EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

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PHYSICS III COMMITTEE

THE OMICRON COLLABORATION

INITIAL PROGRAMME FOR THE OMICRON SPECTROMETER

In the past the Omicron Collaboration has presented a list of possible experiments, the common factor being the requirement of a large magnet containing suitably disposed planes of wire chambers We have recently concentrated attention on a smaller group of experiments from which we expect to choose our initial programme. These experiments have been chosen as a balance between their physics content, the prejudices of the people concerned, and the technical difficulty envisaged.

This document gives details of these experiments which are :

(A) π, μ scattering at backward angles on light nuclei.

(B) Double charge exchange (π^+, π^-).

(C) The branching ratio of the rare decay $\pi^0 \to e^+e^-$.

(D) Electron-positron decay of pionic atoms and β-decay form factor.

(E) Low energy pion production $\pi\, p \to \pi\, \pi\, N$.

For most of these experiments, detailed calculations of rates, background, etc, have been made, and it is now clear what beam is required, what arrangement of detectors will be necessary in the magnet, how the equipment should be triggered, and what sort of analysis will be required. Work is still in progress on two of the experiments for which precise layouts are not yet available.

At the present time the collaboration sees its initial programme as follows :

1st experiment :	A above
2nd experiment :	B above
3rd experiment :	C above.

Detailed layouts have been prepared for all these experiments. The logic is to start with an experiment which is not too demanding technically and which will allow the smooth running-in of the apparatus; hopefully this could be finished fairly quickly. We could then proceed to a nuclear physics experiment of more physics interest, and then pass to the very difficult $\pi^0 \to e^+e^-$ to which attaches a very high level of interest.

It could be, however, that further considerations of rates or backgrounds of these or other experiments, or the arrival of new results from other laboratories will convince us over the next year that this programme should be modified.

FOR THE OMICRON COLLABORATION

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SCATTERING OF PIONS (AND MUONS) FROM LIGHT NUCLEI IN THE BACKWARD DIRECTION

In the comparison between theory and experiments on pion elastic scattering by light nuclei $(A \leq 20)$ up to 280 MeV, three separate energy regions must be distinguished.

1) Low Energies ($\leq$ 40 MeV)

   A velocity-dependent potential[1] turns out to be rather well suited for explaining data even at large angles, due to the smallness of πN scattering amplitudes and the dominant single scattering[2,3].

2) Intermediate Energies (40 to 120 MeV)

   The velocity-dependent potential fails to fit data for scattering angles larger than about 60°. Other models such as the "effective radius" and the "matter radius" behave no better.

3) Energies in the Region of the (3,3) Resonance: 120 MeV $\leq T_{\pi} \leq$ 280 MeV

   Data can be more reasonably explained in terms of a Glauber-type analysis[4] or even of more simple optical models (viz. the "effective-radius" model). Such models however feel the effect of their high-energy derivation and rapidly become unsuccessful whenever energies are not large enough to neglect the nuclear-structure contribution.

In the intermediate-energy region, a phase shift analysis carried out for the ^{12}C target nucleus[5] by adjusting iteratively the parameters of the various partial waves can yield a satisfactory interpretation of the experimental results. In this case, however, different sets of phase shifts fit equally well the existing experimental data up to 120°, while they deviate at larger angles: at 180° this difference may reach two orders of magnitude. Therefore large angle and mainly backward-scattering cross sections must be measured in order to be able to choose the correct set of phase shifts.

Concerning the inelastic pion scattering, since the pion-nucleus elastic scattering is reasonably well described by an optical model, it seems that a multi-channel extension of the model should equally reproduce the inelastic scattering, at least from the lowest excited states. The most serious problem lies in the choice of the pion nucleus interaction, since it is not uniquely determined by the pion-nucleon t-matrix on the energy shell. Wilkin[6] and Fäldt[7] have independently proposed a pion-nucleus optical potential which, on the energy shell, becomes identical to the Kisslinger one. Nishiyama and Otsubo[8], with plane wave impulse approximation (PWIA) examined the inelastic scattering of pions from ^{12}C and found a qualitative agreement between the theory and the experiment. Lee and McManus[9] investigated the excitation of low-lying levels 2^+ and 3^- of ^{12}C by π^- in the distorted wave impulse approximation (DWIA). The conclusions of the comparison of these calculations with the experimental data was that, although the qualitative features of inelastic scattering cross sections can be explained by the simple PWIA approximation, absolute values and the angular distributions depend upon the initial and final state interaction of the pion, and the DWIA gives a good fit to experimental data for excitation of levels with $\vec{T} = 0$. The wide angle data however (where they exist), sometimes exceed the predicted cross sections by as much as an order of magnitude.

From the experimental point of view, the most extensively studied target nucleus was ^{12}C, due to the relatively large spacing between the ground and low-lying excited states. Old measurements are due to Kane[10] with π^+ of 31.5 MeV, Baker et al.[11] with π^- of 80 MeV and Edelstein et al.[12] with π^- of 69.5 and 87.5 MeV. Indeed, the most complete and careful experiment performed up to now was done at CERN by Binon et al.[13], with π^- in the region of the (3,3) resonance ($120 \leqslant T\pi \leqslant 230$ MeV). They used a double achromatic spectrometer with an overall momentum resolution of less than 1%. Measurements were extended up to 140°.

Very recently measurements of differential cross sections for elastic and inelastic scattering of π^- in the intermediate energy region ($60 \leqslant T\pi \leqslant 90$ MeV) on ^{12}C in the angular range 160°-180° were done at Frascati by Barbini et al.[14] This group intend to extend the measurements to lower energies of pions and to a wider angular range; indeed they cannot perform experiments in the region of the (3,3) resonance due to the limitation in the energy of the pion beam[15].

It seems interesting from the physical point of view and quite easy from the experimental side to carry out measurements of elastic and inelastic scattering of π^- from ^{12}C at the same energies as the experiment of the MSS Group[13], and from 130° to 180°, using the Omicron spectrometer.

Concerning the muon scattering by nuclei, Ericson[16] emphasized the great interest of carrying out measurements of elastic scattering of μ^+ and μ^- from a relatively simple nucleus like ^4He, in order to measure the nuclear polarizability. This effect would introduce a difference between the differential cross-sections for μ^+ and μ^- at large momentum transfers, and for muons of low energy. This was initially evaluated to be ~ 5% (at 300 MeV) at scattering angles around 180^0. This number was evaluated by extrapolating the information obtained from the experiments[17] on μ^- capture at rest by ^4He. Further insight into the problem, done by Bermabeu, Ericson and Wilkin, led to the conclusion that the extrapolation from the bound state region to the positive energy region could be dangerous, and the difference between the differential cross-sections for μ^+ and μ^- reduced to the order of 1%. This last difference is obviously harder to be measured, not essentially for counting statistic reasons but because of possible systematic errors that could affect the data and obscure any effect completely. It thus seems difficult to propose this measurement as one of the first Omicron experiments. A complete and careful knowledge of the operation of the apparatus is in fact needed before starting the experiment. Indeed, it could be possible to do some preliminary measurements on μ scattering by nuclei with a more modest physics objective, and for preparation of the future harder experiments on μ scattering.

The less ambitious physics interest could be the study of elastic and inelastic scattering to some well-known excited states (e.g. the first low-lying levels in ^{12}C) in order to test the radiative corrections that must inevitably be applied in the analysis of electron inelastic scattering. Muon scattering cannot of course compete with electron scattering as a tool for extracting nuclear structure information, if we compare the beam intensities and the resolution of the spectrometers for electrons and muons. But a drawback of the electron inelastic scattering is the need for the subtraction of the radiative tails, which can introduce considerable errors in the absolute values of the differential cross-section. The amount of the radiative tail for muon scattering is expected to be two orders of magnitude less than for electron scattering; thus the comparison between the differential cross-sections for the electromagnetic excitation of a well-known nuclear state by electrons and muons could be used as a check of the radiative tail corrections for electrons.

Let us discuss now how the Omicron spectrometer could be used to measure pion backward scattering by ^{12}C. Fig.Al shows a sketch of the lay-out. Beams from 200 to 400 MeV/c can be used essentially with the same geometrical arrangement of the chambers, by increasing the magnetic field up to the maximum value of 10 KGauss and displacing the position of the target and the angle of incidence of the beam.

The figure is related to the arrangement for 200 MeV/c. Pions entering the magnet are signalled by a coincidence of the beam telescope scintillators $S_1*(\overline{A_1+A_2})*S_2*S_3*S_4$ and the Disc counter. Their momenta are measured by the set of multiwire proportional chambers labelled as C1, C2, C3 and C4 in the figure. Scattered pion momenta are measured by the set of multiwire and drift chambers labelled as C5, C6, C7 and C8. The final trigger will be given by an $S_1*(\overline{A_1+A_2})*S_2*S_3*S_4*\overline{A}_3*S_5*S_6$ coincidence. The total energy resolution on the nuclear levels must be of the order of 2 MeV to allow the separation of the elastic and inelastic peaks. The intrinsic momentum resolution of each arm of the spectrometer would be better than 0.5%, thus some care is needed in the choice of the target thickness. Due to lack of the knowledge of the interaction point in the target, the thickness of a graphite target would not exceed 0.5 gm / cm^2 for the quoted resolution of 2 MeV. Even if the expected magnitude of the cross-section is not such as to impose the use of thicker targets, an improvement will be the utilization of a plastic (CH)n scintillator as target. The total energy loss (before and after interaction) of the pion could be measured by the pulse height in the scintillator and a target thickness of ~ 2 gm /cm^2 (Carbon equivalent) could be used. Scattering of pions by protons of the scintillator would not constitute a source of background, since the kinematics is quite different in the backward direction, but on the contrary a useful check for the normalization of the cross-sections. Furthermore, the use of a live target could be a benefit for the study of highly excited states, which can decay by charged particle emission, whose energy could be roughly determined by the pulse height analysis.

