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PROTON STRIPPING AND PION PRODUCTION IN RELATIVISTIC DEUTERON-NUCLEUS COLLISIONS

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A B S T R A C T

We construct a Glauber theory for the proton stripping and pion production in relativistic deuteron-nucleus collisions, taking into account the relativistic dilation of the internal deuteron wave function. The theory is applied to the experiments performed at the Bevalac with 1.05 and 2.1 GeV/nucl incident deuterons. The observed broadening of the stripped proton spectrum is explained as coming from the relativistic dilation effect. The production of fast pions is described as initiated from nucleons of the incident deuteron, whose momenta are large, again due to the relativistic dilation. Both the momentum shapes and the A-dependence of the cross-sections for the different reactions are well-reproduced

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1. INTRODUCTION

In the last few years very interesting results have been obtained at the Berkeley-Bevalac by accelerating deuterons at relativistic energies[1,2].

Here, we are particularly interested in the following reactions:

$$\begin{aligned} &a) \quad d + A \rightarrow p + X \text{ (proton stripping)} \\ &b) \quad d + A \rightarrow \pi + X \text{ (pion production)} \end{aligned} \tag{1.1}$$

In both (a) and (b) A denotes a nuclear target (we shall always use the same symbol A also for the total number of nucleons in the target).

Data for these two reactions exist, either in published form[1,2] or as a thesis[3]; the targets were Be, C, Cu and Pb nuclei; the incident deuteron energies are 1.05 and 2.1 GeV/nucl kinetic energy. Up to now data have been published only at the fixed angle of 2.5^o in the L.S. (nuclear target frame).

These data show a number of interesting features. Below we shall summarize them separately for the two reactions (a) and (b) together with a short description of the proposed interpretation.

a) Proton stripping

As can be seen from Figs. 1-6, the invariant cross-section $E(d^3\sigma/d\vec{k}_p)$ for reaction (a) shows a peak centred at the value of one-half of the deuteron incident momentum as a function of k_p (the proton momentum measured in the L.S.). This peak is quite narrow and its shape is reminiscent of the (square of the) momentum-space deuteron wave function.

A number of other relevant features are:

- the peak is asymmetric, showing an indication of a possible plateau for small k_p at the left side of the maximum, while it keeps decreasing at the right side;
- the A-dependence of the cross-section at the peak is roughly proportional to $A^{2/3}$; the peak cross-section decreases with the incident energy by about a factor of 3 going from 1.05 to 2.1 GeV/nucl.

These features are suggestive of the following physical description: Consider the internal motion of the proton inside the deuteron, described in the deuteron rest frame by the momentum space deuteron wave function. When the deuteron is broken by some external potential (like in the photodisintegration or in the meso-disintegration) the proton (spectator) momentum distribution is a measure of the deuteron wave function.

If now one boosts the deuteron with a relativistic speed and scratches out its neutron with an external potential, like a heavy nucleus which has the property of collimating the projectiles forward, the momentum distribution of the separated proton will again be described by the deuteron wave function. However, due to the relativistic motion of the deuteron, this wave function will be Lorentz-dilated.

Of course, in both cases one has to correct this simple description by including multiple scattering effects, final state interactions, and so on.

Therefore we are going to describe the proton stripping (reaction a) with a theory based on Glauber's multiple scattering expansion in which all the possible interactions are correctly taken into account, and the Lorentz dilation effect on the wave function is included.

This treatment is not new; in fact, both the physical idea and the inclusion of the Lorentz dilation of the wave function have already been discussed by the authors of the experiment[2], and fits based on similar methods have been produced. Also the general theoretical background has been discussed elsewhere[4,5]. What is new in the present work in this respect is:

- the correct treatment of the multiple scattering expansion, using the correct eikonal operator and using unitarity; moreover the constraint that the integral over the differential distribution has to reproduce the integrated cross-section, which on the other side can be exactly computed from closure, has been included;
- the use of the correct kinematics when dealing with the Lorentz dilation of the deuteron wave function;
- the use of a realistic deuteron wave function (including both the S and D components) instead of the too-simple Hulthèn wave function.

The theory will be described in detail in Section 2, and the application to the Berkeley experiment will be discussed in Section 4. Here, however, we anticipate the explanation of the origin of the different effects just discussed above:

- the increase of the width of the distribution with the incident energy and its insensitivity to A are a consequence of the fact that the width is essentially coming from the Lorentz dilation of the deuteron wave function;

- the theory has been constructed in such a way to exclude the inelastic interaction initiated by the proton of the incident deuteron. It is, however, very likely that protons of small momentum have indeed lost energy through inelastic interactions, while protons whose momentum is quite larger than the peak momentum have only experienced elastic collisions. Our theoretical description is, therefore, expected to correctly describe only the right-hand part of the proton momentum distribution and, at the most, a small portion of the left-hand part - in any case excluding the plateau which is very likely associated with production from the proton;
- the A-dependence of the peak cross-section has two origins. One effect is due to a cancellation between different terms in the multiple scattering expansion which, separately, have a different A-behaviour (they correspond to the total and summed differential nucleon-nucleus cross-section): the second effect comes from a normalization factor, which has to be included in order that the integrated cross-section coincides with the one obtained using the closure relation of the proton-neutron states.

The energy dependence of the peak cross-section is essentially a consequence of two facts. The first is that the experiment is at fixed non-zero angle so that the transverse momentum increases with the value of k_p and therefore of the incident energy; a larger value of the transverse momentum means a bigger reduction factor. The second fact comes from the broadening of the peak; since the integrated cross-section is fixed, this also lowers the peak cross-section. Our prediction will be that on a given target the cross-section at 0^o, integrated over the proton momentum, should be energy independent (barring small effects that could arise from possible energy dependence of the total and elastic nucleon-nucleon cross-section, which are the parameters entering in the theory).

b) Pion production

The experimental results for the reaction (1.1b) are reported in Figs. 7-10. The main features shown by the data are the following: Considering the cross-section $d\sigma^2/d\Omega dK_\pi$ at fixed angle $\theta = 2.5^o$ as a function of the pion momentum $\vec{K}_\pi$ (in the nuclear target frame), the most striking feature is the copious production of pions beyond the kinematical limit of the reaction $p + A \to \pi + X$ in which the incident proton momentum is taken to be $\vec{k}_d/2$. Moreover, the difference between the maximum energy of the observed pions and the kinematical limit for $pA \to \pi X$ increases with the incident deuteron energy. The A-dependence of the cross-section[1,2] is consistent with $A^{1/3}$ for $x_\pi = K_\pi/K_{\pi max} \gtrsim \frac{1}{2}$, and increases to a larger power of A, of the order of 0.5, for smaller x_π.

In our model these results have the following interpretation: as a consequence of the Lorentz dilation of the internal motion of the nucleons inside the deuteron there is a large probability for the proton and the neutron to have a momentum quite larger than $k_d/2$; these "fast nucleons" can then produce "fast pions". The actual pion distribution will be computed averaging over the deuteron dilated squared wave function the inclusive cross-section for pion production of nucleon-nucleus collisions at the proper incident energy.

The A-dependence of the inclusive production in dA collision is then simply a reflection of the A-dependence of inclusive production in nucleon-nucleus collisions, which in the proton fragmentation region shows a behaviour for the A-dependence similar to the one discussed for dA collisions[1,2,5].

Again, a similar model has been proposed and applied by the authors of the experiment[2] and discussed elsewhere[5]. Our contribution here consists mainly of the formal proof, given in Section 3, that one must sum the inclusive distributions from incident protons and neutrons, without any interference or double scattering effect, and in the use of a more realistic deuteron wave function.

2. PROTON STRIPPING

We describe here the theoretical formalism for the first reaction (1.1a) -- the proton stripping. As we have already said, we describe the reaction using the Glauber multiple scattering theory.

As discussed in Ref. 4, to compute the relevant cross-section we have to add two contributions: the disintegration cross-section, which corresponds to the situation in which both the proton and the neutron of the deuteron suffer only elastic collisions, plus the neutron absorption cross-section in which the neutron suffers inelastic collisions (but the proton keeps scattering only elastically). The amplitudes for the two reactions

$$\begin{aligned} & d + A \to p + n + A' \\ & d + A \to p + (n + X) + A' \end{aligned} \tag{2.1}$$

are given by

$$F_{dis} = \frac{i k_d}{2\pi} \int d_2 B \, e^{i \vec{Q} \cdot \vec{B}} \langle d, A | \left[1 - (1 - \Gamma_p)(1 - \Gamma_n) \right] | d', A' \rangle \tag{2.2a}$$

$$F_{abs} = \frac{i k_d}{2} \int d_2 B \, e^{i \vec{Q} \cdot \vec{B}} \langle d, A | (1 - \Gamma_p) \Gamma_n^x | x, d', A' \rangle \tag{2.2b}$$

In this formula k_d is the incident deuteron momentum, $\vec{B}$ the deuteron transverse coordinate, $\vec{Q}$ the momentum transferred to the final proton-neutron system, $|A\rangle, |A'\rangle$ are the initial and final nuclear state (where $|A'\rangle$ can also coincide with $|A\rangle$ in the coherent reaction), $|d\rangle$ is the incident deuteron state and $|d'\rangle \neq |d\rangle$ is the final proton-neutron state. Γ_p and Γ_n are the total profile functions for the proton-nucleus and nucleon-nucleus amplitudes, while Γ_n^x is the total profile function for the production of the state $|x\rangle$ in the neutron-nucleus interaction.

The cross-section for reaction (2.1a) is obtained by squaring separately the amplitudes (2.2a) and (2.2b), summing over the nuclear states $|A'\rangle$, integrating over the neutron angular distribution and summing over all the possible produced states $|x\rangle$.

The sum over the nuclear states gives no difficulty since we can use closure relation in the form $\sum_{A'} |A'\rangle\langle A'| = 1$. The sum over the neutron angular distribution and over X is more delicate, so we discuss it in some detail.

