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CP Violation from 5-dimensional QED

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ABSTRACT

It has been shown that QED in $(1+4)$-dimensional space-time, with the fifth dimension compactified on a circle, leads to CP violation (CPV). Depending on fermionic boundary conditions, CPV may be either explicit (through the Scherk–Schwarz mechanism), or spontaneous (via the Hosotani mechanism). The fifth component of the gauge field acquires (at the one-loop level) a non-zero vacuum expectation value. In the presence of two fermionic fields, this leads to spontaneous CPV in the case of CP-symmetric boundary conditions. Phenomenological consequences are illustrated by a calculation of the electric dipole moment for the fermionic zero-modes.

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I. INTRODUCTION

The physics of grand unified theories has been plagued by fundamental difficulties to accommodate different mass scales within a single theory, the so-called hierarchy problem. For a long time supersymmetric models had the commendable feature of being able to solve this problem. More recently, non-supersymmetric higher-dimensional models were proposed [1], [2], which solve the hierarchy problem provided that an appropriate space-time geometry is realized. Though in the original models only gravity was present outside a 4-dimensional slice of the compactified space, this is not an inescapable restriction. In fact, models where all fields propagate throughout the compactified space-time are natural and phenomenologically viable [3], [4]. In this letter we consider quantum electrodynamics (QED) in 5 dimensions (5D) focusing on the possibility that it naturally generates small but non-trivial CP-violating effects[♯1].

II. THE MODEL

We will consider an Abelian model in 5D, with coordinates $x^M,\ M = 0, \ldots, 4$ and $x^4 = y$ compactified to a circle of radius L. We assume the presence of two fermionic fields ($\psi_{1,2}$) interacting with the U(1) gauge field A_M according to the Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4}F^2_{MN} + \sum_{i=1,2} \bar{\psi}_i \left(i\gamma^M D_M - M_i \right) \psi_i + \mathcal{L}_{gf}\,, \tag{1}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$, the covariant derivative is given by $D_M = \partial_M + ie_5 q_i A_M$, where q_i denotes the charge of ψ_i in units of e_5, and $\mathcal{L}_{gf}$ stands for a gauge-fixing term. We will assume that the gauge fields are periodic in y, but we will allow the fermions to obey "twisted" boundary conditions (BCs):

$$\psi_i(x^\mu, y + L) = T\left[\psi_i(x^\mu, y)\right] \equiv e^{i\alpha_i}\psi_i(x^\mu, y)\,, \tag{2}$$

where $x^\mu,\ \mu = 0, \ldots, 3$ denote the coordinates of the 4D Minkowski space-time ($\mathcal{M}_4$) and T is the twist operator. We will also assume that the fermionic mass parameters M_i are

[♯1] For earlier attempts to obtain CP violation within extra-dimensional extensions of the Standard Model (SM) of electroweak interactions, see Refs. [5], [6].

positive and choose a convention where the Dirac matrices γ^M in 5D are the usual ones for $M \neq 4$ while $\gamma_{M=4} = i\gamma_5$; we will also use the metric diag$(1, -1, -1, -1, -1)$.

The action is invariant under the local U(1) transformation

$$\psi_i(x, y) \to e^{-ie_5 q_i \Lambda(x,y)} \psi_i(x, y), \qquad A_M(x, y) \to A_M(x, y) + \partial_M \Lambda(x, y)\,. \tag{3}$$

In addition, the Lagrangian is symmetric under the 5D CP transformations [7]

$$x^M \to \epsilon^M x^M, \quad A^M \to -\epsilon^M A^M, \quad \psi_i \to \eta_i \gamma^0 \gamma^2 \psi_i^\star, \quad |\eta_i| = 1\,, \tag{4}$$

where $\epsilon^{0,4} = -\epsilon^{1,2,3} = +1$ and there is no summation over M.