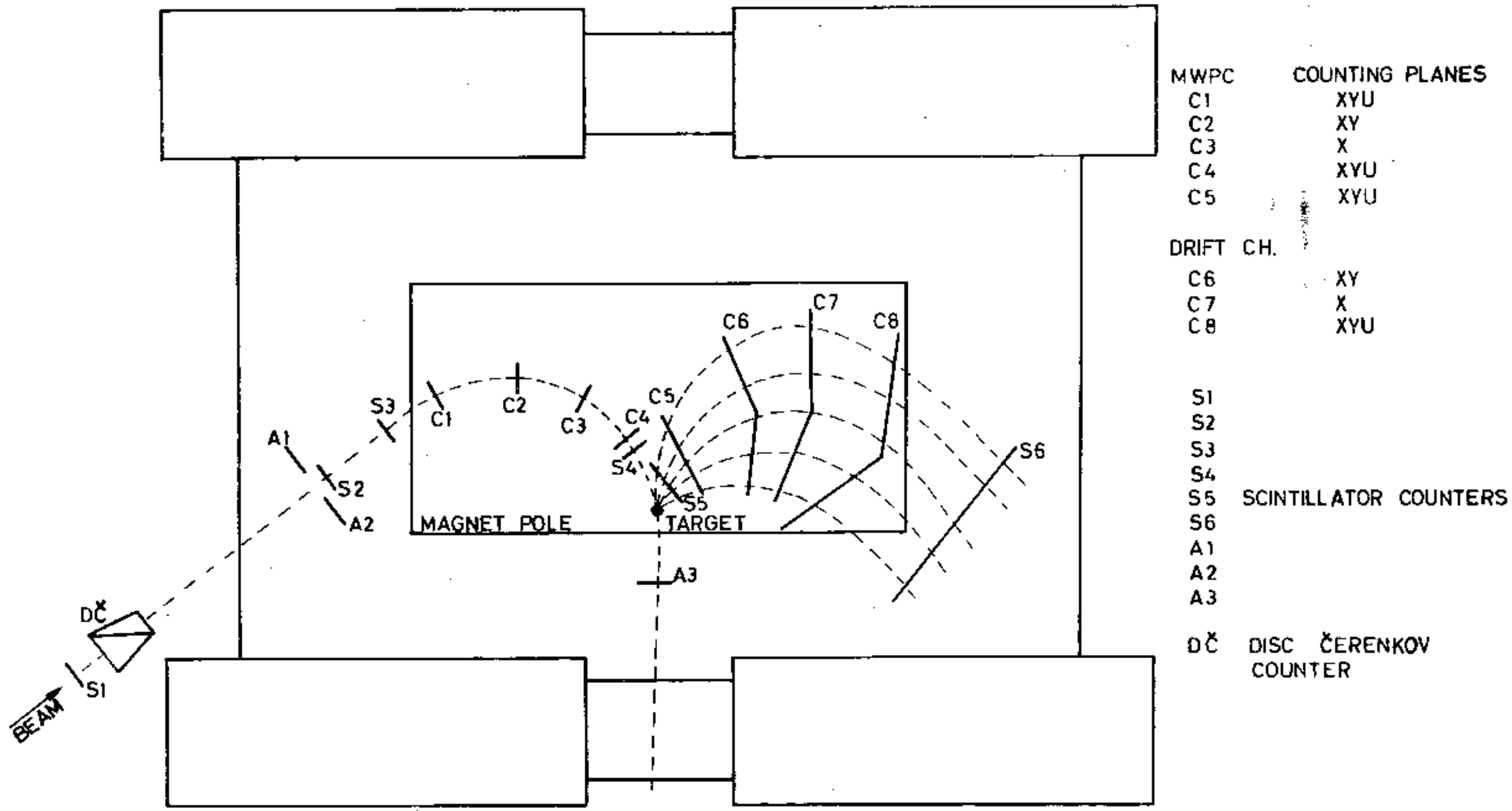
We have calculated by a Monte Carlo simulation of the experiment the angular acceptance for the chamber configuration of Fig.A1. Fig.A2a) shows the acceptance for the azimuthal angle ϕ, integrated over all the θ values. Fig.A2b) shows the total acceptance (i.e. taking into account cuts on ϕ in correspondence of each interval of θ) as a function of the scattering angle θ. Events corresponding to scattering angles from 130° to 180° and on the whole excitation spectrum of ^{12}C will be recorded simultaneously. Assuming an incident beam of $3 \times 10^6 \pi^-$/sec., a target thickness of 1 gm /cm^2, a detection efficiency of 100%, extrapolating the MSS data for the elastic scattering from 140° to 180° and using the angular acceptance value inferred from Fig. 2b), we would obtain ~ 8 events/sec. in the elastic peak at $T\pi$ = 120 MeV and ~ 0.8 events/sec. at $T\pi$ = 200 MeV. Similiar counting rates are expected for the inelastic peaks. It is clear that good statistics could be collected in a short running time.

The above calculations were done on rather pessimistic hypotheses about the beam intensity and the values of the cross-sections.

For muon scattering to the 4.43 MeV excited states in ^{12}C, at momentum transfers around 1 fm^{-1} (angles around 40^o for $T\mu \simeq 100$ MeV), assuming an incident beam of $10^6 \mu^+$/sec., counting rates in the order of 0.5 events/minute are foreseen.

Our final remark concerns the competitivity of the Omicron spectrometer with the high-resolution spectrometers that will be soon in operation at the pion factories of LAMPF and SIN. Omicron cannot complete for resolution; counting rates are on the contrary comparable, due to the large solid angle of detection obtained by placing the target in a big magnetic volume. The resolution is indeed enough to separate single levels in light nuclei (^{12}C, ^{16}O). High-resolution spectrometers on the other hand cannot cover the extreme backward angles (160^o-180^o), without additional arrangements of magnets containing the target. It does not seem probable that the first experiments of the high-resolution spectrometers will be devoted to back-scattering of pions. Thus, if the above measurement will be done in the first stage experimentation with Omicron, it would be the first in the field. A further, and maybe stronger, argument for doing this experiment at the beginning of the operation of Omicron, lies in the fact that it is the easiest one concerning counting rates, analysis of the data, trigger etc. It could thus be considered as an operational test of the whole apparatus.

No competition is expected for muon scattering. The difference with pion scattering lies essentially in the lower counting rates. Some care must thus be devoted to the trigger, even if no particular difficulties are expected. This measurement must be done before starting the design of the more ambitious and difficult lay-out of Omicron able to measure the μ^+, μ^- back-scattering difference.



LAYOUT FOR PION BACKWARD SCATTERING

Fig.A1 : Lay-out of the Omicron spectrometer for pion backward scattering measurement

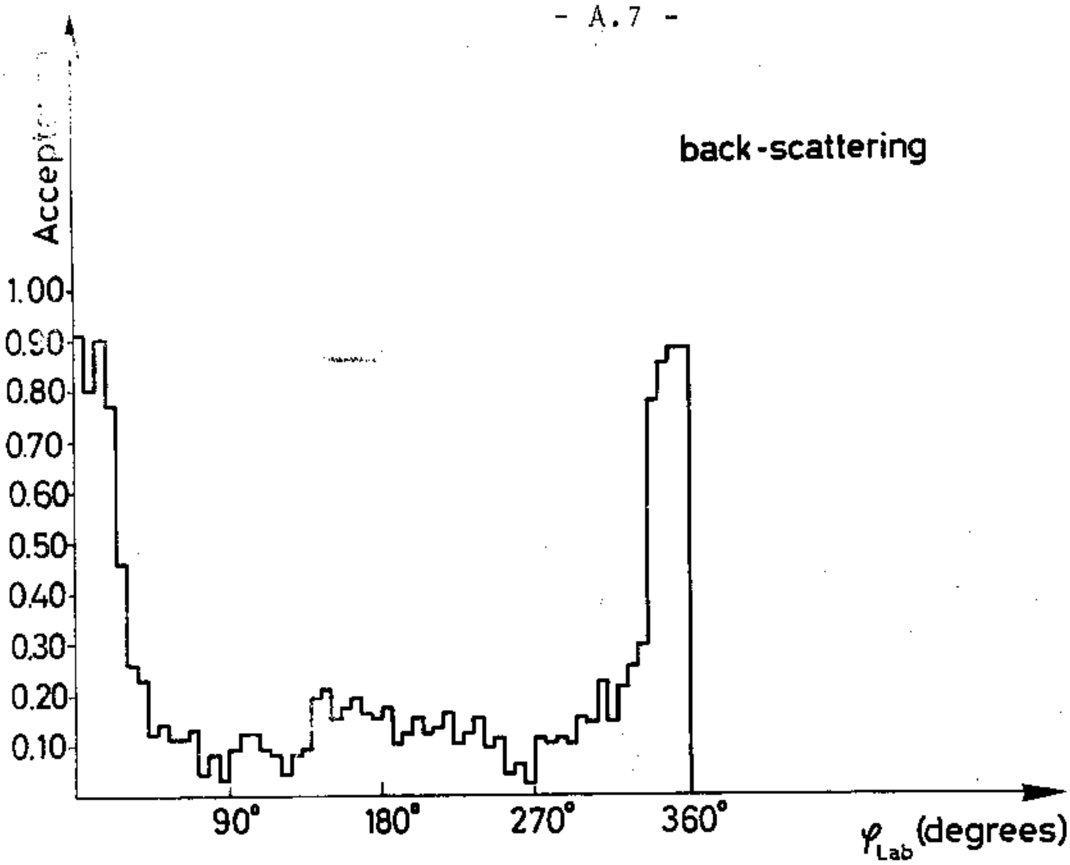


Fig.A2a) : Acceptance for the azimuthal angle ϕ, integrated over all the θ values, calculated by the Monte Carlo simulation of the experiment.

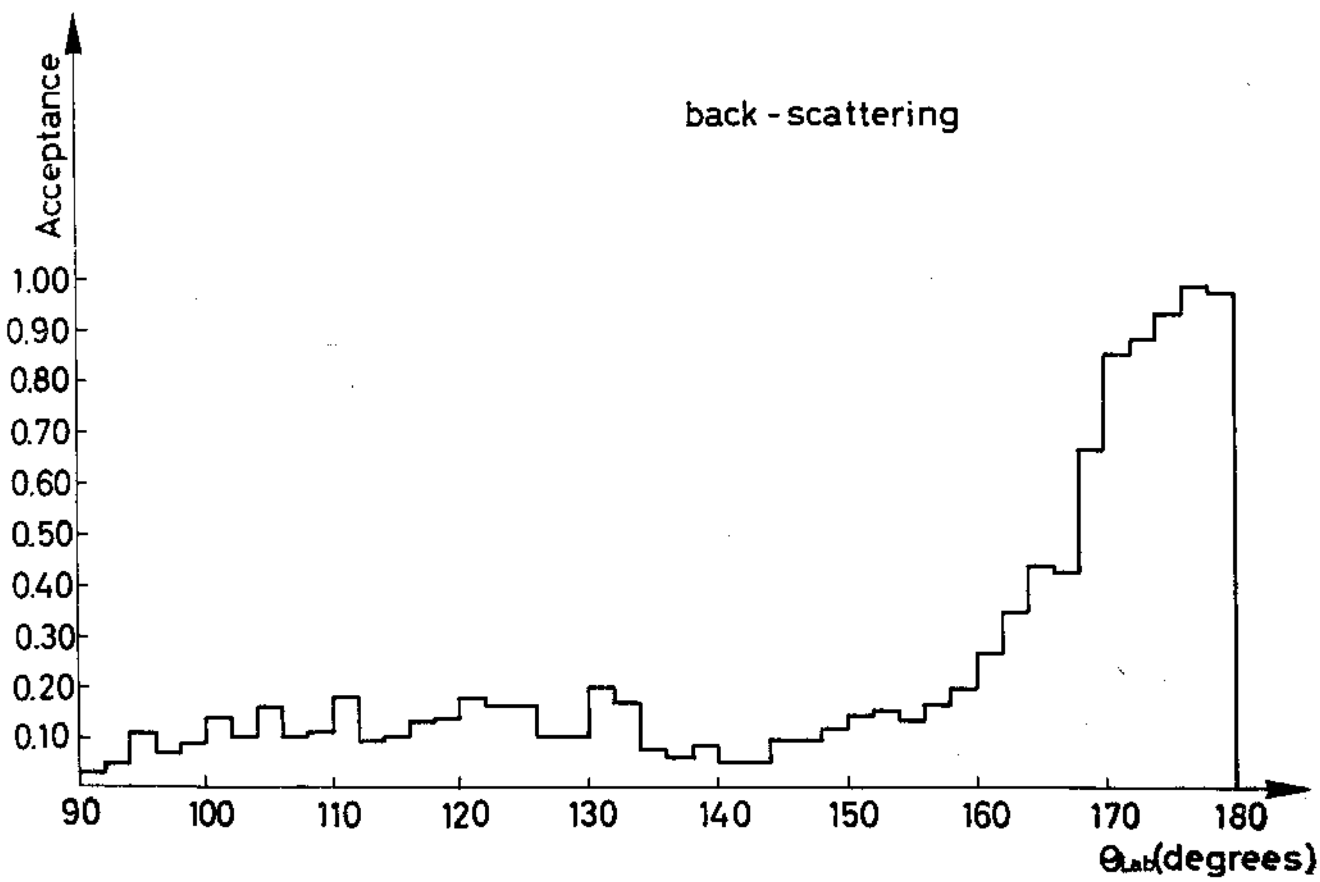


Fig.A2b) : Acceptance for the scattering angle θ, evaluated using the cuts on ϕ corresponding to each θ interval.

