS :

Grahame Rees	RAL	Layout Lattice
Tino Pusterla + Coll.: SOLIANI G., MANCARELLA G., LEO & LEO	Padova / Lecce	Mathematical Approach
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Maria R. Masullo	—"—	Transverse Painting
Harold Butler	LANL	

TRACKING CODES USED :

LONGAD by Shane Koscielniak (TRIUMF) RAL

ACCSIM by Fred JONES and C. MACKENZIE (TRIUMF)

(Spacecharge by H. SCHÖNAUER)

Some long. painting strategies were studied by E. COLTON (LANL) with 'PAINT' code.

CHARGE EXCHANGE ('H⁻') INJECTION INTO EHF BOOSTER

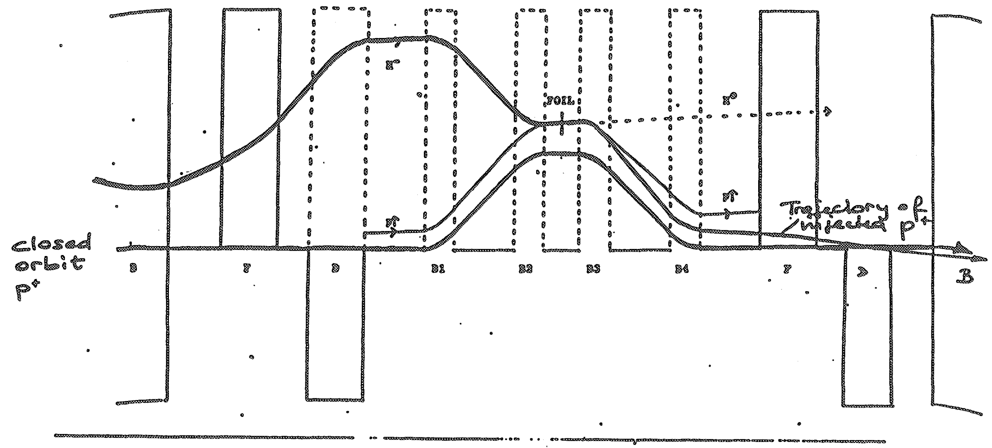
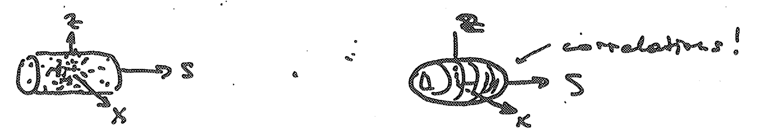


Figure 14.1: Schematic of charge exchange injection. The orbit labelled $p\uparrow$ is the closed orbit for polarized particles (B field reduced), while the orbit through the centre of the quadrupoles is for unpolarized protons.

PAINTING

Objective :

BUILD BUNCHES OF CYLINDRICAL (ELLIPSOIDAL?) SHAPE IN 3-D PHYSICAL SPACE (from ~ 400 µBunches)



(1) with minimum charge density in the centre (or elsewhere)

(2) with uniformly (regularly) (symmetry!) filled phase space

(3) which should be STATIONARY (AVOID BUNCH SHAPE OSCILLATIONS)

PAINTING

"PAINTING": Charge exchange injection as a nearly lossless technique to transform a continuous stream of linac microbunches into a number of (much larger) bunches characteristic for a circular machine.

Goal: 'Paint' a (6-dim.) phase space distribution that results in the **minimum** physical (charge) density in the centre of the bunch

Why: Space-charge detuning (inc.) is THE LIMIT to intensity at 1.2 GeV

This detuning is prop. to density:

Betatron coord. → $\ddot{y} + Q^2 \omega_{rev}^2 y = \dfrac{F_y}{m_0 \gamma}$ smooth appr. to eqn. of mot.

$$F_y = \frac{\partial F}{\partial y} y + \underline{\frac{1}{2}\frac{\partial^2 F_y}{\partial y^2} y^2} + \ldots\ldots$$

$$\doteq e\frac{\partial E}{\partial y} y \frac{1}{\gamma^2} \quad \text{near beam axis}$$

$$\operatorname{div} \vec{E} = \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} = \frac{\rho(0,0,s)}{\varepsilon_0}$$

Problem: Programmes for :

1) linac Bunch Offset in ΔE, $\Delta\phi$

2) injection orbit bumps in x, z

Space-Charge Detuning :

Maximum at peak line density :

$$\hat{\Delta Q}_{sc,V} = \frac{\hat{\lambda}}{\bar{\lambda}} \cdot \frac{N \; r_p}{\pi \; \beta^2 \gamma^3 \; \epsilon_V} \cdot F \cdot G \cdot H_V$$

ϵ_V: "physical" emittance

$$\frac{\hat{\lambda}}{\bar{\lambda}} = \frac{1}{B_f} = \frac{\text{peak line density}}{\text{average}}$$

B_f: "Bunching Factor (< 1)"

F = 1 + image contribution

G = Form Factor to describe x,y distribution = peak density / average density for 100 % emittances

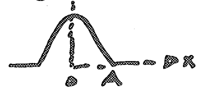
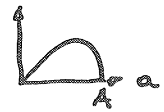
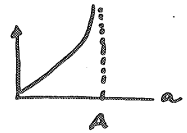
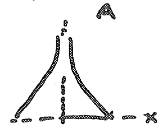
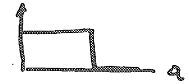
$G = 1$ for K-V beam (uniform)

H_V = Aspect Ratio $\left\langle \frac{1}{1+\frac{a}{b}} \right\rangle$

$$H_H + H_V = 1$$

LONGITUDINAL PAINTING

Some examples :

Bunch shape (projected density)	$g(x)$	↔	Amplitude distribution (painting programme) $f(a)$	
"natural" 	$A^2 - x^2$		$a\sqrt{A^2-a^2}$	
"ideal" 	$\left.\frac{1}{\theta}\right\}$		$\frac{a}{\sqrt{A^2-a^2}}$	
	$\cosh^{-1}\frac{A}{x}$		$\left.\frac{1}{\theta}\right\}$	 ↑ linear painting

↔ ... 1-dim. Abel Transform

For transformation pairs: P. Krempl, CERN MPS/Int.BR/74-1

⚠ The "ideal" bunch shape may not be that ideal because of long. instabilities (local loss of Landau damping)

— Programme for the bunch tandem less obvious !

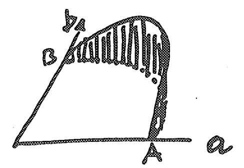
TRANSVERSE PAINTING:

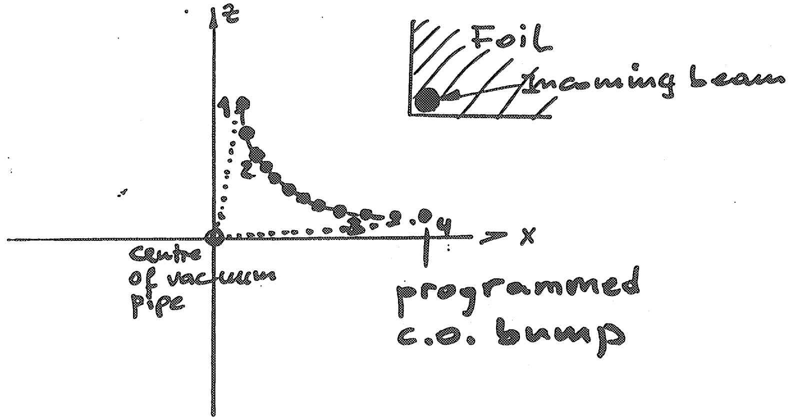
IDEAL distribution well-known:
Uniform density over elliptical cross-section (K-V distribution); linear space-charge forces, i.e. only Q-shift (no spread)

$\rho(x,y)$ ... phys. density $\Leftrightarrow$ Amplitude distr. $g(a,b)$

$\frac{1}{\pi AB}$ ... $\frac{x^2}{A^2}+\frac{z^2}{B^2} \le 1$ $\qquad \frac{8}{(AB)^2}\, ab\, \delta\left[1-\frac{a^2}{A^2}-\frac{b^2}{B^2}\right]$

