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OFF SHELL STATES IN THE DUAL MODEL+)

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A B S T R A C T

We present a class of Lorentz invariant, gauge invariant off-shell amplitudes for the ordinary Veneziano and Ramond-Neveu-Schwarz dual models. They correspond to interactions with the string which are local in space-time.

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The dual theory of relativistic strings provides the only explicit model of quantum mechanical extended objects in more than two dimensions. It is also the only framework which, at least formally, satisfies the general requirements of unitarity, crossing symmetry and Lorentz invariance outside of conventional quantum field theory. One long standing difficulty in the model has been the construction of off-shell currents consistent with the structure of the model, in particular the gauge invariance. These currents are necessary to couple a dual model of strong interactions with the electromagnetic, weak and gravitational interactions. They may also be essential in understanding the spontaneous symmetry breakdown of the theory due to the presence of tachyons.

The covariant off-shell amplitudes[1,2)] for the ordinary Veneziano (OV) model which have previously been obtained are only satisfactory in 16 dimensions, in which case the model itself is not unitary. The analogous set of local off-shell amplitudes constructed directly in the 24 dimensional transverse space of the string[3)], while providing an appealing physical picture of the coupling of currents of Refs. 1) and 2), does not correspond to an off-shell state of pure angular momentum. There is one candidate[2)] for a consistent off-shell scheme for the Ramond-Neveu-Schwarz (RNS) model, although it has not been shown to be completely dual or to have a simple string interpretation.

In this paper we will consider off-shell states of closed strings. We will show that there is a completely consistent set of covariant amplitudes for these off-shell states in both the OV and RNS models. Although we shall be concerned primarily with scalar and vector off-shell states in this paper there is an obvious procedure for generalizing to states of arbitrary angular momentum[4)].

We are motivated by the construction of Ref. 3) to look for closed string states which are local in space-time. States of the closed string are described by two sets of transverse operators A_{ni} and A'_{ni} satisfying the commutation relations

$$\begin{gathered}[A_{mi}, A^{\dagger}_{nj}] = [A'_{mi}, A'^{\dagger}_{nj}] = \delta_{ij}\,\delta_{m,n} \\ [A_{mi}, A'^{\dagger}_{nj}] = 0\,, \end{gathered} \tag{1}$$

i,j = 1, ... ; D − 2.

The position operator has transverse components

$$X_j(\sigma) = x_j + \sum_{n=1}^{\infty} n^{-1/2}\left(A_{nj} e^{in\sigma} + A'_{nj} e^{-in\sigma} + h.c.\right) . \tag{2}$$

We thus have the condition for a transversely localized state:

$$(A_{nj} + A'^{\dagger}_{nj}) | c \rangle = 0 \tag{3}$$

which is solved by

$$| c \rangle = e^{-\sum A^{\dagger}_{ni} A'^{\dagger}_{ni}} | 0 \rangle . \tag{4}$$

It turns out that this state, unlike the analogous state for the open string considered in Ref. 3), is also localized in X^-:

$$X^-(\sigma) | c \rangle = x^- | c \rangle \tag{5}$$

and so the state is truly pointlike. It is then simple to show that the state transforms like a scalar under an arbitrary Lorentz transformation. The obvious candidate for a covariant generalization of Eq. (4) is the state

$$| d \rangle = e^{-\sum A^{\dagger\mu}_{n} A'^{\dagger}_{n\mu}} | 0_{A,A'}, q \rangle \tag{6}$$

where the Greek suffixes denote a D component vector and $|0_{A,A'},q\rangle$ is the vacuum in A and A' but has momentum q. For any momentum q, this state satisfies the gauge relations for all n:

$$(L_n - L'_{-n}) | d \rangle = 0 \tag{7}$$

which are the relevant off-shell conditions for the Pomeron sector of the OV model[5,6]. These conditions hold in any dimension (although D = 26 is required for the model to be unitary) unlike the case discussed in Refs. 1) and 2). We can therefore obtain a consistent amplitude for coupling this state to Reggeon trees by considering the factorization exhibited in Fig. 1. The factors are identical to the planar loop amplitude except that the state $|d\rangle$, which couples the Pomeron to the current, is a state for which $X^{\mu}(\sigma) = x^{\mu}$. For the planar loop, $|d\rangle = \exp(+\Sigma A_n^{\mu\dagger} A'^{\dagger}_{n\mu})|0\rangle$ is the state for which $P(\sigma) = 0$ and only satisfies the gauge conditions [Eq. (7)] when $q = 0$. A comparison of the world surfaces of the planar loop and our off-shell amplitude is given in Fig. 2, in which the difference in boundary conditions is apparent.

As the gauge conditions [Eq. (7)] are unchanged, so is the Pomeron propagator $\int dr\, r^{-3+L_0+L_0'+q^2/4} f^2(r^2)$, and the only change in going from the planar loop to the current amplitude is the minus sign in the exponential in $|d\rangle$. The result is*)

$$F_N(k_1,\ldots,k_N) = c\, g^N \int_0^1 \frac{dr}{r}\, r^{-2+q^2/4} f^2(r^2)\, \phi_{0+}^{-D}(r^2)$$
$$\times \left[\frac{f(r^4)}{\phi_-(r^4)}\right]^{2N} \int \prod_2^N d\nu_i \prod_{i<j} \left[\frac{\theta_1(\nu_i-\nu_j|2\tau)}{r^{1/2}\theta_4(\nu_i-\nu_j|2\tau)}\right]^{k_i\cdot k_j} \tag{8}$$

where[5] $r = e^{i\pi\tau}$ and

$$\begin{aligned} f(r^2) &= \prod_{n=1}^{\infty} (1-r^{2n}) \\ \phi_{0+}(r^2) &= \prod_{n=0}^{\infty} (1+r^{2n}) \\ \phi_-(r^2) &= \prod_{n=0}^{\infty} (1-r^{2n+1}) . \end{aligned} \tag{9}$$

*) The notation is that of Ref. 5).

The functions $\theta_1(\nu|\tau)$, $\theta_4(\nu|\tau)$ are Jacobi theta functions[7]. The angular variables ν_i are integrated over the range

$$0 = \nu_1 \leq \nu_2 \leq \ldots \leq \nu_N \leq 1. \tag{10}$$

This amplitude differs from the planar loop amplitude in two crucial respects. First it has no normal thresholds in any channel and is therefore meromorphic. Secondly it has no momentum conservation δ function and is therefore defined for an arbitrary momentum passing through the Pomeron propagator in Fig. 1. It is thus a completely consistent off-shell amplitude.

The generalization to the RNS model is straightforward. There are two types of positive G-parity off-shell states available:

$$|d^\pm\rangle = e^{-\Sigma[A_n^{\dagger\mu} A'^{\dagger}_{n\mu} \mp B_n^{\dagger\mu} B'^{\dagger}_{n\mu}]} |0_{A,A',B,B'},q\rangle \tag{11}$$

where $B^\dagger_{n\mu}$ and $B'^\dagger_{n\mu}$ are $\frac{1}{2}$ integer moded and satisfy the usual anticommutation relations of the Neveu-Schwarz modes[5]. The state $|d^+\rangle$ has the same B and B' structure as for the ordinary planar positive G-parity Neveu Schwarz loop whereas $|d^-\rangle$ has a sign reversed in the coefficient of $\sum_n B^\dagger_{n\mu} B'^\dagger_{n\mu}$ in the exponent. The amplitudes that result from this are:

$$\begin{aligned} F_N^\pm(k_1,\ldots,k_N) = {} & c\, g^N \int \frac{dr}{r}\, r^{-1+q^2/4}\, \frac{f^2(r^2)}{\phi_\mp^2(r^2)} \\ & \times \left[\frac{\phi_{0+}(r^2)}{\phi_\pm(r^2)}\right]^{-D} \left[\frac{f(r^4)}{\phi_-(r^4)}\right]^N \int \prod d\nu_i \\ & \times \prod_{i<j} \left(\Theta_1(\nu_i-\nu_j|2\tau)/r^{1/2}\Theta_4(\nu_i-\nu_j|2\tau)\right)^{k_i\cdot k_j} \\ & \times \left(\sum_{P\varepsilon P'} (-)^P \prod_{j=1}^{N/2} k_{i_{2j-1}}\cdot k_{i_{2j}}\, \chi^\pm(e^{2\pi i(\nu_i-\nu_j)}, r^2)\right) \end{aligned} \tag{12}$$

with[5]:

$$\phi_{\pm}(r^2) = \prod_{n=0}^{\infty} (1 \pm r^{2n+1})$$

$$\chi^{+}(e^{2\pi i\nu}, r^2) = \frac{i}{2}\,\frac{\theta_2(0|\tau)\,\theta_4(0|\tau)\,\theta_3(\nu|\tau)}{\theta_1(\nu|\tau)} \tag{13}$$

$$\chi^{-}(e^{2\pi i\nu}, r^2) = \frac{i}{2}\,\frac{\theta_2(0|\tau)\,\theta_3(0|\tau)\,\theta_4(\nu|\tau)}{\theta_1(\nu|\tau)} .$$

The types of partition functions entering the first bracket of Eq. (12) are determined by the gauge properties of the states $|d^{\pm}\rangle$. These states satisfy the same gauge conditions as the ordinary Pomeron sector constructed out of loops of fermions and bosons, respectively[4].

Whereas F^- appears to be a satisfactory closed string current amplitude, F^+ contains an obvious divergence as $r \to 1$. Upon Jacobi transforming Eq. (12) this divergence is exhibited in the form $\int w^{-1}\, dw \times h(w, \ldots)$, where h is a power serie in w. Note that there are no factors of log r $[= 2\pi^2\ (\log w)^{-1}]$ or r or w^{q^2} in this expression. The divergent term, $\int w^{-1}\, dw \times h(0, \ldots)$, must satisfy the gaug conditions by itself and we may therefore take h(0) to be a new off-shell amplitude H_0. This amplitude turns out to have a structure very reminiscent of the open string pion amplitude of Ref. 2). However, our off-shell state has even G-parity and H_0 does not have any singularities in q^2. A more interesting (vector) current can be constructed by using the state:

$$|d\rangle = e^{-A^{\dagger}A'^{\dagger} + B^{\dagger}B'^{\dagger}}\, \Upsilon\, |\rho\rangle \tag{14}$$

where $|\rho\rangle$ is the usual "ρ" state in the $\mathcal{F}_1$ formalism. As in F^- (Eq. 12) there is a divergence at r = 1 in the amplitudes involving this state. The integrand at w = 0 once again provides a finite gauge-invariant current H_1^{μ}. For the coupling to two pions this amplitude is:

$$H_1^{\mu} = (k_1 - k_2)^{\mu} \frac{2q^2}{q^2-2} \int_0^1 \frac{du}{u} \left[\left(\frac{1-u}{1+u} \right)^{\frac{1}{2}q^2-1} - 1 \right] . \tag{15}$$

The form of the integral, although very similar to that occurring in the ground state form factor of the OV model in Refs. 1) and 2), emerges automatically in a finite form, and requires no subtraction. This current is not the same as the ρ current proposed in Ref. 2).

Since Eqs. (8) and (12) were constructed by means of a manifestly factorized formalism satisfying all the gauge conditions there is in principle a procedure (analogous to that for multi-loop amplitudes) for generalizing to amplitudes involving more than one external off-shell state. It is probable, however, that new divergences will arise in multi-current amplitudes due to partition functions as in multi-loop amplitudes and the open string current amplitudes[8)].

