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Neutrino masses from $U(1)$ symmetries and the Super-Kamiokande data

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Abstract

Motivated by the Super-Kamiokande data, we revisit models with $U(1)$ symmetries and discuss the origin of neutrino masses and mixings in such theories. We show that, in models with just three light neutrinos and a hierarchy of neutrino masses, large (2-3) mixing fixes the lepton doublet $U(1)$ charges and is thus related to the structure of the charged lepton mass matrix. We discuss the fermion mass structure that follows from the Abelian family symmetry with an extended gauge group. Requiring that the quark and lepton masses be ordered by the family symmetry, we identify the most promising scheme. This requires large, but not necessarily maximal, mixing in the $\mu\tau$ sector and gives $e\mu$ mixing in the range that is required for the small angle solution of the solar neutrino deficit.

1 Introduction

Recent reports by the Super-Kamiokande collaboration [1], indicate that the number of ν_μ in the atmosphere is decreasing, due to neutrino oscillations. These reports seem to be supported by the recent findings of other experiments [2], as well as by previous observations [3]. The data indicates that the number of ν_μ is almost half of the expected number, while the number of ν_e is consistent with the expectations. $\nu_\mu - \nu_\tau$ oscillations, with

$$\delta m^2_{\nu_\mu\nu_\tau} \approx (10^{-2} \text{ to } 10^{-3}) \text{ eV}^2 \quad (1)$$

$$sin^2 2\theta_{\mu\tau} \geq 0.8 \quad (2)$$

match the data very well, while dominant $\nu_\mu \to \nu_e$ oscillations are disfavoured by Super-Kamiokande [1] and CHOOZ [4].

On the other hand, the solar neutrino puzzle can be resolved through matter enhanced oscillations [5] with either a small mixing angle:

$$\delta m^2_{\nu_e\nu_\alpha} \approx (3-10)\times 10^{-6}\ \mathrm{eV}^2 \tag{3}$$
$$sin^2 2\theta_{\alpha e} \approx (0.4-1.3)\times 10^{-2} \tag{4}$$

or a large mixing angle:

$$\delta m^2_{\nu_e\nu_\alpha} \approx (1-20)\times 10^{-5}\ \mathrm{eV}^2 \tag{5}$$
$$sin^2 2\theta_{\alpha e} \approx (0.5-0.9) \tag{6}$$

or vacuum oscillations:

$$\delta m^2_{\nu_e\nu_\alpha} \approx (0.5-1.1)\times 10^{-10}\ \mathrm{eV}^2 \tag{7}$$
$$sin^2 2\theta_{\alpha e} \geq 0.67 \tag{8}$$

where α is μ or τ[1].

If neutrinos were to provide a hot dark matter component, then the heavier neutrino(s) should have mass in the range $\sim (1-6)$ eV, where the precise value depends on the number of neutrinos that have masses of this order of magnitude [7]. Of course, this requirement is not as acute, since there are many alternative ways to reproduce the observed scaling of the density fluctuations in the universe.

Finally, let us note that there is another indication of neutrino mass. The collaboration using the Liquid Scintillator Neutrino Detector at Los Alamos (LSND) has reported evidence for the appearance of $\bar{\nu}_\mu - \bar{\nu}_e$ [8] and $\nu_\mu - \nu_e$ oscillations [9]. Interpretation of the LSND data favours the choice

$$\begin{aligned} 0.2\,\mathrm{eV}^2 &\leq \delta m^2 \leq 10\,\mathrm{eV}^2 \\ 0.002 &\leq \sin^2 2\theta \leq 0.03 \end{aligned} \tag{9}$$

The experiment KARMEN 2 [10] (the second accelerator experiment at medium energies) is also sensitive to this region of parameter space and restricts the allowed values to a relatively small subset of the above region.

The implications of these measurements are very exciting, for non-zero neutrino mass means a departure from the Standard Model and neutrino oscillations indicate violation of lepton family number, again lying beyond the Standard Model. The first question that needs to be answered, is why are the neutrino masses so small. In this paper we will follow what we believe to be the most promising explanation, namely that neutrino masses are small due to the "see-saw" mechanism [11] in which the light neutrino are suppressed by a very large scale associated with

[1]Best fit regions for solutions to the solar neutrino deficit have been identified in [6] .

the onset of new (unified?) physics. The "see-saw" mechanism follows naturally in the case that right-handed neutrinos exist.

Suppose that there is no weak isospin 1 Higgs field and hence there are no mass terms of the $\nu_L \nu_L$ type. In this case there are two possible neutrino masses

$$m_{Dirac}\overline{\nu_L}\nu_R + M_{Majorana}\nu_R\nu_R \equiv \begin{pmatrix} \overline{\nu_L} & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix} \begin{pmatrix} \overline{\nu_L} \\ \nu_R \end{pmatrix} \tag{10}$$

The Dirac mass is similar to an up quark mass and one's naive expectation is that they should have similar magnitude. On the other hand, the Majorana mass term is invariant under the Standard Model gauge group and does not require a stage of electroweak breaking to generate it. For this reason, one expects the Majorana mass to be much larger than the electroweak breaking scale, perhaps as large as the scale of the new physics beyond the Standard Model; for example, the Grand Unified scale or even the Planck scale. Diagonalising the mass matrix gives the eigenvalues

$$\begin{aligned} m_{Heavy} &\simeq M_M \\ m_{Light} &\simeq \frac{m_D^2}{M_M} \end{aligned} \tag{11}$$

The see-saw mechanism generates an effective Majorana mass for the light neutrino (predominantly ν_L) by mixing with the heavy state (predominantly ν_R) of mass M_M. It is driven by an effective Higgs $\Phi^{I_W=1}$ made up of $H^{I_W=1/2}H^{I_W=1/2}/M_M$ (hence the two factors of m_D in eq.(11)). eq.(11) shows that a large scale for the Majorana mass gives a very light neutrino. For example with $M_M = 10^{16}$ GeV and m_D taken to be the top quark mass gives

$$m_{Light} \simeq 3.10^{-3} \text{ eV}$$

This estimate shows that it is quite natural to have neutrinos in a mass range appropriate to give, for example, solar neutrino oscillations. However, in many cases, larger masses capable of explaining the other oscillation phenomena are possible because the Majorana mass for the right handed neutrinos is often smaller than the Grand Unified mass. A Majorana mass for the right-handed neutrino requires a Higgs carrying right-handed isospin 1 (in analogy with the left-handed case when it needed left-handed isospin 1). If this field is not present (for example in level one string theory this is always the case) one may get a double see-saw because the Majorana mass for the right-handed neutrino is also generated by an effective Higgs, made up of $H^{I_{W,R}=1/2}H^{I_{W,R}=1/2}/M'$, where M' denotes a scale of physics beyond the Grand Unification scale. Taking this to be the Planck scale (probably the largest reasonable possibility) and $< H^{I_{W,R}=1/2} >$ to be the Grand Unified scale (it breaks any Grand Unified group) one finds

$$M_M \simeq \frac{(10^{16})^2}{10^{19}} \text{ GeV}$$

giving

$$m_{Light} \simeq 1 \text{ eV}$$

Thus, one sees that the see saw mechanism naturally gives neutrino masses in the range relevant to neutrino oscillation measurements. Moreover, as the neutrino mass is proportional to the Dirac mass squared, taking the Dirac mass of each family of neutrinos to be of the order of the equivalent up quark mass, one obtains a large hierarchy between different families of neutrino. This is what is required if one is to explain several oscillation phenomena, for it allows the existence of several mass differences.

In what follows, we will concentrate on the possibility that there is a minimal extension of the Standard Model involving just three new right-handed neutrino states and that the mass structure of the neutrinos is intimately related to that of the charged leptons and quarks. This implies that the three different indications for neutrino oscillations discussed above cannot be simultaneously explained, because three neutrino masses allow only two independent mass differences. To explain all three observations requires another (sterile) light neutrino state. However, introducing such a state breaks any simple connection between neutrino masses and those of the other Standard Model states and here we wish to explore whether the apparently complex pattern of quark and lepton masses and mixing angles can be simply understood. In this, while the structure of the see-saw mechanism leads naturally to light neutrino masses in a physically interesting range, it does not by itself explain the pattern of neutrino masses and mixing angles.

To go further requires some family symmetry capable of relating the masses of different generations. The recent Super-Kamiokande measurements have triggered a large amount of work studying the implications for neutrino masses in extensions of the fundamental theory [12]. Actually, the origin of fermion masses and mixing angles, including those of neutrinos, has been studied in numerous publications [13, 14, 15]. An obvious possibility is that the various hierarchies arise due to some symmetry at a higher scale. An indication that additional symmetries exist, has been provided by the observation that the fermion mixing angles and masses have values consistent with the appearance of “texture” zeros in the mass matrices. In this framework, the predictions for neutrino textures in models have been studied in [16, 17, 18, 19]. In many cases, a large mixing angle is not easy to reproduce, principally because of the constrained form of the Dirac mass matrices [17]. However, in certain cases, the Dirac sector may lead naturally to such a large mixing, as we showed in [18] (similar conclusions were recently discussed in [20]). Here, we revisit these models in the light of the recent results and study the expected predictions in more detail. In order to avoid the hierarchy problem that is associated with the large mass scale necessary for the see-saw mechanism, we assume the Standard Model descends from a supersymmetric theory with a low scale of supersymmetry breaking.

