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Study on chemical equilibrium in nucleus-nucleus collisions at relativistic energies

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Abstract. We present a detailed study of chemical freeze-out in nucleus-nucleus collisions at beam energies of 11.6, 30, 40, 80 and 158A GeV. By analyzing hadronic multiplicities within the statistical hadronization approach, we have studied the chemical equilibration of the system as a function of center of mass energy and of the parameters of the source. Additionally, we have tested and compared different versions of the statistical model, with special emphasis on possible explanations of the observed strangeness hadronic phase space under-saturation.

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1. The Statistical Hadronization Model

One of the main results of the study of high energy A-A collisions is a surprising success of the statistical-thermal models in reproducing essential features of particle production (see for example [1, 2]). In this paper we study nucleus-nucleus collision within the statistical model in the energy range that is believed [3] to cover the threshold for creation of Quark-Gluon Plasma (QGP) in the early stage of Pb-Pb collisions.

The main idea of the SHM is that hadrons are emitted from regions at statistical equilibrium. No hypothesis is made about how statistical equilibrium is achieved; this can be a direct consequence of the hadronization process. In a single collision event, there might be several clusters with different collective momenta, different overall charges and volumes. However, Lorentz-invariant quantities like particle multiplicities are independent of clusters momenta.

If the system size is sufficiently large [4], the analysis can be done in grand-canonical ensemble in which the mean *primary* multiplicity of the $j^{\rm th}$ hadron with mass m_j and spin J_j reads:

$$\langle n_j \rangle = \frac{(2J_j+1)V}{(2\pi)^3} \int \mathrm{d}^3\mathrm{p} \, \left[\mathrm{e}^{\sqrt{\mathrm{p}^2+m_j^2}/T + \boldsymbol{\mu}\cdot\mathbf{q}_j/T} \pm 1 \right]^{-1} \tag{1}$$

where T is the temperature, V the scaling volume, $\mathbf{q_j} = (Q_j, B_j, S_j)$ is a vector having as components the electric charge, baryon number and strangeness of the hadron and $\boldsymbol{\mu} = (\mu_Q, \mu_B, \mu_S)$ is a vector of the corresponding chemical potentials; the upper sign applies to fermions, the lower to bosons.

In order to correctly reproduce the data, it is also necessary to introduce at least one non-equilibrium parameter suppressing hadrons containing valence strange quarks, $\gamma_S \neq 1$ [5]. With this supplementary parameter, hadron multiplicity is as in Eq. (1) with the replacement: $\exp[\boldsymbol{\mu}\cdot\mathbf{q}_j/T] \to \exp[\boldsymbol{\mu}\cdot\mathbf{q}_j/T]\gamma_S^{n_s}$ where n_s stands for the number of valence strange quarks *and* anti-quarks in the hadron j.

Finally, the overall multiplicity to be compared with the data, is calculated as the sum of primary multiplicity (1) and the contribution from the decay of heavier hadrons: $\langle n_j \rangle = \langle n_i \rangle^{\rm primary} + \sum_k \mathrm{Br}(k \to j)\langle n_k \rangle$, where the branching ratios are taken from the latest issue of the Review of Particle Physics [6].

2. Experimental Data Set And Analysis Results

The bulk of the experimental data consists of measurements made by NA49 collaboration in central Pb-Pb collisions at beam momenta of 30, 40, 80 and 158A GeV [7, 8]. As far as AGS data at 11.6A GeV [9, 10, 11, 12] is concerned, we have used both multiplicities measured by the experiments and extrapolations of measured rapidity distributions (for details, see [15]).

The analysis has been carried out by looking for the minima of the $\chi^2 = \sum_i \frac{(n_i^{\rm exp} - n_i^{\rm theo})^2}{\sigma_i^2}$. The fitted parameters within the main scheme SHM(γ_S) are shown in table 1. The observed differences in the fit parameters between two independent analyses A and B are of the order of the fit errors.

A major result of these fits is that γ_S is significantly smaller than 1 in almost all cases, that is strangeness seems to be under-saturated with respect to a completely chemically equilibrated hadron gas. This confirms previous findings [12, 13, 14].

2.1. Strangeness correlation volume

To account for the observed under-saturation of strangeness, a picture has been put forward in which strangeness is supposed to be exactly vanishing over distances less than those implied by the overall volume V [16], i.e. all clusters or fireballs emerge with $S = 0$ and they are not allowed to share non-vanishing net strangeness.

Assuming, for sake of simplicity, that all clusters have the same typical volume V_c and that treating baryon number and electric charge (but not strangeness) grand-canonically, gives a good description of the system, one can perform the analysis in *strangeness canonical ensemble* (for details, see [15]).