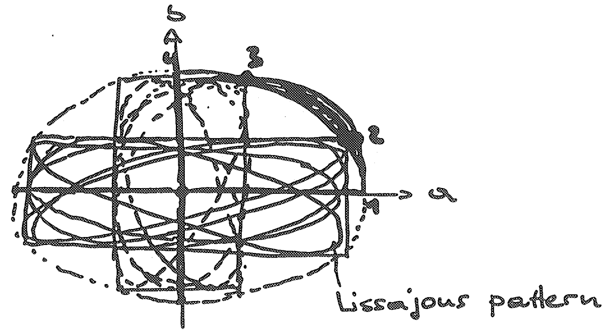
0 ... " > 1



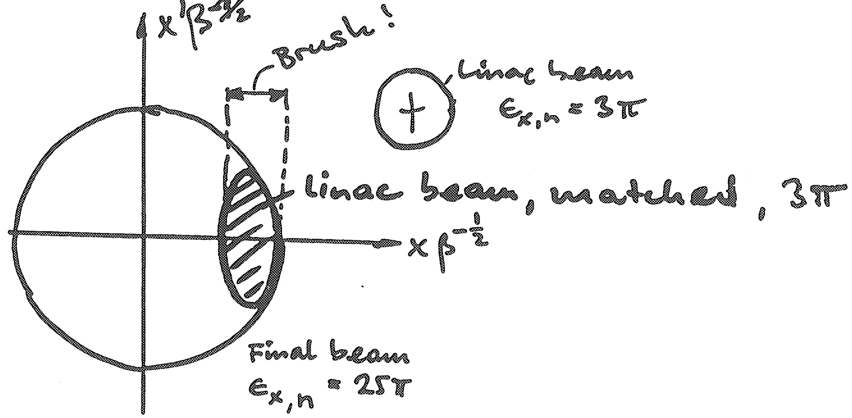


Foil lifetime: not critical for EHF

How to approach a K-V distribution:

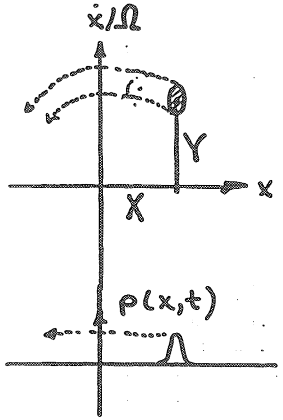


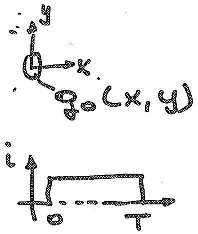
Matching of the injection line



1-dimensional Painting

GOAL: For given projected density (= bunch shape), find the trajectory for one (or two ... EMF) "BRUSH" of elliptical (or general) shape.





$g_0(x,y)$ ... Brush function

... intensity $i(t)$

Brush Trajectory ... $X(t), Y(t)$

$$g(x,y;t) = \int_{-\infty}^{t} i(t')\,dt'\, g_0\big(\underbrace{x'}_{}-X(t'),\ y'-Y(t')\big)$$

phase space density

$x'(x,y;t,t')$ "retarded coordinates"

$$x = x(x',y';t,t')$$
$$y = y(x',y';t,t')$$

Simplification: particles move on circular orbits, with $\Omega = \Omega(r)$:

$$x = r\cos\Omega(t-t'+\varphi'), \qquad x' = r\cos\varphi'$$
$$y = r\sin\Omega(t-t'+\varphi'), \qquad y' = r\sin\varphi'$$
$$r^2 = x'^2 + y'^2$$

$p(x,t) = \int dy\, g(x,y,t)$ projected density

WANTED:

1) $X(t), Y(t)$ for $p(x,t)$
Brush Trajectory

2) $p(x,t) = p(x)$ for $t > T$, T... end of injection
"Stationarity conditions"

Impossible to fulfil in the general case

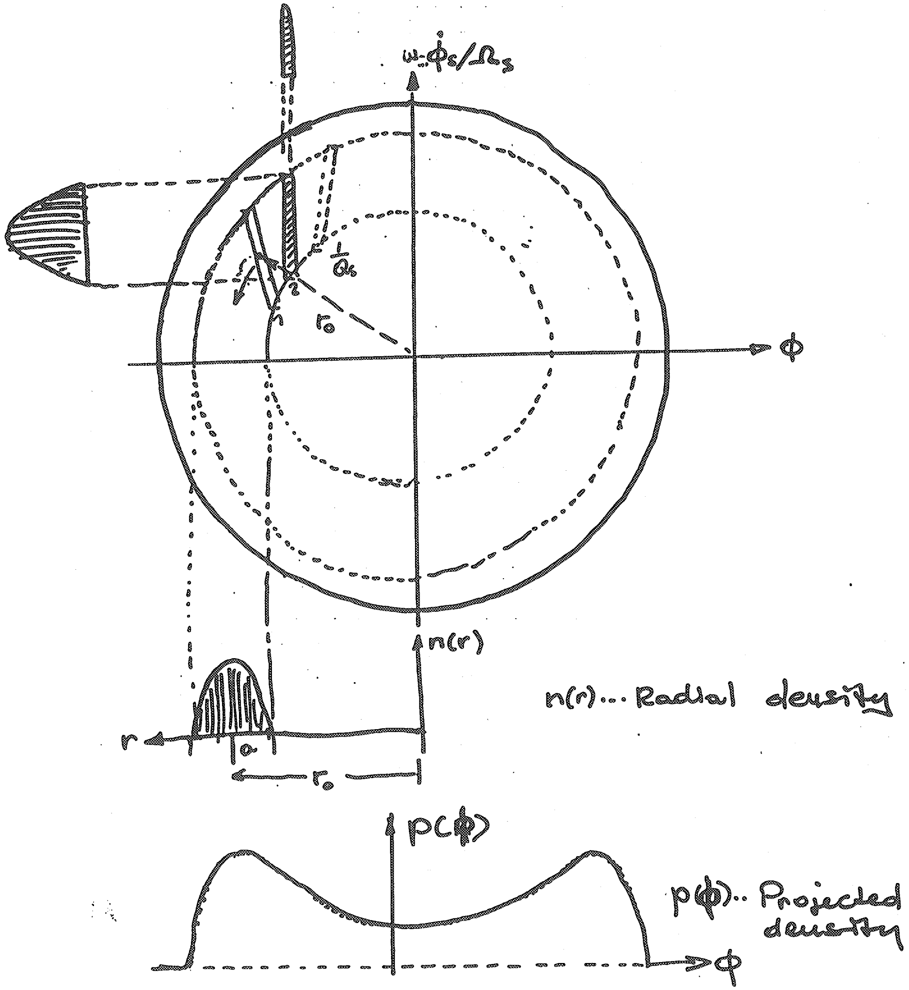
- But can be minimized in a l.m.sq. sense
- can be achieved or minimized for certain conditions → find conditions