What one would like to have after the integration over the neutron angular distribution is an expression which is diagonal in the neutron impact parameter (namely in the squared amplitudes Γ_n and Γ_n^* are functions of the same impact parameter b_n). If this is the case, both in the absorption and disintegration cross-sections the sum of the two will contain the production profile function only in the form

$$\begin{aligned}
&\left[1-(1-\Gamma_p(\vec{b}_p))(1-\Gamma_n(\vec{b}_n))\right]\left[1-(1-\Gamma_p^*(\vec{b}_p'))(1-\Gamma_n^*(\vec{b}_n))\right] + (1-\Gamma_p(\vec{b}_p))(1-\Gamma_p^*(\vec{b}_p')) \cdot \\
&\cdot \sum_x |\Gamma_n^x(\vec{b}_n)|^2 = 1 + (1-\Gamma_p(\vec{b}_p))(1-\Gamma_p^*(\vec{b}_p')) - (1-\Gamma_p(\vec{b}_p))(1-\Gamma_n(\vec{b}_n)) - \\
&- (1-\Gamma_p^*(\vec{b}_p'))(1-\Gamma_n^*(\vec{b}_n)) + (1-\Gamma_p(\vec{b}_p))(1-\Gamma_p^*(\vec{b}_p')) \cdot \left[|\Gamma_n(\vec{b}_n)|^2 - \right. \\
&\left. - \Gamma_n(\vec{b}_n) - \Gamma_n^*(\vec{b}_n) + \sum_x |\Gamma_n^x(\vec{b}_n)|^2 \right]
\end{aligned} \qquad (2.3)$$

Using then unitarity for the neutron-nucleus profile functions in the form

$$\Gamma_n(\vec{b}_n) + \Gamma_n^*(\vec{b}_n) = |\Gamma_n(\vec{b}_n)|^2 + \sum_x |\Gamma_n^x(\vec{b}_n)|^2$$

one gets for the total operator of the (summed-over nuclear states) stripping cross-sections the expression

$$\Gamma_n(\vec{b}_n) + \Gamma_n^*(b_n) + \Gamma_p(\vec{b}_p)\cdot\Gamma_p^*(\vec{b}_p{}') - \Gamma_n(\vec{b}_n)\cdot\Gamma_p(\vec{b}_p) - \Gamma_n^*(\vec{b}_n)\cdot\Gamma_p^*(\vec{b}_p{}') \quad (2.4)$$

which is only a function of the elastic proton-nucleus and neutron-nucleus profile functions.

However, in order to get this diagonalization of the neutron impact parameter the wave function $|d'\rangle$ must be factorized in the proton and neutron coordinates and, moreover, be complete in the neutron space. Such a wave function is clearly the system of plane waves for the proton-neutron system. This wave function is, however, not orthogonal*) to the deuteron-bound state wave function and this property leads to an over-estimation of the stripping integrated cross-section, as discussed in detail in Ref. 5. In fact, it is immediately found that if one uses the plane waves (which form a complete set of states) and compute the sum over the integrated disintegration cross-section plus all the absorption (proton, neutron, and neutron plus proton) cross-sections

$$\begin{aligned}
\sigma_{\rm dis} &\sim \left|1-\left(1-\Gamma_p(\vec{b}_p)\right)\left(1-\Gamma_n(\vec{b}_n)\right)\right|^2 \\
\sigma_n^{\rm abs} &\sim \left|1-\Gamma_p(\vec{b}_p)\right|^2 \sum_{X_1} \left|\Gamma_n^{X_1}(\vec{b}_n)\right|^2 \\
\sigma_p^{\rm abs} &\sim \left|1-\Gamma_n(\vec{b}_n)\right|^2 \sum_{X_2} \left|\Gamma_p^{X_2}(\vec{b}_p)\right|^2 \\
\sigma_{n,p}^{\rm abs} &\sim \sum_{X_1} \left|\Gamma_n^{X_1}(\vec{b}_n)\right|^2 \cdot \sum_{X_2} \left|\Gamma_p^{X_2}(\vec{b}_p)\right|^2
\end{aligned} \qquad (2.5)$$

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*) A wave function which is orthogonal to the deuteron wave function and forms together with it a complete system has been used by Akhiezer and Sitenko[6] in the form

$$|d'\rangle = \left[e^{i\vec{K}\cdot\vec{r}} - \frac{1}{\alpha - iK}\,\frac{e^{-iKr}}{r}\right]\cdot\frac{1}{(2\pi)^{3/2}} \quad ; \qquad |d^0\rangle = \left(\frac{\alpha}{2\pi}\right)^{1/2}\frac{e^{-\alpha r}}{r}$$

where $|d^0\rangle$ is the asymptotic deuteron wave function, $\vec{K}$ is the relative proton neutron momentum and $\vec{r}$ the relative coordinate. This form however, does not serve our purpose since it is not factorized in the proton and neutron coordinates.

one gets, using the unitarity of the proton-nucleus and neutron-nucleus reactions

$$2\,\mathrm{Re}\,\Gamma_p = |\Gamma_p|^2 + \sum_{X_2} |\Gamma_p^{X_2}|^2 \quad ; \quad 2\,\mathrm{Re}\,\Gamma_n = |\Gamma_n|^2 + \sum_{X_1} |\Gamma_n^{X_1}|^2$$

the results

$$\sigma_{dis} + \sigma_n^{abs} + \sigma_p^{abs} + \sigma_{n,p}^{abs} \sim 2 - 2\,\mathrm{Re}\,(1 - \Gamma_p)(1 - \Gamma_n) \ , \qquad (2.6)$$

namely the profile function for the total deuteron-nucleus cross-section[7].

This is of no surprise since the Glauber theory is unitary and we have used a complete set of states for the proton-neutron system.

The expression (2.6) clearly over-estimates the cross-section since the expression on the left-hand side should be given by the total cross-section minus the sum of the cross-sections of all the reactions in which there is a bound deuteron in the final state. In other words, one should subtract from the total cross-section the coherent and incoherent elastic scattering plus the coherent and incoherent elastic production, in the terminology of Ref. 7.

$$\sigma_{dis} + \sigma_n^{abs} + \sigma_p^{abs} + \sigma_{n,p}^{abs} = \sigma_T^{d,A} - \left(\sigma_{c,el} + \sigma_{I,el} + \sigma_{c,el,prod} + \sigma_{I,el,prod}\right) \qquad (2.7)$$

An effective way of overcoming this difficulty, still preserving the advantages of the factorized wave function, is the following. We use a plane-wave function for the proton-neutron system which, however, is not normalized to unity but to a coefficient $C_d < 1$.

The coefficient C_d is fixed by the condition that the sum of the cross-sections of all the reactions in which the deuteron is broken is correctly given by (2.7) which gives

$$C_d = \left(\sigma_T^{d,A} - \sigma_{c,el} - \sigma_{I,el} - \sigma_{c,el,prod} - \sigma_{I,el,prod}\right) / \sigma_T^{d,A}$$

Now, as the sum over all the elastic channels is dominated by the elastic scattering, as shown by Faeldt and Pilkuhn[7], while the elastic production is only a small correction, we can write to a good approximation:

$$C_d = \left(\sigma_T^{d,A} - \sigma_{c,el} - \sigma_{I,el}\right)/\sigma_T^{d,A} = \left(\sigma_T^{d,A} - \sigma_1\right)/\sigma_T^{d,A} ; \qquad \sigma_1 = \sigma_{c,el} + \sigma_{I,el} \tag{2.8}$$

The expression for the stripping differential cross-section in this way becomes

$$\left(\frac{d\sigma}{d\vec{q}_p}\right)_{stripp} = \frac{C_d}{(2\pi)^3}\int d_2 b_p \, d_2 b_p' \, d_2 b_n \, dz \, dz' \, \varphi_d^*(\vec{b}_p - \vec{b}_n, z)\, \varphi_d(\vec{b}_p{}' - \vec{b}_n, z') \cdot e^{-i\vec{t}_p(\vec{b}_p - \vec{b}_p{}')}\, e^{-i l_p (z-z')} \cdot \langle A | \Big(2\,\mathrm{Re}\,\Gamma_n(\vec{b}_n) + \Gamma_p(\vec{b}_p)\cdot\Gamma_p^*(\vec{b}_p{}') - \Gamma_n(\vec{b}_n)\cdot\Gamma_p(\vec{b}_p) - \Gamma_n^*(\vec{b}_n)\cdot\Gamma_p^*(\vec{b}_p{}')\Big) | A \rangle \tag{2.9}$$

where $\vec{t}_p$ and l_p are the transverse and longitudinal momentum of the detected proton in the target nucleus frame. Here we want to stress the difference between formula (2.9) and the corresponding expression used in Refs. 4 and 5. In Ref. 4 the system of plane waves with $C_d = 1$ has been used but the unity in the eikonal expression

$$1 - (1 - \Gamma_p)(1 - \Gamma_n)$$

for the disintegration amplitude has been omitted on the grounds that in fact the correct $|d'\rangle$ should be orthogonal to $|d\rangle$. When, however, one is using a non-orthogonal wave function, like the plane waves, this procedure is quite arbitrary and also dangerous since it can give rise to infinite quantities in the integrated cross-sections. On the other side in Ref. 5 a different empirical recipe has been proposed: namely, always using plane wave with $C_d = 1$, the operator era-

$$2\,\mathrm{Re}\,\Gamma_n(\vec{b}_n) - \Gamma_n(\vec{b}_n)\,\Gamma_p(\vec{b}_p) - \Gamma_n^*(\vec{b}_n)\,\Gamma_p^*(\vec{b}_p{}')$$

has been neglected in (2.9).

Since, as we shall see, these terms give rise to a large cancellation, the result of this approximation turned out to be reasonable on light nuclei;

however, the integrated cross-section is not correct, and the result on heavy nuclei is quite wrong.

The recipe we are proposing with the introduction of C_d has at least the virtue of reproducing the correct theoretical integrated cross-section, keeping the correct operator and lumping the lack of orthogonality in an over-all normalization factor.