2 Family symmetry and hierarchical quark and lepton masses

It has been observed [13, 14] that the hierarchical structure for the fermion mass matrices strongly suggests it originates from a spontaneously broken family symmetry. In this approach, when the family symmetry is exact, only the third generation will be massive corresponding to only the (3,3) entry of the mass matrix being non-zero. When the symmetry is spontaneously broken, the zero elements are filled in at a level determined by the symmetry. Suppose a field θ which transforms non-trivially under the family symmetry acquires a vacuum expectation value, thus spontaneously breaking the family symmetry. The zero elements in the mass matrix will now become non-zero at some order in $<\theta>$. If only the 2-3 and 3-2 elements are allowed by the symmetry at order θ/M, where M is a mass scale to be determined, then a second fermion mass will be generated at $O((\theta/M)^2)$. In this way one may build up an hierarchy of masses.

$$\mathcal{M} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & <\theta>/M \\ 0 & <\theta>/M & 1 \end{pmatrix} \tag{12}$$

How do these elements at $O(\theta/M)$ arise? A widely studied approach communicates symmetry breaking via an extension of the "see-saw" mechanism mixing light to heavy states - in this context it is known as the Froggatt Nielsen mechanism [13]. To illustrate the mechanism, suppose there is a vector-like pair of quark states X and $\overline{X}$ with mass M and carrying the same Standard Model quantum numbers as the c_R quark, but transforming differently under the family symmetry, so that the Yukawa coupling $h\overline{c_L}XH$ is allowed. Here H is the Standard Model Higgs responsible for giving up quarks a mass. When H acquires a vacuum expectation value (vev), there will be mixing between $\overline{c_L}$ and $\overline{X}$. If in addition there is a gauge singlet field θ transforming non-trivially under the family symmetry so that the coupling $h'\overline{X}c_R\theta$ is allowed, then the mixing with heavy states will generate the mass matrix.

$$\begin{pmatrix} \overline{c_L} & \overline{X} \end{pmatrix} \begin{pmatrix} 0 & h<H> \\ h'<\theta> & M \end{pmatrix} \begin{pmatrix} c_R \\ X \end{pmatrix}$$

Diagonalising this gives a see-saw mass formula

$$m_c \simeq \frac{hh'<H><\theta>}{M} \tag{13}$$

This mass arises through mixing of the light with heavy quarks.

A similar mechanism can generate the mass through mixing of the light Higgs with heavy Higgs states. Suppose H_X, $\overline{H}_X$ are Higgs doublets with mass M. If H_X has family quantum numbers allowing the coupling $H\overline{H}_X\theta$, there will be mixing between H and H_X. If the family symmetry

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_2	H_1
$U(1)_{FD}$	α_i	β_i	γ_i	b_i	c_i	d_i	$-\alpha_3-\beta_3$	$-\alpha_3-\gamma_3$

Table 1: $U(1)_{FD}$ charges

also allows the coupling $\overline{c_L}c_RH_X$, the light-heavy Higgs mixing induces a mass for the charm quark of the form given in eq.(13).

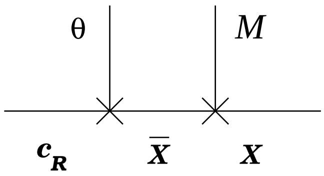

Figure 1: Generation of non-renormalisable operators through quark mixing

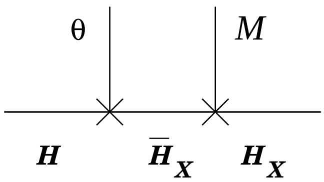

Figure 2: Generation of non-renormalisable operators through Higgs mixing

2.1 Abelian family symmetry

How difficult is it to find a family symmetry capable of generating an acceptable fermion mass matrix? The surprising answer is “Not at all difficult” and the simplest possibility using an Abelian family symmetry group [15, 21] works very well.

As a bonus, such symmetries can also give texture zeros simultaneously in the (1,1) and (1,3) positions, generating good predictions relating masses and mixing angles [15]. The basic idea is that the structure of the mass matrices is determined by a flavour dependent family symmetry, $U(1)_{FD}$. The most general charge assignments of the various states under this symmetry are given in Table 1. If the light Higgs, H_2, H_1, that generate the up-quark (Dirac neutrino) and down-quark (charged lepton) masses respectively, have $U(1)$ charge so that only the (3,3) renormalisable Yukawa coupling to H_2, H_1 is allowed, then only the (3,3) element of the associated mass matrix will be non-zero. The remaining entries are generated when the $U(1)$ symmetry is broken. This breaking is taken to be spontaneous via Standard Model singlet fields carrying family charge acquiring vacuum expectation values (vevs). For example the fields, θ, $\bar{\theta}$, with

$U(1)_{FD}$ charge -1, +1 respectively [2], may acquire equal vevs along $D-$flat directions. After this breaking, the structure of the mass matrices is generated. Let us discuss, as an example, the origin of the (3,2) entry in the up quark mass matrix. This appears at order $\epsilon^{|\alpha_2-\alpha_3|}$ because U(1) charge conservation allows only the non-renormalisable operator $c^c t H_2(\theta/M_2)^{\beta_2-\alpha_3},\ \beta_2 > \alpha_3$ or $c^c t H_2(\bar{\theta}/M_2)^{\alpha_3-\beta_2},\ \alpha_3 > \beta_2$. Here $\epsilon = (<\theta> /M_2)$ and M_2 is the unification mass scale which governs the higher dimension operators. As we discussed above this is most likely to be the mass of the heavy quark or heavy Higgs which, on spontaneous breaking of the family symmetry, mixes with the light states.

2.2 Abelian family symmetry and large lepton mixing

Let us consider in more detail the 2×2 heavier sector of the theory, relevant to the atmospheric neutrino oscillations for the case only one mass squared difference contributes. The charged lepton matrix constrained by the U(1) family symmetry has the form

$$\frac{\mathcal{M}_\ell}{m_\tau} = \begin{pmatrix} c\left(\frac{\theta}{M}\right)^{q_L}\left(\frac{\theta}{M}\right)^{q_R} & a\left(\frac{\theta}{M}\right)^{q_L} \\ b\left(\frac{\theta}{M}\right)^{q_R} & 1 \end{pmatrix} \tag{14}$$

where the origin of the intermediate mass scale, M, will be discussed shortly. The parameters $a,\ b,\ c$ are constants of O(1), reflecting the unknown Yukawa couplings and $q_L = b_2 - b_3$, $q_R = c_2 - c_3$[3].

It is instructive to write this in the form

$$\frac{\mathcal{M}_\ell}{m_\tau} = E\left((\frac{\theta}{M})^{q_L}\right)\cdot A\cdot E\left((\frac{\theta}{M})^{q_R}\right) \tag{15}$$

where

$$E(x) = \begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix},\quad A = \begin{pmatrix} c & a \\ b & 1 \end{pmatrix}, \tag{16}$$

The matrix A is determined by the Yukawa couplings only. If the only symmetry restricting the form of the mass matrices is the Abelian family symmetry there is no reason to expect correlations between the elements of A and so we expect $Det(A) = O(1)$. This is the situation we will explore in this paper. Given this we may see that $\mathcal{M}_\ell^D$ has the form

$$\mathcal{M}_\ell = V_{\ell L}\cdot\mathcal{M}_{\ell,Diagonal}\cdot V_{\ell R}^T \tag{17}$$

[2]In some models only fields with one sign of family charge acquire vevs. In this case holomorphic zeros may occur in the mass matrices as is discussed below.

[3]Here, for simplicity, we assume b_i, c_i, q_L, q_R are all positive. The analysis also applies without this restriction for the case $<\theta>\approx<\overline{\theta}>$. We will discuss what happens when these conditions are not satisfied later.

where

$$\frac{\mathcal{M}_{\ell,Diagonal}}{m_\tau} = \begin{pmatrix} r(\frac{\theta}{M})^{q_L}(\frac{\theta}{M})^{q_R} & 0 \\ 0 & 1 \end{pmatrix} \tag{18}$$

and

$$V_{\ell L} = V\left(r'(\frac{\theta}{M})^{q_L}\right), \quad V_{\ell R} = V\left(r''(\frac{\theta}{M})^{q_R}\right) \tag{19}$$

with $r,\ r', r'' = O(1)$ and $V(x) = \begin{pmatrix} 1 & x \\ -x & 1 \end{pmatrix}$. The lepton analogue [22] of the CKM mixing matrix for quarks is given by

$$V_{MNS} \approx V_{\nu L}^{\dagger} \cdot V_{\ell L} \tag{20}$$

The important point to note is that the left-handed lepton mixing matrix contribution is determined entirely by the *left-handed* lepton doublet family symmetry charges while the eigenvalues are determined by both the left-handed and right-handed charges.

A similar analysis may be applied to the neutrino sector. We have

$$\begin{aligned} \mathcal{M}_\nu^{effective} &= \mathcal{M}_\nu^D \cdot (\mathcal{M}_\nu^M)^{-1} \cdot \mathcal{M}_\nu^{DT} \\ &= (V_{\nu L} \cdot \mathcal{M}_{\nu,Diagonal}^D \cdot V_{\nu R}^{(D)T}) \cdot (\mathcal{M}_\nu^M)^{-1} \cdot (V_{\nu L} \cdot \mathcal{M}_{\nu,Diagonal}^D \cdot V_{\nu R}^{(D)T})^T \\ &\equiv V_{\nu L} \cdot V_{\nu R} \cdot \mathcal{M}_{\nu,Diagonal}^{effective} \cdot V_{\nu R}^T \cdot V_{\nu L}^T \end{aligned} \tag{21}$$

We see that there are two contributions to V_{MNS} in eq.(20) coming from the neutrino sector. The first is

$$V_{\nu L} = V\left(s(\frac{\theta}{M'})^{q_L}\right) \tag{22}$$

where $s = O(1)$ and we have allowed for a different intermediate scale M' (see below). It is determined by the same *left-handed* lepton doublet family symmetry charges that determine $V_{\ell L}$.