T. PUSTERLA: $g_0(x,y) \propto e^{-\alpha(x^2+y^2)}$
$i(t) = \Sigma_k\, i_k\, \delta(t-t_k)$

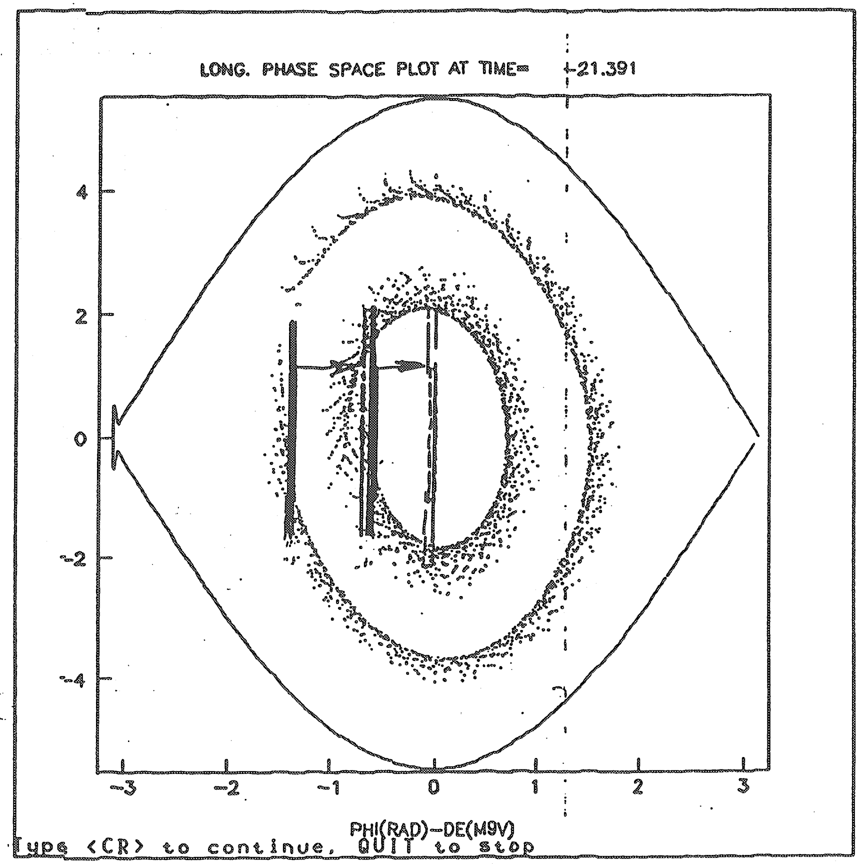
Results:
- FAIRLY HANDY ANALYTIC EXPR.
- STATIONARITY POSSIBLE WITH >1 BUNCH OR k>1 δ-pulses of i

New results presented at this workshop!
(Pusterla, Soliani et al.)

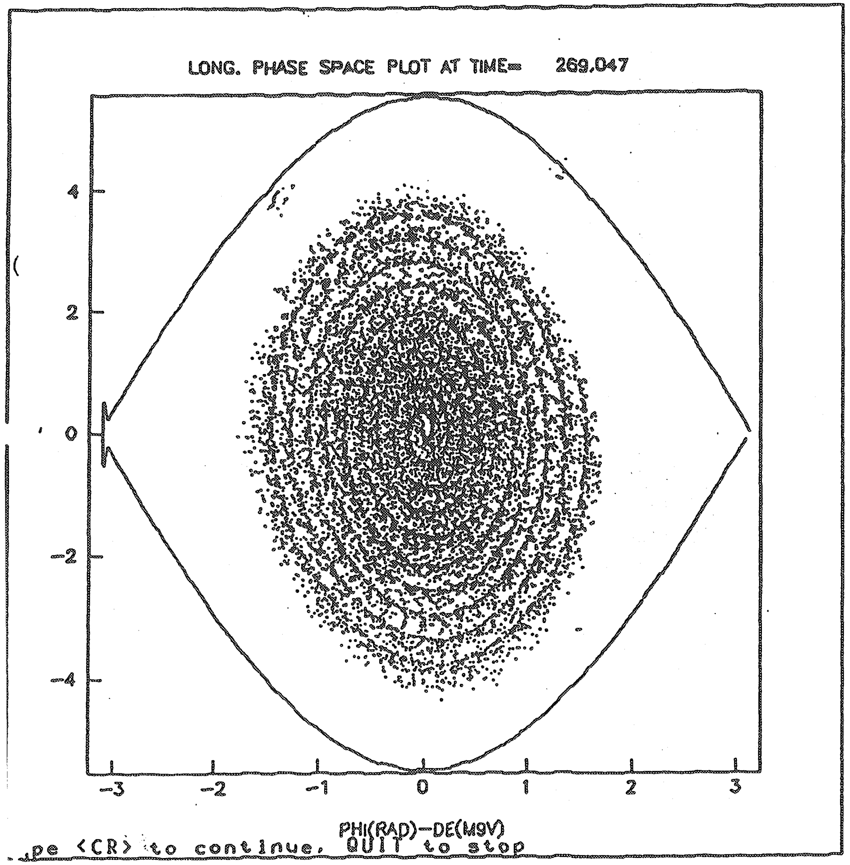
More pedestrian approach: figure how annuli are formed by small, short bunches from linac and synchrotron motion:



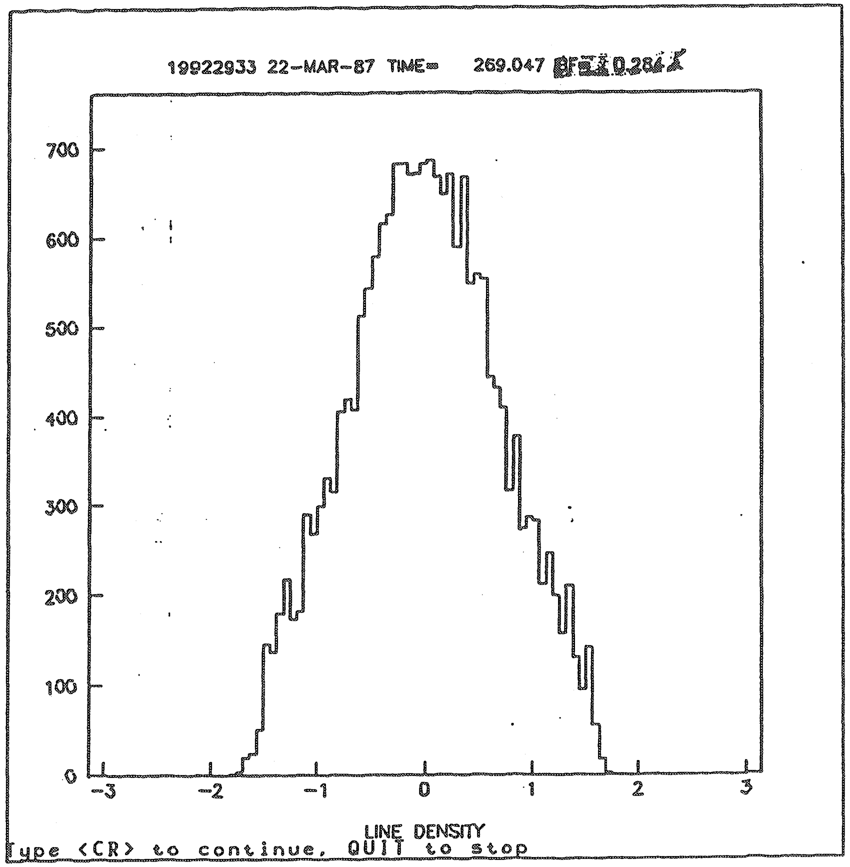
$$p(\phi) = \frac{1}{\pi}\int_{\phi}^{R} \frac{n(r)\,dr}{\sqrt{r^2-\phi^2}}$$



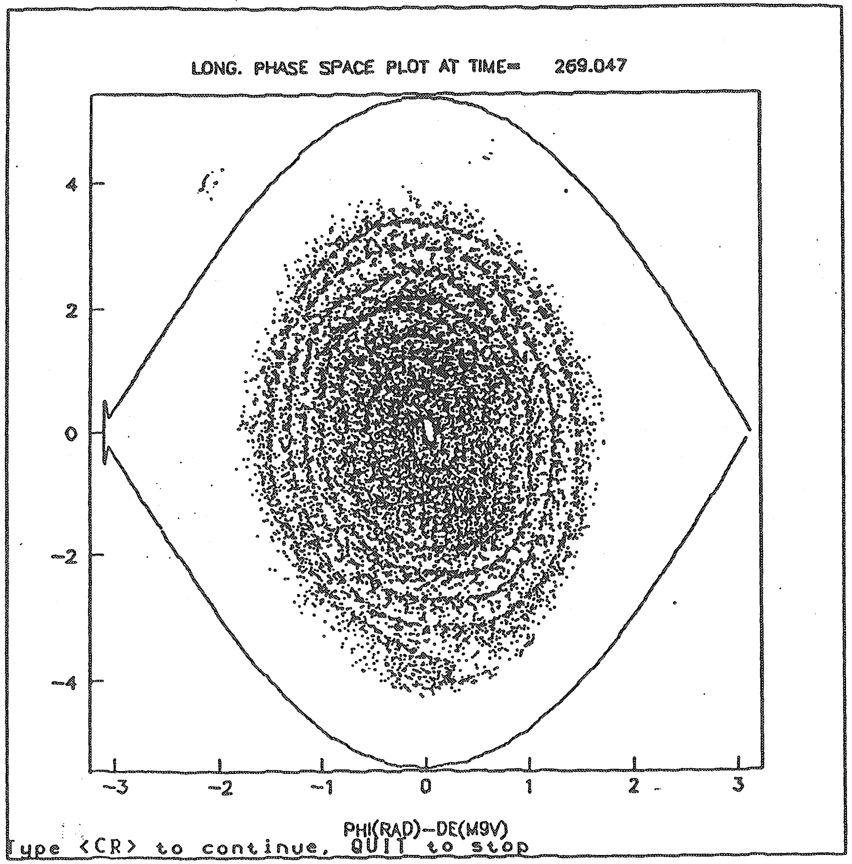
one can fill the bucket in a regular way —