It is straightforward to expand the fields in Fourier series, leading to an infinite tower of fields propagating in $\mathcal{M}_4$,

$$\psi_i(x, y) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} \psi_{i,\,n}(x)\, e^{i\bar{\omega}_{i,\,n} y}, \quad A^M(x, y) = \frac{1}{\sqrt{L}} \left[\sum_{n=-\infty}^{\infty} A_n^M(x)\, e^{i\omega_n y} + a\delta_4^M \right], \tag{5}$$

where $\omega_n = 2\pi n/L$ and $\bar{\omega}_{i,\,n} = \omega_n + \alpha_i/L$. The fields associated with the $M = 4$ component of the gauge field become 4D scalars, which raises the interesting possibility that $A_{M=4}$ may acquire a non-zero vacuum expectation value; this, in fact, is known to occur [8]. In this case (4) suggests that this is also a sign of spontaneous CPV [♯2], an expectation that is indeed confirmed, as we will see in the following section.

It should also be emphasized that the BCs (2) are not symmetric under CP (unless $\alpha_i = 0, \pm\pi$), and this is an *additional* source of explicit[♯3] CPV, present even if $\langle A_4 \rangle = 0$. Note that the twist operator T [9] does not commute with CP (which is a symmetry of (1)), therefore the CP violation by the boundary terms is an example of the Scherk–Schwarz breaking mechanism [9].

Let us first focus on the fermionic piece of the Lagrangian (1). Integrating over the y coordinate we find

$$\mathcal{L}_\psi = \sum_{in} \bar{\psi}_{i,\,n} \left[i\gamma^\mu \partial_\mu - M_i + i\gamma_5 \mu_{i,\,n} \right] \psi_{i,\,n} - e \sum_{i,l,n} q_i \bar{\psi}_{i,\,l} \left(A_{l-n} + iA_{l-n}^4 \gamma_5 \right) \psi_{i,\,n}\,, \tag{6}$$

♯2 An attempt to generate CPV in a similar spirit has also been considered in Refs. [6].

♯3 In order to see that the BCs generate explicit CPV, it is sufficient to reformulate the theory in terms of the periodic field $\psi^{'}(y) \equiv e^{-i\alpha y/L}\psi(y)$. Then the BCs preserve CP but CP-violating interactions appear explicitly in the 5D Lagrangian.

where $\mu_{i,n} \equiv [2\pi n + (\alpha_i + eq_i La)]/L$, with $e \equiv e_5/\sqrt{L}$ the 4D gauge coupling. In order to diagonalize the fermion mass term we define the angles $\theta_{i,n}$ by

$$\tan(2\theta_{i,n}) = \frac{\mu_{i,n}}{M_i}; \quad |\theta_{i,n}| \leq \pi/4 \tag{7}$$

and replace[♯4] $\psi_{i,n} \to \exp(i\gamma_5\theta_{i,n})\psi_{i,n}$. From this we find that the physical fermion masses are $m_{i,n} = \sqrt{M_i^2 + \mu_{i,n}^2}$, while the interactions with the gauge fields read

$$\mathcal{L}_{A\psi} = -e\sum_i q_i\Big\{A_\mu \sum_k \bar{\psi}_{i,k}\gamma^\mu\psi_{i,k} + \sum_{k\neq l} A_{\mu\,k-l}\bar{\psi}_{i,k}\Gamma^{(v)}_{i,kl}\gamma^\mu\psi_{i,l}\Big\}, \tag{8}$$

$$\mathcal{L}_{\varphi\psi} = -e\sum_i q_i\Big\{\varphi \sum_k \bar{\psi}_{i,k}\Gamma^{(\varphi)}_{i,k}\psi_{i,k} + \sum_{k\neq l} A_{4\,k-l}\bar{\psi}_{i,k}\Gamma^{(s)}_{i,kl}\psi_{i,l}\Big\}, \tag{9}$$

where $\varphi \equiv A_{4\,0}$, $A_\mu \equiv A_{\mu\,0}$ and

$$\Gamma^{(\varphi)}_{i,k} \equiv -i\gamma_5 e^{2i\gamma_5\theta_{i,k}}, \qquad \Gamma^{(s)}_{i,kl} \equiv -i\gamma_5 e^{i\gamma_5(\theta_{i,k}+\theta_{i,l})}, \qquad \Gamma^{(v)}_{i,kl} \equiv e^{i\gamma_5(\theta_{i,k}-\theta_{i,l})}. \tag{10}$$

It is evident that A_μ corresponds to the 4D photon. However, the field φ is a new, physical, low-energy degree of freedom whose Yukawa couplings appear to be CP-violating. As we will show shortly, this naive conclusion is incorrect in general. The couplings of $A_{4\,n}$ and $A_{\mu\,n}$ also appear to violate CP.