The second contribution, $V_{\nu R}$, is sensitive to the right-handed neutrino family charges. However in the case the light neutrinos have a hierarchical mass pattern (necessary if we are to explain both the atmospheric and solar oscillations) this contribution cannot be large. To see this note that if the elements of $V_{\nu R}$ are all of $O(1)$ and one neutrino mass, m_1, dominates then the elements of the matrix $V_{\nu R} \cdot \mathcal{M}_{\nu,Diagonal}^{effective} \cdot V_{\nu R}^T$ are all of $O(m_1)$ but its determinant is $<< O(m_1^2)$. This matrix is also given by $(\mathcal{M}_{\nu,Diagonal}^D \cdot V_{\nu R}^{(D)T}) \cdot (\mathcal{M}_\nu^M)^{-1} \cdot (\mathcal{M}_{\nu,Diagonal}^D \cdot V_{\nu R}^{(D)T})^T$. As discussed above, the Abelian family symmetry cannot give correlations between the Yukawa couplings determining different matrix elements of $\mathcal{M}_{\nu,Diagonal}^D$ and $\mathcal{M}_\nu^M$. Thus, its determinant cannot be of a different order than the product of its diagonal elements, in contradiction with the conclusion that follows if the neutrinos are hierarchical in mass. The implication is that large mixing can only come from the right-handed neutrino sector if there are two nearly degenerate neutrinos. If we are to describe solar neutrino mixing too, this has to be extended to three nearly degenerate neutrinos [23] and since an Abelian symmetry alone cannot generate this structure we dismiss this possibility here. As a result, we require a hierarchical neutrino mass pattern and this implies $V_{MNS} \approx V_{\nu L}^T \cdot V_{\ell L}$ giving

$$\sin\theta_{\mu\nu_\tau} \approx r'(\frac{\theta}{M})^{q_L} - s(\frac{\theta}{M'})^{q_L} \tag{23}$$

with the implication that $q_L = 0$ for near maximal mixing.

At this point, it is important to discuss what are the expansion parameters in the various sectors, i.e. what are M, and M' . As discussed above, the most reasonable origin of the higher dimension terms $\propto \left(\frac{\theta}{M_i}\right)^a$ is via the Froggatt-Nielsen mechanism [13], through the mixing of the lepton states or the Higgs states. In the case of the mixing responsible for V_{MNS}, the former is irrelevant for in this case the mixing arises via heavy states which belong to $SU(2)$ doublets and hence are closely degenerate ($M' = M$). In this case, the contributions to eq.(20) or (23) cancel. We conclude that *the relevant mixing is generated through the Higgs states*. Thus, M should be interpreted as the mass of the heavy Higgs states mixing with H_1, generating the down quark and charged lepton masses, while M' is the mass of the Heavy Higgs state mixing with H_2, generating the up quark masses. Consequently, the expectation is that $M_1 > M_2$ because the same expansion parameters govern the hierarchy of quark masses, and typically one needs a smaller expansion parameter in the up-quark sector to explain the larger hierarchy of masses in that sector. This in turn implies that *the lepton mixing comes primarily from the charged lepton sector*. Of course, this conclusion depends on the relative up and down-quark charges - we will return in a discussion of this shortly.

Although we have argued that the mixing matrix V_{MNS} is determined by the left-handed charges only, the mass eigenvalues are sensitive to the right-handed charges. In particular the Majorana mass has a similar form to that in eq.(15)

$$\mathcal{M}_\nu^M \propto E\left((\frac{\theta}{M''})^{q_R}\right) \cdot B \cdot E\left((\frac{\theta}{M''})^{q_R}\right) \tag{24}$$

where we have allowed for a different intermediate mass scale, M'', in the right-handed sector and B is a matrix of Yukawa couplings of $O(1)$. This gives

$$Det(m_{eff}) = \frac{[Det(M_\nu^D)]^2}{Det(M_\nu^R)} \propto \frac{(\frac{\theta}{M'})^{2q_R}(\frac{\theta}{M})^{2q_L}}{(\frac{\theta}{M''})^{2q_R}} \tag{25}$$

To summarise, the choice $q_L = 0$ leads to $O(1)$ mixing, although there is no reason for the mixing to be really maximal i.e. $\pi/4$ (for this, a non-Abelian symmetry is necessary [23]). The lepton mass may be adjusted by the choice of $q_{\ell R}$, while the neutrino masses may be adjusted by the choice of $q_{\nu R}$. Thus, a $U(1)$ family symmetry is readily compatible with an hierarchical neutrino mass matrix and a large mixing angle in the lepton sector although it is unlikely to be maximal.

At this point it is perhaps useful to comment how *maximal* mixing *can* be obtained from the right-handed neutrino sector [19, 24, 25] via an Abelian symmetry. This may be arranged using the holomorphic structure of the superpotential in supersymmetric theories. Suppose the $<\overline{\theta}>=0$ i.e. only the positive family charge field θ acquires a vev. Suppose further the family charges of the heavy lepton doublets are given by $q_{\mu_L}=n$, $q_{\tau_L}=0$ while the right-handed neutrinos have charge given by $q_{\nu_\mu}=(p+r)$ $q_{\nu_\tau}=-p$ with n, p and r positive and $n\geqslant p$. Then we have

$$\begin{aligned}
\mathcal{M}_\nu^D &\propto \begin{pmatrix} (\frac{\theta}{M})^{n+p+r} & (\frac{\theta}{M})^{n-p} \\ (\frac{\theta}{M})^{p+r} & 0 \end{pmatrix} \\
\mathcal{M}_\nu^M &\propto \begin{pmatrix} (\frac{\theta}{M'})^{2p+r} & 1 \\ 1 & 0 \end{pmatrix}
\end{aligned} \tag{26}$$

The zeros here arise because the net charge in the $(2,2)$ element is negative (the case not allowed in the discussion above). For the simple case $M=M'$ we have

$$\begin{aligned}
\mathcal{M}_\nu^{effective} &= \mathcal{M}_\nu^D \cdot (\mathcal{M}_\nu^M)^{-1} \cdot \mathcal{M}_\nu^{DT} \\
&\propto \begin{pmatrix} (\frac{\theta}{M})^n & 1 \\ 1 & 0 \end{pmatrix}
\end{aligned} \tag{27}$$

(a similar structure applies for a range of M/M'). This gives maximal mixing and two nearly degenerate neutrinos. Thus if we wish to describe solar as well as atmospheric oscillations it is necessary to add a sterile neutrino [25, 26]. We do not consider such schemes here.

3 Gauge unification constraints

While an Abelian family symmetry provides a promising origin for an hierarchical pattern of fermion masses, in order to go further it is necessary to specify the charges of the quarks, charged leptons and neutrinos. As we discussed in the last section, it is straightforward to fit all the observed masses and mixing angles by the choice of the $U(1)$ charges not constrained by the Standard Model gauge symmetry. However, the structure of the Standard Model is suggestive of an underlying unification which may relate quark and lepton multiplets. The success of the unification of the gauge couplings also supports this picture. Thus, we think it of interest to consider whether realistic quark mass structures are consistent with the constraints on an Abelian family symmetry that result from some underlying unified gauge symmetry.

3.1 $SO(10) \times U(1)$

Consider first the possibility that the family symmetry commutes with an $SO(10)$ GUT. In this case, all quark and lepton charges for the left- and charge conjugate right-handed fields in a given generation are the same. This gives rise to a left-right symmetric mass matrix with similar structure for the up quark, the down quark, the charged lepton and the Dirac neutrino mass matrices. The only difference between these sectors is due to the possibility the expansion parameters can be different. Thus $SO(10)$ provides a specific realisation of the first model of lepton masses that has been discussed in reference [15] for the case $b = 0$. Following the discussion of Section 2.2 we note that the expansion parameters determining V_{MKS} are principally those arising from Higgs mixing. Since the same Higgs is responsible for the structure of the down quark mass and the charged lepton mass this leads to the prediction

$$V_{\mu\tau} \approx V_{cb} \tag{28}$$

Clearly this is in gross conflict with observations so to rescue it it is necessary for the coefficients of $O(1)$ associated with the down quark and leton sectors to differ. In our analysis, we are going to discard solutions that, in order to match the observations, require the existence of either cancellations that are not predicted by the Abelian family symmetry, or coefficients with magnitude comparable to that of the expansion parameter ordering the elements of the mass matrices [4]. For this reason we do not consider this $SO(10)$ possibility further. We also apply these criteria to the analysis in the rest of this section.

We should further stress that in our analysis we use the GUT structure only in order to constrain the $U(1)$ flavour charges of the light fields. In particular we assume that all terms allowed at a given order by the family symmetry do in fact occur. This condition can be avoided if the heavy fields responsible for the Froggatt Nielsen mixing have restricted $U(1)$ family charges. A simple example of this mechanism appears in [15]; viable $SO(10)$ examples appear in [27, 28]. We do not consider such a possibility here, because we wish to explore whether the $U(1)$ family symmetry structure of the light fields alone is sufficient to determine the pattern of light fermion masses and mixings.

3.2 $SU(5) \times U(1)$

We turn to the possibility that the family symmetry commutes with an $SU(5)$ GUT. This is, of course, consistent with an underlying $SO(10)$ structure but to avoid the bad relation of eq.(28) it is necessary for the Abelian family symmetry to have a component along the $SO(10)$ neutral

[4] This requirement is unreasonable in the case the down quark and lepton couplings are *predicted* to differ by the underlying GUT. We consider such a possibility for the case of $SU(5)$ in the next Section but choose not to pursue it for $SO(10)$.

generator $\propto (B-L)$ which commutes with $SU(5)$. There are only three $U(1)$ family charges needed for each family. These are given by

$$\begin{aligned} Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\ Q_{(l,d^c)_i} &= Q_i^{\bar{5}} \\ Q_{(\nu_R)_i} &= Q_i^{\nu_R} \end{aligned} \tag{29}$$

From the above it immediately follows that :

(i) The up-quark mass matrix is symmetric.

(ii) the charged lepton mass matrix is the transpose of the down quark mass matrix.