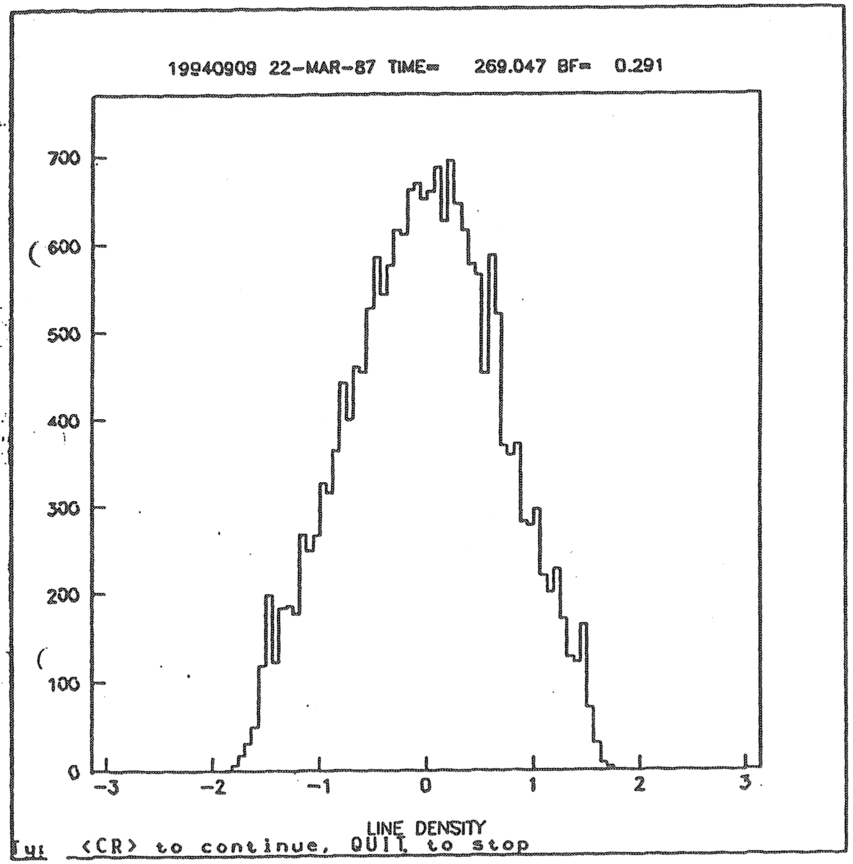
but the bunching factor may not be satisfactory!



Space charge deforms slightly the previous result

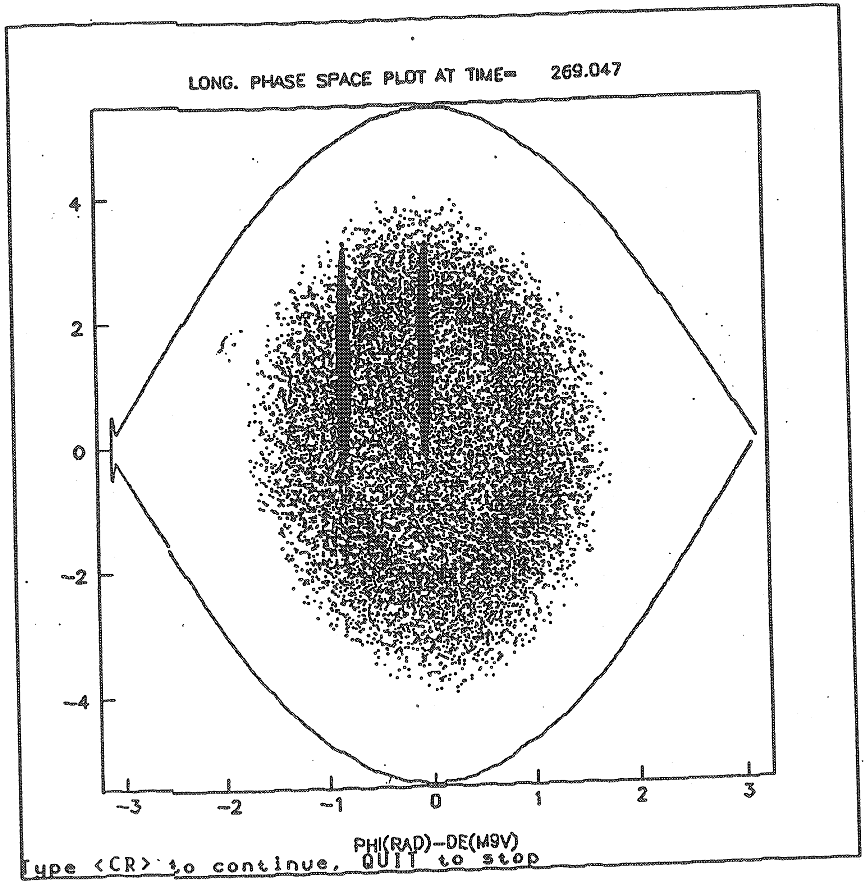


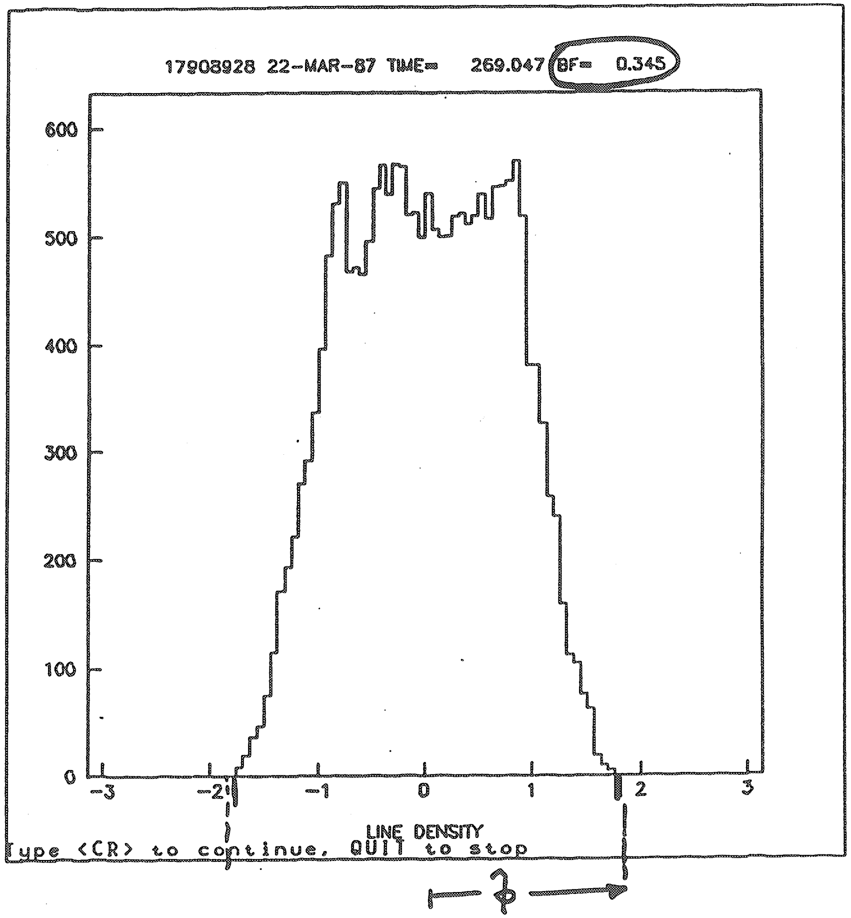
but does not affect the bunching factor.