We now proceed by assuming that $\Gamma_p = \Gamma_n$ and, moreover, that these profile functions are purely real (a quite good approximation in the scattering on nuclei). The expression (2.9) is then transformed into

$$\frac{d\sigma}{d\vec{q}_p} = C_d \Big[\varphi_d^2(\vec{t}_p, l_p)\, \sigma_{N,A} - 2\varphi_d(\vec{t}_p, l_p) \int d_2q\, \varphi_d(\vec{q}, l_p) \cdot$$

$$\cdot\, \sigma_{N,A}^{N}(\vec{t}_p - \vec{q}) + \int d_2q\, \varphi_d^2(\vec{q}, l_p)\, \sigma_{N,A}^{N}(\vec{t}_p - \vec{q}) \Big] \qquad (2.10)$$

where $\sigma_{N,A}$ is the total nucleon-nucleus cross-section and $\sigma_{N,A}^{N}(\vec{q})$ the summed elastic (coherent + incoherent) nucleon-nucleus differential cross-section.

We will now discuss the problem of the Lorentz boost of the internal motion of the proton inside the flying deuteron.

In order to compute the Lorentz transformation of the internal motion of a bound system, one should use a fully covariant description of the bound state itself, like for instance the one given by the solution of the Bethe-Salpeter equation. One can, however, try a simplifying hypothesis and assume that the internal nucleon motion follows the same transformation laws as those of free nucleons. This hypothesis is likely to be not very wrong, at least when the internal motion itself is non-relativistic.

Our procedure of the calculation would then be the following: Let $\psi(p)$ be the momentum space deuteron wave function in the deuteron rest frame [we normalize $\psi(p)$ as $\int d^3p/E_p |\psi(p)|^2 = 1$ so that $|\psi(p)|^2$ is an invariant quantity]. As the wave function $\psi(p)$ is only a function of p^2, we shall express p^2 in the deuteron frame as a function of invariant quantities, which are then computed in terms of the proton momenta measured in the target nucleus frame (lab frame).

The momentum p of the proton in the deuteron frame can be expressed in terms of t, the square of the four-momentum difference between the deuteron and the neutron, as

$t = (p_d - p_n)^2$; $\quad p^2 = (m^2 - t)/2 - \chi^2$; $\chi^2 = mB$; B deuteron binding energy.

In this relation the neutron is taken on mass shell since the correspondence is between the non-relativistic wave function and the relativistic 3-point function with only one particle off-shell[8]. We now express t as a function of $\vec{t}_p$ and l_p, obtaining

$$p^2 = k_d l_p + E_d \left(k_d^2 + t_p^2 + l_p^2 - 2 k_p l_p + m^2\right)^{\frac{1}{2}} - \left(E_d^2 + k_d^2\right)/2 \; - \chi^2 \tag{2.11}$$

with $E_d = (k_d^2 + m_d^2)^{\frac{1}{2}}$. This expression, as it should, vanishes for $\vec{k}_p = \vec{k}_d/2$, up to terms of the order χ^4.

To compute the cross-section (2.10) we insert the following expression for the deuteron wave function $\phi_d(\vec{t}_p, l_p)$:

$$\psi\left(p^2(\vec{t}_p, l_p, \vec{k}_d)\right) = \varphi_d\left(\vec{t}_p, l_p\right)$$

with $p^2(\vec{t}_p, \ell_p, \vec{k}_d)$ given by (2.11) and where, for $\psi(p^2)$, we take the non-relativistic (S + D) expression of the deuteron wave function.

With this treatment we have neglected all the spin properties of the nucleons, treating the wave function as a scalar quantity; it is however very likely that the spin properties will not play a substantial role in the problem and that this approximation will not result in serious troubles.

3. PION PRODUCTION

In full analogy with the theory described in Section 2, we treat here the reaction

$$d + A \rightarrow \pi + X \tag{3.1}$$

with the Glauber theory, again taking into account the relativistic dilation of the deuteron internal motion.

We can classify the possible eikonal operators for reaction (3.1) as follows:

$$
\begin{aligned}
&\text{a)} \quad \left(1-\Gamma_p(\vec{b}_p)\right)\Gamma_n^{x_n,\pi}(\vec{b}_n,\vec{b}_{x_n},\vec{b}_\pi)\\
&\text{b)} \quad \Gamma_p^{x}(\vec{b}_p,\vec{b}_x)\;\Gamma_n^{x_n,\pi}(\vec{b}_n,\vec{b}_{x_n},\vec{b}_\pi)\\
&\text{c)} \quad \left(1-\Gamma_n(\vec{b}_n)\right)\Gamma_p^{x_p,\pi}(\vec{b}_p,\vec{b}_{x_p},\vec{b}_\pi)\\
&\text{d)} \quad \Gamma_n^{x}(\vec{b}_n,\vec{b}_x)\;\Gamma_p^{x_p,\pi}(\vec{b}_p,\vec{b}_{x_p},\vec{b}_\pi)
\end{aligned}
\tag{3.2}
$$

Here $\vec{b}_i$ refers to the impact parameters of the particles or groups of particles. They correspond to the following physical processes:

a) The proton scatters only elastically, with a total proton-nucleus elastic profile function $\Gamma_p(\vec{b}_p)$. The neutron produces the most general possible state, in which one distinguishes the π with momentum $\vec{K}_\pi$ from the other produced particles, denoted by x_n, and the corresponding total neutron-nucleus production function is $\Gamma_n^{x_n,\pi}(\vec{b}_n,\vec{b}_{x_n},\vec{b}_\pi)$.

b) The proton produces the most general state x [with profile function $\Gamma_p^{x}(\vec{b}_p,\vec{b}_x)$] while the neutron [as in (a)] produces the pion with momentum $\vec{K}_\pi$ and the state x_n.

c) and d) can be obtained from a) and b) by exchanging the rôles of the proton and the neutron.

Since all the final states in a-d are orthogonal, the cross-section will be obtained by summing the squares of the corresponding amplitudes; one then sums over the nuclear states using closure, over the proton-neutron states using a now complete set of states which also includes the deuteron bound state (no proton is detected so the deuteron can also remain bound in the experiment). Therefore the pion production is not affected by the problem of the lack of orthogonality of the wave functions, discussed when dealing with the break-up.

Integrating over all the unobserved states x, and summing over x, one gets

$$\frac{d\sigma}{d\vec{K}_\pi\, d\vec{q}_{x_1}} \sim \left(\left|1-\Gamma_p(\vec{b}_p)\right|^2+\sum_x\left|\Gamma_p^x(\vec{b}_p,\vec{b}_x)\right|^2\right)\cdot\left(\Gamma_n^{x_1,\pi}(\vec{b}_n,\vec{b}_{x_1},\vec{b}_\pi)\right)\left(\Gamma_n^{x_1,\pi}(\vec{b}_n,\vec{b}'_{x_1},\vec{b}'_\pi)\right)^*$$

$$+\left(\left|1-\Gamma_n(\vec{b}_n)\right|^2+\sum_x\left|\Gamma_n^x(\vec{b}_n,\vec{b}_x)\right|^2\right)\cdot\left(\Gamma_p^{x_1,\pi}(\vec{b}_p,\vec{b}_{x_1},\vec{b}_\pi)\right)\left(\Gamma_p^{x_1,\pi}(\vec{b}_p,\vec{b}'_{x_1},\vec{b}'_\pi)\right)^* =$$

$$= \left(\Gamma_n^{x_1,\pi}(\vec{b}_n,\vec{b}_{x_1},\vec{b}_\pi)\right)\left(\Gamma_n^{x_1,\pi}(\vec{b}_n,\vec{b}'_{x_1},\vec{b}'_\pi)\right)^*+\left(\Gamma_p^{x_1,\pi}(\vec{b}_p,\vec{b}_{x_1},\vec{b}_\pi)\right)\left(\Gamma_p^{x_1,\pi}(\vec{b}_p,\vec{b}'_{x_1},\vec{b}'_\pi)\right)^* \qquad (3.3)$$

where in the last line unitarity of the proton-nucleus and neutron-nucleus scattering has been used. Summing now over all the states x_1 one defines the inclusive cross-section in d + A collisions in terms of the inclusive cross-sections for pion production in nucleon-nucleus collisions as

$$\left(E_\pi\frac{d\sigma}{d\vec{K}_\pi}\right)_{dA} = \int\frac{d_3K}{E_K}\,\phi_d^2(\vec{K},\vec{k}_d)\left[\sigma_{pA\to\pi X}(\vec{K}_\pi,\vec{K})+\sigma_{nA\to\pi X}(\vec{K}_\pi,\vec{K})\right] \qquad (3.4)$$

where all the inclusive cross-sections are invariant ones. In this formula $\sigma_{pA\to px}(\vec{K}_\pi,\vec{K})$ is the invariant inclusive cross-section for the production of pions of momentum $\vec{K}_\pi$ from protons of momentum $\vec{K}$ incident on the nucleus A. The wave function $\phi_d(\vec{K},\vec{k}_d)$ has the same meaning as in Section 2.

It is important to notice that as a result of the unitarity condition there is no interference between the production from protons and from neutrons, nor any double scattering term, although all the multiple scattering terms have been included in the starting formula (3.2).

From formula (3.4) it appears clearly how one can produce pions with momenta larger than the maximum kinematical limit allowed in pA collisions when the proton has ½ of the deuteron momentum; in fact, fast pions are produced by nucleons whose momenta are larger than $k_d/2$ owing to the Lorentz dilation of the internal motion.

4. APPLICATION TO THE EXPERIMENTAL RESULTS

We now apply the theoretical formalism, developed in Sections 2 and 3, to the Bevalac experiments described in the Introduction.

We start from proton stripping.

To actually compute the cross-section given by (2.10) we must know:

- the deuteron wave function;
- the nucleon-nucleus total and summed elastic differential cross-sections;
- the sum of the coherent and incoherent elastic integrated cross-sections for deuteron-nucleus collisions, together with the total deuteron-nucleus cross-section, to compute the factor C_d.

For the deuteron wave function we have used a multigaussian fit[9] for the deuteron momentum space wave function, including both the S and D components. We have tried different wave functions like the Gartenhaus or Reid; the results of the calculations turned out to be essentially the same up to values of k_p such that $(k_p - (k_p)_{peak}) \sim 1$ GeV/c at T = 1.05 and $(k_p - (k_p)_{peak}) \sim$ 1.5 GeV/c at T = 2.1 GeV/nucl. For higher k_p the Reid wave function gives somewhat higher (from 20% to a factor 3) cross-sections.