An alternative way of picturing the off-shell emission amplitudes is illustrated in Fig. 2. Fig. 2c represents the planar loop amplitude of Fig. 2a in the light-like orthonormal gauge (the convention is defined in Ref. 9)) with the loop represented by a horizontal slit. In Fig. 2d we illustrate the equivalent representation of our amplitude for emission of a zero momentum off-shell state. [We have checked that the Neumann function for this problem is the same as for the closed string emission problem[4)] of Eq. (8) but have not yet obtained the measure explicitly within the string picture.] We see directly that Fig. 2d is just the functional integral for the ordinary four-point amplitude with the added constraint that a section of string (of length $\sigma_1 - \sigma_2$) collapses to a point at time τ_0 (with σ_1, σ_2 and τ_0 then integrated over). Our currents may therefore be viewed as coupling to point-like momentum densities on the string.

The r = 1 end point of the integral for the closed string currents [Eqs. (8) and (12)] corresponds to one end of the "slit" touching a boundary of the string. Therefore the currents, H_0 and H_1^{μ}, which were extracted from this end point should be viewed as coupling to the spatial end of the string (see Fig. 2c). In the OV model the leading behaviour of the integrand at r = 1 gives exactly the open string currents discussed in Refs. 1)-3). However, this end point does not have a divergence in the OV model and therefore its contribution need not be gauge invariant by itself.

The insertion of arbitrary numbers of "slits" at zero momentum constitutes a modification of the original free string action which reweights histories for which a finite amount of p^+ is at one point in space at some instant of time. Such a mechanism may be connected with spontaneous breakdown of the vacuum that is expected because of the existence of tachyons in the present formulation of the theory.

A third way of viewing the off-shell amplitudes is strongly suggested by analogy with the treatment for open string currents (given in Ref. 2)). This introduces c_n modes which are ½-integer commuting modes satisfying[2)]:

$$[c_n^\nu, c_m^{\nu\dagger}] = g^{\mu\nu}\delta_{n,m} \qquad n,m = \tfrac{1}{2},\ldots,\infty. \tag{17}$$

The analogous construction of a closed loop of c-moded "propagators" coupling to two trees shown in Fig. 1b yields an expression identical to our Eq. (8) apart from undetermined factors of various partition functions*).

Since the "propagators" constructed out of c modes do not carry momentum (there are no zeroth modes), we see immediately that the current amplitude, Fig. 1b, has no normal thresholds.

In conclusion, we have off-shell states consistent with factorization, gauge conditions and Lorentz invariance, for both the ordinary Veneziano and Ramond-Neveu-Schwarz models. These currents couple to internal points on the string in a manner truly localized in space-time. We feel they may be very useful in discussing spontaneous symmetry breaking, as well as making contact with the Green functions of quantum field theory.

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*) Such factors may be expected to arise from a correct elimination of spurious states circulating in the loop.

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Figure captions

Fig. 1 : a) Factorization of general Reggeon tree with current emission. (The notation is from Ref. 5)).

b) Equivalent construction of Fig. 1a from a loop of c-moded propagators.