The expansion parameters in the various sectors can be different (depending on whether the non-renormalisable contributions are due to fermion or to Higgs mixing, or a to combination of the two). However, as discussed in Section 2.2, a single expansion parameter describing H_1 mixing determines the down quark and charged lepton *mixing*, and similarly a single expansion parameter describing H_2 mixing determines the up quark and Dirac neutrino *mixing*. The fact that the right-handed neutrino charges are unconstrained means the neutrino mass spectrum is not restricted but, again as discussed in Section 2.2, the mixing angle in the $\mu\tau$ sector is insensitive to these charges and is determined primarily by $Q_{\mu,\tau}^{\bar{5}}$.

At first sight this charge structure seems to offer an immediate explanation for the difference between large mixing angle observed in atmospheric neutrino mixing and the small quark mixing angles. This is because the former is determined by $Q_{\mu,\tau}^{\bar{5}}$ while the corresponding quark mixing matrix element, V_{cb}, is determined by Q_i^{10}. However the main difficulty in using this freedom to describe both mixings arises from the associated correlations between the eigenvalues of the charged lepton and the down quark mass matrices due to structure (ii) above. Indeed, if the eigenvalues of the down mass matrix (with expansion parameter e) are given by a sequence $1, e, e^k$ the eigenvalues for the leptons (with expansion parameter $\tilde{\epsilon}$) are $1, \tilde{\epsilon}, \tilde{\epsilon}^k$. The down quark masses are well described by the choice $e \simeq \bar{\epsilon}^2 \simeq 0.04$ and $k=2$. while for the leptons the hierarchies are well described by $\tilde{\epsilon} \simeq \bar{\epsilon} \simeq 0.2$ and $k=5$. This is clearly inconsistent with the pattern coming from the family symmetry which requires the same k in the down quark and lepton sectors.

One way to reconcile the two forms for the mass matrix, originally advocated by Georgi and Jarlskog [29], is to have different Yukawa couplings in the quark and lepton sectors. These couplings are determined by the underlying $SU(5)$ gauge group. If the mass comes from the coupling to a 5 of Higgs then $m_{d_i} = m_{l_i}$ while if the mass comes from the coupling to a 45 of Higgs then $m_{d_i} = 3m_{l_i}$.The observed hierarchy for the lepton masses is well described by the eigenvalues $1, 3\bar{\epsilon}^2, \bar{\epsilon}^4/3$. Georgi and Jarlskog achieved this by restricting the mass matrices by family symmetries to have the form

$$\begin{aligned} \mathcal{M}_d &= \begin{pmatrix} 0 & a' < H^5 > & 0 \\ a < H^5 > & c < H^{45} > & 0 \\ 0 & 0 & b < H^5 > \end{pmatrix} \\ \mathcal{M}_\ell &= \begin{pmatrix} 0 & a' < H^5 > & 0 \\ a < H^5 > & 3c < H^{45} > & 0 \\ 0 & 0 & b < H^5 > \end{pmatrix} \end{aligned} \tag{30}$$

While this gives an acceptable pattern of masses it clearly does not give the large mixing angle in the $\mu\tau$ sector. Here we will determine whether it is possible in more general schemes. To do so, we look explicitly at the forms of the matrices. Given that the top quark is very heavy it is reasonable to assume that it is given by an $O(1)$ renormalisable contribution. Then, the up-quark mass matrix is specified to be:

$$\mathcal{M}_u \propto \begin{pmatrix} \epsilon^{|2x|} & \epsilon^{|x+b|} & \epsilon^{|x|} \\ \epsilon^{|x+b|} & \epsilon^{|2b|} & \epsilon^{|b|} \\ \epsilon^{|x|} & \epsilon^{|b|} & 1 \end{pmatrix} \tag{31}$$

where $x = Q_1^{10} - Q_3^{10}$ and $b = Q_2^{10} - Q_3^{10}$. Then $\frac{m_c}{m_t} = e^{|2b|}$ and $e^{|b|} \approx 0.045$ gives a good fit. The contribution of the up sector to V_{cb} is given by $V_{cb}^{up} = e^{|b|} = 0.045$.

What about the light up-quark hierarchies? We have

$$\frac{m_u}{m_c} = max(\frac{e^{|x+b|}}{e^{|2b|}}, \frac{e^{|x|}}{e^{|b|}}), \tag{32}$$

indicating that either

$$e^{\ |x+b|-2|b|} = \mathcal{O}(10^{-6}) \tag{33}$$

or

$$e^{|x|-|b|} = \mathcal{O}(10^{-6}) \tag{34}$$

We now pass to the down-quark and charged lepton hierarchies. We consider the case that the (3,3) element of the quark and lepton mass matrices is allowed by the family symmetry and the difference between the top and the bottom quark masses is largely due to $tan\beta \equiv < H_1 > / < H_2 >$ being large. In this case the charge of H_2 is fixed to be the same as of H_1. Then, the down quark and charged lepton textures have the form :

$$M_\ell \propto \tilde{\epsilon}^{|Q_3^{\bar{5}} - Q_3^{10}|} \begin{pmatrix} \tilde{\epsilon}^{|x+y|} & \tilde{\epsilon}^{|y+b|} & \tilde{\epsilon}^{|y|} \\ \tilde{\epsilon}^{|x+a|} & \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|a|} \\ \tilde{\epsilon}^{|x|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix}, \quad M_{down} \propto \bar{\epsilon}^{|Q_3^{\bar{5}} - Q_3^{10}} \begin{pmatrix} \bar{\epsilon}^{|x+y|} & \bar{\epsilon}^{|x+a|} & \bar{\epsilon}^{|x|} \\ \bar{\epsilon}^{|y+b|} & \bar{\epsilon}^{|a+b|} & \bar{\epsilon}^{|b|} \\ \bar{\epsilon}^{|y|} & \bar{\epsilon}^{|a|} & 1 \end{pmatrix} \tag{35}$$

where $a = Q_2^{\bar{5}} - Q_3^{\bar{5}}$ and $y = Q_2^{\bar{5}} - Q_3^{\bar{5}}$.

From the above matrices, we see that the eigenvalues for the lepton mass matrix are

$$1, \tilde{\epsilon}^{|a+b|}, max(\tilde{\epsilon}^{|x+y|}, \frac{\tilde{\epsilon}^{|x+a|}\tilde{\epsilon}^{|y+b|}}{\tilde{\epsilon}^{|a|}\tilde{\epsilon}^{|b|}}), \tag{36}$$

while for the down quarks

$$1, \bar{\epsilon}^{|a+b|}, max(\bar{\epsilon}^{|x+y|}, \frac{\bar{\epsilon}^{|x+a|}\bar{\epsilon}^{|y+b|}}{\bar{\epsilon}^{|a|}\bar{\epsilon}^{|b|}}) \tag{37}$$

Clearly, irrespective of the choice of expansion parameters, this form will not simultaneously generate the correct ratios for down quarks and leptons without requiring the lepton Yukawa couplings are different from the quark Yukawa couplings. Following the suggestion of Georgi and Jarlskog we assume the Higgs responsible for the (2,2) entry is a 45 representation of $SU(5)$. This generates a relative factor of 3 in the (22) entry of the lepton mass matrix. Then provided that the (11) entry is smaller than the (12) entry, the smallest eigenvalue is suppressed by a factor of 3. For this to occur, we need: $xy > 0$, while $|y+b| + |x+a| - |a| - |b|$ has to be smaller than $x + y$.

Can we reconcile these constraints with an acceptable pattern of mixing angles? Let us first consider the most predictive case which has texture zeros in the $(1,1)$ and $(1,3)$ positions (see Section 4 and ref. [15]). In this case the left-handed quarks have charges $-4, 1, 0$ and V_{CKM}^{12} has to arise from the down sector. If we want near-maximal lepton mixing, the two heavier *right*-handed down charges (which are the same as those of the two heavier *left*-handed lepton charges respectively) have to be zero. In this case, the second generation left-handed quark charge, $|\alpha_2| = 1$, give an unacceptably large m_s/m_b.

To obtain the correct m_s/m_b for the case of maximal lepton mixing we must give up the texture zero structure and choose $|\alpha_2| = 2$ and $|\alpha_3| = 0$. Then, to obtain the correct (12)-quark mixing from the down sector (note that to get such a large mixing from the up sector is more difficult, as it would lead to a large up-quark mass), we need $|\alpha_1| = 3$. What does this imply for M_u? For $\alpha_1 = 3$, the mass of the up-quark is given by

$$\frac{m_{up}}{m_c} \approx \frac{M_u(1,1) - M_u^2(1,2)/M_u(2,2)}{M_u(2,2)} \approx O(\epsilon^6 - \epsilon^6) \tag{38}$$

where now $\epsilon = 0.23$ (this is in order to obtain the correct charm mass, for our choice $|\alpha_2| = 2$). Thus we see that there must be a cancellation between the two terms, or the introduction of small coefficients to obtain the correct ratio for these masses.

Let us now go back to the down mass matrix. The only charge undetermined is that for d_1^c. Fixing this to $+1$, gives the correct mass for the down quark.

The obtained mass matrices are:

$$\frac{M_u}{m_t} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^5 & \bar{\epsilon}^3 \\ \bar{\epsilon}^5 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}, \frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \tag{39}$$

This is the choice of charges that appears in [30] (with the exception of the down and lepton charges of the first generation). To summarise, we see that within the framework of $SU(5)$, viable solutions only exist provided (c.f. eq.(38)) the $O(1)$ coefficients have a special form not guaranteed by the Abelian symmetry applied to the light fields alone (c.f the discussion in Section 3.1). Since here we are concerned to explore how much of the fermion mass patterns can be generated by the Abelian symmetry alone, we will not discuss these $SU(5) \times U(1)$ models further.

3.3 Flipped $SU(5) \times U(1)$

In the case of the flipped $SU(5)$, the fields Q_i, d_i^c and ν_i^c belong to 10 of $SU(5)$, while u_i^c and L_i belong to a $\overline{5}$. Finally the e_i^c fields belong to singlet representations of $SU(5)$.