For the total and summed nucleon-nucleus cross-sections we have proceeded in the following way. We have used the Glauber theory to compute the total cross-section, using the Woods-Saxon density for the nucleus and a total nucleon-nucleon cross-section of 44 mb. This value represents a reasonable average of the pp and pn total cross-section both at 1.05 and 2.1 GeV.

The summed nucleon-nucleus differential cross-section heen approximated by the Gaussian form

$$\sigma^{N}_{N,A}(q^2) = \sigma^{summed}_{integr} \cdot \frac{B_A}{\pi} \exp(-B_A q^2)$$

where the integrated summed nucleon-nucleus cross-section σ^{summed}_{integr} has been computed from the Glauber theory using the value of 24 mb for the inelastic nucleon-nucleon cross-section, while the slope B_A, for ^{9}Be and ^{12}C has been taken from the fit to the experiment of Bellettini et al.[10]. While this experiment is at 19.3 GeV/c it is very plausible that the slope B_A is energy independent, depending essentially upon nuclear parameters. We have preferred to use this parametrization instead of computing directly the q^2 dependence of $\sigma^{summed}_{NA}(q^2)$ from the Glauber theory since the differential quantities can be rather sensitive to nuclear details for light nuclei, in contrast with the integrated ones. In the case of lead, as B_A has not been determined experimentally, we have tried different values for B_A (B_A = 200, 250, 300 GeV/c^{-2}), checking that the result is indeed independent of B_A for heavy nuclei. The reason is that for B_A so large $\sigma^{summed}_{NA}(q^2)$ is peaked forward so strongly, that the deuteron wave function can be considered as almost constant with respect to the integration variable in the second and third terms of (2.10) and we are left with the integrated summed cross-section which, with our normalization, does not depend upon B_A.

To obtain the constant C_d we have computed the total deuteron-nucleus cross-section σ_T^{dA} with the Glauber theory[7], while σ_1, the sum of the coherent and incoherent elastic deuteron-nucleus cross-sections, has been taken for the different nuclear targets from the paper of Tékou[11]. The values of C_d obtained in this way are

$$C_d(^{9}Be) = 0.53 \; ; \quad C_d(^{12}C) = 0.54 \; ; \quad C_d(^{208}Pb) = 0.32 \; .$$

The results of our theory are shown in Figs. 1-6, as compared with the experimental results, for the three nuclear targets (^{9}Be, ^{12}C, ^{208}Pb) and the two incident energies (1.05 and 2.1 GeV/nucl) for which data are available. In compiling these curves the Reid wave function has been used.

On these results we can make the following remarks: Our theory reproduces in general, quite reasonably the experimental results (remember that the theory is parameter-free); the momentum shape of the cross-section is well reproduced, except sometimes for a few points at very large k_p (as for instance for k_p = 3.8 GeV/c in Fig. 4). This effect might depend upon details of the deuteron wave function. Moreover, there is a certain tendency of the experimental shape for heavy nuclei to be a little narrower than for light nuclei (compare,for example, Figs. 3 and 5), while in our theory the shape is essentially A-independent. We do not have a clear explanation of this fact. A possible origin could, however, be found in the decoupling which occurs in the Glauber theory between the transverse and the longitudinal coordinates, on the ground that all the transverse momenta are small. Since the present experiment is at fixed non-zero angle, large k_p imply large transverse momenta, where the decoupling is probably no more correct. It is then likely that the effect of this approximation would depend upon A. If this is the case, the effect should not be there in an experiment at 0°.

Another interesting remark is that the shape would not be reproduced at all, except for very small $(k_p - k_p)_{peak}$), had we neglected the D component of the deuteron wave function.

The energy-dependence of the shape is also well reproduced, and this result proves, in our opinion, that the effect of the Lorentz dilation of the wave function is there. The absolute normalization of the cross-section at the peak and its A-dependence are also in general well-reproduced (with the only exception of the Be target at the lower energy; we do not have an explanation of this anomaly). The prediction of the correct normalization and ratio for the peak cross-section going from light to heavy nuclei would not have been obtained omitting the factor C_d which, as we have said, varies from 0.53 to 0.32. It is reassuring that when enforcing the correct integrated cross-section for all the break-up reactions (including also the production from the proton) one gets both the correct normalization and A-dependence for the stripping differential cross-section.

We turn now to pion production.

The ingredients to be inserted in the theoretical formula are:

- the Lorentz-dilated deuteron wave function, as for the stripping;
- the inclusive cross-sections for pion production in proton-nucleus and neutron-nucleus collisions. These are needed in an energy interval of the order of the Lorentz dilation of the average proton momentum in the deuteron wave function around the value $k_d/2$. Using charge symmetry, $nA \to \pi^{\pm}$ is related to $pA \to \pi^{\mp}$. In order to perform the integral, it is useful to use an analytic fit for the inclusive cross-section in nucleon-nucleus collisions, as a function of the incident nucleon energy and the pion momentum, for the different nuclei. We have used fits of the following forms[12,5]:

a scaling fit of the form

$$\frac{E_\pi}{K_\pi^2}\frac{d^2\sigma}{d\Omega\, dK_\pi} = A\,(1-x')^{\beta} \quad ; \quad x' = (K_\pi/K_{\pi,max})_{c.m.}$$

for $p + A \to \pi^- + X$;

a non-scaling fit of the form

$$\frac{E_\pi}{K_\pi^2}\frac{d^2\sigma}{d\Omega\, dK_\pi} = \frac{c(T)}{x^{3/2}}\exp\left[-a(T)\,\left|x - x(T)\right|^{A(T)}\right]$$

where $x = K_\pi/T$ (T = incident kinetic energy), and A(T), A(T), c(T) and x(T) are polynomials of the form

$$\sum_{n=0}^{3} b_n T^{-n}$$

for $n + A \to \pi^- + X = p + A \to \pi^+ + X$;

the parameters A, β, a(T), A(T), C(T), and x(T) have been obtained for every nuclear target fitting the experimental distributions in $p + A \to \pi + X$ measured by the Berkeley group itself[3]. For the transverse momentum distribution we have chosen a universal form $\exp(-6|\vec{p}_\perp|)$.

The integral in (3.4) has been computed limiting the integration range inside a region whose boundary is given by the condition of the over-all energy conservation in $d + A \rightarrow \pi + X$.

The results of the calculations are shown in Figs. 7-10 for different nuclear targets, plotting on the same graph the data and the theoretical curves for the two different incident energies (1.05 and 2.1 GeV/nucl).

Again, the over-all agreement between theoretical curves and data is reasonably good (also here our theory is parameter-free). However, there is sometimes a tendency for the theoretical curves to lie below the experimental points.

As a general conclusion, we can assert that the theory we have presented describes quite well the experimental results in the range of energies in which data presently exist, both for the stripping and for pion production. It would be desirable to check the theory against new data, possibly varying the observation angle and the incident energy.

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Figure captions

Fig. 1 : The invariant cross-section, in mb GeV/str/(GeV/c)^3, for the reaction d + A $\rightarrow$ p + X at fixed proton angle (2.5^o), as a function of the proton momentum in GeV/c. The incident deuteron kinetic energy is T = 1.05 GeV/nucl, the target is ^{9}Be. The triangles are the experimental results from Ref. 3, the curve is the theoretical prediction.

Fig. 2 : The same as for Fig. 1, for T = 2.1 GeV/nucl and for ^{9}Be.

Fig. 3 : The same as for Fig. 1, for T = 1.05 GeV/nucl and for ^{12}C.

Fig. 4 : The same as for Fig. 1, for T = 2.1 GeV/nucl and for ^{12}C.

Fig. 5 : The same as for Fig. 1, for T = 1.05 GeV/nucl and for ^{208}Pb.

Fig. 6 : The same as for Fig. 1, for T = 2.1 GeV/nucl and for ^{208}Pb.

Fig. 7 : The cross-sections in mb/sr/(GeV/c) at fixed pion angle (2.5^o) for two different incident deuteron energies (left curve and data for T = 1.05 GeV/nucl, right for T = 2.1 GeV/nucl), as a function of the pion momentum in GeV/c. The points indicate that the experimental results for the reactions observing π^+ (● and Δ) and π^- (□ and ○). The curves are the theoretical predictions (the same for π^+ and π^-). The arrows indicate the kinematical limits in pA collisions at 1.05 GeV/nucl (↙) and 2.1 GeV/nucl (⫽).

Fig. 8 : The same as Fig. 7 for ^{12}C;

Fig. 9 : The same as Fig. 7 for ^{64}Cu.

Fig. 10 : The same as Fig. 7 for ^{208}Pb.