BEST OF 'SIMPLE' PAINTING SCENARIOS

2a



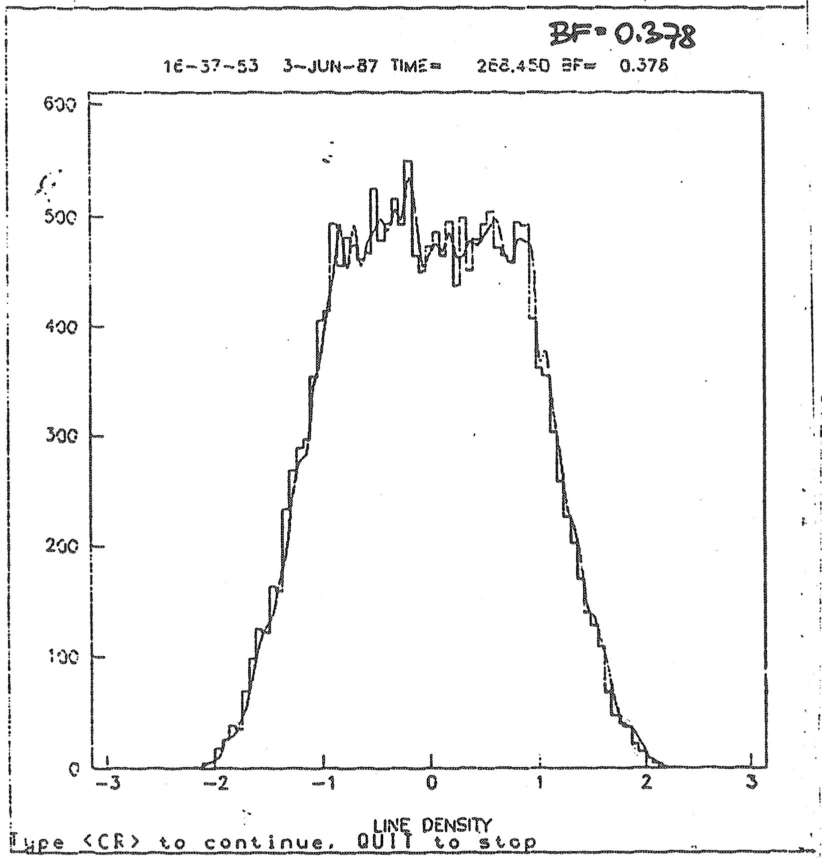


$\phi = 108° \approx 1.9$ rad

2sc

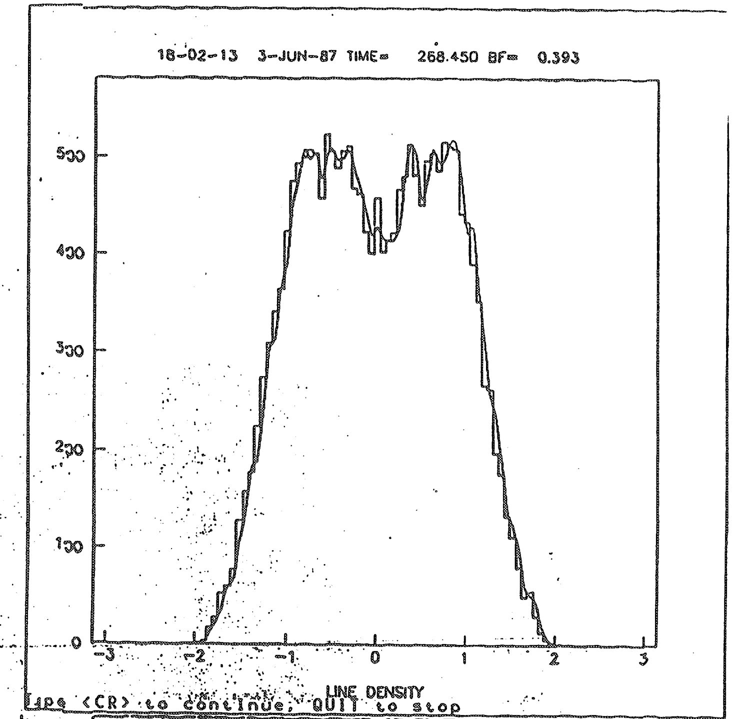
2sc

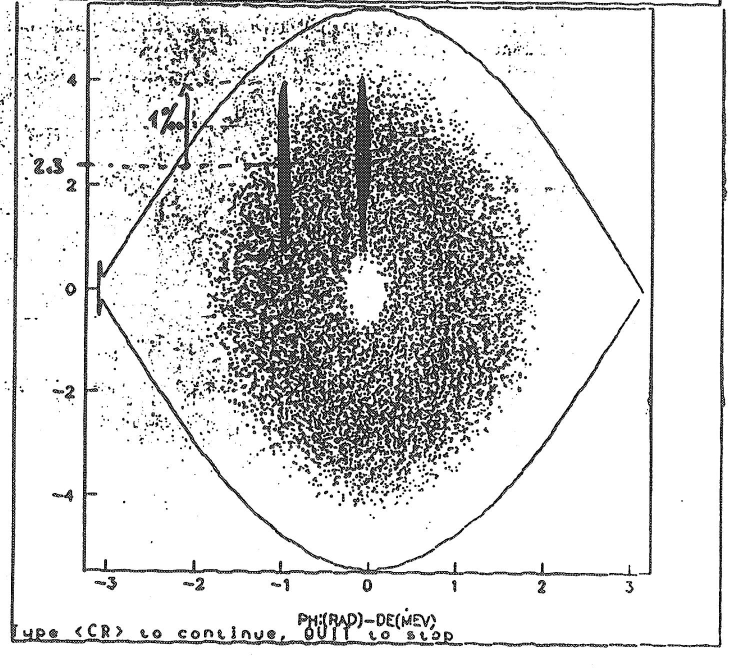
Best flat-topped distribution with
2-bunch "no-point" technique



2L

Finally adopted:
for
EHF Booster





Filling strategies for ONE BUNCH:

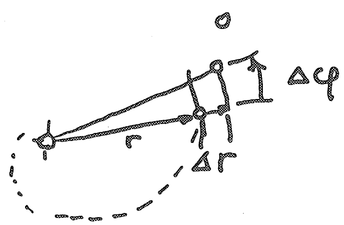
Plot points on a spiral such that their density is proportional to a prescribed rotationally symmetric density $\rho(r)$ [normalized to $2\pi\int \rho(r)\, r\, dr = 1$]

Then the increment of r between two consecutive points is

$$\Delta r = \frac{1}{N\, 2\pi r\, \rho(r)}$$

N ... # of points (bunches)

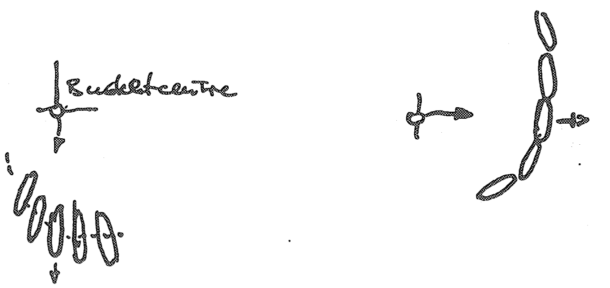
independent of $\Delta\varphi$.