The 5D gauge transformation (3) in terms of KK modes implies $A^4_k \to A^4_k + i\omega_k\Lambda_k$, where $\Lambda(x,y) = L^{-1/2}\sum_{n=-\infty}^{+\infty}\Lambda_n(x)e^{i\omega_n y}$, which shows that, while $A^4_{k\neq 0}$ can be removed by an appropriate gauge choice, $\varphi = A_{4\,0}$ is a gauge singlet[♯5]. Because of this, even though the φ mass m_φ vanishes at tree level (see (13) below), it will receive calculable finite corrections at higher orders in perturbation theory.

It is worth noting that even if $A_4(x,y) = \varphi(x)$ by a choice of gauge, there still remains a residual y-dependent discrete gauge freedom

$$A_4 \to A_4 + \frac{2\pi n_i}{e_5 q_i L}, \qquad \psi_i \to e^{-i\frac{2\pi n_i}{L}y}\psi_i, \quad n_i = 0, \pm 1, \cdots, \tag{11}$$

[♯4] The chiral rotation of the fermions induces an $\epsilon_{\mu\nu\sigma\rho}F_{\mu\nu}F^{\sigma\rho}$ term in the Lagrangian; however, in the Abelian case considered here, this is a total derivative and it can be dropped.

[♯5] This is a consequence of the compactification of the x^4 direction; in an uncompactified space one could always choose the $A^4(x,y) = 0$ gauge. Note also that in the case of compactification on the orbifold S^1/Z_2, φ disappears as a consequence of the requirement of the antisymmetry of A_4 under Z_2: $y \to -y$. Therefore CP cannot be violated spontaneously; however, if BCs are not symmetric under CP, $A_{\mu\,n}$ and $A_{4\,n}$ would still have CP-violating couplings with fermions.

provided q_1/q_2 is a rational number. In this case there exist some discrete constant values of φ $(= 2\pi n_i/(e_5 q_i L))$ that can be removed completely. Note also that $\alpha_i + e_5 q_i A_4$ is invariant under (11); this will be relevant when we discuss the one-loop effective potential for $\langle\varphi\rangle$.

The physical content of KK excitations for A_4 can be easily revealed by adopting the following generalization of the 4D R_ξ gauge [10]:

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial^\mu A_\mu - \xi\partial_y A_4\right)^2 . \tag{12}$$

Decomposing into KK modes, one can write the kinetic part of the Lagrangian density in the following form

$$\mathcal{L}_A + \mathcal{L}_{gf} \;=\; \frac{1}{2}\sum_n \left\{A^\mu_n\left[(\Box + \omega_n^2)g_{\mu\nu} - (1-\xi^{-1})\partial_\mu\partial_\nu\right]A^\nu_{-n} - A^4_n(\Box + \xi\omega_n^2)A^4_{-n}\right\} . \tag{13}$$

It is then clear that A^4_n $(n \neq 0)$ are the would-be Goldstone bosons that become a longitudinal component of A^μ_n, while $\varphi = A_{4\,0}$ is a physical massless scalar. Note that even for $\langle A_4\rangle \neq 0$, the 4D $U(1)$ gauge symmetry remains unbroken, so that the 4D photon $A_{\mu\ 0}$ remains massless.

III. THE EFFECTIVE POTENTIAL

The above discussion raises the possibility that φ will acquire a non-vanishing vacuum expectation value $a \equiv \langle\varphi\rangle$. In order to determine the conditions under which this occurs, we evaluate the corresponding effective potential to one loop. We will adopt dimensional regularization for the d^4p integral together with a summation over the infinite tower of KK modes.