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DOUBLE CHARGE EXCHANGE REACTIONS

Among the pion-induced reactions one of the less experimentally studied is double charge exchange (DCX):

$$\pi^{\pm} + {}^{A}_{Z}X \rightarrow \pi^{\mp} + {}^{A}_{Z\pm 2}X \qquad (1)$$

The interest in studying these reactions was firstly pointed out by De Shalit and Drell, while a comprehensive revision of both experimental and theoretical aspects has recently been done by Becker and Batusov[1]. The main feature of DCX is the change of two units of the third component of the isotopic spin of the nucleus $\Delta T_3 = \pm 2$.

Of the many possible final channels of the DCX reaction, the more interesting seem to be those leading to nuclear bound states. These events are unfortunately rather infrequent, but are those that give the greatest information about nuclear structure.

Production of bound states in $^7B^*$ and $^9C^*$ was observed at CERN[2] in the reactions:

$$\pi^+ + {}^7Li \rightarrow {}^7B^* + \pi^- \quad ; \quad \pi^+ + {}^9Be \rightarrow {}^9C^* + \pi^- \qquad (2)$$

at 0^o and $T\pi$ = 195 MeV. However, no bound state production was observed in a subsequent experiment done at Berkeley, at 8.5^o and 16^o and the same energy. There are no explanations for this experimental discrepancy. A firm conclusion about the existence and the properties of nuclei with large proton excess could only be made with systematic experiments, not at a fixed angle, but with angular distributions.

If we consider the rather poor energy resolution of the experiments mentioned above (about 10 MeV), and the intensities of the pion beams used to induce the reactions ($\sim 5 \times 10^5 \pi$/sec. for the CERN experiment), we think that the SC2 and the Omicron spectrometer are in a good potential for solving the open questions about these reactions and these types of nuclei.

Another very stimulating property of the DCX lies in the possibility of exciting double analogue states in nuclei. These could be reached by elastic DCX (i.e. with $\Delta\vec{T} = 0$) or by inelastic DCX (i.e. with $\Delta\vec{T} \neq 0$). This last case has been examined by Ericson[4] for nuclei with $\vec{T} = T_3$. As an example, the DCX reaction on ^{12}C could give:

$$\pi^- + {}^{12}C \rightarrow \pi^+ + {}^{12}Be \quad \text{or} \quad \pi^+ + {}^{12}C \rightarrow \pi^- + {}^{12}O \qquad (3)$$

Thus from the nucleus ^{12}C, which in the ground state is a pure $\vec{T} = 0$, $T_3 = 0$ state, we could obtain the extreme elements of a $\vec{T} = 2$ multiplet, with $T_3 = -2$ and $T_3 = +2$ respectively. They will be the analogue states of a $\vec{T} = 2$, $T_3 = 0$ excited state in ^{12}C.

Searches for double analogue states have been made [3,5] for ^{51}V and ^{90}Zr, with π^+ of 195 MeV and 31 MeV. The results of these experiments were rather discouraging; differential cross-sections for the production of double analogue states were small, and only upper limits for their values were reported; they are 1 μb/sr for the experiment at 195 MeV and 0.5 μb/sr for that at 31 MeV. However these quite low values could be due to an unhappy choice of the energy of the pions and/or to the nuclear targets used and/or to the angle of emission of the outgoing pions.

Theoretical predictions are in fact less discouraging than the experimental results. In particular a very careful recent work[6] is devoted to the calculation of the differential cross-section for the production of the ground state of ^{18}Ne in the reaction:

$$\pi^+ + {}^{18}O \rightarrow \pi^- + {}^{18}Ne \tag{4}$$

This reaction is an interesting example of elastic DCX in which the nuclear final state (the ground state of ^{18}Ne) is $\vec{T} = 1$, $T_3 = 1$ and is the analogue of the ^{18}O ground state ($\vec{T} = 1$, $T_3 = -1$). The theoretical differential cross-sections for this reaction are shown in Fig.B1 and Fig.B2, which represent the angular distributions in the worst and best situations in the energy range between 100 and 450 MeV (which is nearly the energy range covered by the SC2 beams).

Fig.B3 shows how the Omicron spectrometer can be arranged in order to measure the DCX reaction. The incoming beam is detected by a coincidence between the scintillators $S1*(\overline{A1 + A2})*S2*S3*S4$ while electrons contaminating the beam and which can give spurious effects are vetoed by the gas Čerenkov counter Č. Uninteresting pions and charged particles leaving the target in a solid angle different from that accepted for the π produced in DCX are detected and vetoed by the scintillator A3 and another set of properly-shaped scintillators, not represented in the figure. The multiwire proportional chambers C1, C2, C3 and C4 allow measurement of the momenta of the incident pions $\pi\pm$, while the proportional and drift chambers C5, C6, C7 and C8 analyze the momenta of the produced pions $\pi\pm$. The total energy resolution of the spectrometer for π of 200 MeV can be pushed to the value of 1 MeV, which is enough to separate, in the most interesting case of the $\pi^+ + {}^{18}O \rightarrow \pi^- + {}^{18}Ne$ reaction, the ground state of ^{18}Ne from the first excited state. The trigger of the experiment is completed by a detector R (which really consists of an assembly of several different detectors) located after the last drift chamber and able to distinguish pions from electrons coming out of the target.

We have calculated by a Monte Carlo simulation of the experiment the angular acceptance for the chamber configuration of Fig.B3. Fig.B4a shows the acceptance for the azimuthal angle ϕ integrated over all the θ values, whereas Fig. B4b shows the total acceptance (i.e. taking into account cuts on ϕ in correspondence to each interval of θ) as a function of the scattering angle θ. Assuming an incident beam of $3 \times 10^6 \pi^+$/sec., a detection efficiency of 100%, the angular acceptance evaluated by the Monte Carlo simulation, the theoretical differential cross-sections of Lin and Franco[6] (Fig.B1 and Fig.B2) and a target thickness of 1 gm/cm^2 of ^{18}O (water with the pure isotope), we obtain counting rates of 1.3×10^{-2} ev/sec. in the worst case ($T\pi$ = 130 MeV) and 1.9×10^{-1} ev/sec. in the most favourable case ($T\pi$ = 450 MeV). The quoted target thickness would not reduce the intrinsic momentum resolution because for DCX in the forward direction there is no influence of the lack of knowledge of the interaction point, but only the effect of the energy loss in the target.

DCX on ^{18}O has been chosen as an example based on the existing theoretical calculations, but the calculated rates demonstrate that all the DCX reactions with differential cross-sections of the order of 0.5 μb/sr could be easily studied at angles from 0^o to 20^o. DCX reaction to bound final states could be seen even if the cross-sections are an order of magnitude lower.

spectrometer it is certainly possible to perform some systematic analysis of DCX reactions. We recall here that with the study of the predominant mechanisms of DCX one could obtain information about nucleon correlations, pairing effects and shell structure in nuclei[1].

The sources of background were discussed in a previous document[7]. A rather realistic evaluation of the electron background coming from annihilation of the protons from the π^0 decays (π^0 are produced by simple charge exchange reactions) which seems to be the most dangerous can be inferred from the data of Ref.3 . These authors detected with a gas Čerenkov counter the electron background in a 10 MeV-wide energy range where DCX events were expected and found a value of 0.005 μb/sr for 98% electron rejection efficiency. Thus we can expect that the electron background in a 1 MeV energy bin amounts to 25 nb/sr. This background level will be further lowered by the pion detector R.

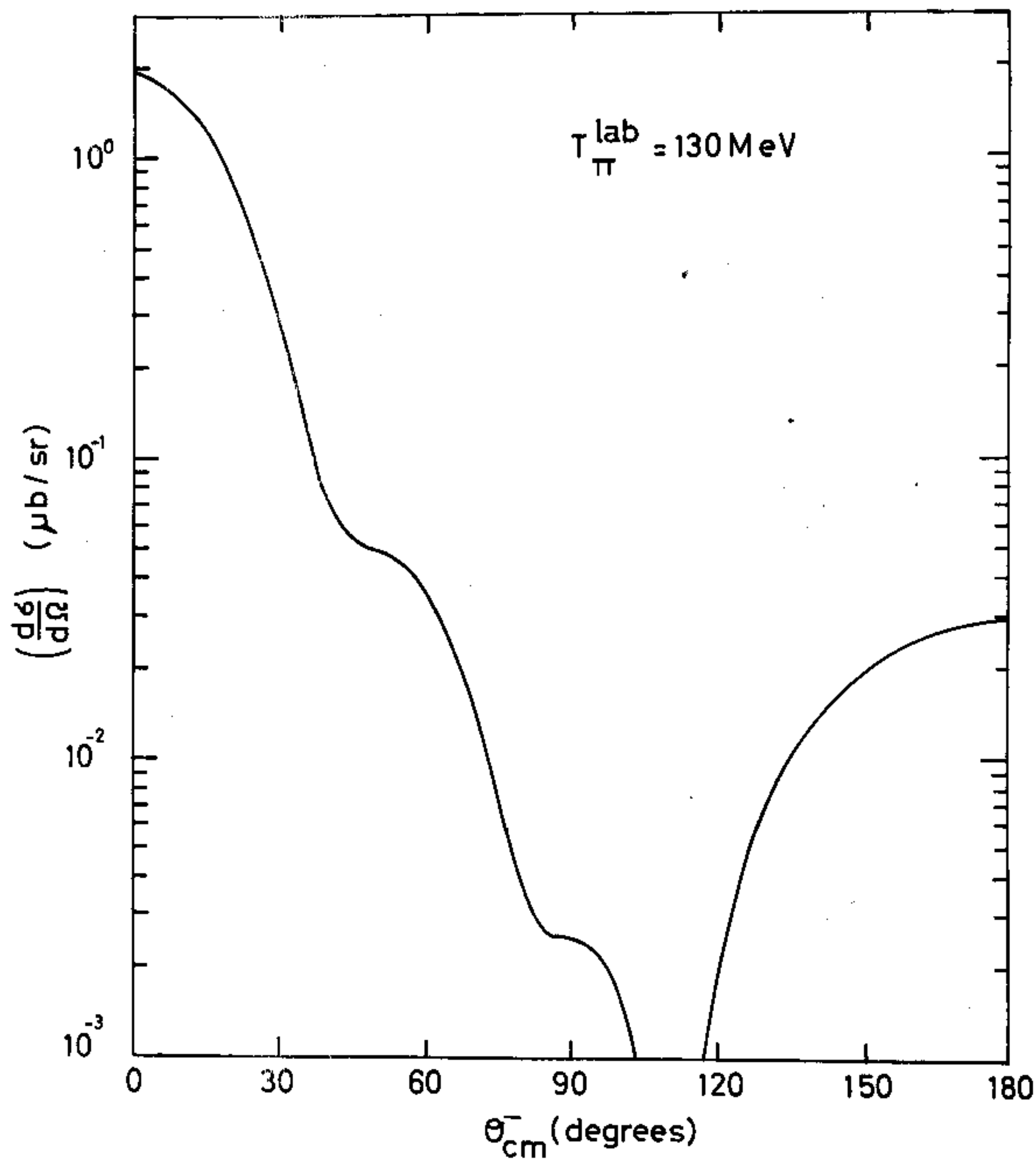


Fig. B1 : Differential cross-section for the $\pi^+ + {}^{18}O \rightarrow \pi^- + {}^{18}N$(g.s.) reaction at $T\pi$ = 130 MeV, following the calculation of Lin & Franco[6]