If V_c is sufficiently small, the multiplicities of strange hadrons turn out to be significantly suppressed with respect to the corresponding grand-canonical ones due to an effect called *canonical suppression.*

We have fitted the data sample of full phase space multiplicities in Pb-Pb collisions at 158A GeV fixing $\gamma_S = 1$. The quality of the fit is worse ($\chi^2/\mathrm{dof} = 37.2/9$) with respect to the SHM(γ_S) model, whilst thermal parameters T=157.9 MeV and μ_B=261.5 MeV are compatible with the main version of the statistical model. Our result suggests that, for the local strangeness correlation to be an effective mechanism, the cluster volume should be of the order of 2.5% of the overall volume.

2.2. Superposition of NN collisions with a equilibrated fireball

In this picture, henceforth referred to as SHM(TC), the observed hadron production is approximately the superposition of two components: one originated from a large fireball at complete chemical equilibrium at freeze-out, with $\gamma_S = 1$, and another component from single nucleon-nucleon collisions. Since it is known that in NN collisions strangeness is strongly suppressed [1], the idea is to ascribe the observed under-saturation of strangeness in heavy ion collisions to the NN component.

With the simplifying assumption of disregarding subsequent inelastic collisions of particles produced in those primary NN collisions, the overall hadron multiplicity can be written then as $\langle n_j \rangle = \langle N_c \rangle \langle n_j \rangle_{NN} + \langle n_j \rangle_V$, where $\langle n_j \rangle_{NN}$ is the average multiplicity of the j^{th} hadron in a single NN collision, $\langle N_c \rangle$ is the mean number of single NN collisions and $\langle n_j \rangle_V$ is the average multiplicity of hadrons emitted from the equilibrated fireball.

To calculate $\langle n_j \rangle_{NN}$ we have used the statistical model and fitted pp full phase space multiplicities measured at the same beam energy by the same NA49 experiment. For np and nn collisions, the parameters of the statistical model determined in pp are retained and the initial quantum numbers are changed accordingly. Theoretical multiplicities have been calculated in the canonical ensemble, which is described in detail in ref. [2].

We have fitted T, V, μ_B of the central fireball and $\langle N_c \rangle$ by using NA49 data in Pb-Pb collisions at 158A GeV. The fit quality, as well as the obtained values of T and μ_B, are comparable to the main fit within the SHM(γ_S) model. The predicted number of "single" NN collisions is about 50 with a 16% uncertainty.

2.3. Non-equilibrium of hadrons with light quarks

In this model [17] two non-equilibrium parameters are introduced for the different types of quarks, γ_q for u, d quarks and γ_s for strange quarks. By defining: $\gamma_S = \frac{\gamma_s}{\gamma_q}$ and $\tilde{V} = V\gamma_q^2$, the Boltzmann limit of average multiplicity reads:

$$\langle n_j \rangle = \frac{(2J_j+1)\tilde{V}}{(2\pi)^3} \gamma_S^{n_s} \gamma_q^{|B_j|} \int \mathrm{d}^3\mathrm{p}\ \exp[-\sqrt{\mathrm{p}^2 + m_j^2}/T + \boldsymbol{\mu} \cdot \mathbf{q}_j/T] \tag{2}$$

where B_j is the baryon number, as long as mesons have two and baryons have three valence quarks. The parameter γ_q has a definite physical bound for bosons which can be obtained by requiring the convergence of the series $\sum_{N=0}^{\infty}(\gamma_q^{n_q N}) \exp(-N\epsilon/T + N\boldsymbol{\mu} \cdot \mathbf{q}_j/T)$ for any value of the energy. If γ_q reaches its bounding value $\gamma_q = \exp(m_{\pi^0}/2T) \simeq 1.5$ for $T \simeq 160$ MeV, a Bose condensation of particles in the lowest momentum state sets in.

It is seen that the absolute χ^2 minimum falls in the region of pion condensation, at $\gamma_q \simeq 1.62$, with $\chi^2 \simeq 13$ and $T \simeq 140$ MeV. This finding is in agreement with what is found in ref. [17]. However, there is also a local minimum at the lower edge $\gamma_q = 0.6$, with a temperature of 187 MeV, which is only one unit of χ^2 higher than the absolute minimum. This indicates that the absolute minimum could be rather unstable against variations of the input data and this is in fact what we find by varying down the pion multiplicities by only 1 σ. For this case, the absolute minimum of χ^2 now lies at $\gamma_q = 0.6$ instead of at the pion condensation point.

In view of the instability of the fit, and of the small *relative* χ^2 improvement in comparison with the main fit, we conclude that there is so far no evidence for the need of this further non-equilibrium parameter.