The above assignment, implies that the down quark mass matrices are symmetric, and therefore they are expected to have the form presented in [15]. Then we obtain viable hierarchies by fixing the down-quark charges to ie $4, -1, 0$ and the expansion parameter in the down mass matrix to be $\bar{\epsilon} = 0.23$.

$$M_{down} \propto \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix} \tag{40}$$

Since the charge conjugate of the right-handed neutrinos have the same charge as the down quarks the Majorana mass matrix will be constrained by this charge assignment. For example, for a zero Σ charge for the Higgs generating this mass term, $\nu_i^R \nu_j^R \Sigma$, the Majorana mass matrix has the same form as eq.(40) although with a different expansion parameter. Moreover, due to the above charge assignments, the Dirac neutrino mass matrix is the transpose of the up-quark mass matrix.

The structure of the up-quark mass matrix will depend on the charges of the right-handed quarks. However as these are the same with the charges of the left-handed leptons the mass matrix will be constrained by the need to generate large mixing for atmospheric neutrinos. Assigning the left-handed leptons charges y, x and 0, and the right-handed leptons charges a, b and 0 we see that maximal (2-3) mixing requires $x = 0$. [5] Then:

[5]Acceptable solutions may also be generated for $x = \pm 1/2$ which gives large but non-maximal mixing. We discuss an example of this in detail later.

$$M_\ell \propto \left(\begin{array}{ccc} \bar\epsilon^{|a+y|} & \bar\epsilon^{|b+y|} & \bar\epsilon^{|y|} \\ \bar\epsilon^{|a|} & \bar\epsilon^{|b|} & 1 \\ \bar\epsilon^{|a|} & \bar\epsilon^{|b|} & 1 \end{array} \right) \tag{41}$$

The lepton eigenvalues are of order $1, \bar\epsilon^{|b|}$ and $max(\bar\epsilon^{|a+y|}, \bar\epsilon^{|a|}\cdot\bar\epsilon^{|b+y|}/\bar\epsilon^{|b|})$. Fitting m_e/m_μ constrains the combined charges a, y and b for a given choice of the expansion parameter.

Now, we are ready to go to the up-quark mass matrix: Its form, for $x = 0$, is:

$$M_{up} \propto \left(\begin{array}{ccc} \epsilon^{|-4+y|} & \epsilon^4 & \epsilon^4 \\ \epsilon^{|1+y|} & \epsilon & \epsilon \\ \epsilon^{|y|} & 1 & 1 \end{array} \right) \tag{42}$$

In order to obtain the correct value for m_c/m we need to make the assignment $\epsilon = \bar\epsilon^4 = 0.23^4$ (where ϵ is the up-matrix expansion parameter and $\bar\epsilon$ the down-quark and the charged lepton one). Note that this is a direct outcome of the requirement to obtain maximal (2-3) charged lepton mixing, which constrained x to zero. Finally we can chose y so as to get the correct m_u/m_c ratio. An obvious choice is to take $y = 2$ (remember that ϵ is now $\bar\epsilon^4$).

Turning to the implications for the mixing angles we see that the contribution from the up quark sector to V_{cb} is very small. Thus from eq.(40) we see that the expectation is that $V_{cb} \simeq \sqrt{m_s/m_b}$. This is too large and requires a very small coefficient in the (2,3) entry of eq.(40). For this reason, we consider that this model seems less promising in the framework of a single $U(1)$ symmetry[6].

3.4 $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)$

This is a particular GUT group which readily emerges from an underlying string theory with an intrinsic $E(6)$ symmetry. In it a single family of quarks and leptons are accommodated in a $(3,3,1) \oplus (\bar3,1,\bar3) \oplus (1,3,\bar3)$ under $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$. The left- and right-handed quarks belong to $(3,3,1)$ and $(\bar3,1,\bar3)$ respectively and thus their $U(1)$ charges are not related. On the other hand the left handed and (charge conjugate) right handed leptons belong to the same $(1,3,\bar3)$ representation and hence must have the same $U(1)$ charge. Thus, the lepton mass matrices have to be symmetric.

This freedom allows us to construct fully realistic mass matrices. Let us start from the lepton mass matrices. Taking the charges

$$b_i = c_i = d_i \;\; = \;\; (-\frac{7}{2}, \frac{1}{2}, 0)$$

[6]Note that in realistic models with more $U(1)$ groups coming from the string, solutions have been found in [31] .

$$
\begin{aligned}
b_i = c_i = d_i &= (\frac{5}{2}, \frac{1}{2}, 0) \\
b_i = c_i = d_i &= (3, 0, 0)
\end{aligned} \tag{43}
$$

leads to the three possible charged lepton matrices

$$
\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^7 & \bar{\epsilon}^3 & \bar{\epsilon}^{7/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{7/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix}, \frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^5 & \bar{\epsilon}^3 & \bar{\epsilon}^{5/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{5/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix}, \frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & 1 & 1 \\ \bar{\epsilon}^3 & 1 & 1 \end{pmatrix}, \tag{44}
$$

We see that the third matrix leads to maximal mixing, however it requires an accurate cancellation in the (2,3) sector in order to get the correct m_μ/m_τ. On the other hand, the other two matrices, lead to natural lepton hierarchies and predict large but non-maximal lepton mixing. We study this case in detail in section 4.

What about the quark mass matrices? The choice of $U(1)$ charges given by

$$
\begin{aligned}
\alpha_i &= (3, 2, 0) \\
\beta_i &= \gamma_i = (1, 0, 0)
\end{aligned} \tag{45}
$$

leads to the mass matrices

$$
\frac{M_u}{m_t} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}
$$

For $\epsilon = \bar{\epsilon}^2$ we obtain viable quark hierarchies. The V_{CKM} mixing is dominated by the contribution from the down quark sector. However the structure of charges chosen here means that $V_{cb} \simeq m_s/m_b$. This is in good agreement with the measured value.

The structure of the neutrino mass matrices is fixed because the left and right handed neutrino charges are determined because they belong to the same $(1, 3, \bar{3})$ representation as the charged leptons. Both the Dirac and Majorana mass matrices have a symmetric form with the Dirac mass matrix of the same form as the charged leptons but with a different expansion parameter. This case is discussed in detail in the next section where we consider symmetric mass matrices in general. The first two forms of eq.(44) lead to large mixing in the atmospheric neutrino sector and generate solar neutrino oscillation with parameters in the small mixing angle range of eq.(2).

3.5 Left-Right symmetric models.

Another gauge structure that has been widely explored is one which is left-right symmetric [32]. The simplest possibility is $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes U(1)$ with a discrete

	Q_i	u^c_i	d^c_i	L_i	e^c_i	ν^c_i	H_2	H_1
$U(1)_{FD}$	α_i	α_i	α_i	b_i	b_i	b_i	$-2\alpha_3$	$-2\alpha_3$

Table 2: Symmetric $U(1)_{FD}$ charges

Z_2 symmetry interchanging the two $SU(2)$ factors and their associated quark and lepton representations. This is readily generalised to larger groups. For example, the case considered above $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)$ may similarly be rendered left-right symmetric through a Z_2 symmetry interchanging the two $SU(3)$ factors. Such a structure is found in the three generation string theories resulting from compactifying on a specific Calabi-Yau manifold [33]. A partial unification based on $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$ was proposed by Pati and Salam [34].

In these models the U(1) family charges are strongly constrained because the Z_2 symmetry requires that the $U(1)$ charges of the left- and right- handed fields be the same. As a result the mass matrices will be symmetric. The left- right- symmetry together with the $SU(2)_L$ symmetry requires that the left- and right-handed components of up and down quarks of each generation should have the same charge and the left- and right-handed components of the charged leptons and neutrinos of each generation should have also have the same charge. This means that only six $U(1)$ charges need to be specified to completely fix the model. Given the interest in left- right- symmetric models and the highly constrained nature of the $U(1)$ we think it of some interest to explore this possibility in some detail. As we shall discuss the fully left- right- symmetric models with symmetric quark, lepton and neutrino masses, are in remarkably good agreement with the measured values of masses and mixing angles, with texture zeros leading to definite relations between masses and mixing angles in good agreement with experiment. However as noted above, particularly in Section 3.4, it may be that only a sub-sector has a symmetric mass matrix. To deal with all these cases in the next Section we turn to a detailed discussion of the implications of an Abelian family symmetry leading to symmetric mass matrices.

4 Symmetric textures and neutrino masses

4.1 Quark Masses

Here we consider in more detail the implications of the symmetric charge assignments discussed in the last section. Although we are primarily interested in neutrino masses, in order to answer the question whether the neutrino masses and mixings fit into the pattern of quark and lepton masses, it is necessary to discuss the latter first. We start with an Abelian family symmetry with the most general symmetric charge assignments given in Table 2. Following the discussion of Section 2.1 we find mass matrices of the form

$$\frac{M_u}{m_t} \approx \begin{pmatrix} h_{11}\rho_{11}\epsilon_a^{|2+6a|} & h_{12}\rho_{12}\epsilon_b^{|3a|} & h_{13}\rho_{13}\epsilon_a^{|1+3a|} \\ h_{21}\rho_{21}\epsilon_b^{|3a|} & h_{22}\rho_{22}\epsilon^2 & h_{23}\rho_{23}\epsilon^1 \\ h_{31}\rho_{31}\epsilon_a^{|1+3a|} & h_{32}\rho_{32}\epsilon^1 & h_{33} \end{pmatrix} \tag{46}$$

$$\frac{M_d}{m_b} \approx \begin{pmatrix} k_{11}\sigma_{11}\bar{\epsilon_a}^{|2+6a|} & k_{12}\sigma_{12}\bar{\epsilon_b}^{|3a|} & k_{13}\sigma_{13}\bar{\epsilon_a}^{|1+3a|} \\ k_{21}\sigma_{21}\bar{\epsilon_b}^{|3a|} & k_{22}\sigma_{22}\bar{\epsilon}^2 & k_{23}\sigma_{23}\bar{\epsilon}^1 \\ k_{31}\sigma_{31}\bar{\epsilon_a}^{|1+3a|} & k_{32}\sigma_{32}\bar{\epsilon}^1 & k_{33} \end{pmatrix} \tag{47}$$

where $\bar{\epsilon} = (\frac{<\theta>}{M_1})^{|\alpha_2-\alpha_1|}$, $\epsilon = (\frac{<\theta>}{M_2})^{|\alpha_2-\alpha_1|}$, and $a = (2\alpha_1 - \alpha_2 - \alpha_3)/3(\alpha_2 - \alpha_1)$, h_{ij}, k_{ij} are Yukawa couplings all assumed to be of $O(1)$ and ρ, σ are related to Yukawa couplings in the Higgs sector (again we expect them to be $O(1)$) and describe Higgs or quark mixing in the way discussed below.