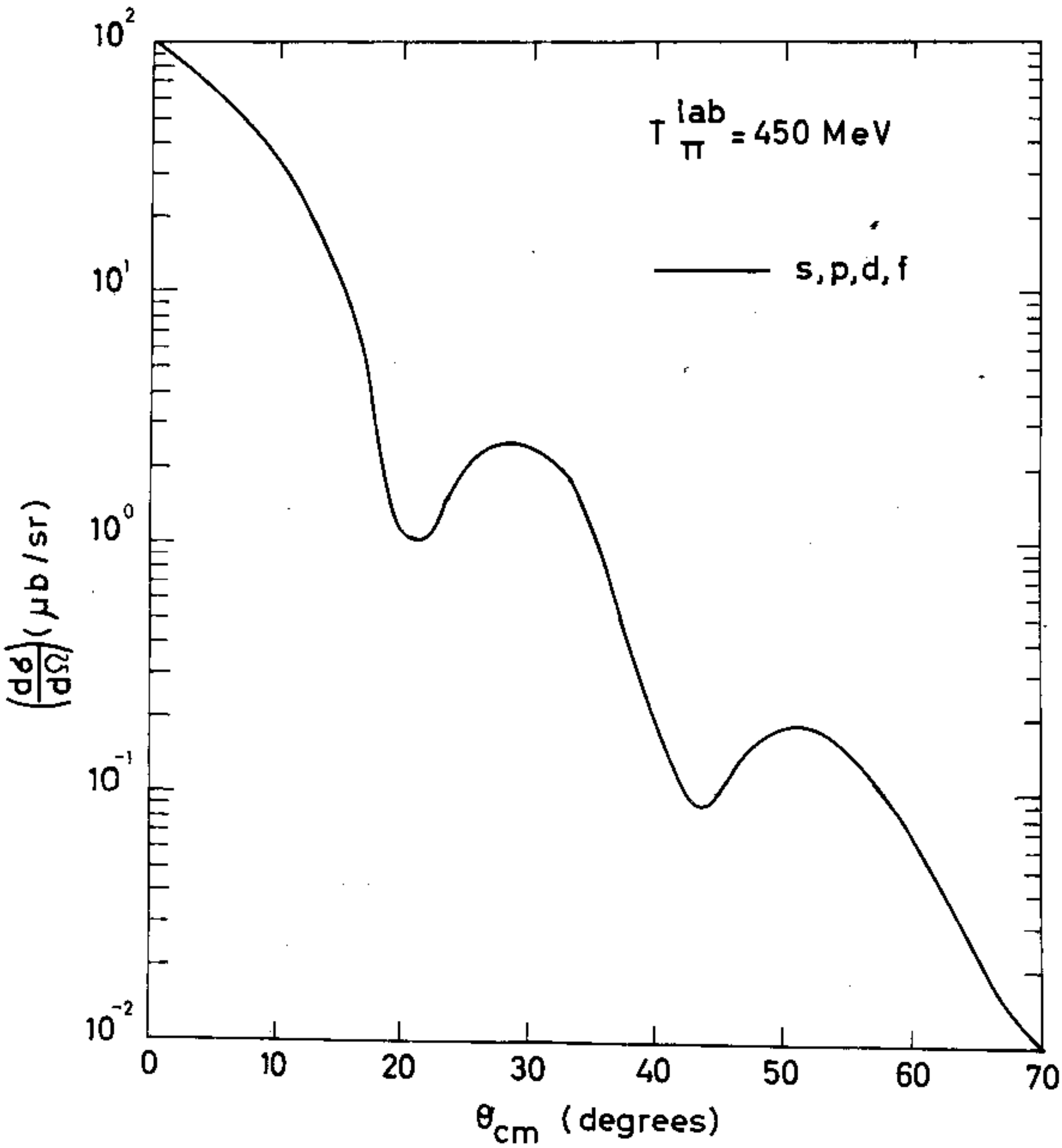


Fig. B2 : Differential cross-section for the $\pi^+ + {}^{18}O \rightarrow \pi^- + {}^{18}N$(g.s.) reaction at $T\pi$ = 450 MeV, following the calculation of Lin & Franco[6]

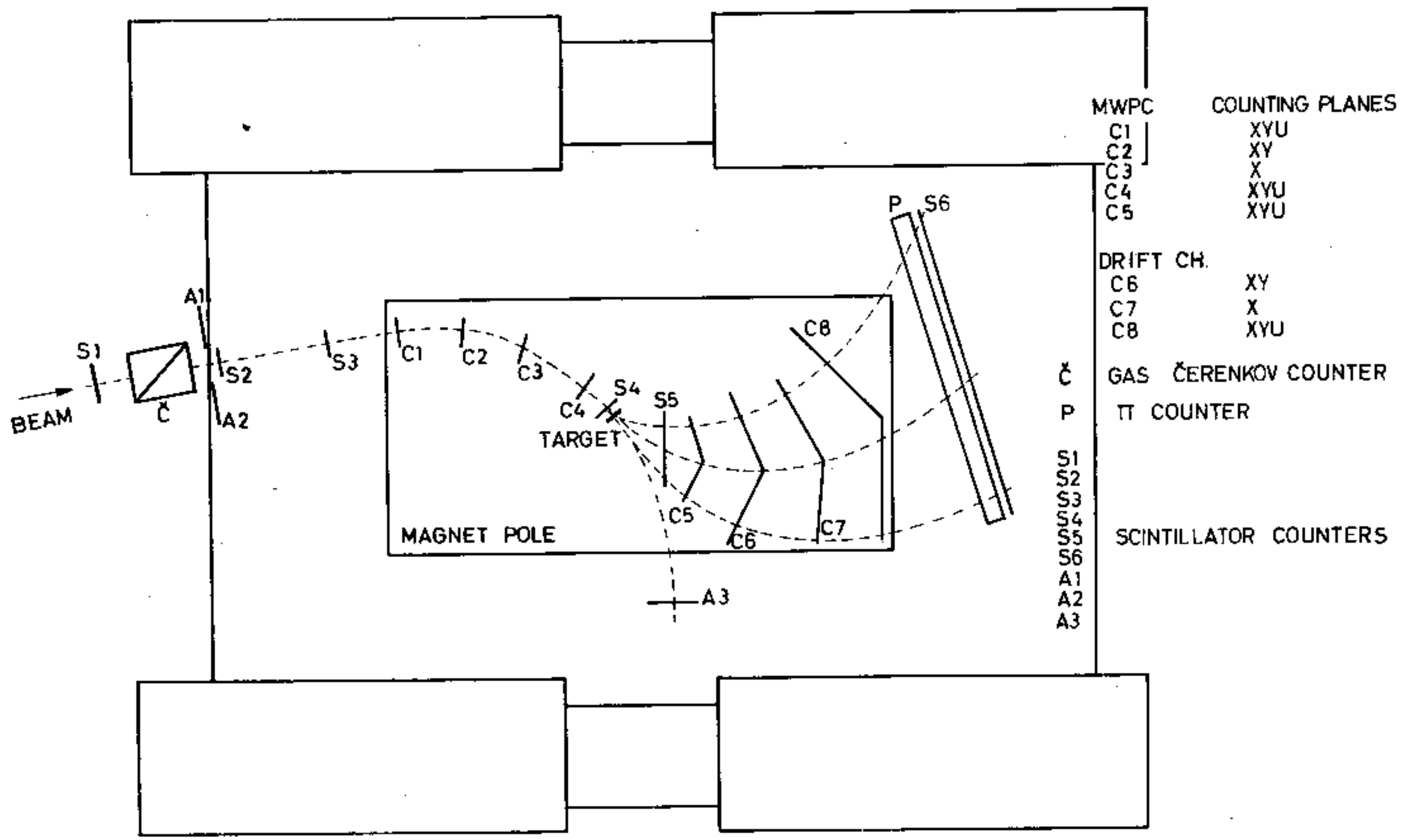
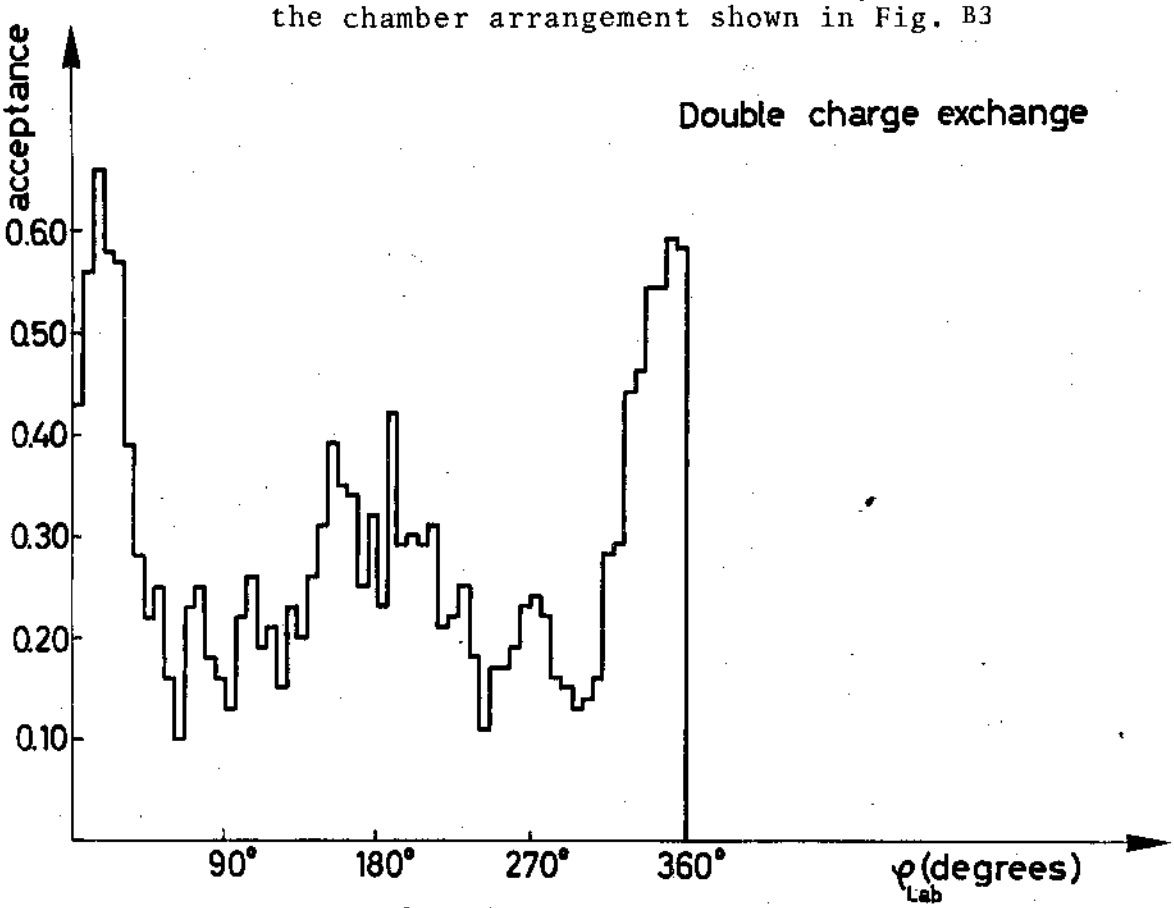
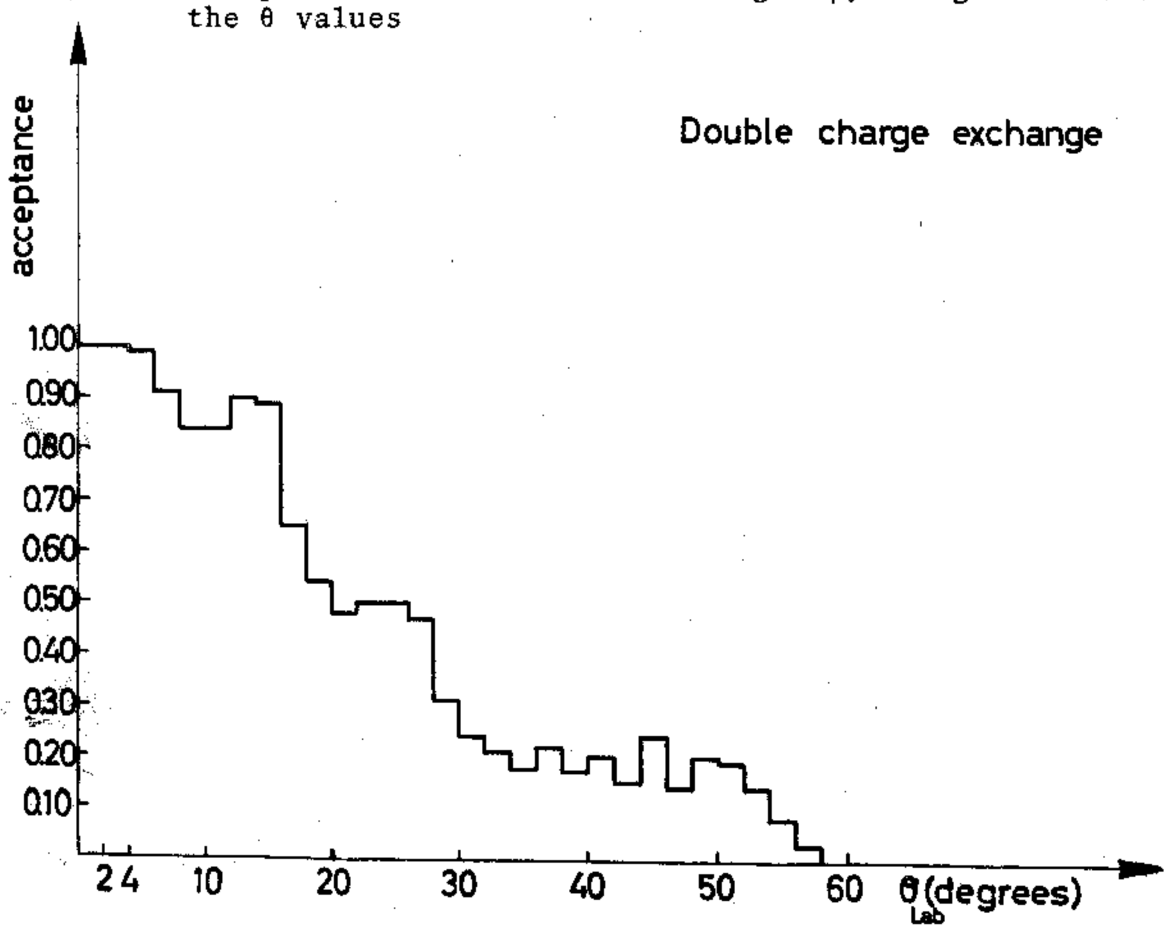