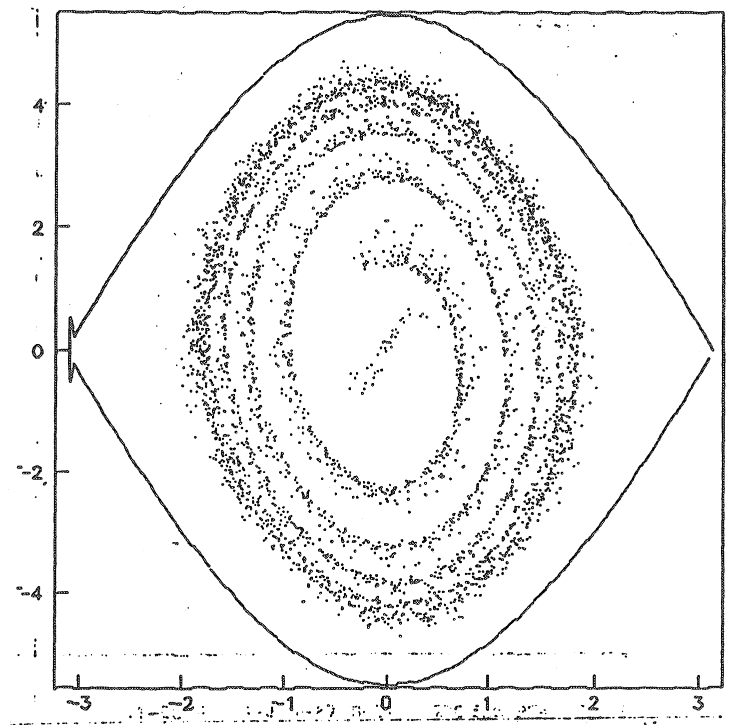


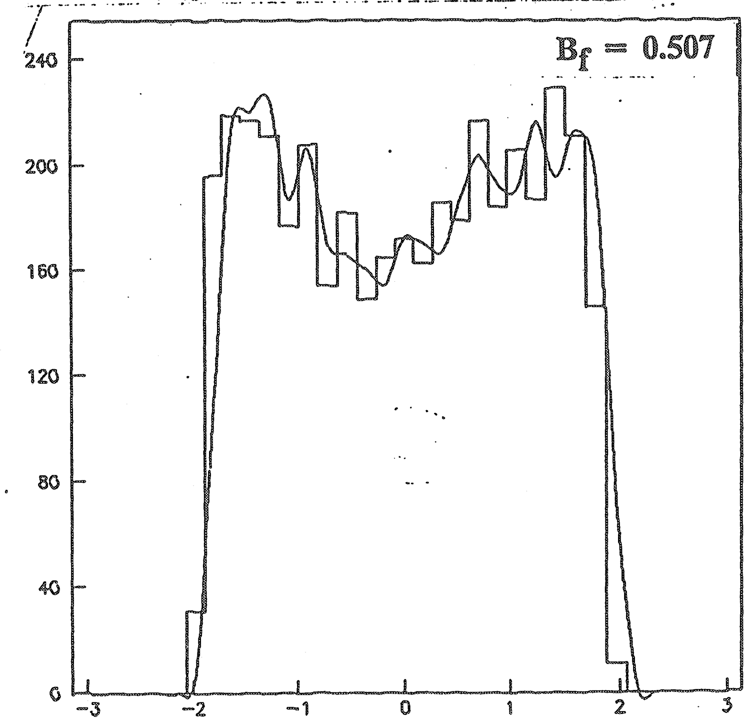
$\Delta\varphi = 2\pi Q_s \simeq \frac{2\pi}{40}$

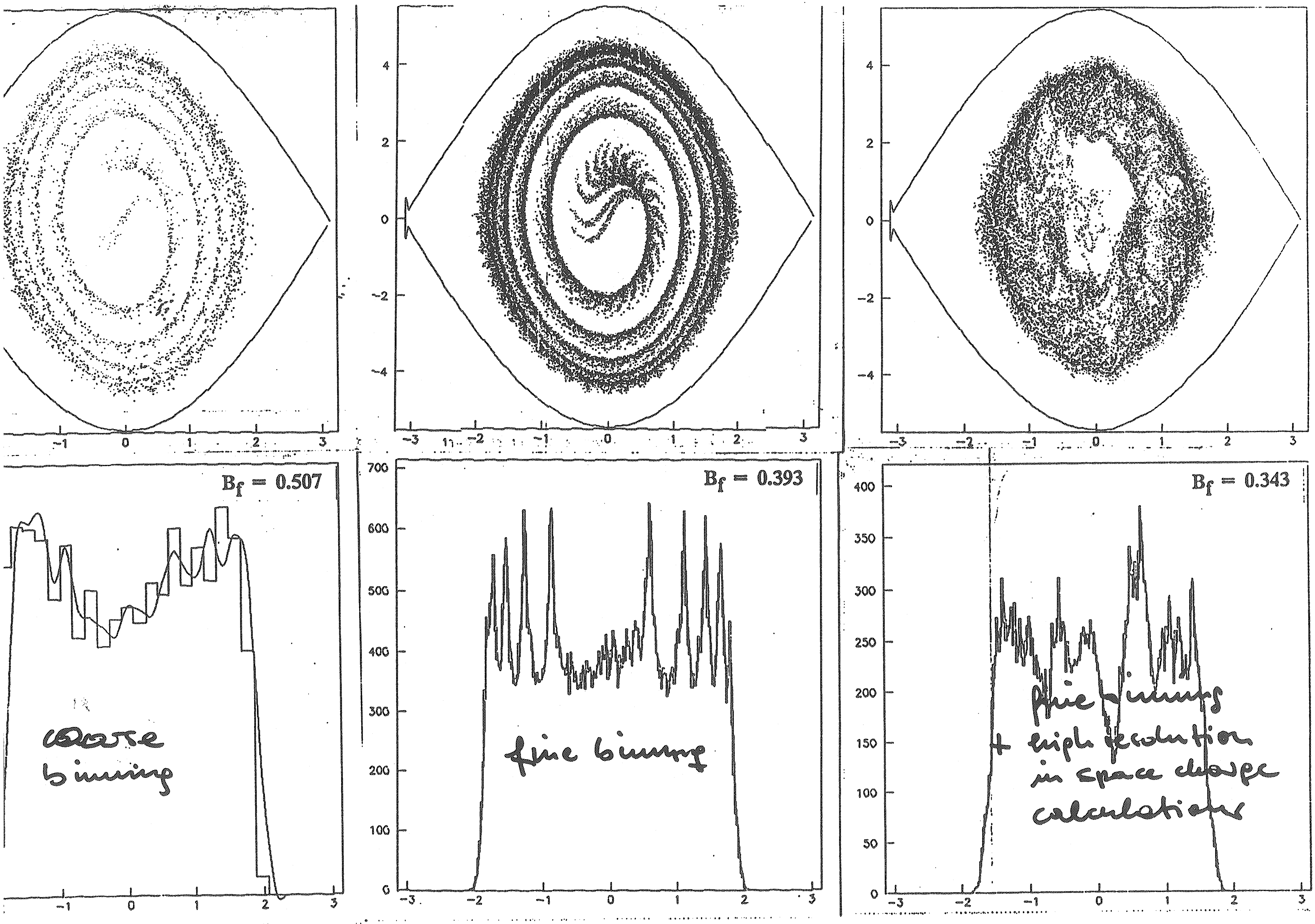
We don't plot points, but bunches like thin, upright ellipses:

There is a difference between energy ramping and phase ramping:









$B_f = 0.507$
coarse binning
$B_f = 0.393$
fine binning
$B_f = 0.343$
fine binning
+ high resolution
in space charge
calculations

resolution in sp.-ch. calculations:
where is the limit ?

# of Fourier components :

Space Charge term : $V_{sc} = \frac{\lambda}{4\pi\varepsilon_0}\left(1 + 2\ln\frac{b}{a}\right)$

Potential at beam axis in a circular pipe

$$\frac{1}{\gamma^2}E_{sc} = -\frac{g_0}{4\pi\varepsilon_0\gamma^2}\frac{\partial\lambda}{\partial s}$$

$$g_0 = 1 + 2\ln\frac{b}{a}$$

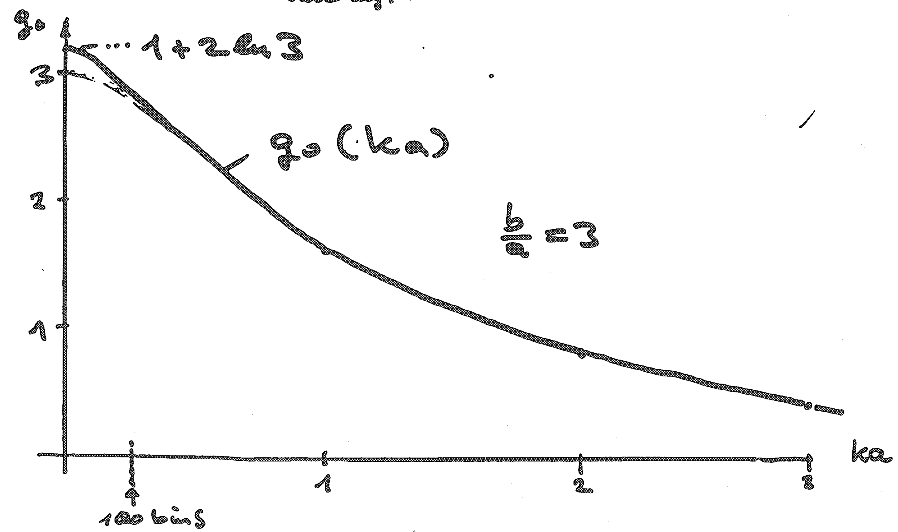
$2b$ $2a$

1) g_0 rolls off for high-frequency density modulation of $\lambda(s)$

$$g_0 = \frac{4}{x^2} - \frac{4}{x}\left[K_1(x) + I_1\frac{K_0(x\frac{b}{a})}{I_0(x\frac{b}{a})}\right]$$

$$x = ka = \frac{2\pi a}{\lambda\gamma} = \frac{a n}{R\gamma} \quad ;$$

wavelength



$$x_{max} = \frac{2\pi a}{\gamma}\frac{N_{bin}/2}{\lambda_{RF}} = \frac{N_{bin}}{2}\frac{ha}{R\gamma} = 0.27 \quad \text{for } a \sim 1\,\text{cm}$$