3. Energy Dependence

The chemical freeze-out points in the $\mu_B - T$ plane are shown in fig. 1. The four points at beam energies of 11.6, 40, 80 and 158A GeV have been fitted with a parabola: $T = 0.167 - 0.153\mu_B^2$, where T and μ_B are in GeV.

A possible indication of deconfinement phase transition in Pb-Pb collisions at the low SPS energies was reported on the basis of the observed energy dependence of several observables [18]. Particularly, the $\langle \mathrm{K}^+ \rangle / \langle \pi^+ \rangle$ ratio shows a peaked maximum at about 30A GeV. One may expect that this anomaly should be reflected in the energy dependence of γ_S parameter fitted within SHM(γ_S) scheme. This dependence is plotted in fig. 2 and in fact a maximum shows up at 30A GeV.

The anomalous increase of relative strangeness production at 30A GeV can be seen also in the Wroblewski variable $\lambda_S = 2\langle \mathrm{s\bar{s}} \rangle / (\langle \mathrm{u\bar{u}} \rangle + \langle \mathrm{d\bar{d}} \rangle)$, the estimated ratio of newly produced strange quarks to u, d quarks at primary hadron level, shown in fig. 2

In order to further study strangeness production features, we have also compared the the measured $\langle \mathrm{K}^+ \rangle / \langle \pi^+ \rangle$ ratio with the theoretical values in a hadron gas along the interpolated freeze-out curve for different values of γ_S (see fig. 1). The calculated dependence of $\langle \mathrm{K}^+ \rangle / \langle \pi^+ \rangle$ on μ_B is non-monotonic with a broad maximum at $\mu_B \simeq 400$ MeV (i.e. $E_{beam} \simeq 30A$ GeV). Taking into account that systematic errors at different energies in Pb-Pb collisions are fully correlated, we can conclude that the data points seem not to follow the constant γ_S lines.

4. Summary and Conclusions

It is found that the main version of the statistical model (with γ_S), fits all the data analyzed in this paper. We have tested a model (SHM(TC)) which can fit the data at 158A GeV very well if the number of independent NN collisions is around 50 with a sizeable uncertainty. A model in which strangeness is assumed to vanish locally [16] yields a worse fit to the data with respect to SHM(γ_S) and SHM(TC). Moreover, we have found that the present set of available data does not allow to establish whether a further non-equilibrium parameter (γ_q) is indeed needed to account for the observed hadron production pattern.

The evolution of the freeze-out temperature and baryon-chemical potential is found to be smooth in the AGS-SPS-RHIC energy range. $\langle \mathrm{K}^+ \rangle / \langle \pi^+ \rangle$ ratio, Wroblewski factor λ_S as well as γ_S parameter calculated within the statistical model suggests that there might be a peak in relative strangeness production at about 30A GeV of beam momenta.

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Parameters	Main analysis A	Main analysis B		Main analysis A	Main analysis B
	Au-Au 11.6A Gev			Pb-Pb 30A GeV	
T (MeV)	118.1±3.5 (4.1)	119.1±4.0 (5.4)		139.5	140.3
μ_B (MeV)	555±12 (13)	578±15 (21)		428.6	428.7
γ_S	0.652±0.069 (0.079)	0.763±0.086 (0.12)		0.938±0.078 (0.13)	1.051±0.103 (0.16)
V'	1.94±0.21 (0.24)	1.487±0.18 (0.25)		6.03±0.50 (0.85)	5.273±0.526 (0.80)
χ^2/dof	4.0/3	5.5/3		5.75/2	4.6/2
	Pb-Pb 40A GeV			Pb-Pb 80A GeV	
T (MeV)	147.6±2.1 (4.0)	145.5±1.9 (3.5)		153.7±2.8 (4.7)	151.9±3.4 (5.4)
μ_B (MeV)	380.3±6.5 (13)	375.4±6.4 (12)		297.7±5.9 (9.8)	288.9±6.8 (11)
γ_S	0.757±0.024 (0.046)	0.807±0.025 (0.047)		0.730±0.021 (0.035)	0.766±0.026 (0.042)
V'	8.99±0.37 (0.71)	8.02±0.34 (0.63)		15.38±0.61 (1.0)	14.12±0.65 (1.1)
χ^2/dof	14.7/4	13.6/4		11.0/4	10.4/4
	Pb-Pb 158A GeV				
T (MeV)	157.8±1.4 (1.9)	154.8±1.4 (2.1)			
μ_B (MeV)	247.3±5.2 (7.2)	244.5±5.0 (7.8)			
γ_S	0.843±0.024 (0.033)	0.938±0.027 (0.042)			
V'	21.13±0.80 (1.1)	18.46±0.69 (1.1)			
χ^2/dof	16.9/9	21.6/9			