Fig. B3 Experimental lay-out of the Omicron spectrometer to measure double charge exchange reactions

Fig. B4 : Monte Carlo evaluation of the angular acceptance for the chamber arrangement shown in Fig. B3



B4a : Acceptance for the azimuthal angle ϕ, integrated over all the θ values



B4b : Acceptance as a function of the emission angle θ, taking into account the cuts on ϕ in correspondence to each interval of θ.

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THE BRANCHING RATIO OF THE RARE DECAY $\pi^0 \to e^+e^-$

1. Introduction

No experiment to measure the decay $\pi^0 \to e^+e^-$ has been reported, but estimates of the branching ratio compared to the two-photon decay mode have been made. The various calculations of the branching ratio assume different mathematical formulations for the pion form factor and it is seen from Table 1 that values from 5×10^{-8} to 22×10^{-8} are obtained. The unitarity limit is model independent and consequently a comparison with an experimental result is very important, since a value less than this limit would imply a phenomenon such as a CP violating current. If, however, the experimental result is in the range of a few times 5×10^{-8}, then it could be used as a test of the model for the pion form factor, or as a guide to the construction of future models.

Table 1

Calculated branching ratios for $\pi^0 \to e^\pm$

Model cut-off parameter	$B_{\pi^0}(\times 10^{-8})$	Reference
Unitarity	4.7	(b)
1.0 m_π	4.7	(a)
1.4 m_π	4.7	(b)
3.16 m_π	6.7	(b)
5.7 m_π	6.4	(c)
6.95 m_π	12.0	(a)
7.6 m_π	6.1	(c)
9.8 m_π	5.7	(b)
10.0 m_π	4.9	(c)
13.9 m_π	22.0	(d)
Baryon loop	14.0	(a)

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It has been suggested that the high cross-section $e^+e^- \rightarrow$ hadrons observed at SPEAR, CEA and Frascati can be explained by electrons having hadronic cores. These could contribute to the decays of pseudoscalar mesons into e^+e^- pairs for which model-independent considerations[1] show that only pseudoscalar and axial vector couplings are possible. Decay rates were then estimated in terms of the corresponding coupling constants, f_{PS} and f_A , and these demonstrated that the measurement of the $\pi^0 \rightarrow e^+e^-$ branching ratio is the most sensitive test. The published data of those experiments having large numbers of π^0's and the means of detecting e^+e^- pairs were then reviewed to obtain an upper limit on B_{π^0} of $< 8 \times 10^{-6}$. This severely limits pure pseudoscalar coupling.

Any experiment will employ a very high intensity beam of some other particle to make the π^0 within a high precision magnetic spectrometer of large solid angle. One will trigger on 2 electrons and measure their momentum and opening angle so as to construct their invariant mass; $\pi^0 \rightarrow e^+e^-$ should then show as a peak super-imposed on the background of internal conversion electron-positron pairs.

Methods for copious π^0 production have been examined elsewhere in detail[2] especially with regard to competing backgrounds. The most serious backgrounds come from

i) $\pi^0 \rightarrow e^+e^- \gamma$.

ii) Any particle combination making a π^0 can also make a γ which may then internally convert. Consider π^- capture at rest; then the low value of the Panofsky ratio , P = 1.6 gives

$$R = \frac{\pi^- p \rightarrow \pi^0 n \rightarrow e^+e^- n}{\pi^- p \rightarrow \gamma n \rightarrow e^+e^- n} = \frac{1}{30}$$

for $E_{e^+e^-} = m_{\pi^0} \pm 1\%$.

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1) H. Burckhardt, R.K.P. Zia and J.D. Davis, J. Physics AZ, 40, 1974.

2) J.D. Davies, J.G. Guy and R.K.P. Zia, RHEL RI-74-092.

Extensive Monte Carlo studies show that even in the optimal case, $\pi^0 \rightarrow e^+e^-$, would give a small enhancement on an enormous background[3]. At present, only $\pi^-p \rightarrow \pi^0 n$ at 180 MeV can give several hundred $\pi^0 \rightarrow e^+e^-$ at the unitarity limit free from large backgrounds. It has the large σ = 48 mb since it is dominated by $\Delta(1236)$ production; background (ii) is suppressed by the favourable 0.6% partial decay fraction of $\Delta(1236) \rightarrow \gamma n$. Measuring foreward angles exploits the different angular distributions of the two processes so that $R \sim 1$ can be expected (see Fig. 1).

2. Event Rate

A liquid hydrogen target 10 cm long converts 2% of π^- at 300 MeV/c to π^0 with a spatial distribution $\approx \left[P_0 (\cos\theta) + P_2 (\cos\theta)\right]$ in the laboratory frame (see Fig. 1). Conservatively we expect to obtain a beam of 3×10^6 π^-/sec yielding 6×10^4 π^0/sec. At the unitarity limit of $B_{\pi^0} = 5 \times 10^{-8}$, there would then be eleven $\pi^0 \rightarrow e^+e^-$ events/hour, distributed over all angles and all momenta.

At forward angles, where the π^0 intensity is a maximum and the background a minimum, the π^0 momentum varies slowly with angle and is close to 300 MeV/c. The decay electrons, isotropic in the π^0 rest frame, are thrown forward in the laboratory frame and have an energy distribution which is nearly rectangular, i.e. all energies for e^- (or e^+) between the minimum of 15 MeV and maximum of 315 MeV are equally likely. Fig. 2 shows a possible arrangement of chambers in the Omicron magnet which accepts electron momenta roughly in the range $\sim 165 \pm 55$ MeV/c. For these momenta the opening angle ψ of the e^+e^- pair falls in the narrow band $\psi = 50 \pm 2^0$, and the angle of the more energetic electron (relative to π^0 direction) varies between 16^0 and 24^0. The π^0, e^+ and e^- are coplanar but this plane is random relative to the $\pi^-\pi^0$ plane.

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3) W.P. Trower, private communication.

The multiwire proportional chamber immediately after the target is intended to accept electrons with polar angles (relative to the π^- beam) up to 60° and all azimuthal angles. Taking the approximation that the more energetic electron follows the π^0 angular distribution and that the opening angle of the pair is always 50°, then an integration gives the geometric acceptance as 18%. The probability of detecting $\theta_>$ of the more energetic electron varies from unity for small $\theta_>$ to 0.36 for $\theta_> = 60^\circ$. The geometric efficiency for pairs varies rather more slowly than linearly with the maximum electron angle allowed by the wire chamber.

The momentum acceptance has been calculated neglecting the spatial correlation of $e^+ e^-$ pairs, which neglect is unlikely to lead to a large error because of the distribution of π^0 direction and the variation of the e^+e^- plane. In Fig.2 the drift chambers have been placed to optimize the acceptance of electrons with the central momentum 165 MeV/c, assuming that drift chambers with a useful vertical height of 70 cm can be accomodated within a magnet gap of 85 cm. Rather surprisingly, the momentum acceptance is not critically dependant on the initial angle (θ,ϕ) of the electron within the geometric limit $\theta < 60^\circ$, and subject to the further condition that the angle of incidence on the drift chambers should be $< 30^\circ$. Generally the range 165 ± 55 MeV/c can be accepted, i.e. about one third of the electron energy spectrum.