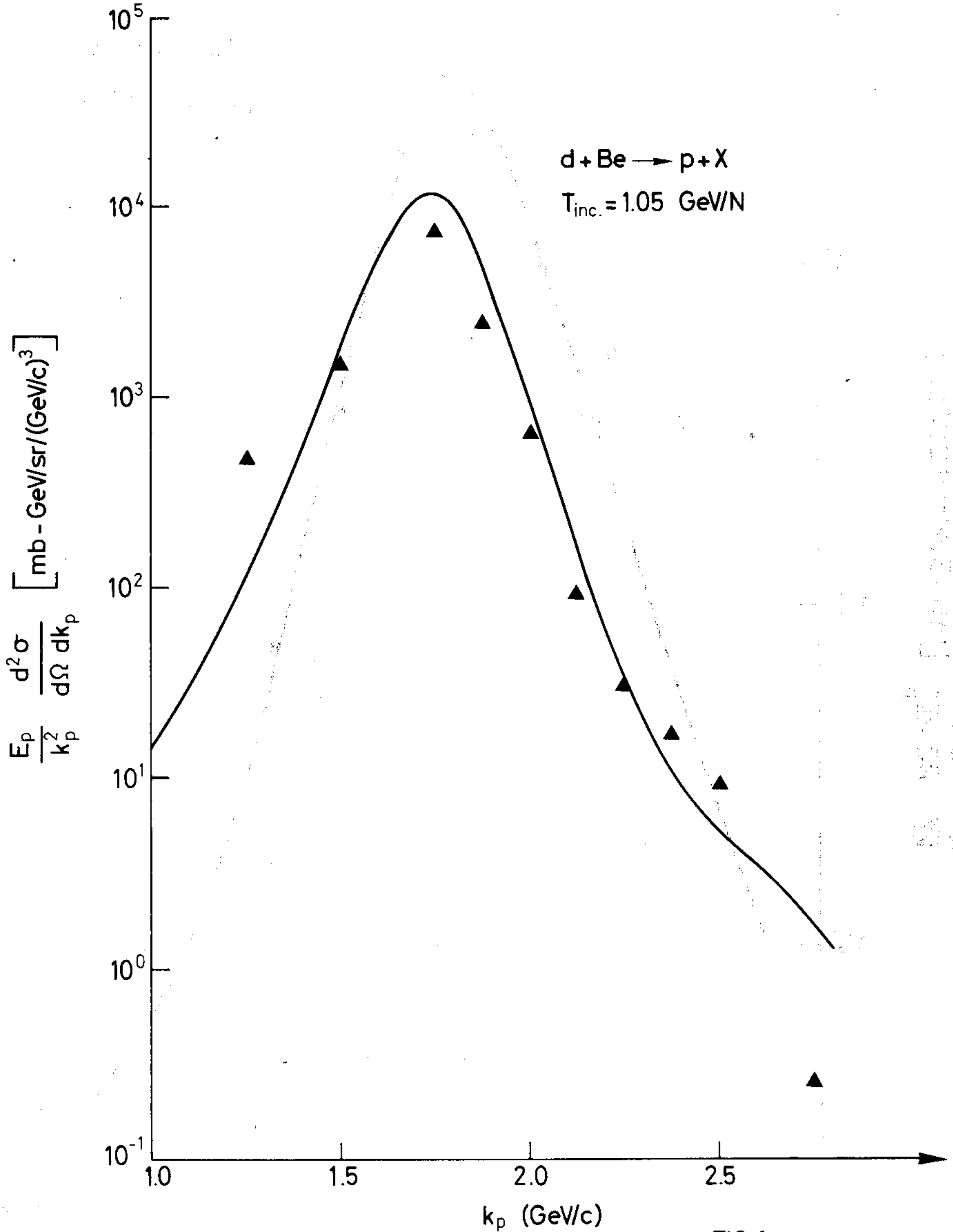
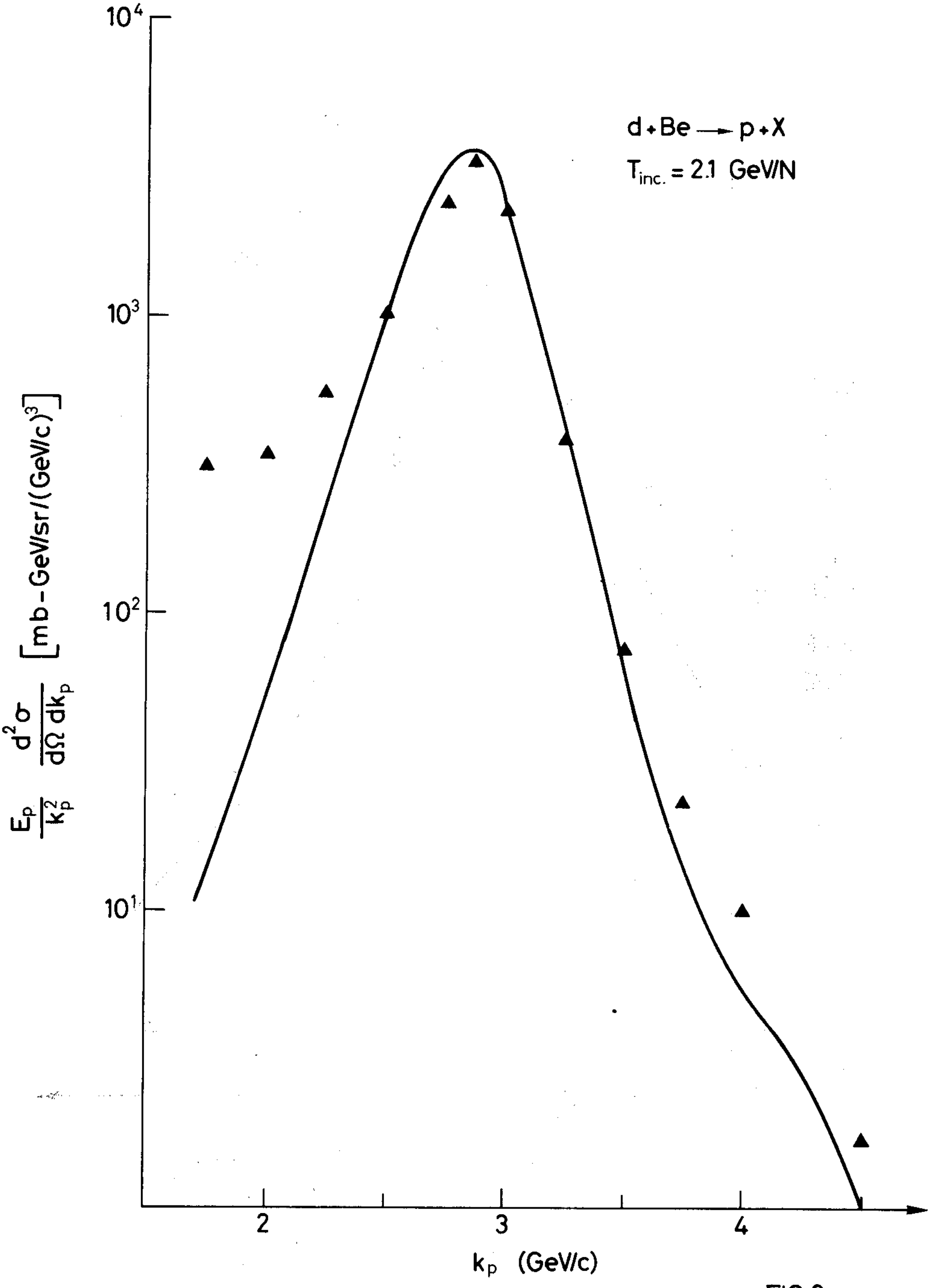
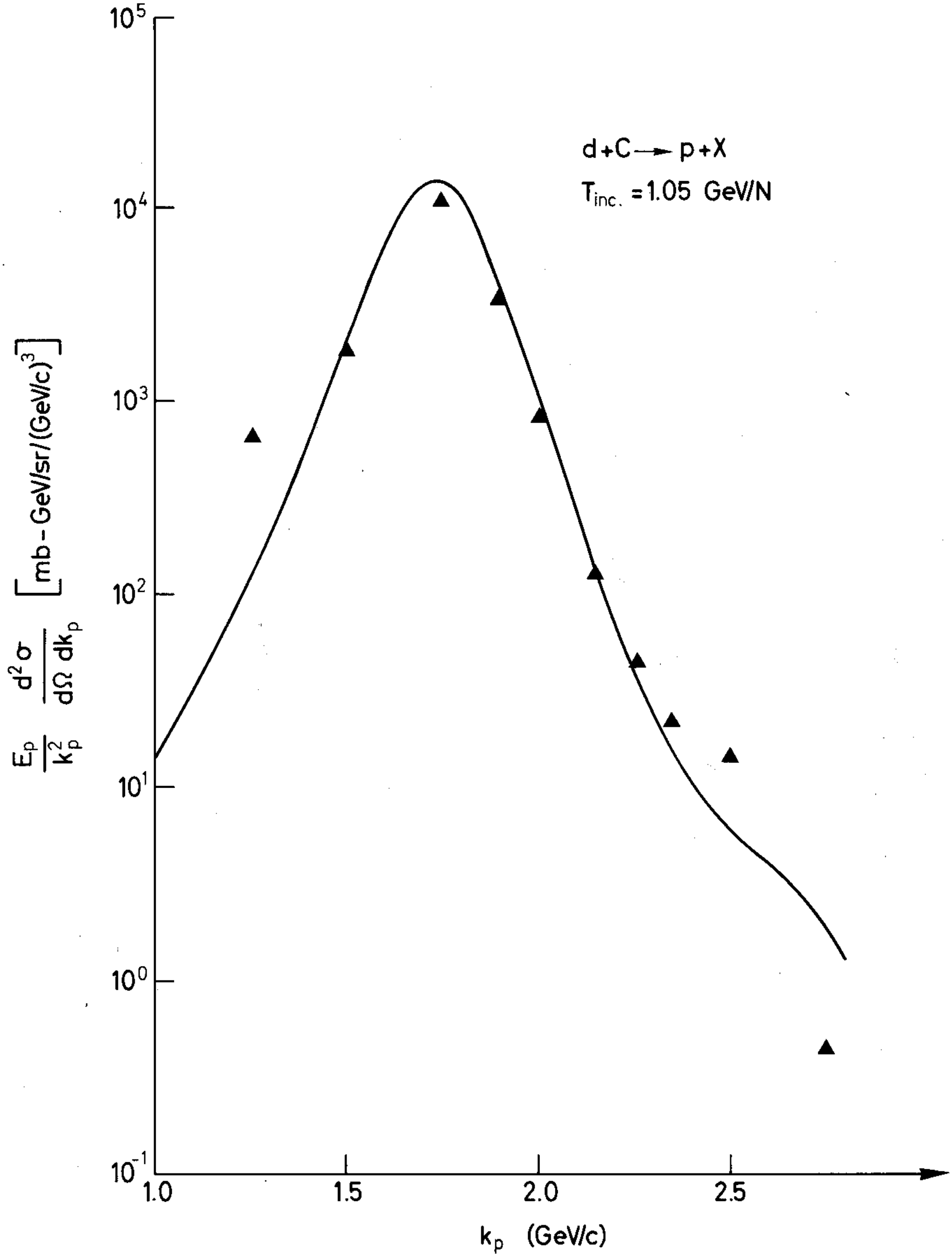

d + Be ⟶ p + X
$T_{inc.} = 1.05$ GeV/N
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$\frac{E_p}{k_p^2} \frac{d^2\sigma}{d\Omega\, dk_p}$ [mb - GeV/sr/(GeV/c)^3]
1.0
1.5
2.0
2.5
k_p (GeV/c)