After dropping an irrelevant constant contribution, and using dimensional regularization for the d^4p integral, we find

$$V(M;\omega) = \frac{1}{32\pi^6 L^4}\left[x^2 Li_3(re^{-x}) + 3xLi_4(re^{-x}) + 3Li_5(re^{-x}) + \text{H.c.}\right] , \tag{14}$$

where $x \equiv LM$, $\omega = (\alpha + eq_\psi La)/L$, $r = \exp(iL\omega)$, and $Li_n(x)$ is the standard polylogarithm function. Note that, as a consequence of the hermiticity, the potential is a symmetric function of ω: $V(M;\omega) = V(M;-\omega)$. We will consider a theory that contains two fermionic fields, so the total effective potential reads

$$V_{eff}(a) = \sum_{i=1,2} V(M_i;\omega_i) . \tag{15}$$

The total effective potential is not periodic in a unless q_1/q_2 is a rational number n_1/n_2, in which case the period is $T = 2\pi n_1/(eq_1 L) = 2\pi n_2/(eq_2 L)$. This property is a consequence of the residual gauge invariance (11) present when q_1/q_2 is rational.

It is worth discussing what would happen if we had just one fermionic field ψ. In this case the minimum of V is at $L\omega = \pi(2l+1)$ for integer l, but since α is defined modulo 2π we can choose the minimum $\alpha + eq_\psi La = \pi$. We can also eliminate α from (2) by the following field redefinition:

$$\psi'(x,y) = e^{-i\alpha y/L}\psi(x,y), \qquad e_5 q_\psi A'_M(x,y) = e_5 q_\psi A_M(x,y) + \alpha\delta_{M,4}\,, \tag{16}$$

so that ψ' and A' are periodic in y with period L. We then expand around the vacuum $e_5 q_\psi \langle A'_M\rangle = (\pi/L)\delta_{M4}$ by shifting the gauge field $e_5 q_\psi A'_M(x,y) \to e_5 q_\psi A'_M(x,y) + (\pi/L)\delta_{M4}$, and again redefine the fermion fields, so that the effect of this shift disappears from the Lagrangian density:

$$\chi(x,y) = e^{i\pi y/R}\psi'(x,y)\,, \tag{17}$$

which is antiperiodic in y, $\chi(x, y+L) = -\chi(x,y)$.

Through this series of field redefinitions, we have shown that the original theory is equivalent to one where the gauge field has a vanishing vacuum expectation value (hence, no spontaneous CPV) and also the fermionic field has CP-invariant BCs; consequently the theory predicts no CP-violating effects. However, in Eqs. (8) and (9) we have noted the presence of CP-violating couplings of φ, $A_{\mu\, n}$ and $A_{4\, n}$ even if only one fermion is present; this therefore deserves further explanation.

At the minimum of V, $\mu_n = \pi(2n+1)/L$ and $m_n = m_{-n-1}$, so that any unitary transformation U acting on the (ψ_n, ψ_{-n-1}) subspace will leave the corresponding kinetic terms invariant. This allows for a generalized definition of the CP transformation:

$$\psi_i \xrightarrow{\text{CP}} U_{ij} C\overline{(\gamma_0\psi_j)}^T\,, i,j = n, -n-1\,, \tag{18}$$

where $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Choosing $U = \sigma^1$ (the usual Pauli matrix) one can easily see that in fact, the couplings of ψ_n, ψ_{-n-1} are invariant under CP as defined in (18).