Apart from trigger and chamber inefficiencies which are likely to be small it should thus be possible to detect 6% of all $\pi^0 \rightarrow e^+e^-$ events which gives a rate of 0.6 events/hour at the unitarity limit. A much higher event rate of $\sim$ 2.4 per hour could be obtained by using a 10 cm ^6LiH target in place of liquid hydrogen but the cost is considerable. The ^6LiH target is 0.10 radiation lengths,(c.f. 0.011 for hydrogen)which would be fairly disastrous for the mass resolution. Furthermore the π^0 momentum and direction become unknown quantities and additional chambers would certainly be required to avoid ambiguities.

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In the case of $\pi^- p \rightarrow \pi^0 n \rightarrow e^+ e^-$ in hydrogen there are a large number of kinematic constraints which can be applied to the electron momenta $\underline{p+}$ and $\underline{p-}$:

$$|P+| + |P-| = E_{\pi^0}$$

$$\underline{P+} + \underline{P-} = \underline{P\pi^0}$$

$\underline{P+}$ is determined uniquely by the angle $(\underline{P\pm}, \underline{P\pi^0})$, the angle $(\underline{P\pi^0}, \underline{P\pi^-})$ having a known distribution.

$\underline{P+}$ and $\underline{P\pi^-}$ determine $\underline{P-}$,

$|P\pi^-|$, $|P+|$ and $|P-|$ are sufficient in principle to determine ψ and hence the invariant mass.

$\underline{P+} \times \underline{P-}$ direction is uncorrelated with $P\pi^0 \times P\pi^-$.

Multiple scattering, plus nuclear inelasticity make ^{6}LiH much less attractive than liquid hydrogen despite the rate.

3. Mass and Momentum Resolution

The most important variable is in the invariant mass x of the electron positron pair which is given by the expression

$$x^2 = 4\ |P+|\ |P-|\ \sin^2\ \psi/_2\ \sim |P+|\ |P-| \psi^2$$

where the electron mass has been neglected. Errors arise from

(a) multiple scattering in the target :
an electron of 165 MeV/c suffers a 10 mr scattering passing through 10 cm of liquid hydrogen;

(b) multiple scattering by the argon filling gas and the wires of the chambers amounting to $\sim$ 1.5 mr for 300 MeV/c π^- and $\sim$ 3 mr for 165 MeV/c electrons.

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(c) spatial resolution of the wire chambers (1 mm wire spacing).

Energy loss in the target is not a serious cause of error, particularly as there are two outgoing particles with an opening angle of 50^o to locate the vertex.

As indicated above the variables observed are not independant. The energy difference $|P+| - |P-|$ and the angle between a single electron and the π^o, are rapidly varying functions of ψ. The latter is plotted in Fig. 3 and it appears that the influence of target multiple scattering can be minimized by determining θ_+ (or θ_-) rather than ψ directly.

Overall it should be possible to determine x^2 to about 2% and the summed energy $|P+| + |P-|$ to about 1%.

4. Background

The sources of background for $\pi^o \rightarrow e^+e^-$ with the π^o generated by $\pi^-p \rightarrow \pi^on$ at 300 MeV/c is discussed in detail in ref. 1.

(a) Single Dalitz Pairs

The branching ratio for

$$\pi^o \rightarrow \gamma e^+e^-$$

is 1.2% giving a total e^+e^- rate from this source of 700 e^+e^-/sec. Characteristically these pairs have near zero opening angle and small invariant mass. The branching ratio[4)] for Dalitz pairs with x^2 within 2% of m_π^2 is about 10^{-9} and therefore negligible as a background.

4) S.D. Drell, Nuovo Cimento 11, 692, 1959

Of the 700 small opening angle pairs per second, about 300/sec will have momenta within the acceptance band of $\sim$165 $\pm$ 55 MeV/c. As these pairs are confined within a cone of half angle 20^o about the π^o momentum, the geometric acceptance will be $\sim$34% giving $\sim$ 100 Dalitz pair events/second which must be recorded as there is no opportunity to make a fact decision (non-computer) on opening angle. In principle Dalitz pairs can be suppressed by discriminating against the summed energy recorded by Cherenkov counters, but with, say, 20% resolution for each of two counters the cut can hardly be placed higher than 0.6 E_{π^o}, in which case about 50 Dalitz pairs/sec. will be recorded, i.e. only a factor two suppression.

The rate of Dalitz pair triggers will be $\sim 10^6$ times the $\pi^o \rightarrow e^+e^-$ triggers which is a nuisance but does not pose very serious problems for the analysis. Given a knowledge of the location of beam particles to $\sim$ 1 mm, a wire chamber resolving time of 100 nsec and and instantaneous beam rate of 3×10^6 π^-/sec (60% duty cycle), then all but 1 in 10^5 of the Dalitz pair triggers can be eliminated assuming that $\sim$ 20% of pairs record like a single particle in the first chamber. The remaining events can be reduced to zero by the requirements that the summed energy is E_{π^o} within a few percent (the maximum for Dalitz pairs is 0.955 E_{π^o}) and/or the opening angle is $50^o \pm 2^o$ and/or the $\pi^-e^+e^-$ vertex is acceptable.

(b) Electrons in the π^- Beam

A 5% e^- contamination of the π^- beam could generate $\sim$ 1 trigger/sec from small angle pairs. This is simply a small addition to the Dalitz pair triggers.

The rate for a Dalitz pair trigger, not spatially resolved in the first chamber, accompanied by a chance scattered electron, resolved from the initial electron trajectory and accepted by the spectrometer momentum band, is of order 10^{-6} per second and therefore negligible without analysis.

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(c) $\pi^- p$ Elastic Scattering

If not otherwise suppressed elastic scattering will generate a pair coincidence trigger rate of several hundred per second. There is a choice between using an absorber ($\sim$ 7 mm Al or $\sim$15 mm CH_2) to stop the protons,or a Cherenkov counter as a means of suppression.

The rate of scattered π^- or protons accepted geometrically by the first chamber is $\sim 10^4$ per second and most of these will not record in other chambers as they have high momentum (> 250 MeV/c). Random coincidences with Dalitz pairs not spatially resolved in the first chamber are $\sim 2 \times 10^{-2}$ per second, 10^2 times the $\pi^o \to e^+e^-$ trigger rate, and must be removed by detailed analysis of opening angle, summed energy and vertex.

(d) Radiative Capture $\pi^- p \to \gamma n$

The cross-section at 300 MeV/c is 0.66 mb yielding 8×10^2 γ/sec from 3×10^6 π^-/sec. onto a 10 cm hydrogen target with an angular distribution as shown in Fig. 1. About 1% of the gammas will be converted to small angle pairs in the target giving $\sim$1 pair/sec into the geometric acceptance. This is, in effect, an addition to the 100/sec small angle Dalitz pairs already discussed, except that the summed energy of the external pairs is within 1% of E_{π^o} and they must be recognized by opening angle.

(e) Internal Pairs by Radiative Capture

The internal pairs are also $\sim 1\%$ of the gammas giving a trigger rate of ~ 1 pair/sec, mostly with small opening angle and near zero invariant mass. But the invariant mass is not necessarily zero, as in the case of external pairs, and can, with a small probability, be equal to m_{π^0} and therefore kinematically indistinguishable from $\pi^0 \rightarrow e^+e^-$ pairs. This problem has been examined in detail by Burkhardt et al [1] who argue that for $\pi^- p \rightarrow n\, e^+e^-$ at 300 MeV/c the contribution from longitudinal virtual photons is small compared with the transverse virtual photons in which case the expression [5] for the differential conversion coefficient integrated over all angles approximates to

$$\frac{d^2\rho}{dx\,dy} = \frac{\alpha}{4\pi} \cdot \frac{1+y^2}{x} \frac{|P_o|}{|P_\gamma|} \quad \text{for } x \gg m_e$$

and the spatial angular distribution is identical with that for $\pi^- p \rightarrow \gamma n$ (see Fig. 1). Here Po and Pγ are momenta of virtual and real photons, ρ is the internal conversion coefficient, α the fine structure constant, m_e the electron mass, and x and y the kinematic variables invariant mass,and energy partition respectively :

$$x^2 = 4\,|P+|\,|P-|\,\sin^2 \psi/_2, \qquad \text{for } P+,\ P- \gg m_e$$
$$\approx P{+}P{-}\ \psi^2$$

where $P\pm$ are the electron momenta and ψ the opening angle, and

$$y = (\,|P+| - |P-|\,)\ /\ |\underline{P+} + \underline{P-}| \qquad \text{for } P+,\ P- \gg m_e$$
$$= (\,|P+| - |P-|\,)\ /\ |Po|$$

5) N.M. Kroll and W. Wada, Phys. Rev. 98, 1355, 1955 /....

and $|Po|$ is the momentum of the virtual photon ($\equiv$ momentum of π^o for $x \equiv m_{\pi^o}$) and is nearly equal to $|P\gamma|$.

The range of y is $-1 \leq y \leq +1$ and of x is $2\, m_e \leq x \leq m_\Delta - m_n$, where m_Δ, m_n are the masses of the $\Delta(1236)^-$ and the neutron, and $\rho(x, y)$ has a maximum (not described by the approximation above), at $x = \sqrt{6}\, m_e$, goes to zero at $x = 2$ m, and is down to half the peak value at $x = 9\, m/\sqrt{2}$, falling like x^{-1}.

The internal conversion coefficient for a range δx near $x = m_{\pi^o}$ and all y is

$$\Delta\rho \sim \frac{2\alpha}{3\pi} \cdot \frac{\delta x}{x} \frac{|Po|}{|P\gamma|}$$

$$= 1.3 \times 10^{-5} \qquad \text{for} \quad \delta x/x = 1\%$$

yielding 38 e^+e^- pairs/hour distributed over all angles and momenta. The comparable number for $\pi^o \to e^+e^-$ is 11 per hour , giving a signal/noise of 0.29. Because of the energy partition dependance $(1 + y^2)$ for internal pairs the momentum acceptance is smaller, by a factor $\sim 7/9$, for internal pairs than for $\pi^o \to e^+e^-$,which is independant of y. The geometric acceptance of pairs of opening angle $\psi = 50^o$ falling within a cone of half angle 60^o is favoured by the π^o angular distribution (Fig. 1) giving a further factor ~ 3.3. The net signal to noise ratio for the geometry of Fig. 2 is expected to be ~ 1.4 for a mass resolution of 1% (2% for x^2).

It is worth noting that the internal pair "noise" can be determined with negligible statistical error by the data for pairs with $x \neq m_{\pi^o}$. In addition some further advantage can be gained by weighting the pair data around $x = m_{\pi^o}$ according to the values of y, the π^o (virtual photon) angle, and the angle between the e^+e^- plane and the $\pi^o\pi^-$ plane [1] to enhance the $\pi^o \to e^+e^-$ signal. This might be worth another factor of two in signal to noise.

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5. Data Acquisition

It is necessary to record $\sim 10^2$ pair triggers per second with the HP21M computer on line. Most of these triggers can be rejected immediately by the requirement that the wire chamber immediately after the target has recorded at least two particles separated by at least 10 mm and that neither member of the separated pair is colinear with a beam particle within the resolution of the chamber. It is necessary for the beam chambers to have a resolving time greater than that of the post-target chamber for this purpose.

The rate for triggers not immediately suppressed should be one or two per minute largely due to a Dalitz pair and a random coincidence, scattered π^-. Crude analysis within the HP21M computer could reduce the data to a few events per hour.