Table 1. Summary of fitted parameters (V'=VT^3 $\exp[-0.7\,\mathrm{GeV}/T]$) at AGS and SPS energies in the framework of the SHM(γ_S) model. The re-scaled errors (see [6, 15]) are quoted within brackets. For Pb-Pb at 30 A GeV data, we have constrained T and μ_B to lie on the fitted chemical freeze-out curve (see fig. 1)

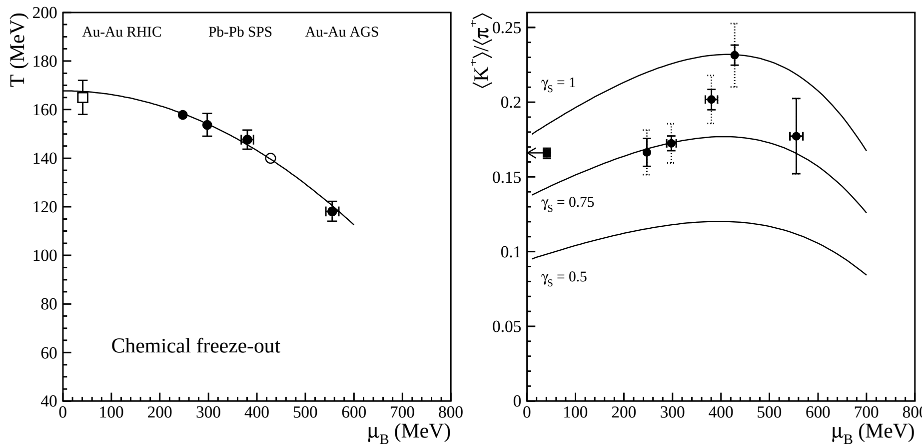

Fig. 1. LEFT: Chemical freeze-out points in the $\mu_B - T$ plane in various heavy ion collisions. The full round dots refer to Au-Au at 11.6 and Pb-Pb collisions at 40, 80, 158A GeV obtained in the analysis A, whilst the hollow square dot has been obtained in ref. [19] by using particle ratios measured at midrapidity in Au-Au collisions at $\sqrt{s}_{NN} = 130$ GeV. The hollow round dot without error bars refers to Pb-Pb collisions at 30A GeV and has been obtained by forcing T and μ_B to lie on the parabola fitted to the full round dots.
RIGHT: Measured $\langle \mathrm{K}^+ \rangle / \langle \pi^+ \rangle$ ratio as a function of the fitted baryon-chemical potential. The full square dot is a preliminary full phase space measurement in Au-Au collisions at $\sqrt{s}_{NN} = 200$ GeV [20]. For the SPS energy points the statistical errors are indicated with solid lines, while the contribution of the common systematic error is shown as a dotted line. Also shown the theoretical values for a hadron gas along the fitted chemical freeze-out curve (left), for different values of γ_S.

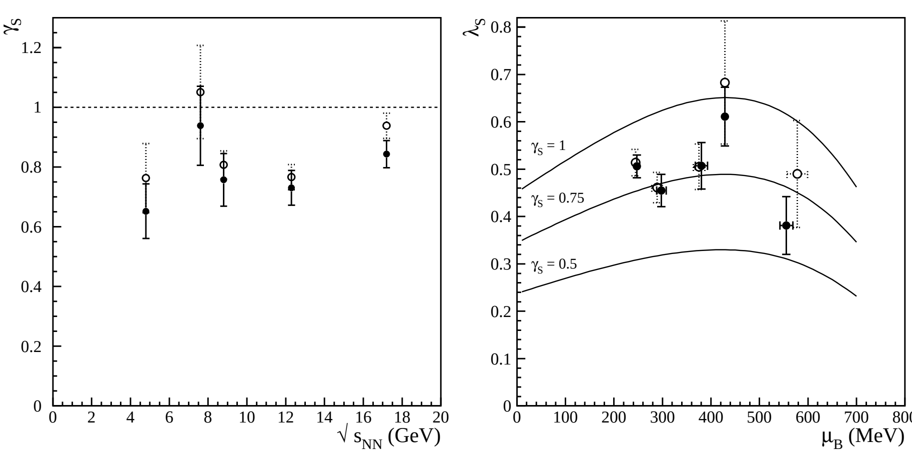

Fig. 2. LEFT: Strangeness non-equilibrium parameter γ_S as a function of the nucleon-nucleon centre-of-mass energy. Full dots refer to fit A, hollow dots fit B.
RIGHT: λ_S estimated as a function of the fitted baryon-chemical potential. Also shown the theoretical values for a hadron gas along the fitted chemical freeze-out curve shown in fig. 1, for different values of γ_S.