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DOES THE $\rho'(1600)$ COUPLE TO $\pi\pi$?
A QUESTION OF ANALYTICITY

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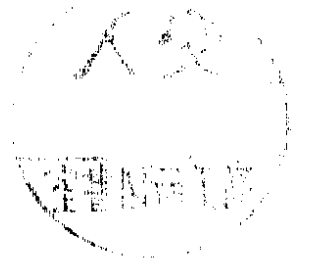
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A B S T R A C T

Analyticity is exploited to distinguish between classes of $\pi^+\pi^-$ partial wave solutions. Fixed t and fixed u dispersion relations determine the over-all phase of the amplitude and clearly select solutions with a $\rho'(1600)$ resonance of 25% elasticity.

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An outstanding question of meson spectroscopy, particularly relevant with the advent of the new ψ particles, concerns the existence of ρ' , ω' and ϕ' vector mesons. Of these, there is so far only information on the ρ' state. Although the Frascati $e^+e^- \rightarrow 2\pi$ or 4π data ¹⁾ is not conclusive, there is definite evidence for a $\rho'(1600)$ from the photoproduction process $\gamma\text{Be} \rightarrow (2\pi \text{ or } 4\pi)\text{Be}$ observed at FNAL ²⁾. The ρ' is also evident in $\pi\pi$ partial wave analyses of the high statistics $\pi^-p \rightarrow \pi^-\pi^+n$ CERN-Munich data ³⁾ and it is this determination of $\rho' \rightarrow \pi\pi$ coupling which concerns us here.

The $\pi\pi$ scattering amplitude extracted from data on one-pion-exchange dominated reactions has two main kinds of ambiguities ^{*}): first the discrete Barrelet type which gives a four-fold multiplicity of amplitudes ^{3),4)} in the energy range up to $M_{\pi\pi} = 1.8$ GeV, and secondly each of these amplitudes has an over-all phase which is undetermined above the inelastic threshold. However, these amplitudes, extracted from the data, do not necessarily satisfy the required unitarity or analyticity properties. Of these, unitarity is most easily studied by decomposing the amplitude into partial waves and checking that they lie on or within the unitarity circle. This is indeed found to be the case, within the errors, for all four possible partial wave solutions. Thus unitarity does not act as a discriminant - although, of course, it does determine the over-all phase of the amplitude in the elastic region below the $K\bar{K}$ threshold, and constrain it in the inelastic region.

On the question of the existence of a p wave $\rho'(1600)$ resonance the solutions divide into two categories: i) solutions B, D [in the notation of Ref. 4] have a ρ' coupling relatively strongly to $\pi\pi$ (elasticity 25%), whereas ii) solutions A, C show no evidence for a ρ' signal (elasticity $\lesssim 4\%$). In terms of Barrelet zeros, these two categories arise because the first zero, $z_1(s)$, entering the physical region has $\text{Im } z_1 \approx 0$ near $\sqrt{s} \equiv M_{\pi\pi} = 1.25$ GeV and so a bifurcation of solution is possible ^{3),4)}. Solutions of type i) and ii) correspond to $\text{Im } z_1 > 0$ and $\text{Im } z_1 < 0$ respectively above this energy.

How can we choose between these two classes, typified by solutions A and B? One way would be to obtain $\pi N \rightarrow \pi\pi N$ data in this energy region for other than $\pi^+\pi^\pm$ scattering; a recent discussion of these possibilities is given in Ref. 5).

^{*}) There are also possible ambiguities associated with the extrapolation of the data to the π exchange pole and ambiguities, particularly near $M_{\pi\pi} = 1.8$ GeV, which arise if the $\pi\pi$ partial wave series is not truncated at $l = 3$.

Here we shall use analyticity to select the physical solution. Above the $\bar{K}\bar{K}$ threshold the data give the modulus of the amplitude $|F(s,t)|$ and phase shift analysis determines the phase $\varphi(s,t)$ of each solution relative to, say, its phase at $t=0$. This is illustrated in Fig. 1 at one typical energy $\sqrt{s}=1.55$ GeV. We see that solutions A and B, though of course having the same $|F|$, have quite different phases especially near the backward direction. The Barrelet zeros, z_i , are indicated in the figure. As we go from the forward to the backward direction the phase φ changes by π or $-\pi$ in the region $z \approx \text{Re } z_i$ depending on whether $\text{Im } z_i < 0$ or $\text{Im } z_i > 0$. The solutions A and B differ only in the sign of $\text{Im } z_1$ above $M_{\pi\pi}=1.25$ GeV and this produces the striking phase difference near the backward direction.

Now analyticity inter-relates the phase and the modulus of an amplitude and, since each solution has the same modulus but a different phase, we may hope analyticity will discriminate between them. From Fig. 1 we expect analyticity at fixed t to determine the over-all phase of each amplitude, but not to choose between solutions, since their real and imaginary parts are very similar near the forward direction. In contrast, near the backward direction, where the phases differ most, fixed u analyticity can be expected to distinguish solutions once the over-all phase is given. As there exists no discrete ambiguity below 1.25 GeV, it is just above this first bifurcation point that we may hope analyticity will most readily make this distinction. So how do we proceed ?