6. Calibration

It will be necessary to establish the momentum scale from the measured field of the magnet and the positions of the wire chambers and to check the computed momentum and geometric acceptance using $\pi^- p$ elastic scattering.

The resolution of the system with respect to invariant mass x can be investigated using the internal and external pairs from $\pi^- p \rightarrow n\gamma$, but only in the region of the peak near $x = 0$ which is rather a long way from $x = m_\pi$.

7. Conclusion

The Omicron spectrometer allows the possibility of observing the decay $\pi^0 \rightarrow e^+e^-$ at the unitarity limit at a rate of ~ 0.6 events per hour with a signal to noise ratio of one or two, using a 300 MeV/c π^- beam of 3×10^6 per sec. and a 10 cm liquid hydrogen target. A well focused (2 or 3 cm) beam would allow a 20 cm target, and a beam intensity of 10^7 π^-/sec. would be acceptable giving an event rate $\sim$4/hour at the unitarity limit and $\sim$ 16/hour for a branching ratio of 2×10^{-7}.

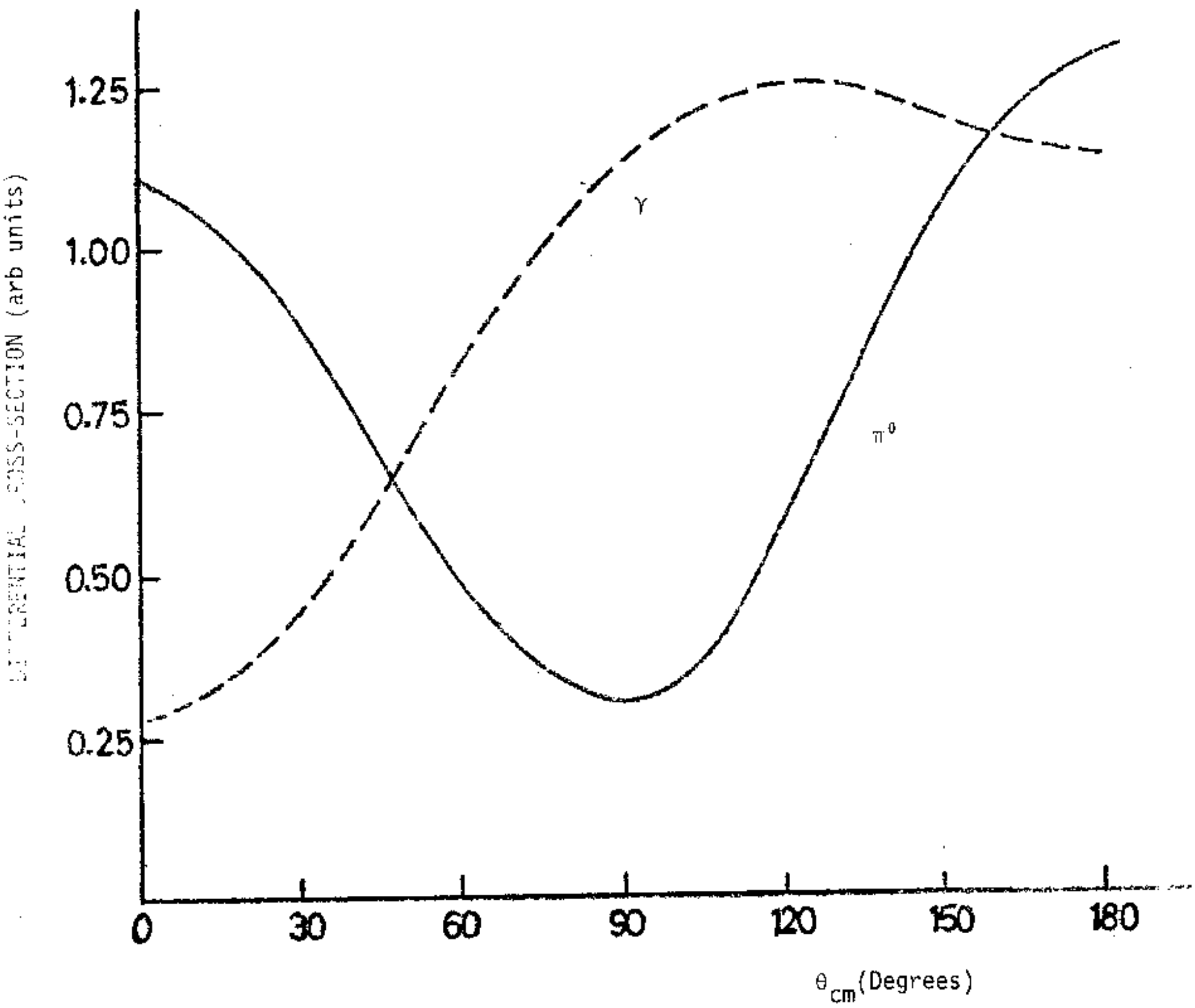

Fig. 1 The angular distributions of photo-emission ($\pi^- p \to \gamma n$) and charge-exchange ($\pi^- p \to \pi^0 n$) processes at T_π = 180 MeV