FIG.1



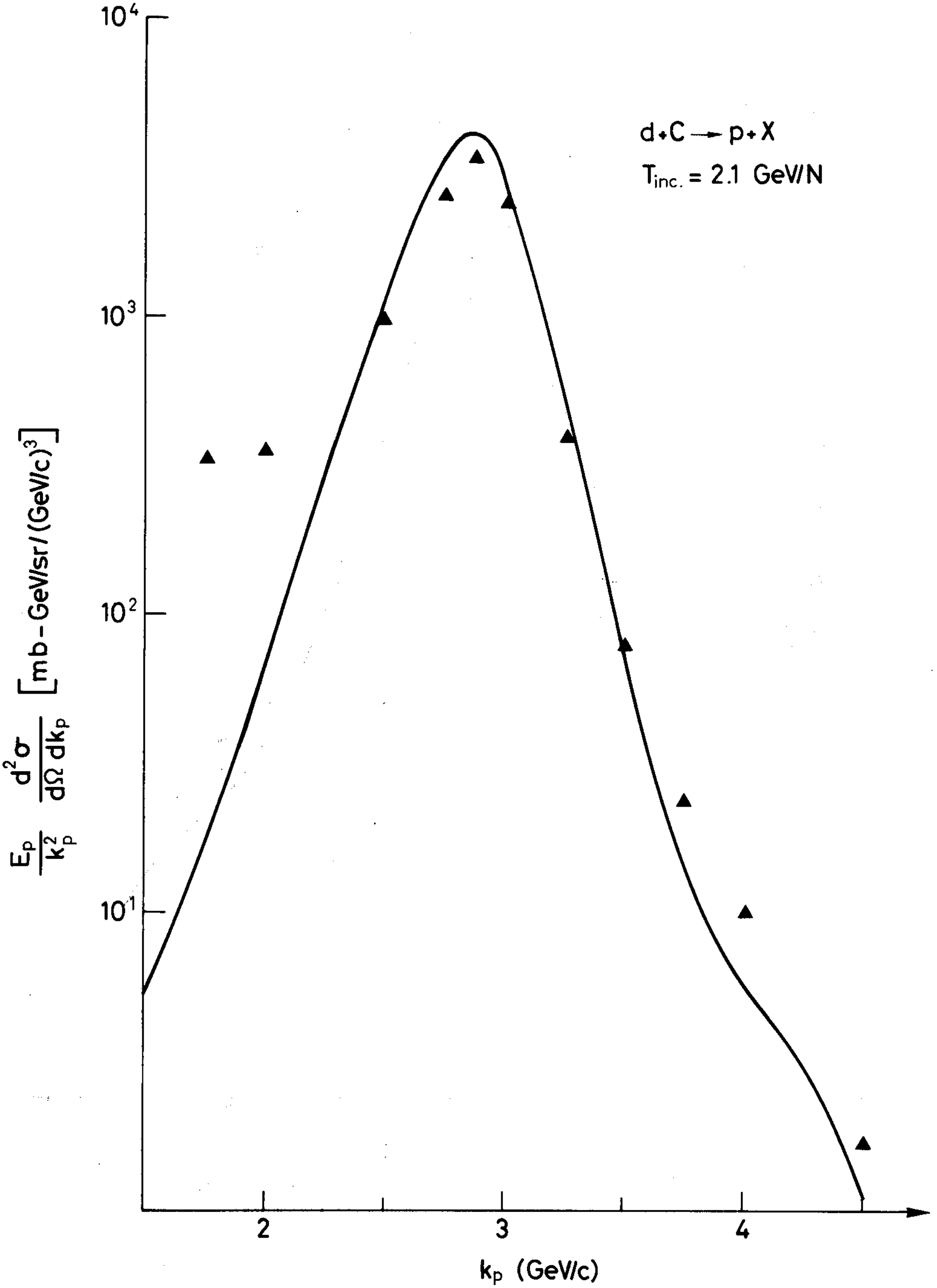
d + Be → p + X
$T_{inc.}$ = 2.1 GeV/N
10^4
10^3
10^2
10^1
$\frac{E_p}{k_p^2} \frac{d^2\sigma}{d\Omega\, dk_p}$ [mb-GeV/sr/(GeV/c)^3]
2
3
4
k_p (GeV/c)

FIG.2



d+C → p+X
$T_{inc.}$ = 1.05 GeV/N
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$\frac{E_p}{k_p^2} \frac{d^2\sigma}{d\Omega\, dk_p}$ [mb - GeV/sr/(GeV/c)^3]
1.0
1.5
2.0
2.5
k_p (GeV/c)

FIG. 3



10^4
10^3
10^2
10^1
d+C → p+X
$T_{inc.}$ = 2.1 GeV/N
$\frac{E_p}{k_p^2} \frac{d^2\sigma}{d\Omega\, dk_p}$ [mb-GeV/sr/(GeV/c)^3]
2
3
4
k_p (GeV/c)

FIG. 4

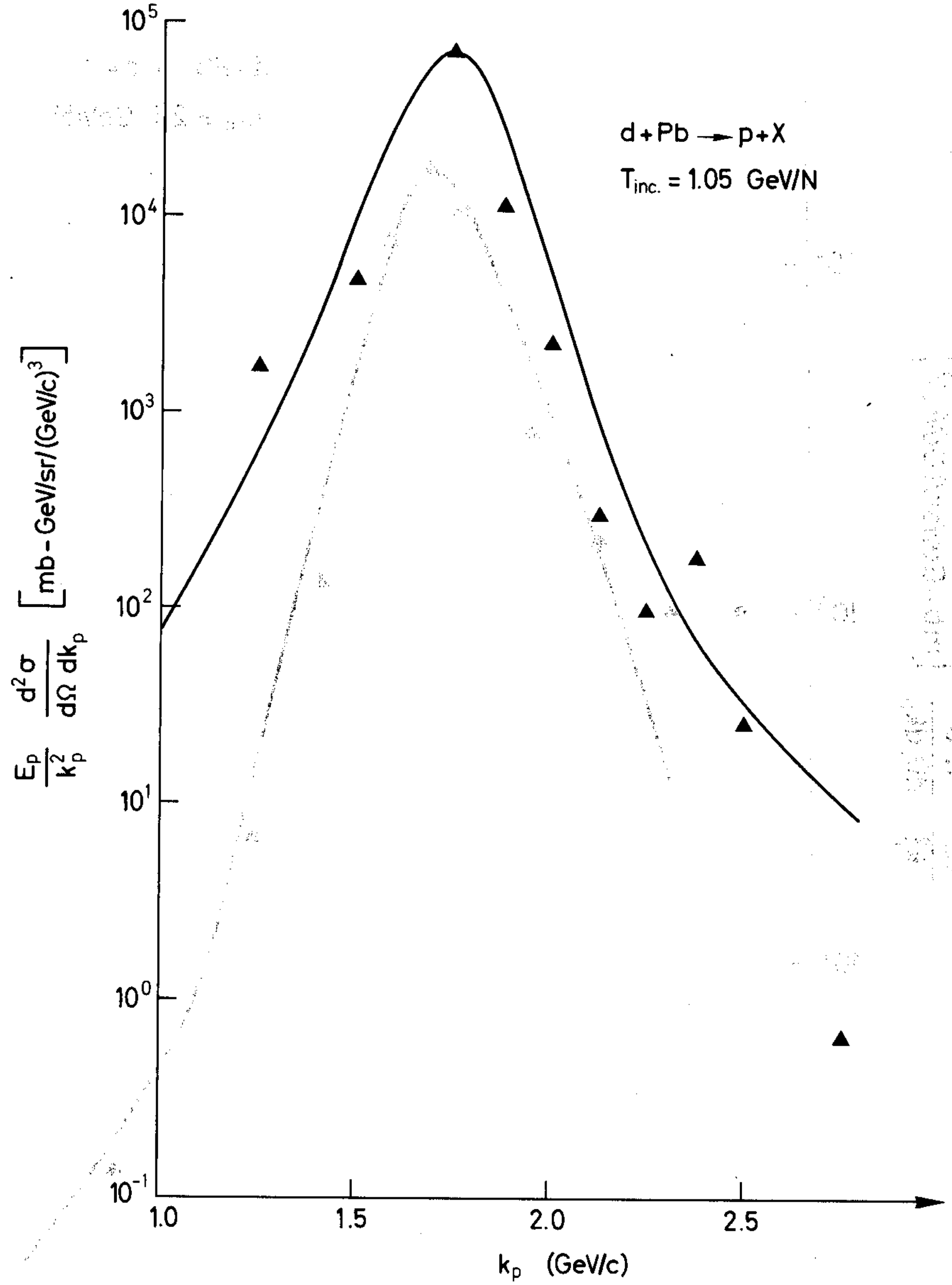

d + Pb → p + X
T_inc. = 1.05 GeV/N
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
1.0
1.5
2.0
2.5
k_p (GeV/c)
E_p/k_p^2 d^2σ/dΩ dk_p [mb - GeV/sr/(GeV/c)^3]

