# COMMENTS

## Elastic Scattering Crossovers from 50 to 175 GeV*

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A comparison of $K^{\pm}p$ and $p^{\pm}p$ elastic scattering is made for incident energy 50 to 175 GeV. Average values of $0.19 \pm 0.04$ and $0.11 \pm 0.02$ GeV$^2$ were found for the invariant–momentum-transfer values of the $Kp$ and $pp$ crossover points, respectively.

High-energy elastic scattering is dominated by diffraction, or in $t$-channel language, by Pomeron exchange. Amplitudes with quantum-number exchange are much smaller, but can be isolated by a careful comparison of closely related reactions. In this Comment we compare particle and antiparticle elastic scattering by protons to obtain information on the odd-charge-conjugation $(C = -1)$ exchange amplitude in the $t$ channel. For $Kp$ and $pp$ scattering, this amplitude is believed to be dominated by nonflip $\omega$ exchange.[1] At momenta $\lesssim 10$ GeV/$c$, this amplitude changes sign near $-t = 0.2$ GeV$^2$, resulting in the crossover point where the differential cross sections for particle and antiparticle scattering are equal.[2,3] The invariant momentum transfer $t_c$ at which this occurs can be related to a typical interaction radius in impact-parameter space for the $C = -1$ amplitude.[4]

The experiment has already been described.[5] It was carried out in the Fermilab M6E high-precision beam line and used the Single Arm Spectrometer Facility to detect scattered particles. The results of the final analysis[6] were used and differ only slightly from those previously published.[5] The uncertainy in the relative normalization of the particle and antiparticle cross sections is estimated to be $\pm 3\%$.

The cross sections were fitted with the form

$$d\sigma/dt = Ae^{Bt + Ct^2} \tag{1}$$

using data up to $-t = 0.8$ GeV$^2$. The optical points, as derived from the total-cross-section measurements[7] and corrected for the real part of the amplitude,[8] were included in the fits at $t = 0$ with $\pm 3\%$ error to account for the uncertainty in the overall normalization of the elastic data. Figure 1 shows the results of the $K^{\pm}p$ and $p^{\pm}p$ fits. In each case, a crossover is found consistent with $-t_c$ in the range 0.1 to 0.2 GeV$^2$, and the values of $t_c$ obtained from the fits are plotted in Fig. 2 as a function of the laboratory momentum. For $Kp$ scattering, the crossover points are consistent with those found at lower energies,[2,3] with an average of $-t_c = 0.19 \pm 0.04$ GeV$^2$ where the error includes both statistical and systematic uncertainties. The nucleon values average to $-t_c = 0.11 \pm 0.02$ GeV$^2$, definitely lower than the value of $0.162 \pm 0.004$ GeV$^2$ reported[2] near 5 GeV/$c$.

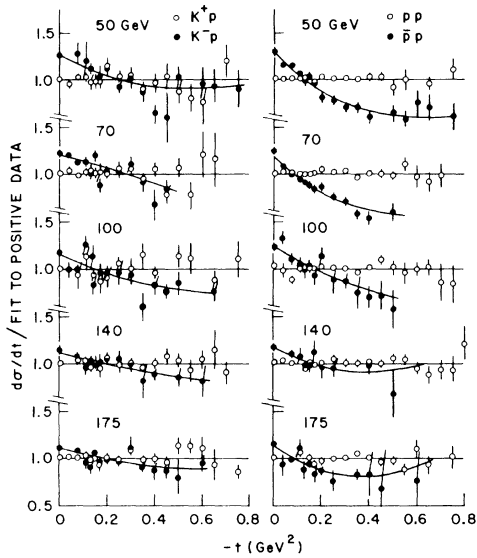These crossover points can be compared with

FIG. 1. Elastic-scattering cross section divided by the fit [Eq. (1)] to the positive-beam data. The lines show the ratio of the negative-beam fits to the positive-beam fits.



FIG. 2. Crossover points as a function of the incident momentum.

those predicted by a model of geometric scaling in which particle and antiparticle elastic scattering on protons differ only in the radial scale.[9] The resulting prediction, $-t_c = 4/(B^+ + B^-)$, where $B^\pm$ are forward slopes for $X^\pm p$ scattering, gives values considerably larger than observed, $-t_c = 0.27$ and $0.19$ GeV$^2$ for $Kp$ and $pp$, respectively, at 100 GeV/$c$. The model thus needs additional assumptions, such as a smaller opacity for the $K^- p$ and $\bar{p}p$ reactions, as compared with the $K^+$ and $pp$ reactions.[9]

The particle-antiparticle cross sections differ from one another because of the interference between $C = +1$ and $C = -1$ exchange amplitudes:

$$\frac{d\sigma(X^\pm p)}{dt} = |F^+ \mp F^-|^2, \tag{2}$$

where the $F^\pm$ amplitudes correspond to $C = \pm 1$ exchange. Since the diffractive amplitude with $C = +1$ dominates, the quantity

$$\Delta = \frac{\sigma^- - \sigma^+}{[8(\sigma^- + \sigma^+)]^{1/2}}, \tag{3}$$

where $\sigma^\pm \equiv d\sigma(X^\pm p)/dt$, isolates to a good approximation that part of $F^-$ with the same phase and spin state as the $C = +1$ amplitude (mainly imaginary nonflip).[1]