It is straightforward now to see how texture zeros occur. For $-3a > 1$ $\epsilon_a = \epsilon_b = \epsilon$ and $\bar{\epsilon}_a = \bar{\epsilon}_b = \bar{\epsilon}$. In this case it is easy to check that there are *no* texture zeros because all matrix elements contribute at leading order to the masses and mixing angles. For $1 > -3a > 0$, ϵ_a, $\bar{\epsilon}_a$ change and are given by $\bar{\epsilon_a} = (\frac{<\bar{\theta}>}{M_1})^{|\alpha_2-\alpha_1|}$, $\epsilon_a = (\frac{<\bar{\theta}>}{M_2})^{|\alpha_2-\alpha_1|}$. In this case texture zeros in the (1,1) and (1,3) positions *automatically* appear for small $<\bar{\theta}>/M_i$. However the (1,2) matrix element is too large (cf. eqs. (46),(47). For $a > 0$ however $\bar{\epsilon}_{a,b} = (\frac{<\bar{\theta}>}{M_1})^{|\alpha_2-\alpha_1|}$, $\epsilon_{a,b} = (\frac{<\bar{\theta}>}{M_2})^{|\alpha_2-\alpha_1|}$, the texture zeros in the (1,1) and (1,3) positions persist, and the (1,2) matrix element can be of the correct magnitude.

Thus we see that the simplest possibility of an additional $U(1)$ gauge family symmetry *requires* texture zeros in the phenomenologically desirable positions for a large range of the single relevant free parameter, a. In addition it generates structure for the other matrix elements which can duplicate the required hierarchical structure of masses and mixing angles. To illustrate the mechanism we consider Froggatt-Nielsen mixing in the Higgs sector masses along the lines mapped out in Section 2.2. After mixing the light Higgs states are given by $H_{33}^2 + \sum \rho_{ij} H_{ij}^2 \epsilon_{a,,b}^{n_{ij}}$ and $H_{33}^1 + \sum \sigma_{ij} H_{ij}^1 \bar{\epsilon}_{a,,b}^{n_{ij}}$ where the powers n_{ij} are those appearing in eq.(47) and ρ, σ are related to Yukawa couplings in the Higgs sector. Similarly mixing in the quark sector can also generate the elements of eq.(47).

As discussed above, for $a > 0$, there are two approximate texture zeros in the (1,1) and (1,3), (3,1) positions. These give rise to excellent predictions for two combinations of the CKM matrix. The magnitude of the remaining matrix elements is sensitive to the magnitude of a and the values of the expansion parameters. Then choosing $a = 1$ the remaining non-zero entries have magnitude in excellent agreement with the measured values. From eqs. (46) and (47), we see that to a good approximation we have the relation [15]

$$\epsilon = \bar{\epsilon}^2 \tag{48}$$

corresponding to the choice $M_2 > M_1$.

Such a choice gives an excellent description of quark masses and mixing angles, after allowing for the unknown coefficients of $O(1)$. It is instructive to determine the magnitude needed for these coefficients to fit all measured masses and mixings. This is clearly an under-determined problem, so the best we can do is to give an illustrative example. The choice

$$\frac{M_u}{m_t} \approx \begin{pmatrix} 0 & \epsilon^3 & 0 \\ \epsilon^3 & i\epsilon^2 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}$$

$$\frac{M_d}{m_b} \approx \begin{pmatrix} 0 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \frac{1}{2}\bar{\epsilon} \\ 0 & \frac{1}{2}\bar{\epsilon} & 1 \end{pmatrix} \tag{49}$$

generates the correct masses and mixing angles with $\epsilon = 0.05$, $\bar{\epsilon} = 0.18$. This requires a coefficient $1/2$ in the $(2,3)$ entry, necessary to give the value of $V_{cb} = 0.04$. The latter is anomalously small due to a cancellation between the contributions of the up and the down quark sectors. Such a cancellation may be expected as there is an approximate $SU(2)_R$ symmetry in the magnitude of the matrix elements following from the very symmetric choice of family charges and in the limit $SU(2)_R$ is exact V_{cb} vanishes.

4.2 Lepton Masses

Now that we have a theory of quark masses it is possible systematically to address the original question whether the large mixing angle found in the neutrino sector is consistent with this theory or whether it requires completely new structure [17, 18, 19].

4.2.1 Charged leptons

The charged lepton masses and mixings are determined in a similar way to that of the quarks. Requiring $m_b = m_\tau$ at unification, sets $\alpha_3 = b_3$ and then the charged lepton mass matrix is

$$\frac{M_\ell}{m_\tau} \approx \begin{pmatrix} \bar{\epsilon}^{|2+6a-2b|} & \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|1+3a-b|} \\ \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|2(1-b)|} & \bar{\epsilon}^{|1-b|} \\ \bar{\epsilon}^{|1+3a-b|} & \bar{\epsilon}^{|1-b|} & 1 \end{pmatrix} \tag{50}$$

where $b = (\alpha_2 - b_2)/(\alpha_2 - \alpha_3)$. A solution with $b = 0$ leads to lepton hierarchies similar to those for the quarks. However in this case the expectation is that $m_s \simeq m_\mu$ and $m_d \simeq m_e$ at the unification scale, in conflict with experiment. In this case we must rely on large coefficients to generate an acceptable mass matrix structure. If instead we require that the explanation of the mass structure of the light generations results from the choice of lepton family charges, i.e. through the choice of b, one is led to take $\beta \equiv 1 - b = \pm 1/2$[7].

Let us first comment on the case with $\beta = -1/2$. This gives the lepton texture [15]

$$\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^5 & \bar{\epsilon}^3 & \bar{\epsilon}^{5/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{5/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix} \tag{51}$$

On the other hand $\beta = 1/2$, leads to [17, 18]

$$\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^7 & \bar{\epsilon}^3 & \bar{\epsilon}^{7/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{7/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix} \tag{52}$$

As we see, both types of textures can give correct predictions for lepton masses (note that when estimating the lightest eigenvalue we have not allowed for cancellations in eq.(51) between the contributions from the (1,1), (2,2) and (1,2), (2,1) elements. Due to the Yukawa couplings of $O(1)$ we do not expect such a cancellation to occur.).

4.2.2 Neutrinos

We may now determine the predictions of the flavour symmetry for neutrino masses. Due to the see-saw mechanism we generate quite naturally light neutrino masses. In this framework, the light Majorana neutrino masses are given by the generalisation of eq.(11)

$$m_{eff} = M_\nu^D \cdot (M_{\nu_R}^M)^{-1} \cdot (M_\nu^D)^T \tag{53}$$

where M_ν^D and $M_{\nu_R}^M$ are the 3×3 Dirac and Majorana mass matrices respectively.

How do we determine these mass matrices? The Dirac mass matrix is actually fixed by the symmetries of the model. Indeed, $SU(2)_L$ fixes the $U(1)_{FD}$ charge of the left-handed neutrino states to be the same as the charged leptons, and then the left-right symmetry fixes the charges of the right-handed neutrinos as given in Table 4.1. Thus the neutrino Dirac mass is given by

$$M_\nu^D \propto \begin{pmatrix} \epsilon^{|2+6a-2b|} & \epsilon^{|3a|} & \epsilon^{|1+3a-b|} \\ \epsilon^{|3a|} & \epsilon^{|2(1-b)|} & \epsilon^{|1-b|} \\ \epsilon^{|1+3a-b|} & \epsilon^{|1-b|} & 1 \end{pmatrix} \tag{54}$$

[7]In some cases, the textures with half-integer b have been simplified, by imposing a residual Z_2 discrete gauge symmetry after the $U(1)$ breaking, by which the electron and muon fields transform by (-1). Then, entries raised in a half-integer power vanish. However this is not a necessary condition: in general, half-integer entries remain present at low energies and may have interesting phenomenological implications.

In unified or partially unified models the large mass scale associated with Froggatt-Nielsen mixing is the same as the one for the up-quarks and so in eq.(54) we have used the up quark expansion parameter.

We turn now to the Majorana masses for the right-handed neutrinos. Such masses arise from terms of the form $\nu_R \nu_R \Sigma$ where Σ is a $SU(3) \otimes SU(2) \otimes U(1)$ invariant Higgs scalar field with $I_W = 0$ and ν_R is a right-handed neutrino. Since we do not know the charge of Σ, we have to consider all possible choices. This allows us to "rotate" the larger coupling to any of the entries of the heavy Majorana mass matrix, generating a discrete spectrum of possible forms [17, 18]. For example, if the Σ charge is the same as that of the $H_{1,2}$ doublet Higgs charges, the larger element of M_ν will be in the (3,3) entry. The rest of the terms will be generated as before through the $U(1)_{FD}$ breaking by $< \theta >$ and $< \bar{\theta} >$.