The situation can be different if a second fermion is present. Then, following the steps described above for the case of a single fermion, it can be shown that without losing any generality we can adopt the BCs $\psi_1(y+L) = \psi_1(y)$, $\psi_2(y+L) = \exp(i\alpha)\psi_2(y)$. The condition for an extremum is

$$\frac{\partial V_{eff}}{\partial a} = e \sum_{i=1,2} q_i \frac{\partial V(M_i;\omega_i)}{\partial \omega_i} = 0 \tag{19}$$

and leads to a CP-conserving vacuum when the minimum of V_{eff} is at $\omega_i = 0, \pi/L$. In general, however, the minimum is located elsewhere, opening a possibility for spontaneous CPV. As we have seen, at least two fermions are necessary to observe CPV in 5D QED compactified on a circle. In this case the KK modes of both fermions will have CP-violating Yukawa couplings to φ, $A_{\mu\, n}$ and $A_{4\, n}$. In Fig. 1 we plot the effective potential for various choices of the twist angle α as a function of $e\, a$. Note that V_{eff} is a symmetric function of $e\, a$ when $\alpha = 0, \pi$ (technically this is a consequence of the symmetry of $V^{(L)}(M_i;\omega)$ under $\omega \to -\omega$). This is the case of CP-symmetric BCs (2) [♯6], since the Lagrangian is invariant under CP; therefore the whole theory is CP-symmetric. Under CP, $a \to -a$, and therefore the observed symmetry of the effective potential is precisely a consequence of CP invariance. Consequently, choosing any of the two degenerate vacuum leads to spontaneous CP violation. For other (i.e. CP-asymmetric) choices of α, the effective potential is not invariant with respect to $a \to -a$; therefore, even though $a \neq 0$ at the minimum, CP is explicitly violated in those cases.

In the case of two fermionic fields there are two observable CP-violating parameters, α and $\langle A_4 \rangle$. In general, for N_f fermions there will be N_f CP-violating parameters: $N_f - 1$ twist angles and $\langle A_4 \rangle$. If, for instance, all the fermions are periodic in y ($\alpha_i = 0$), only $\langle A_4 \rangle$ will parametrize all CP-violating effects.

IV. PHENOMENOLOGY

The most striking consequence of CPV in our model will be a prediction for a non-zero fermionic electric dipole moment (EDM) d defined through the following effective $\gamma\bar{\psi}\psi$ vertex $\langle p'|j^\mu_{EM}|p\rangle = -(d/e)\bar{u}(p')\sigma^{\mu\nu}\gamma_5(p'-p)_\nu u(p)$ where $p,\ p'$ are on shell and the limit $p' \to p$ is assumed. In our model a non-zero EDM is generated already at the one-loop level (in the

[♯6] While it is trivial for $\alpha = 0$, one can, adopting similar arguments as above, show that also for $\alpha = \pi$ there are no CP-non-invariant interactions in the effective theory.

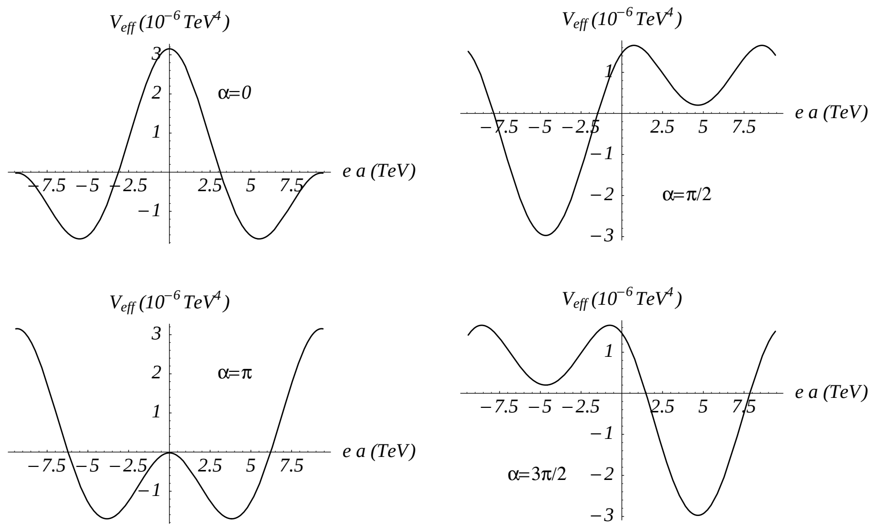


FIG. 1: The effective potential V_{eff} in units of 10^{-6} TeV^4 for $L^{-1} = 0.3$ TeV, $M_1 = 0.2$ TeV, $M_2 = 0.005$ TeV, $q_1 = 2/3$, $q_2 = -1/3$ and four choices of the twist angle $\alpha = 0, \pi/2, \pi, 3\pi/2$ (staring from the upper left plot and moving clockwise) is plotted as a function of $e\,a$ in units of TeV.