The energy dependence of $\Delta$ for $p^\pm p$ scattering has been studied using the form
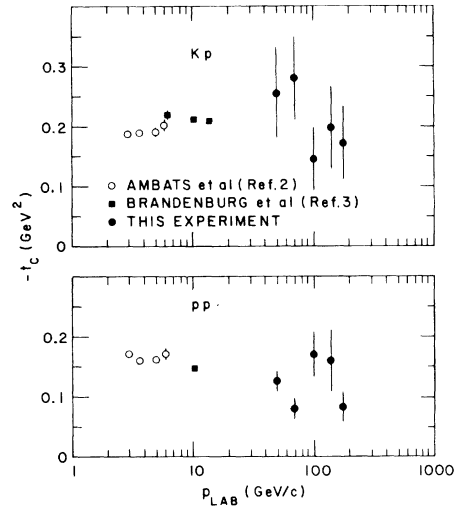
$$\Delta(t) = c(t) s^{\alpha(t)-1}, \tag{4}$$

where $\alpha(t)$ is the effective Regge trajectory, presumably of the $\omega$. Fits to our data [Eq. (1)] together with the 10.4-GeV/$c$ results,[3] both evaluated at $t = -0.4$ GeV$^2$, yield $\alpha(-0.4) = 0.25 \pm 0.07$, consistent with the value found at lower energy.[2] It is slightly higher than the value $\alpha(-0.4) = 0.14$ found[10] for $\pi^- p \to \pi^0 n$, which is expected to be mainly $\rho$ exchange.

The shape of $\Delta(t)$ resembles that of the Bessel function $J_0(R_- \sqrt{-t})$, suggesting that the $C = -1$ amplitude is strongly absorbed with most of the contribution coming from the periphery of the interaction radius.[4] If one associates the experimentally determined location of the crossover point with the first zero of the Bessel function, a typical radius for the source of the $C = -1$ amplitude in impact-parameter space can be defined as

$$R_- = 0.475/\sqrt{-t_c} \quad \text{F} \tag{5}$$

for $t_c$ in GeV$^2$. This is compared in Table I with a typical radius for the $C = +1$ amplitude derived

TABLE I. Typical radii $R_+$ and $R_-$ in impact parameter space for $C = +1$ and $C = -1$ exchange amplitudes, as defined by Eqs. (5) and (7). The uncertainties in $R_+$ are typically $\pm 2\%$ except for $R_-$ at 100 GeV/$c$ ($\pm 10\%$).

| | $p$ (GeV/$c$) | $R_+$ (F) | $R_-$ (F) | $R_-/R_+$ |
|---|---|---|---|---|
| $Kp$ | 4 | 1.00 | 1.09 | 1.09 |
| | 100 | 1.15 | 1.09 | 0.95 |
| $pp$ | 4 | 1.33 | 1.18 | 0.89 |
| | 100 | 1.33 | 1.43 | 1.08 |

from the forward logarithmic slope $B$ of the quantity

$$\Sigma = \tfrac{1}{2}[d\sigma(X^{-}p)/dt + d\sigma(X^{+}p)/dt]. \tag{6}$$

Using the black-disk approximation, $B = R_{+}^{2}/4$; or, for $B$ in $GeV^{-2}$,

$$R_{+} = 0.395\sqrt{B} \quad F. \tag{7}$$

With these definitions, $R_{-}/R_{+} = 1$ to within about 10% for both $Kp$ and $pp$ scattering from 4 to 100 GeV.

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# Particle Line Shapes in Heavy-Ion Reactions

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The kinetic energy of excited heavy ions produced at high velocity in a nuclear reaction can be modified in flight as a result of emission of $\gamma$ rays. It is shown that the resulting broadened ion-energy line shapes can be expressed analytically and have a pattern which depends on the relative $m$-substate populations produced in the reaction. Previously unexplained spectra are interpreted in this manner and the derived $m$-substate populations compared with calculations.

Heavy-ion–induced reactions in thin targets often produce excited nuclei which decay by $\gamma$ radiation in flight before being detected. If the heavy-ion detector has sufficiently good resolution a broadened line reflecting the Doppler shift of the emitted $\gamma$ rays is observed (see, for example, Erskine et al.[1] and Zisman et al.[2] The details of these broadened shapes provide information about the spin orientation of the ensemble of nuclear states with respect to the momentum vector of the recoiling nuclei. Consequently, we suggest that in favorable cases this additional information which is relevant to reaction mechanism studies can be obtained without recourse to, for example, particle–$\gamma$-ray coincidence techniques.

Consider an excited heavy ion leaving the reaction region, and let its momentum vector define the $z$ axis. If it is produced in a state of spin $S_b$ there will be $2S_b + 1$ substates $m_b$ quantized with respect to the $z$ axis. The orientation of the ensemble of nuclei produced in the reaction is specified by the components $\rho_{m_b m_b'}$ of the nuclear density matrix. If the state $S_b$ decays by the emission of a $\gamma$ ray of energy $E_\gamma$ to a state of spin $S_c$, the kinetic energy of the ion is shifted away from its nominal value $E_0$ by an amount

$$E_0 - E \sim \beta E_\gamma \cos\theta, \tag{1}$$

where $\beta$ is the velocity (in units of $c$) of the ion with energy $E_0$, and $\theta$ is the angle between the ion and photon momentum vectors. The dependence of the energy shift on $\theta$ clearly implies that the broadened line shape will depend on the angular distribution of emitted $\gamma$ rays, which in turn depends on $\rho_{m_b m_b'}$.

It is easily shown that the shape of the Doppler-broadened line as a function of energy shift, including effects of finite experimental resolution,