Among the cases that naturally generate the correct lepton hierarchies those that are also of interest for Super-Kamiokande are mainly $\beta \equiv 1 - b = \pm 1/2$ which lead to large (2-3) lepton mixing [8]. Restricting the discussion to these cases the general forms for the heavy Majorana mass textures (allowing for the various choices of the Σ charge), appear in Table 3. The form of m_{eff}, its eigenvalues and the mixing matrices for $\beta = 1/2$, are presented in Table 4 and for $\beta = -1/2$ in Table 5. Here, $\tilde{m}_{eff} = m^D_{\nu,diag} \cdot R_D^T \cdot (M_\nu^R)^{-1} \cdot R_D \cdot m^D_{\nu,diag}$, where R_D is the neutrino Dirac mixing matrix. This combination has been chosen because it contains all the information necessary to determine the mass eigenvalues and also exhibits the contribution to the mixing angles that is sensitive to the mixing in the Majorana mass matrix. The full effective Majorana mass matrix is then given by $m_{eff} = R_D.\tilde{m}_{eff}.R_D^T$. A word of caution is in order here. The mass hierarchies are quite sensitive to the order unity coefficients that are not be predicted by the $U(1)$ symmetry due to the fact that the inverse of the Majorana mass matrix must be taken and the product of several mass matrices are involved. Indeed a small difference in a coefficient in the neutrino Dirac mass matrix, may lead to a large difference in the eigenvalues of m_{eff}. Thus the estimate given in Tables 4 and 5 for m_{eff} should be viewed as only a rough estimate. On the other hand the sensitivity of the mixing angles to the $O(1)$ coefficients is much less because, as discussed above, it largely comes from the Dirac mass matrix alone. Even with this cautionary word we see from Tables 4 and 5 we see that in all cases large mass hierarchies between the neutrino masses are expected to arise. The lightest neutrino eigenvalue is very suppressed compared to the other two.

4.2.3 Neutrino Masses

Given the results of Tables 4 and 5 we may now determine the expectation for the magnitude of the neutrino masses. As discussed in Section 1, the double see-saw gives the largest mass

[8] We discussed the first case in [18].

$\begin{pmatrix} \bar{\epsilon}^{2\|3\alpha+\beta\|} & \bar{\epsilon}^{3\|\alpha\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} \\ \bar{\epsilon}^{\|3\alpha\|} & \bar{\epsilon}^{2\|\beta\|} & \bar{\epsilon}^{\|\beta\|} \\ \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{\|\beta\|} & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{3\|2\alpha+\beta\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{\|3\alpha+2\beta\|} \\ \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{\|\beta\|} & 1 \\ \bar{\epsilon}^{\|3\alpha+2\beta\|} & 1 & \bar{\epsilon}^{\|\beta\|} \end{pmatrix}$
$\begin{pmatrix} \bar{\epsilon}^{2\|3\alpha+2\beta\|} & \bar{\epsilon}^{\|3\alpha+2\beta\|} & \bar{\epsilon}^{3\|\alpha+\beta\|} \\ \bar{\epsilon}^{\|3\alpha+2\beta\|} & 1 & \bar{\epsilon}^{\|\beta\|} \\ \bar{\epsilon}^{3\|\alpha+\beta\|} & \bar{\epsilon}^{\|\beta\|} & \bar{\epsilon}^{2\|\beta\|} \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{\|\beta\|} & 1 \\ \bar{\epsilon}^{\|\beta\|} & \bar{\epsilon}^{3\|\alpha+\beta\|} & \bar{\epsilon}^{3\|\alpha+2\beta\|} \\ 1 & \bar{\epsilon}^{3\|\alpha+2\beta\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} \end{pmatrix}$
$\begin{pmatrix} 1 & \bar{\epsilon}^{\|3\alpha+2\beta\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} \\ \bar{\epsilon}^{\|3\alpha+2\beta\|} & \bar{\epsilon}^{2\|3\alpha+2\beta\|} & \bar{\epsilon}^{3\|2\alpha+\beta\|} \\ \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{3\|2\alpha+\beta\|} & \bar{\epsilon}^{2\|3\alpha+\beta\|} \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{\|3\alpha+2\beta\|} & 1 & \bar{\epsilon}^{\|\beta\|} \\ 1 & \bar{\epsilon}^{\|3\alpha+2\beta\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} \\ \bar{\epsilon}^{\|\beta\|} & \bar{\epsilon}^{\|3\alpha+\beta\|} & \bar{\epsilon}^{\|3\alpha\|} \end{pmatrix}$

Table 3: General forms of heavy Majorana mass matrix textures. Interesting textures arise for $\alpha = 1, \beta \pm 1/2$.

	$\tilde{m}_\nu^{eff}$	$m_\nu^{eff,Diag}$	m_ν^{eff}	R_ν^{eff}
1	$\begin{pmatrix} e^{30} & e^{18} & e^{15} \\ e^{18} & e^{10} & e^{5} \\ e^{15} & e^{5} & 1 \end{pmatrix}$	$\begin{pmatrix} e^{26} & & \\ & e^{10} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{26} & e^{15} & e^{13} \\ e^{15} & e^{4} & e^{2} \\ e^{13} & e^{2} & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & e^{11} & -e^{13} \\ -e^{11} & 1 & e^{2} \\ e^{13} & -e^{2} & 1 \end{pmatrix}$
2	$\begin{pmatrix} e^{25} & e^{17} & e^{12} \\ e^{17} & e^{9} & e^{4} \\ e^{12} & e^{4} & e^{-1} \end{pmatrix}$	$\begin{pmatrix} e^{25} & & \\ & e^{9} & \\ & & e^{-1} \end{pmatrix}$	$\begin{pmatrix} e^{25} & e^{14} & e^{12} \\ e^{14} & e^{3} & e \\ e^{12} & e & e^{-1} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{11} & -e^{13} \\ -e^{11} & 1 & e^{2} \\ e^{13} & -e^{2} & 1 \end{pmatrix}$
3	$\begin{pmatrix} e^{24} & e^{16} & e^{11} \\ e^{16} & e^{8} & e^{3} \\ e^{11} & e^{3} & e^{-2} \end{pmatrix}$	$\begin{pmatrix} e^{24} & & \\ & e^{8} & \\ & & e^{-2} \end{pmatrix}$	$\begin{pmatrix} e^{24} & e^{13} & e^{11} \\ e^{13} & e^{2} & 1 \\ e^{11} & 1 & e^{-2} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{11} & -e^{13} \\ -e^{11} & 1 & e^{2} \\ e^{13} & -e^{2} & 1 \end{pmatrix}$
4	$\begin{pmatrix} e^{47} & e^{23} & e^{20} \\ e^{23} & e^{-1} & e^{-4} \\ e^{20} & e^{-4} & e^{-7} \end{pmatrix}$	$\begin{pmatrix} e^{33} & & \\ & e^{13} & \\ & & e^{-7} \end{pmatrix}$	$\begin{pmatrix} e^{15} & e^{6} & e^{4} \\ e^{6} & e^{-3} & e^{-5} \\ e^{4} & e^{-5} & e^{-7} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{9} & -e^{11} \\ -e^{9} & 1 & e^{2} \\ e^{11} & -e^{2} & 1 \end{pmatrix}$
5	$\begin{pmatrix} e^{40} & e^{16} & e^{13} \\ e^{16} & e^{-8} & e^{-11} \\ e^{13} & e^{-11} & e^{-14} \end{pmatrix}$	$\begin{pmatrix} e^{40} & & \\ & e^{-8} & \\ & & e^{-14} \end{pmatrix}$	$\begin{pmatrix} e^{8} & e^{-1} & e^{-3} \\ e^{-1} & e^{-10} & e^{-12} \\ e^{-3} & e^{-12} & e^{-14} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{9} & -e^{7} \\ -e^{9} & 1 & e^{2} \\ e^{7} & -e^{2} & 1 \end{pmatrix}$
6	$\begin{pmatrix} e^{48} & e^{24} & e^{21} \\ e^{24} & e^{4} & e^{-1} \\ e^{21} & e^{-1} & e^{-6} \end{pmatrix}$	$\begin{pmatrix} e^{32} & & \\ & e^{16} & \\ & & e^{-6} \end{pmatrix}$	$\begin{pmatrix} e^{20} & e^{9} & e^{7} \\ e^{9} & e^{-2} & e^{-4} \\ e^{7} & e^{-4} & e^{-6} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{11} & -e^{13} \\ -e^{11} & 1 & e^{2} \\ e^{13} & -e^{2} & 1 \end{pmatrix}$

Table 4: Masses and mixing angles for the light neutrino components, and for $b = 1/2$.