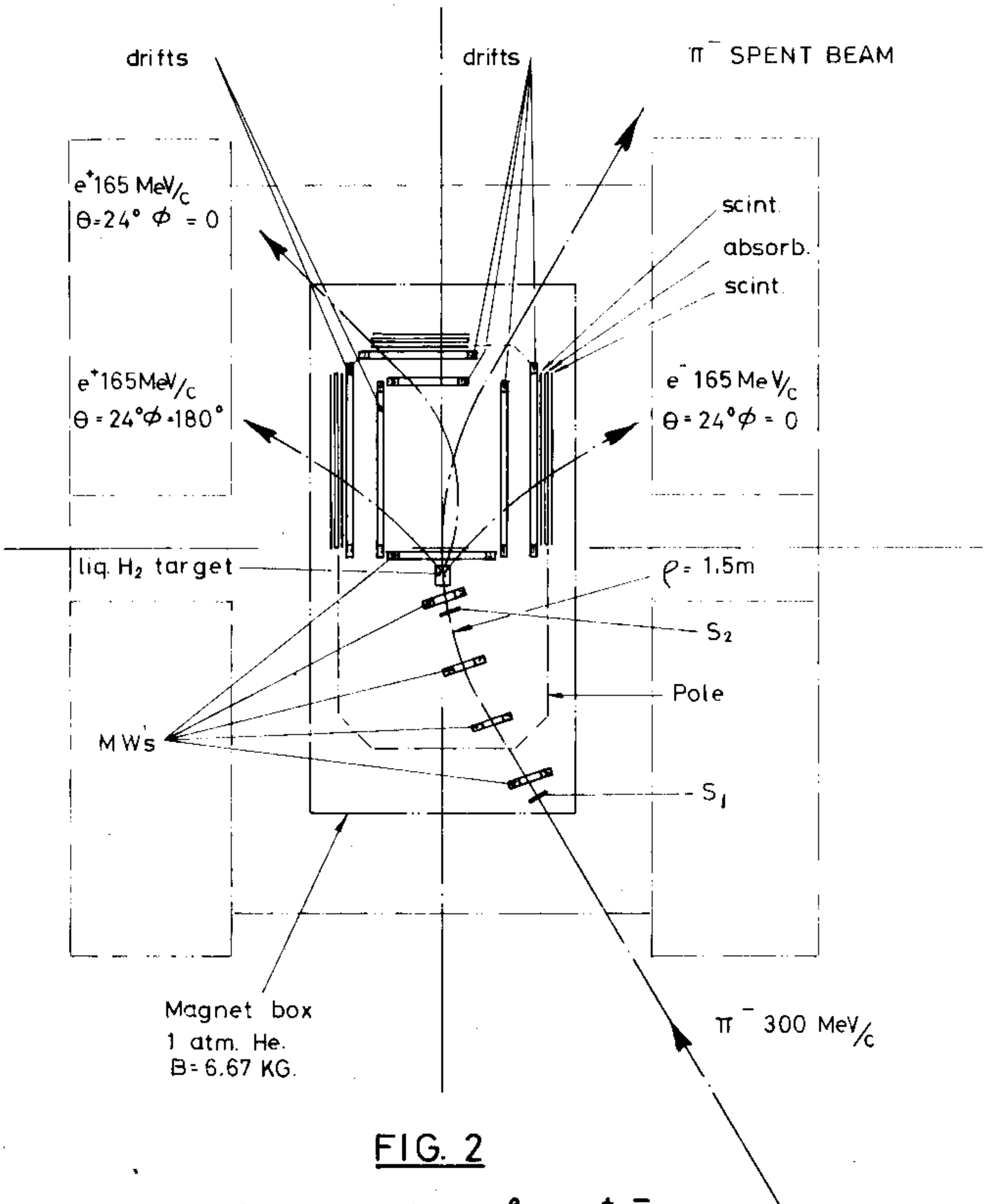

FIG. 2

Layout for $\pi^0 \rightarrow e^+e^-$
in the OMICRON Spectrometer

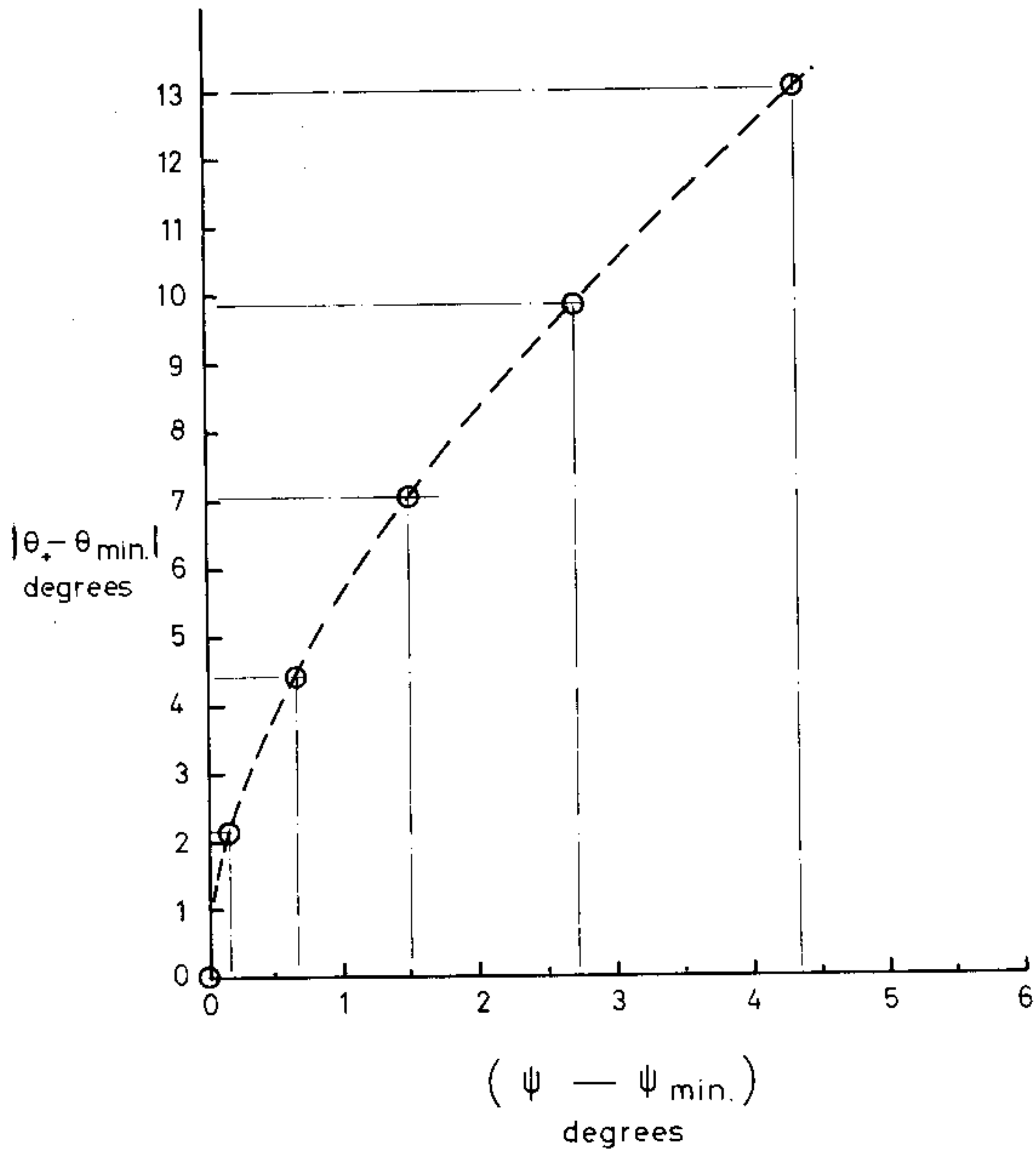

FIG. 3

Variations of Θ_+ with the opening angle ψ in $\pi^0 \to e^+e^-$

ELECTRON - POSITRON DECAY OF PIONIC ATOMS AND β-DECAY FORM FACTOR

The radiative capture of low energy pions in light nuclei is a well established probe for certain aspects of nuclear structure[1]. The basic process $\pi^- p \rightarrow n\gamma$ near the threshold is sufficiently understood that we may deduce from it an effective Hamiltonian for the interaction which can be used to calculate absorption on bound single protons in nuclei via the impulse approximation. As a typical example of the type of physics this leads to,consider the predicted Panofsky ratio in ^{3}He. In its most simplified form this is

$$P(^3\mathrm{He}) = (\mathrm{Kinematics}) * \frac{|F_V\ (q^2=0.054\ m_\pi^2)\ /\ g_V^2\ |^2}{|F_A\ (q^2=0.954\ m_\pi^2)\ /\ g_A{}^2|^2} \quad * P(^1\mathrm{H}) \qquad (1)$$

where g_A/g_V is the ratio of the axial to vector coupling constants for the nucleus, and F_V and F_A the vector and axial form factors of the ^{3}He nucleus, evaluated at the momentum transfers appropriate for π charge exchange and radiative capture respectively. Essentially then,the experiment can be taken as a measure of the axial form factor of ^{3}He at the point $q^2 \sim m_\pi{}^2$. Note that the same matrix element occurs in β-decay at $q^2 \sim 0$ and in muon-capture at $q^2 \sim m_\mu{}^2$.

Suppose now that instead of detecting a photon from these reactions we observe an (e^+e^-) pair. It was shown many years ago by Kroll and Wada [2] that the branching ratio for this internal conversion is of the order of 0.5%, and that in the vast majority of the events the virtual photon is almost real. As will be discussed later, this suggests that even in overall rates, the internal pairs provide a competitive means of measuring the radiative capture, since the photon need not be converted. In addition though, it is expected that about 3% of the events will have $m_\gamma{}^2 > \frac{2}{3}\, m_\pi{}^2$ and hence $q^2 < \frac{1}{3}\, m_\pi{}^2$. The physics of the (e^+e^-) and the γ capture being so similar, by applying equation (1) to the (e^+e^-) events we can hope to measure the axial form factor of the nucleus as a function of the momentum transfer, rather than at a few isolated points.

All the arguments that can be given for radiative capture, e.g. inducing transitions into the giant resonance region via the electric-dipole part of the effective Hamiltonian (1), apply equally well for the internal pairs. The latter have the advantage that even the polarisation of the virtual photon is measurable from the energy partition and opening angle of the pair. In particular 1 - 2% of the events will correspond to excitations with longitudinal photons, which may lead to states which cannot be seen with real photons. To exploit the transverse polarization information from a stopped pion we must clearly detect another particle to provide a reference direction. Although such a triple coincidence is not high on our list of priorities, Omicron is the ideal instrument for it, and for example $\pi^- d \rightarrow e^+e^- nn$ might yield extra checks on the nn scattering length.

A further motivation for this sort of measurement is the suggestion[3)] that the longitudinal photons might be more prolific if the pion density inside the nucleus is anomalously high, as it could be if there were a pion condensate inside nuclei. This topic has received a good deal of attention recently.

As is mentioned above, the yield of electron-position pairs from internal conversion is expected to be of order 0.5% of the radiative capture rate; typical numbers for this latter rate are about 0.03 per stopped pion in light nuclei, 0.3 for deuterum and 0.5 for hydrogen. Of the e^+e^- pairs it is estimated that 3% will have large opening angles ($q^2 < \frac{1}{3} m_\pi^2$, say) and it is these events which will provide the most interesting information on the axial form factor, the events with small opening angles carrying no more information than radiative capture, unless another particle is detected in coincidence.

Overall there should thus be of order 5 interesting events per million stopped pions on light nuclei, and an order of magnitude more on deuterum or hydrogen (where the prolific production of π^0 might be a problem, but which has great theoretical importance). These interesting events are those in which the e^+ and e^- are of similar momenta but opposite in direction, and they will be distributed over the whole 4π solid angle because the π^- producing them is at rest.

In Omicron the geometric acceptance for these events should not be less than 5%, and so assuming a target thickness of 1 gm/cm^{-2} a stopped pion rate of 10^5 per gm cm^{-2} per second, the resulting event rate is about 100 per hour for light nuclei and an order of magnitude more for D or H. Such stop rates should be attainable at the new SC.

Backgrounds have not been studied in detail yet, but it should be possible to trigger only on e^+ e^- going opposite directions. An energy resolution of 1 MeV will be possible, being limited mainly by multiple scattering in the target and the detectors. The experimental arrangement is still being studied, but at present it is proposed to use a thin slab of target inside the magnet, inclined at a small angle to the incoming pions such that they see a total thickness of about 1 gm cm^{-2}. The electrons emitted approximately normal to this slab thus see very little material in escaping from the target. The way in which the stopping π^- beam should be produced is also still under discussion (e.g. wedge absorber before the pions enter the magnet).

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(3) T.E.O. Ericson & C. Wilkin, paper in preparation.

LOW ENERGY PION PRODUCTION

$\pi\ p \rightarrow \pi\ \pi\ N$

The $\pi\pi N$ system is the first inelastic channel in πp scattering. Because of the simple spin structure vis-à-vis $NN\pi$ it is the cleanest three body system to be studied in particle physics, so that in the foreseeable future one might hope to have some kind of understanding for the reaction mechanism, at least at low energies where there are not many partial waves. There have been many experimental investigations of the different charge states but mainly at high energies where the cross-section is healthy (many mb.). At low energies the primary motivation for the experiments is to compare with the soft pion prediction of Weinberg[1] or Schwinger[2]. We shall confine ourselves here to a discussion along these lines. The basic formulae at threshold are as follows [3] [4]

$$(\pi p \rightarrow \pi_a\ \pi_b\ N_c)$$

$$= \ |a\ (\pi_a\ \pi_b\ N_c)|^2\ Q^2 x \text{ (Phase Space)}$$

where Q is the cm momentum of the incident pion, and the phase space is modified by a factor of two if the two pions have the same charge.

The $\pi\ \pi$ scattering lengths in soft pion theory are given by

$$2\ a_0 - 5\ a_2 \ = \ \{68/f_\pi\}^2$$

and

$$a_0/a_2 \ = \{\tfrac{5}{2} \times \psi - 7\} \ / \ \{\psi + 2\}$$

where ψ is a free parameter, being ψ = 0 for Weinberg and ψ = 1 for Schwinger.

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Taking the pion decay constant f_π = 82 MeV, the pion production amplitudes are

$$\begin{aligned} a\,(-\;+\;n) &= -\,1.36 + 0.6\,\psi \\ a\,(0\;0\;n) &= \;\;2.11 - 0.3\,\psi \\ a\,(+\;+\;n) &= \;\;2\,a\,(+\;0\;p) \\ &= \;\;2\,a\,(-\;0\;p) \\ &= \;\;1.51 + 0.6\,\psi \end{aligned}$$

To show the order of magnitude of the cross-sections involved, we reproduce below the prediction from the Weinberg model[4)]

TABLE I

Kinetic Energy MeV	$\sigma_{\pi^-p \to \pi^-\pi^+n}$	$\sigma_{\pi^-p \to \pi^0\pi^0n}$	$\sigma_{\pi^+p \to \pi^+\pi^+p}$	$\sigma_{\pi^\pm p \to \pi^\pm\pi^0p}$
210	0.020	0.040	0.012	0.004
280	0.156	0.236	0.096	0.028

Soft pion cross-section for $\pi N \to \pi\pi N$ in mb.

However, in addition to the soft pion effects there are contributions from hard pions, but these latter are expected to become small near threshold. One therefore has the choice of working at very low energy with small cross-sections where the theory is clean, or with the larger cross-sections at high energies and trying to estimate the corrections. In practice an extrapolation as a function of energy is the most reasonable approach, and at present an experiment is in progress at Los Alamos which detects only one of the produced pions. This should lead to the determination of the energy dependance of the integrated cross-section with good statistics.

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On the basis of the present data, the most complete being rather old Russian measurements[5] on $\pi^- p \rightarrow \pi^- \pi^+ n$, a value of ψ closer to that of Weinberg seems to be preferred but to test the predictions completely, good measurements are required for all charge states.

Much more information can be obtained from a system such as Omicron. The soft pion amplitudes are connected with definite angular momentum states of the final particles (viz, S-states) so that if we can project out these states from the fully constrained data, the extrapolation to threshold will be less uncertain.

As will be seen from the table, the cross-section near threshold is of order 10 μb for the case of two charged pions in the final state. For a target of liquid hydrogen of thickness 3 mm, and assuming an incident pion flux of 3×10^6 /sec, this results in an event rate of about 0.4/sec for all angles and momenta. The secondary pions have energies of 10 to 30 MeV for an incident pion energy of 210 MeV that is, momenta from about 50 to 100 MeV/c; the range of such particles is from about 3 to 20 cm in the target. The pions are produced in a cone of about 60^o half-angle in the forward direction so that for this experiment it is clear that the target will need to be at the centre of the magnet, and the experimental arrangement will need to be very similar to that in the experiment $\pi^0 \rightarrow e^+e^-$ (see section C).

Such slow pions will cause difficulties. For example the decay length is of order 3 m, but the loss within the $\sim$ 1 m dimensions of Omicron will be tolerable. The multiple scattering will be serious but the required resolution is poor in this experiment and a reduced number of detection planes can probably be used. Estimating the total momentum and geometric acceptance of the desired events as 5%, (although it could be a good deal higher), the final event rate will be a few per minute.

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Background should not be a problem because of the search for such low energy pions, but this particular aspect is still being studied.

The other reactions , $\pi^{\pm} p \rightarrow \pi^{\pm} \pi^{0} p$ or $\pi^{-} p \rightarrow \pi^{0} \pi^{0} n$, are very much more difficult to study than those with two charged pions in the final state because of the short range of the proton, and the low efficiency of π^{0} and neutron counters. It is not intended to study these reactions.

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