FIG.5

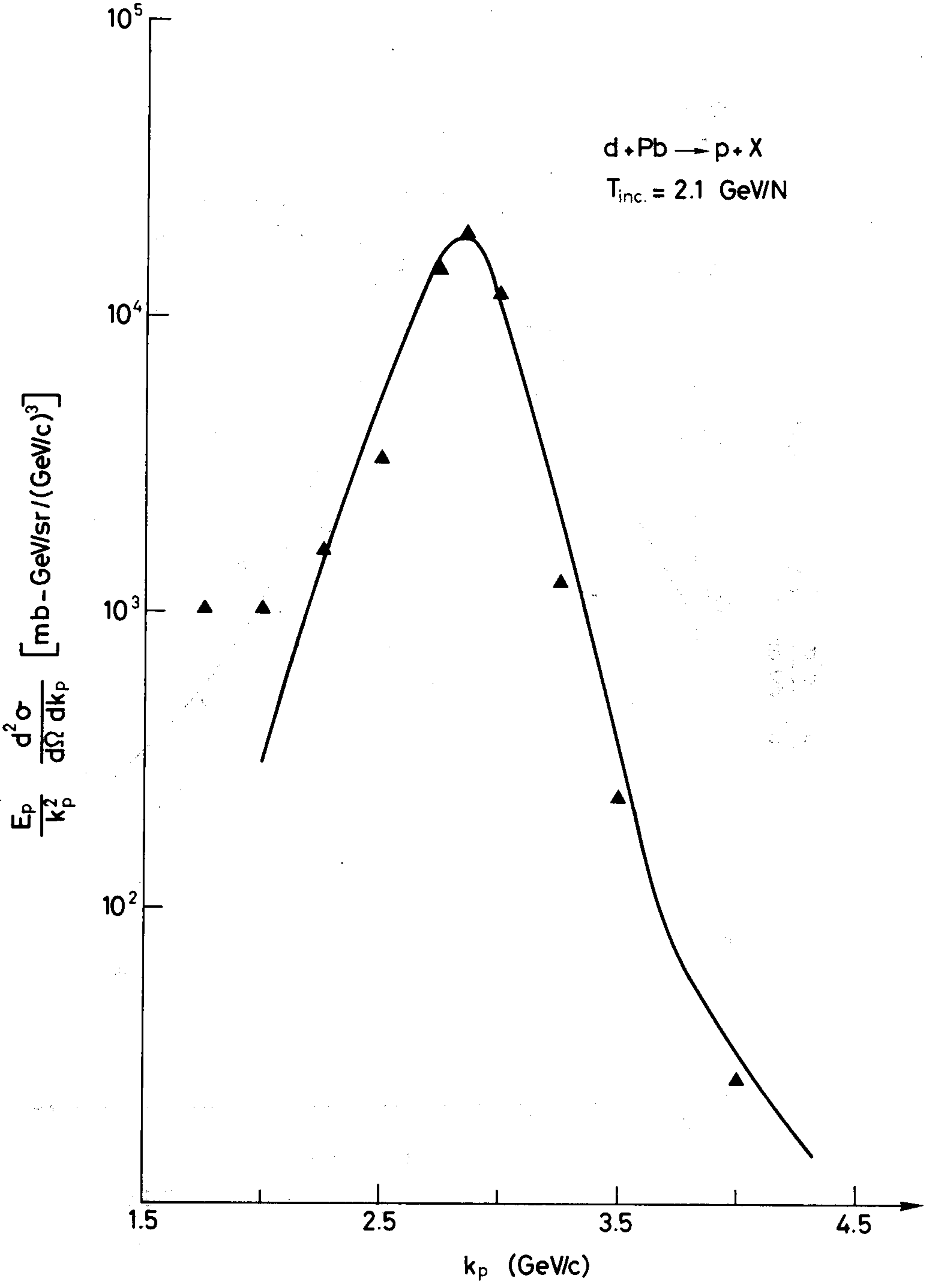
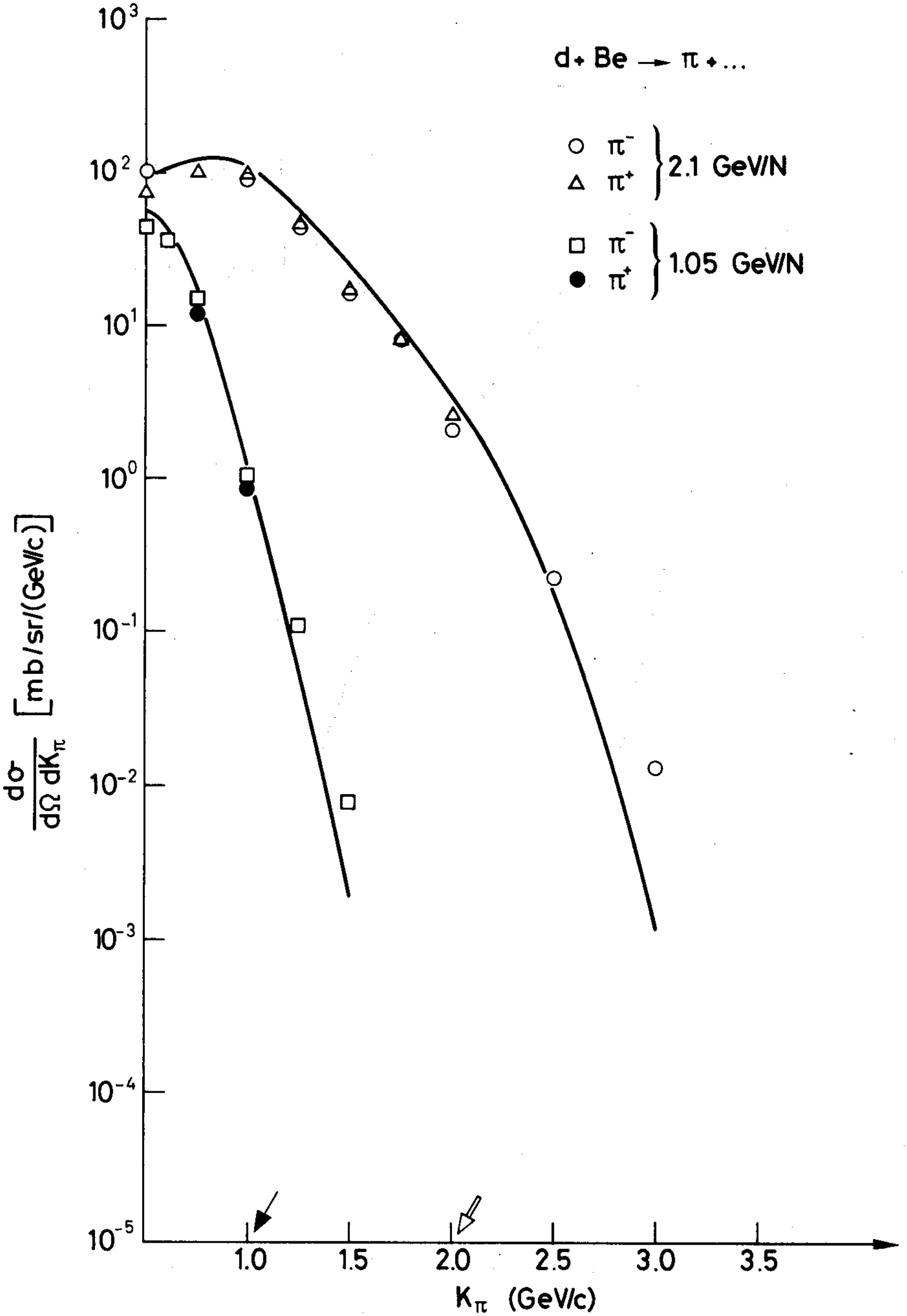
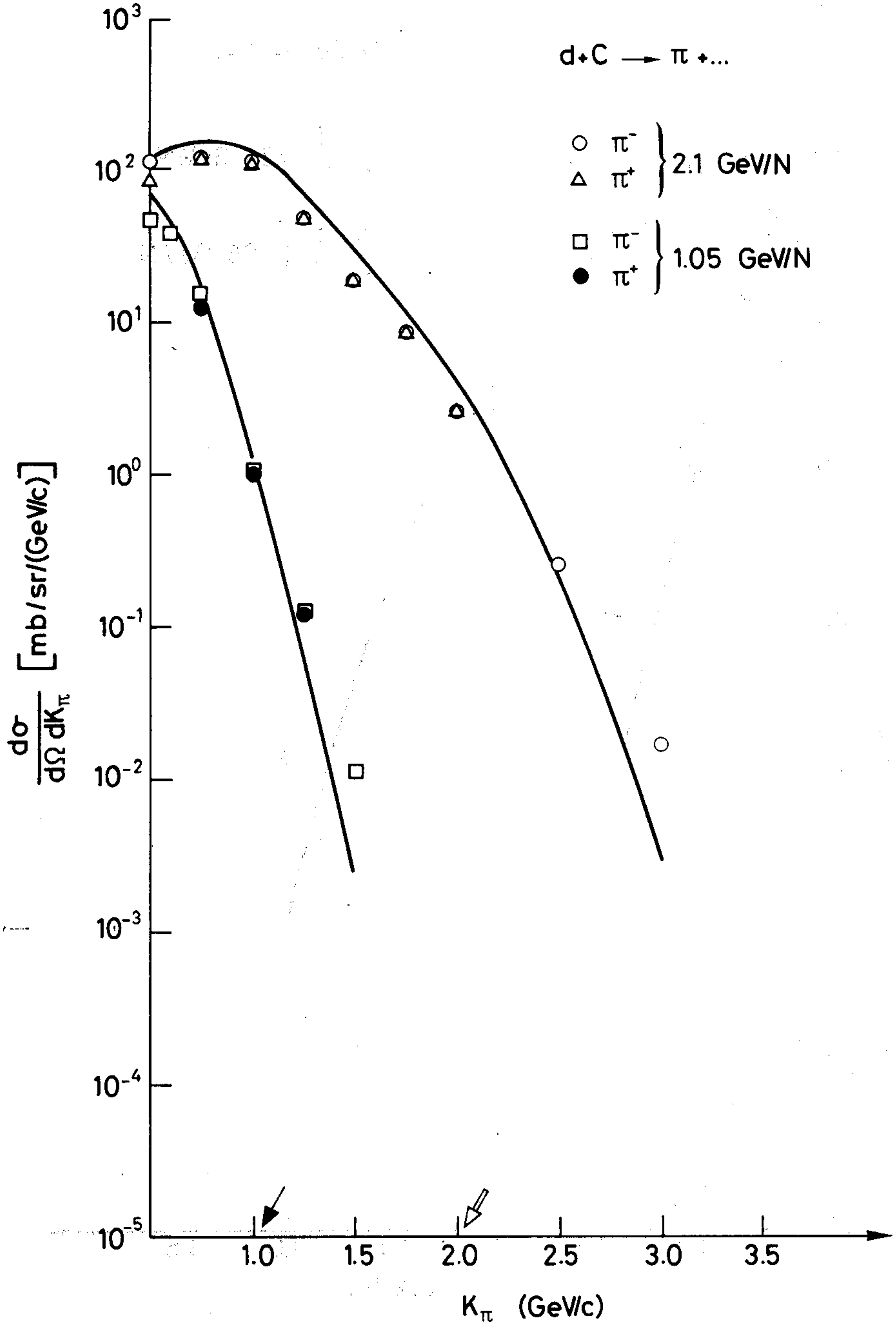

d + Pb → p + X
T_inc. = 2.1 GeV/N
10^5
10^4
10^3
10^2
E_p/k_p^2 d^2σ/dΩ dk_p [mb-GeV/sr/(GeV/c)^3]
1.5
2.5
3.5
4.5
k_p (GeV/c)