	$\tilde{m}_\nu^{eff}$	$m_\nu^{eff,Diag}$	m_ν^{eff}	R_ν^{eff}
1	$\begin{pmatrix} e^{30} & e^{18} & e^{15} \\ e^{18} & e^{6} & e^{3} \\ e^{15} & e^{3} & 1 \end{pmatrix}$	$\begin{pmatrix} e^{30} & & \\ & e^{6} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{20} & e^{12} & e^{10} \\ e^{12} & e^{4} & e^{2} \\ e^{10} & e^{2} & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$
2	$\begin{pmatrix} e^{31} & e^{19} & e^{16} \\ e^{19} & e^{7} & e^{4} \\ e^{16} & e^{4} & e \end{pmatrix}$	$\begin{pmatrix} e^{31} & & \\ & e^{7} & \\ & & e \end{pmatrix}$	$\begin{pmatrix} e^{21} & e^{13} & e^{11} \\ e^{13} & e^{5} & e^{3} \\ e^{11} & e^{3} & e \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$
3	$\begin{pmatrix} e^{36} & e^{22} & e^{17} \\ e^{22} & e^{8} & e^{5} \\ e^{17} & e^{5} & e^{2} \end{pmatrix}$	$\begin{pmatrix} e^{32} & & \\ & e^{8} & \\ & & e^{2} \end{pmatrix}$	$\begin{pmatrix} e^{22} & e^{14} & e^{12} \\ e^{14} & e^{6} & e^{4} \\ e^{12} & e^{4} & e^{2} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$
4	$\begin{pmatrix} e^{39} & e^{23} & e^{20} \\ e^{23} & e^{7} & e^{4} \\ e^{20} & e^{4} & e \end{pmatrix}$	$\begin{pmatrix} e^{35} & & \\ & e^{11} & \\ & & e \end{pmatrix}$	$\begin{pmatrix} e^{21} & e^{13} & e^{11} \\ e^{13} & e^{5} & e^{3} \\ e^{11} & e^{3} & e \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$
5	$\begin{pmatrix} e^{40} & e^{20} & e^{15} \\ e^{20} & 1 & e^{-5} \\ e^{15} & e^{-5} & e^{-10} \end{pmatrix}$	$\begin{pmatrix} e^{40} & & \\ & 1 & \\ & & e^{-10} \end{pmatrix}$	$\begin{pmatrix} e^{10} & e^{2} & 1 \\ e^{2} & e^{-6} & e^{-8} \\ 1 & e^{-8} & e^{-10} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$
6	$\begin{pmatrix} e^{44} & e^{24} & e^{19} \\ e^{24} & e^{4} & e^{-1} \\ e^{19} & e^{-1} & e^{-6} \end{pmatrix}$	$\begin{pmatrix} e^{36} & & \\ & e^{12} & \\ & & e^{-6} \end{pmatrix}$	$\begin{pmatrix} e^{14} & e^{6} & e^{4} \\ e^{6} & e^{-2} & e^{-4} \\ e^{4} & e^{-4} & e^{-6} \end{pmatrix}$	$\begin{pmatrix} 1 & e^{8} & -e^{10} \\ -e^{8} & 1 & e^{2} \\ e^{10} & -e^{2} & 1 \end{pmatrix}$

Table 5: Masses and mixing angles for the light neutrino components, and for $b = -1/2$.

in the range required to explain the atmospheric neutrino oscillation. Given that the Abelian family symmetry fixes the ratio of masses can we simultaneously accommodate atmospheric and solar neutrino oscillations? Remarkably this proves to be easy for 4 of the 12 cases of Tables 4 and 5 lead to the second heaviest neutrino in the range needed to explain solar neutrino oscillations. In case 5 of Table 4 and cases 1, 2 and 3 of Table 5 we see that the ratio of the two heaviest eigenvalue is $O(e^6) \simeq 10^{-2}$. Thus if the heaviest neutrino has mass $0.1eV$, consistent with atmospheric neutrino oscillation, the next neutrino will have mass $O(10^{-3}$ eV). Given the uncertainties due to the coefficients of $O(1)$, this is certainly in the range needed to generate solar neutrino oscillations via the small angle MSW solution, eq.(3).

4.2.4 Lepton Mixing Angles

What about the mixing angles associated with atmospheric and solar oscillations? The former is governed by the $(2-3)$ mixing angle. From Tables 4 and 5 we see the contribution to this mixing angle from the neutrino sector is always of $O(e^2 = \bar{\epsilon})$. Further the contribution to this from $\tilde{m}_{eff}$ is only of $O(e^5)$ and so the dominant contribution is from the rotation R_D needed to diagonalise the Dirac neutrino mass matrix. This is determined entirely by the left-handed neutrino charge (which is the same as the associated charged lepton). The contribution to the $(2,3)$ mixing angle from the charged lepton sector may be read from eq.(51) and eq.(52). It is of $O(e = \sqrt{\bar{\epsilon}})$ and thus is larger than the contribution from the neutrino matrix. Whether these two contributions are of the same sign depends on the phases of the mass matrix elements which is not determined by the Abelian family symmetry. In the case that the two sources of mixing act constructively, a $(2-3)$ mixing with $sin\theta$ up to $\sqrt{\bar{\epsilon}} + \bar{\epsilon} \approx 0.7$ is obtained. Given the uncertainties of the coefficients of $O(1)$ one must conclude this is cconsistent with the present range observed in atmospheric neutrino oscillation, eq.(2)! The conclusion is that the mixing angles in the lepton sector are much larger than in the quark sector results from the need to choose different family charges to account for the relative enhancement observed for the ratio m_μ/m_s compared to m_τ/mb. It is this fact that allows the large mixing observed in atmospheric neutrino oscillation to be accommodated with the quark masses and mixings within the context of a very simple Abelian family symmetry.

Remarkably the family symmetry also leads to an excellent prediction for the mixing angle in the solar neutrino sector. We observed in the previous section that four of the twelve possible structures of Tables 4 and 5 give the second heaviest neutrino in the mass range required to explain the solar neutrino deficit. We may see from eq.(50) and Tables 4 and 5 that the $(1-2)$ mixing relevant to the solar neutrino oscillations is dominated by the mixing in the charged lepton sector and is of $O(\bar{\epsilon}^2 \simeq 0.03)$. This is in excellent agreement with the range for the small angle MSW solution, eq.(3), which is the one selected by the neutrino mass estimates given above.

4.2.5 Renormalisation group stability

Let us finally make some comments on the stability of our solutions with respect to radiative corrections. Note that for large $tan\beta$, renormalisation group effects tend to amplify the (2-3) mixing angle in m_{eff}. The running of the mixing angle is given by [35]

$$16\pi^2 \frac{d}{dt}\sin^2 2\theta_{23} = -2\sin^2 2\theta_{23}(1-\sin^2 2\theta_{23})(Y_{E3}^2 - Y_{E2}^2)\frac{m_{eff}^{33}+m_{eff}^{22}}{m_{eff}^{33}-m_{eff}^{22}} \qquad (55)$$

Due to the effect of the τ Yukawa coupling, m_{eff}^{33} decreases more rapidly than m_{eff}^{22}, so if at the GUT scale $m_{eff}^{22} < m_{eff}^{33}$, the mixing becomes larger. This can be easily shown by semi-analytic equations [31]. In fact, in solutions with m_{eff}^{22} close to m_{eff}^{33}, we can end up either amplifying or destroying the mixing at the GUT scale (the later would occur if m_{eff}^{22} becomes equal to m_{eff}^{33} at an intermediate scale). However, in the case discussed here, we get large hierarchies between m_{eff}^{22} and m_{eff}^{33} due to the splitting in the Dirac mass matrices. This means that our solutions are stable under the RGE runs, even for large $tan\beta$ (for small $tan\beta$ the running of h_τ is so slow, that unless m_{eff}^{22} and m_{eff}^{33} are very close to start with, they never become equal).

5 Summary and Conclusions

The measurement of neutrino oscillations interpreted as evidence for non-zero neutrino mass has the dramatic implication that the Standard Model in its original form is dead. However the simplest modification needed to allow for neutrino masses, the introduction of right-handed neutrinos, is relatively modest and has the aesthetic advantage of restoring the symmetry between the left-handed and right-handed multiplet structure. Moreover, the see-saw mechanism offers a very plausible explanation for the smallness of the neutrino masses, with the heaviest neutrino quite naturally in the range needed to explain the atmospheric neutrino oscillation in the case a double see-saw is operative.

The Standard Model thus extended with three right-handed neutrinos is able to generate both atmospheric and solar neutrino oscillations. Remarkably, the neutrino masses and mixing angles needed fit quite comfortably with a theory of quark and lepton masses based on an Abelian family symmetry with left- right- symmetric charges, spontaneously broken at a very high mass scale close to the gauge unification scale. In this model the reason the mixing angles are very large in the $(2-3)$ lepton sector and less so in the $(2-3)$ quark sector may be traced to the fact that the ratio m_μ/m_s is much larger than that of m_τ/m_b. To fit this, requires a choice of the lepton family charges which in turn gives rise to the expectation of large mixing angles. The size of the lepton mixing in the $(2-3)$ sector relative to that in the quark sector (generating V_{cb}) may be further enhanced if in the quark sector there is a cancellation between the rotations needed in the up and the down quark matrices, while in the lepton sector the up and down mixings add. This is certainly possible within the framework of an Abelian family symmetry, but is not guaranteed because the Abelian family symmetry does not determine the relative phases of these terms. The family symmetry *does* determine the order of magnitude of the neutrino mass differences, and readily generates a mass consistent with the small angle MSW solution to solar neutrino oscillations. Again it is remarkable that the associated expectation for the mixing angle is in the correct range to accommodate the small angle solution.

While the left- right- symmetric models are of particular interest because the $U(1)$ family charges are so strongly constrained, there are further possibilities of interest involving a combination of the family symmetry with an extension of the Standard model gauge group. We explored various possibilities to see if the large mixing angle observed in atmospheric neutrino oscillation was compatible with the quark and charged lepton masses and mixings without requiring large coefficients or cancellation between terms unrelated by the Abelian symmetry. The most interesting possibility we identified involved the gauge group $SU(3)^3$. This has the merit that the (small) value of V_{cb} is not related to the large neutrino mixing angle. In this the freedom to choose different left- and right- handed charges for the down quarks allows the construction of a model with $V_{cb} \approx m_s/m_b$. However the left- and right-handed leptons belong to the same representation of $SU(3)^3$ and this leads to the same symmetric lepton mass matrices as in the solutions found in the fully left-right symmetric case, thus maintaining the good prediction for large atmospheric and small solar neutrino mixing.

We find it quite encouraging that the simplest possible family symmetry is able to correlate so many different features of quark, charged lepton and now neutrino masses and mixings. Due to the unknown Yukawa couplings of O(1) it is difficult to make precise predictions (apart from those arising from texture zeros) and this makes the scheme difficult to establish definitively. However there are general characteristic features in the neutrino sector that will be tested in the future. For example, although the mixing is expected to be large in the $(2, 3)$ sector, it is quite unlikely to be maximal. Also, if the indication for neutrino masses coming from the LSND collaboration proves correct, it will be necessary to add at least a further sterile neutrino component. The information on neutrino masses and mixing is very important in testing theories of fermion mass and we look forward to the extensive new data in this area that will be forthcoming with the new generation of detectors.

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