2) Propagation of EM waves in beam pipe

cutoff of lowest TE mode

$\lambda_c = 3.41\, b$

$$N_{bins} = 2n_c = 2\frac{\gamma\lambda_{RF}}{\lambda_c} = \frac{\gamma\, 4\pi R}{3.41\, b\, h}$$

for $b \sim 3$ cm, 1.2 GeV .. $N_{bins} \simeq 240$

... this is the dominant effect !

Increase in # of bins → less super-part. per bin

As fluctuations $\sim \sqrt{N_{super}}$:

$N_{super} \sim N_{bins}^2$

LINEAR COUPLING: BASIC FACTS

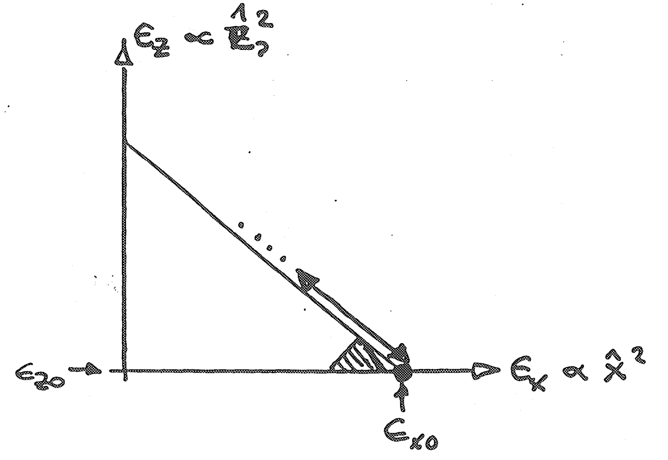
For not too strong coupling (and $\epsilon_{z0} = 0$)

$$\epsilon_x = \epsilon_{x0} \left\{ 1 - \frac{1}{1+\vartheta^2} \sin^2\left(\sqrt{1+\vartheta^2}\,|Q|\,\Omega_{rev} t \right) \right\}$$

$\vartheta = \frac{\delta}{2|Q|}$; $\delta = q_H - q_V$ ... distance from diagonal

For one SKEW Quad:
$$\begin{cases} Q = -\frac{\sqrt{\beta_H \beta_V}}{4\pi} (K_1 L)\, e^{iq} \qquad K_1 = \frac{B'}{(B\rho)} \\ q \text{ ... contains: both } \beta\text{tron phases H and V, position of skew quad} \end{cases}$$

$$\boxed{\epsilon_x + \epsilon_z = \epsilon_{x0} + \underbrace{\cancel{\epsilon_{z0}}}_{\text{here}=0} = \text{Const.}}$$



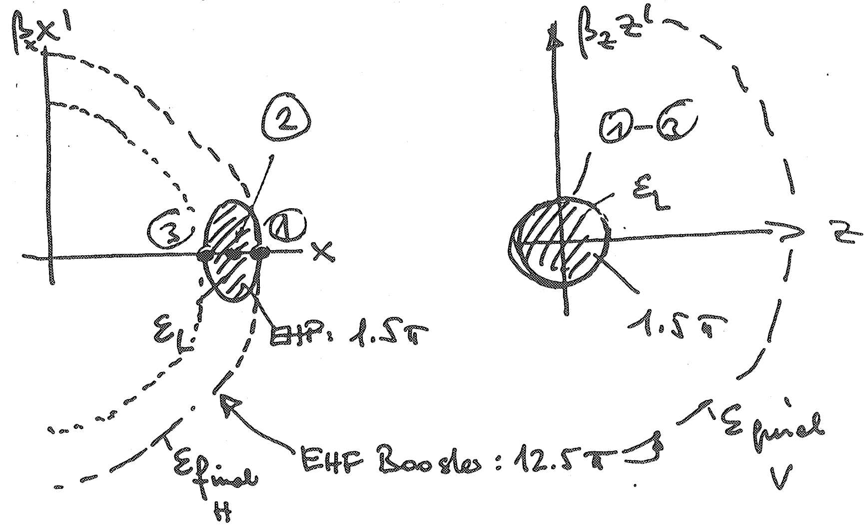
For the more general case of $\epsilon_{z0} > 0$ it's somewhat more complicated:

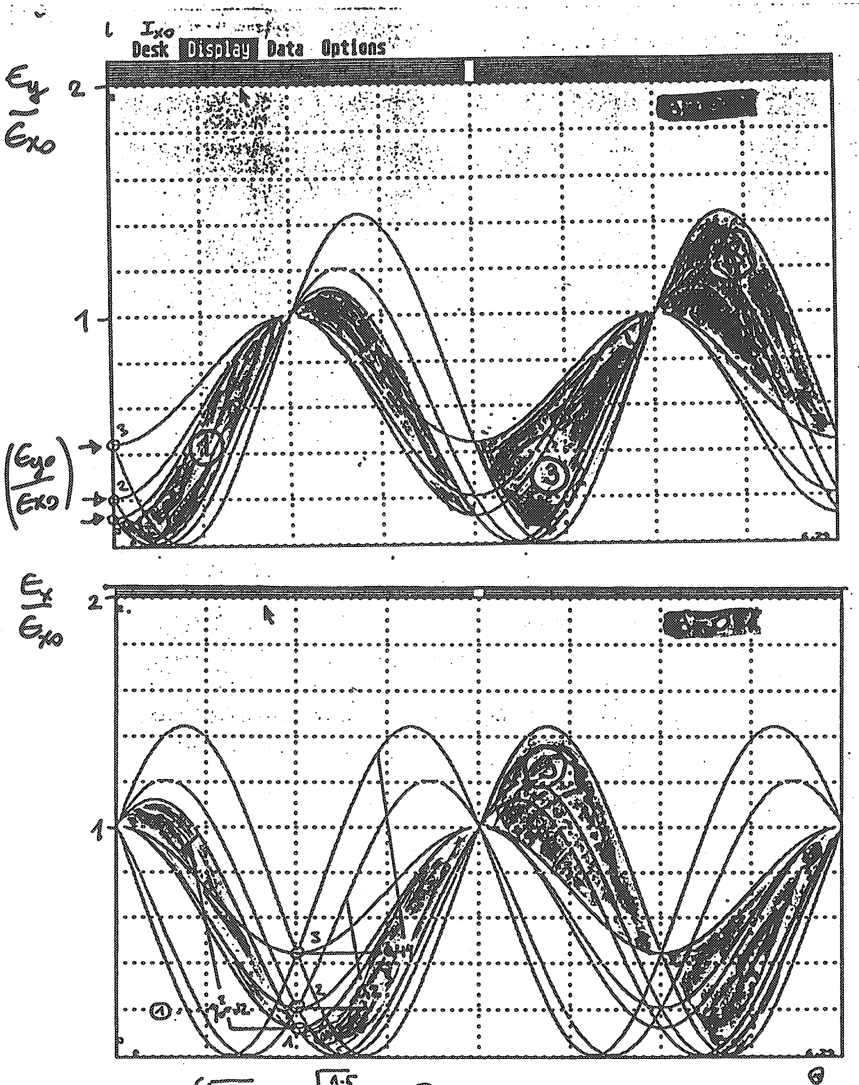
$$\epsilon_x = \epsilon_{x0} + \epsilon_{x0} \frac{1}{1+\vartheta^2} \times$$
$$\times \left\{ \left[\left(\frac{\epsilon_{z0}}{\epsilon_{x0}}\right) - 1 + 2\vartheta \left(\frac{\epsilon_{z0}}{\epsilon_{x0}}\right)^{\frac{1}{2}} \cos\phi \right] \sin \Theta + 2 \left(\frac{\epsilon_{z0}}{\epsilon_{x0}}\right)^{\frac{1}{2}} \sqrt{1+\vartheta^2} \sin\phi \quad \cos \Theta \right\}$$

$$\Theta = \sqrt{1+\vartheta^2}\,|Q|\,\Omega_{rev} t \qquad \vartheta = \frac{\delta}{2|Q|}$$

$$\boxed{\epsilon_x + \epsilon_z = \epsilon_{x0} \left(1 + \left(\frac{\epsilon_{z0}}{\epsilon_{x0}}\right) \right)}$$