FIG. 6



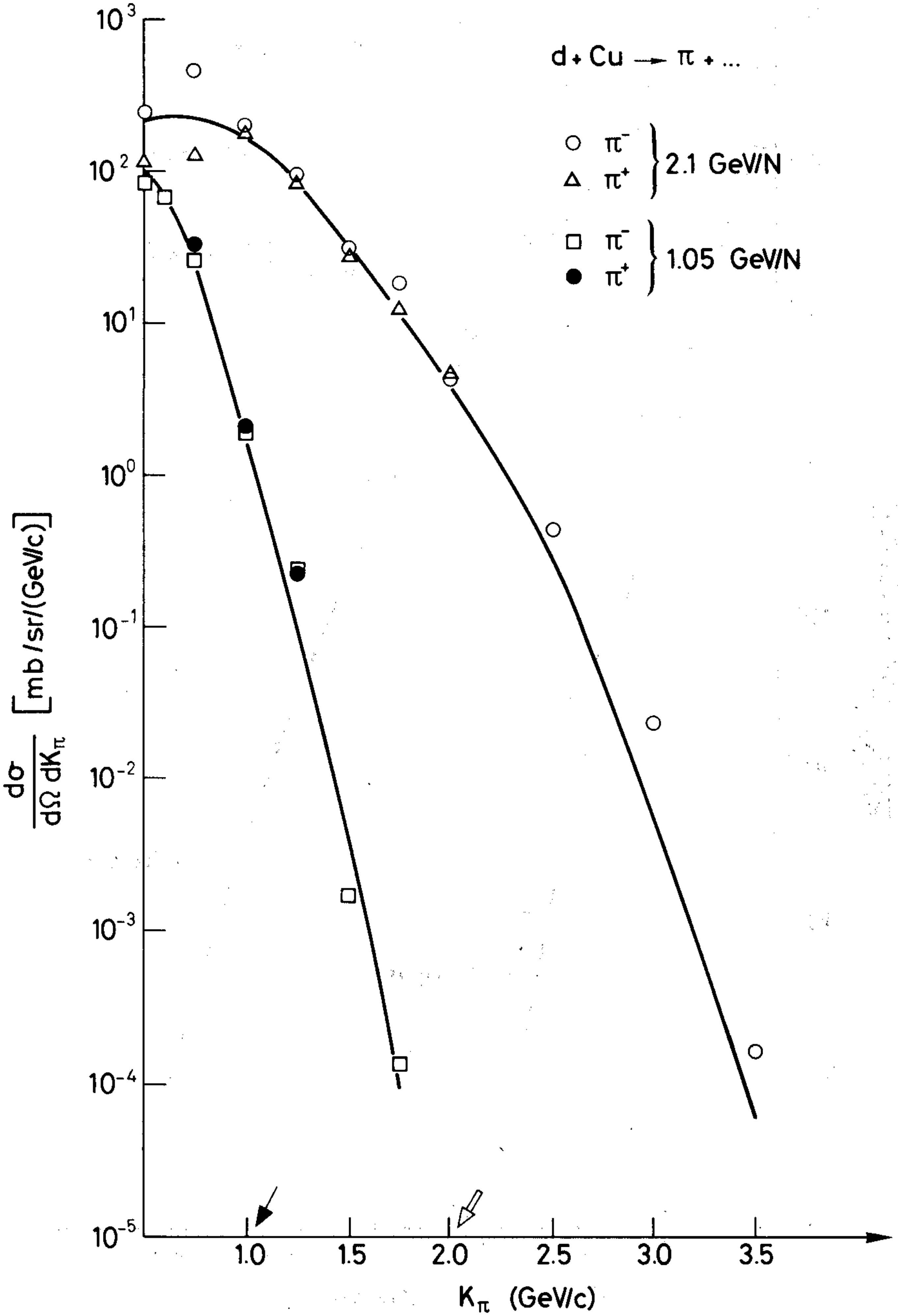
d + Be → π + ...
○ π⁻
△ π⁺
2.1 GeV/N
□ π⁻
● π⁺
1.05 GeV/N
dσ/dΩ dKπ [mb/sr/(GeV/c)]
10³
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
1.0
1.5
2.0
2.5
3.0
3.5
Kπ (GeV/c)

FIG.7



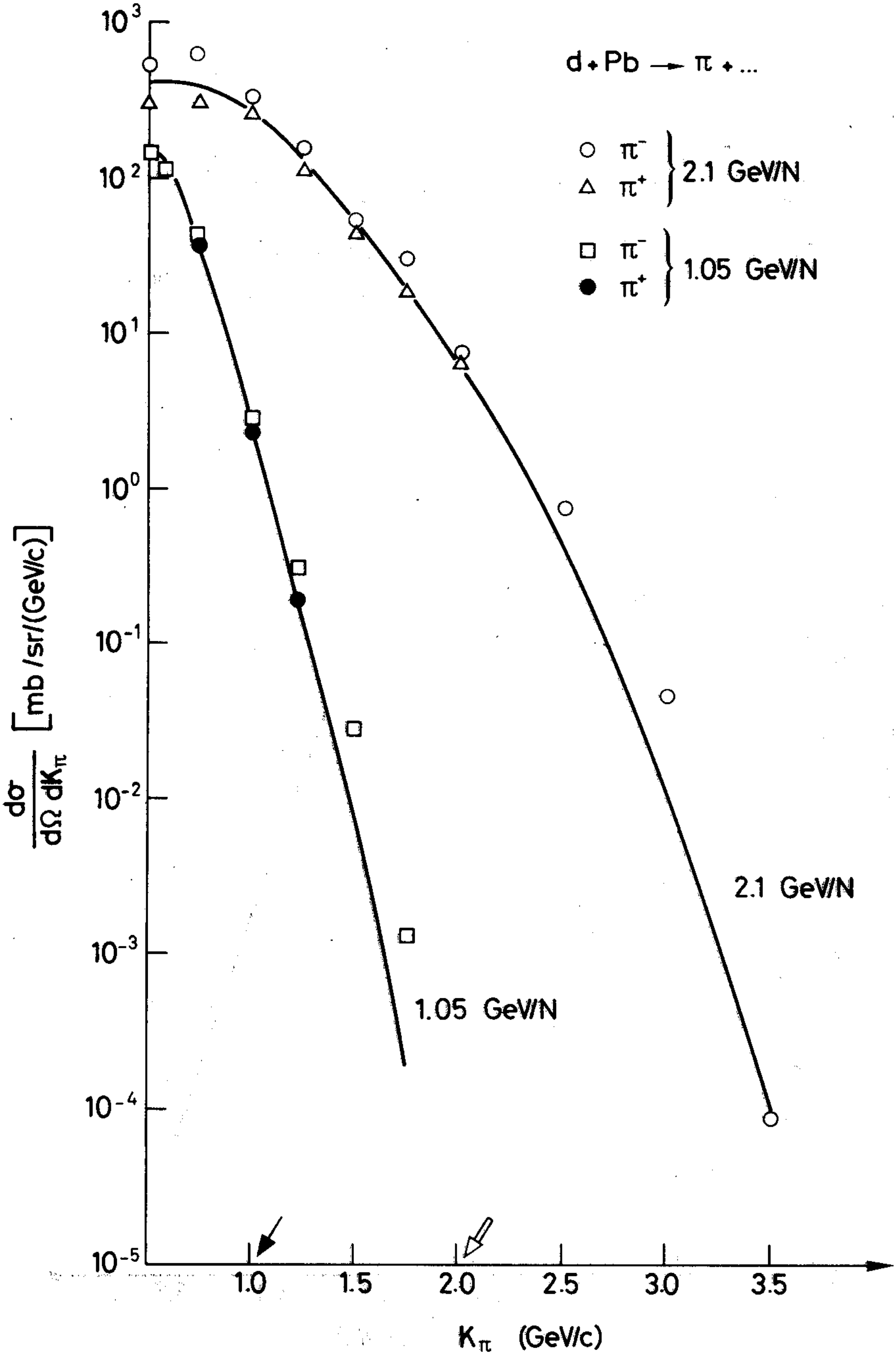
d+C → π +...
○ π⁻
△ π⁺
2.1 GeV/N
□ π⁻
● π⁺
1.05 GeV/N
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\frac{d\sigma}{d\Omega\, dK_\pi}$ [mb/sr/(GeV/c)]
1.0
1.5
2.0
2.5
3.0
3.5
K_π (GeV/c)

FIG. 8



d + Cu → π + ...
○ π⁻
△ π⁺
2.1 GeV/N
□ π⁻
● π⁺
1.05 GeV/N
$\frac{d\sigma}{d\Omega\, dK_\pi}$ [mb/sr/(GeV/c)]
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1.0
1.5
2.0
2.5
3.0
3.5
K_π (GeV/c)

FIG. 9



d + Pb → π + ...
π⁻
π⁺
2.1 GeV/N
π⁻
π⁺
1.05 GeV/N
2.1 GeV/N
1.05 GeV/N
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\frac{d\sigma}{d\Omega\, dK_\pi}$ [mb/sr/(GeV/c)]
1.0
1.5
2.0
2.5
3.0
3.5
K_π (GeV/c)

FIG. 10