SM at least three loops are required). For the fermion ψ_i, the diagram involving φ yields[♯7]

$$d_{i\,0} = -\frac{(eq_i)^3 c^{(+)}_{i,\,0}}{16\pi^2 m_{i,\,0}} J^{(s)}(m^2_\varphi/m^2_{i,\,0}, 1)\,, \tag{20}$$

while the contributions from the n-th modes circulating in the loop equal

$$\begin{aligned} d^{(v)}_{i,\,n} &= \frac{(eq_i)^3 c^{(-)}_{i,\,n}}{4\pi^2} \frac{m_{i,\,n}}{m^2_{i,\,0}} J^{(v)}(x_{i,\,n}, y_{i\,n}) \qquad \text{from } A^\mu_n \text{ exchange}\,, \\ d^{(s)}_{i,\,0} &= -\frac{(eq_i)^3 c^{(+)}_{i,\,n}}{16\pi^2} \frac{m_{i,\,n}}{m^2_{i,\,0}} J^{(s)}(x_{i,\,n}, y_{i\,n}) \qquad \text{from } A^4_n \text{ exchange}\,, \end{aligned} \tag{21}$$

where $c^{(\pm)}_{i,\,n} = \pm M_i(\mu_{i,\,n} \pm \mu_{i,0})/(m_{i,\,n} m_{i,\,0})$, $x_{i\,n} = (\omega_n/m_{i,\,0})^2$, $y_{i\,n} = (m_{i,\,n}/m_{i,\,0})^2$, and

$$\begin{aligned} J^{(s)}(x,y) &= 1 + \frac{x-y+1}{2} \ln\left(\frac{y}{x}\right) + \left(\frac{2x}{\rho} - \rho\right)\Theta\,, \\ J^{(v)}(x,y) &= -1 + \frac{y-x}{2} \ln\left(\frac{y}{x}\right) + (\rho - \cot\Theta)\Theta\,, \end{aligned} \tag{22}$$

with $\rho^2 \equiv 4xy - (x+y-1)^2$ and $\tan\Theta \equiv \rho/(x+y-1)$.

[♯7] An analogous contribution appears within the Two Higgs Doublet Model (2HDM), see Refs. [11, 12].

The total EDM of the i-th zero-mode fermion is then

$$d_i = d_{i,0} + \frac{(eq_i)^3}{16\pi^2 m_{i,0}^2} \sum_{n\neq 0} m_{i,n} \left[4c_{i,n}^{(-)} J^{(v)}(x_{i,n}, y_{i\,n}) - c_{i,n}^{(+)} J^{(s)}(x_{i,n}, y_{i\,n})\right] . \tag{23}$$

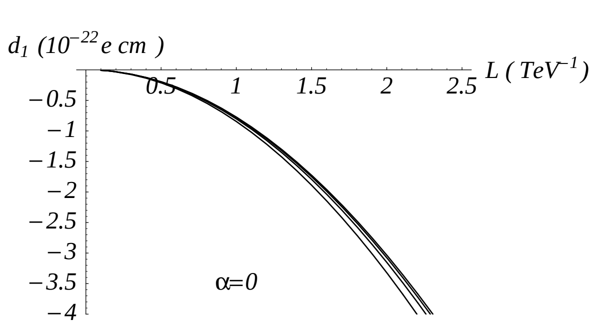
Note that for large n, $c^{(\pm)} J^{(s/v)}(x_{i,n}, y_{i\,n}) \sim 1/n + \mathcal{O}(1/n^2)$, so that d_i will be finite after symmetric summation over n. It follows that the EDM is finite and therefore insensitive to the cut off of the 5D theory.