We deal with something like



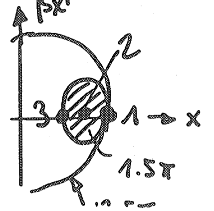


$$\eta = \begin{cases} \sqrt{0.12} = \sqrt{\frac{1.5}{12.5}} & \dots (1) \\ \sqrt{0.2} = \sqrt{\frac{1.5}{7.3}} & \dots (2) \\ \sqrt{0.44} = \sqrt{\frac{1.5}{3.4}} & \dots (3) \end{cases}$$

$\omega = Q_c \sqrt{1+\left(\frac{\delta}{2Q_c}\right)^2}\,\theta$

for $\frac{\delta}{2} \gg Q_c$

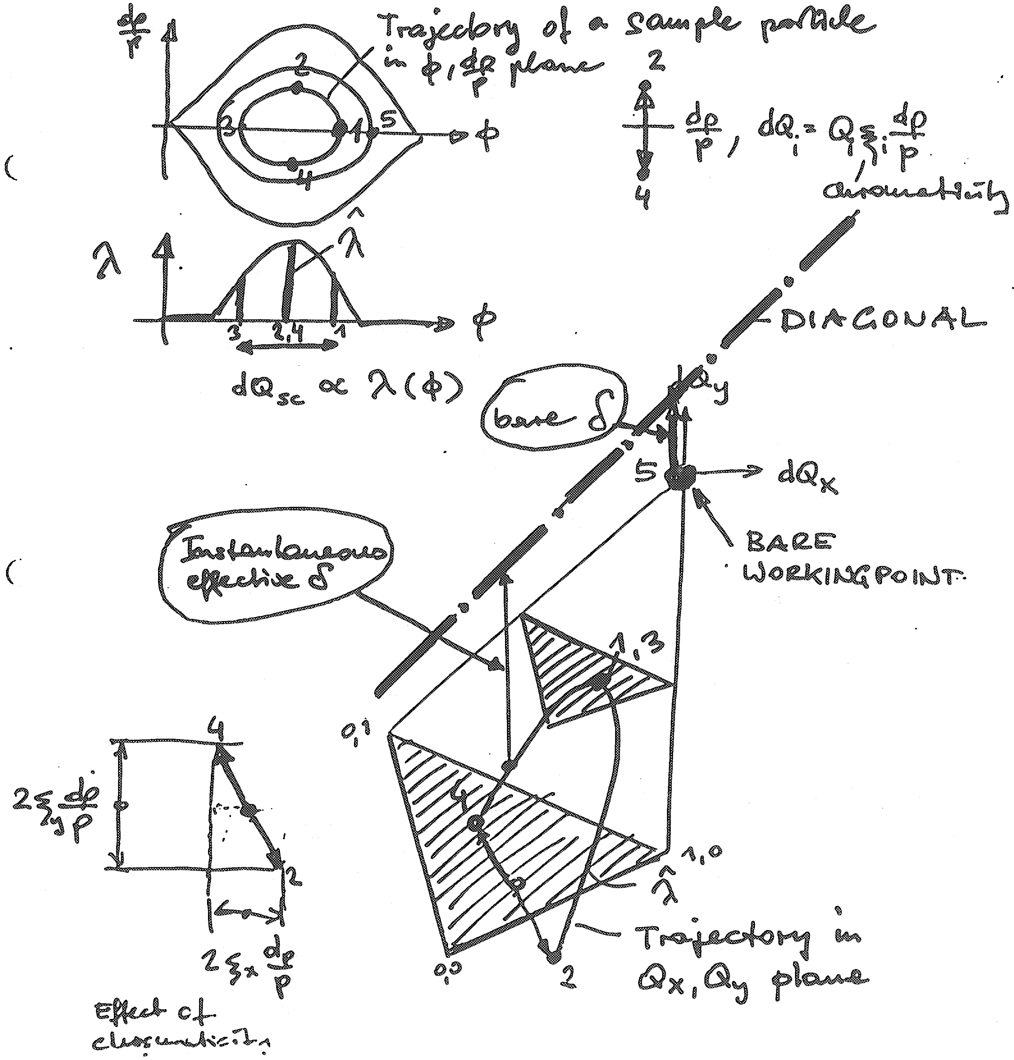
$\omega \approx \frac{\delta}{2}\Theta$



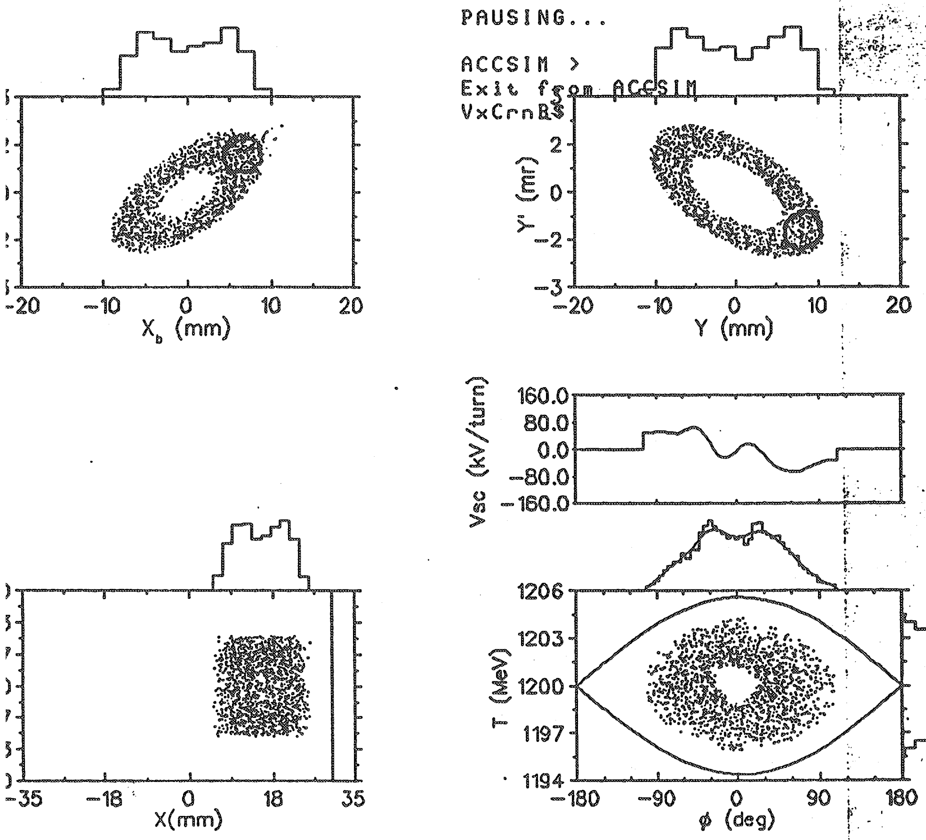
The actual evolution of an ensemble of beating particles is hard to predict

Remember : Crucial parameter $\vartheta = \frac{\delta}{2|Q|}$

$\vartheta \propto \delta$ varies with $2\omega_s$ and amplitude.



NO COUPLING



WITH COUPLING