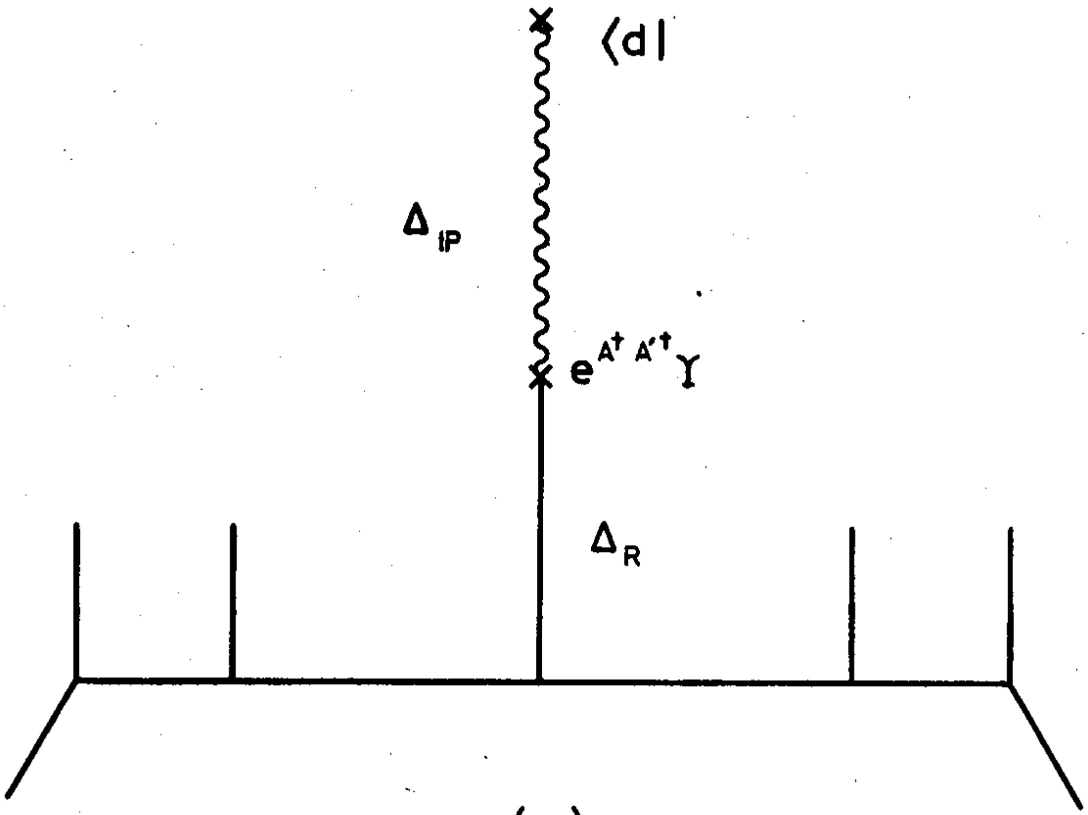
Fig. 2 : a) World sheet of planar loop viewed as the emission of the Pomeron into the vacuum.

b) World sheet for the emission of an off-shell closed string.

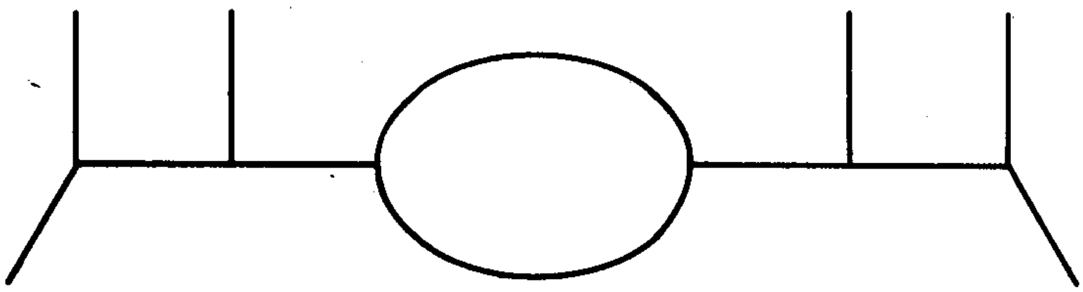
c) The planar loop in the orthonormal light-like gauge.

d) The emission of a zero momentum off-shell state as viewed in the orthonormal light-like gauge.

e) An open string current as derived from the limit of a closed string current.