In Fig. 2 we plot the fermionic EDM as a function of the compactification scale L. The parameters have been adjusted in such a way that the model has the mass scales and the coupling constants of the same order as those that are present in the SM. We have chosen for illustration to plot the EDM of the zero-mode of the $i = 1$ fermion. Note that the mass of the zero-mode depends on L; for parameters adopted here (with $\alpha = 0$, i.e. for the case of spontaneous CP), it varies from ~ 37 TeV for $L = 0.1$ TeV^{-1}, to ~ 1.5 TeV when $L = 2.5$ TeV^{-1}; for these values, m_φ ranges from ~ 88 GeV to ~ 3.5 GeV. The leading contribution to d_i comes from the φ exchange; the contribution of the non-zero modes is of opposite sign and smaller by a factor $\mathcal{O}(5)$.

In Fig. 2 the positive vacuum expectation value of A_4 was chosen (see Fig. 1) for $\alpha = 0$. It is worth noticing that EDM, as a CP-odd quantity, would be of the opposite sign if the other (negative) vacuum expectation value was chosen.

V. CONCLUSIONS

We have shown that QED in $(1 + 4)$-dimensional space-time, with the fifth dimension compactified on a circle, leads to CP violation. Depending on fermionic boundary conditions, CPV may be either explicit, or spontaneous via the Hosotani mechanism. The new possibility of CP breaking by fermionic, twisted boundary conditions has been emphasized and demonstrated explicitly by derivation of CP-violating effective couplings. The fifth component of the gauge field acquires (at the one-loop level) a non-zero vacuum expectation value. We have shown that in the presence of two fermionic fields, this leads to spontaneous CPV in the case of CP-symmetric boundary conditions. The one-loop effective potential for $A_{4\,0}$ has been calculated and its features have been discussed in the presence of two fermionic fields.



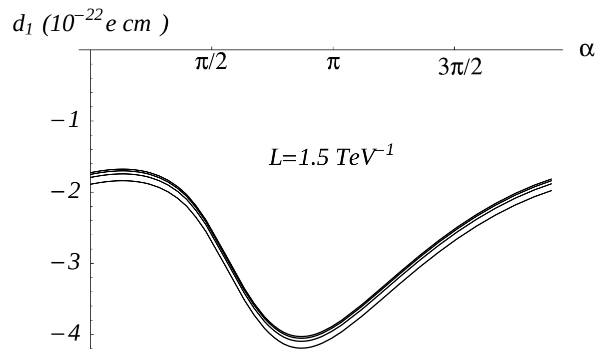


FIG. 2: Left: The fermionic EDM, d_1, in units of 10^{-22} e cm for the zero-mode of the fermion $i = 1$ as a function of the compactification length L in units of TeV^{-1} for $\alpha = 0$. Right: Same, as a function of α for $L = 1.5$ TeV^{-1}. The curves from the bottom to the top correspond to the number of modes included in (23) varying from $|n| = 1$ to $|n| = 4$; the fast convergence of the series is evident. Note that the mass of the zero-mode also varies with L. We used $e = \sqrt{4\pi\alpha_{QED}}$ and the same parameters as in Fig. 1.

The most striking feature of the model considered here is the presence of the light scalar φ, which has CP-violating Yukawa couplings similar to those present in the scalar sector of the 2HDM model. The presence of CP-violating couplings leads to a non-zero EDM, which was calculated at the one-loop level for a zero-mode fermion. This effect can be used to test the mechanism for CPV present in our model.

There are several other observables, originally developed to investigate extended Higgs sectors, which can also be used to detect the presence of a light scalar (regardless of whether its couplings conserve or violate CP) such as φ. For example, aside from the fermionic electric and magnetic dipole moments, one also has $\Gamma[\Upsilon \to \varphi\gamma]$ and $BR[b \to \varphi s]$. Experimental constraints on all such quantities would impose some restrictions on the parameters of the model. We will present the results of such an investigation in a separate publication, where we will consider a more realistic non-Abelian theory.

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