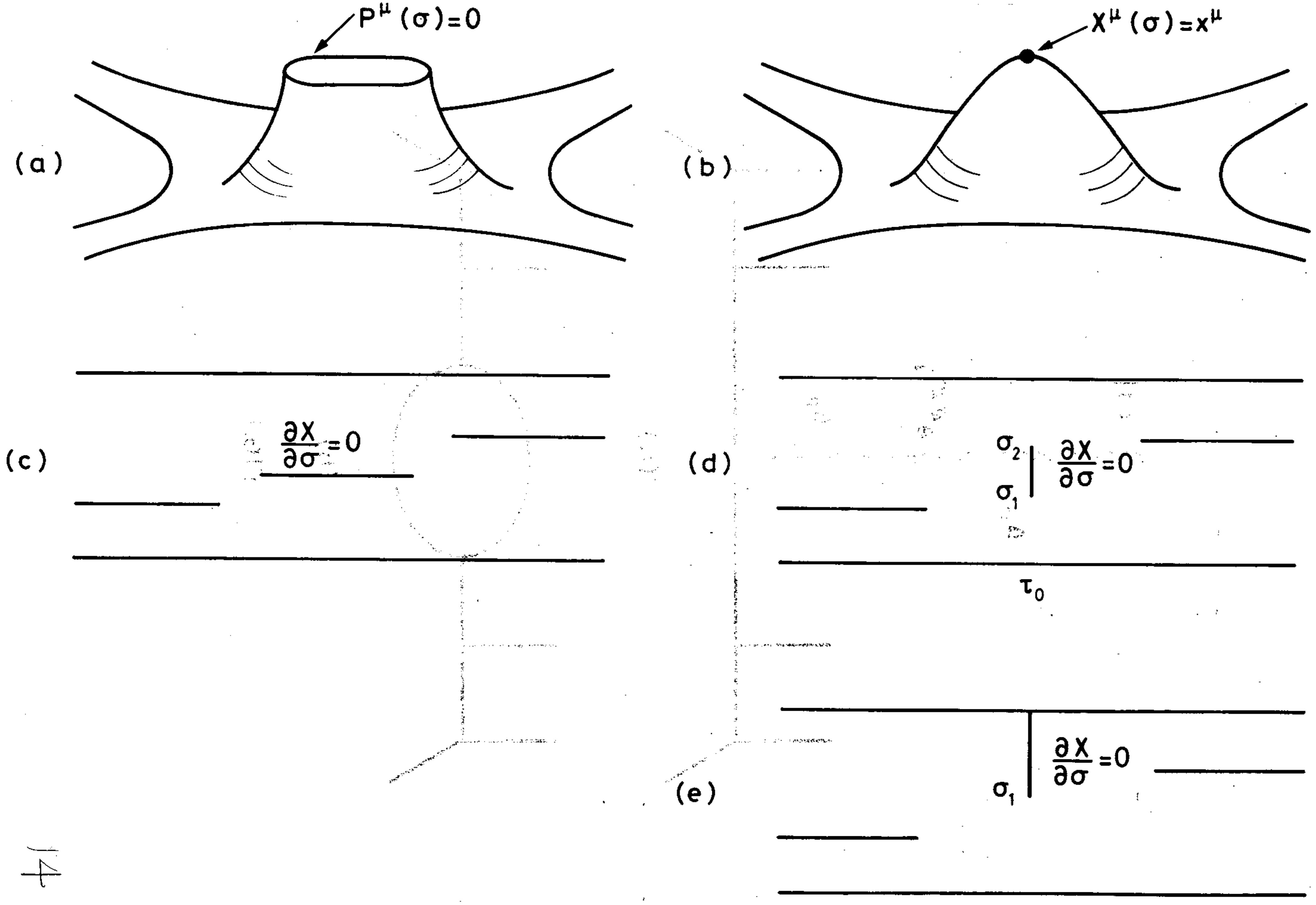


(a)



(b)

FIG.1



$P^{\mu}(\sigma)=0$
$X^{\mu}(\sigma)=x^{\mu}$
(a)
(b)
(c)
(d)
(e)
$\frac{\partial X}{\partial \sigma}=0$
σ_2
σ_1
$\frac{\partial X}{\partial \sigma}=0$
τ_0
σ_1
$\frac{\partial X}{\partial \sigma}=0$
τ_0

FIG. 2