A previous attempt to use fixed t and u analyticity has been made by Froggatt and Petersen ⁷⁾. Following Pietarinen ⁸⁾ they conformally map the right- (and left-) hand cut s plane into the unit circle. At each fixed t (fixed u) they parametrize the amplitude as a polynomial in the new variable and fit to the data for $|F|$, including a certain penalty function to ensure smoothness. They find a solution very similar to B. However, there are two notable difficulties associated with their procedure. Firstly they explicitly include the ρ, f and g resonance poles in the full amplitude so as to leave a smoother function for polynomial parametrization. This would appear to bias the analysis in favour of the solution (B) with a ρ' resonance under the g , since it is only for one form of the residue of the pole that this daughter can be eliminated. Secondly, the high energy behaviour of the amplitude expanded in terms of large order polynomials in the unit circle is far from transparent and difficult to control. It may lead in practice to oscillations in the behaviour of the amplitude between the end of the data and $|s| = \infty$. While such oscillations do not violate analyticity they are at variance with our expectations for high energy forward scattering.

In order to avoid these difficulties, we evaluate conventional fixed t and fixed u dispersion relations. Phase modulus ^{9),10)} dispersion relations immediately come to mind, but they are unsuitable in the backward direction, which we have seen is the crucial region for resolving the phase shift ambiguity, since there the number and the location of zeros in the complex plane is unknown. We therefore simply use the Cauchy representation for the scattering amplitude at fixed t and fixed u in order to impose the necessary cut s , plane analyticity. The integration contour we choose extends only to finite energies so as to avoid subtraction parameters, and simple Regge pole behaviour is assumed to hold for the amplitude beyond the region of existing data, that is for $M_{\pi\pi} > 1.8$ GeV.

Our input for the fixed t and fixed u dispersion integrals for the $\pi^+\pi^-$ amplitude, $F(s,t)$, is listed A)-E) below.

- A) For $M_{\pi\pi} < 0.6$ GeV we use the solution ¹¹⁾ of the Roy equations for the s and p waves with an $I=0$ s wave scattering length $a_0^0 = 0.3$ in pion mass units since this matches on with the data we use ³⁾ most straightforwardly.
- B) For $0.6 < M_{\pi\pi} < 1.8$ GeV in the s and t channels, we reconstruct the $\pi^+\pi^-$ amplitude from the known elastic s , p and d wave phases below the $K\bar{K}$ threshold ¹²⁾. Above this threshold we input $|F(s,t)|$, which is reconstructed from the solutions of Ref. 4) in exactly the same way as described in Ref. 7). In this inelastic region the phase $\varphi(s,t=0)$ is free. In practice we set this equal to the phase shown in Ref. 2) (assuming Breit-Wigner f and g resonance forms) to which we add a free phase $\varphi_0(s)$ to be determined by analyticity.
- C) In the u channel from 0.6 - 1.3 GeV we input the $\pi^+\pi^+$ data of Ref. 13) and beyond that up to 1.8 GeV a smooth interpolation to the Regge form is used. The $\pi^+\pi^+$ data contribute only to the fixed t dispersion relation and then only to the left-hand cut and so despite their large uncertainties they give only a very small contribution to the dispersion integrals evaluated for $\sqrt{s} > 0.6$ GeV.
- D) The contribution from the circular contour in the fixed t dispersion relations is calculated assuming Pomeron, f and ρ exchange of the form

$$\begin{aligned}
 F(\nu, t) = & \frac{\sigma^{\pi\pi}(\infty)}{32\pi\alpha'} e^{bt} \xi_+^P (\alpha'\nu)^{\alpha_P(t)} \\
 & + \frac{1}{3} \beta_f \frac{\alpha_f(t)}{\alpha_f(0)} \xi_+^f (\alpha'\nu)^{\alpha_f(t)} \\
 & + \frac{1}{2} \beta_\rho \frac{\alpha_\rho(t)}{\alpha_\rho(0)} \xi_-^\rho (\alpha'\nu)^{\alpha_\rho(t)}
 \end{aligned} \tag{1}$$

with $\nu = (s-u)/2$, $\alpha' = 0.9 \text{ GeV}^{-2}$ and where $\alpha_P(t) = 1 + 0.2t$ and $\alpha_{\rho, f}(t) = 0.5 + \alpha't$ with t in GeV^2 , and where the signature factors are normalized so that $\text{Im} \xi = 1$. The slope b is chosen in the range $(1.5, 4.5) \text{ GeV}^{-2}$. The asymptotic cross-section, $\sigma^{\pi\pi}(\infty)$, and β_f are free parameters to be determined by the fit. The ρ exchange having odd signature contributes to $\text{Re} F$ much less than the Pomeron and f for s much less than $(1.8 \text{ GeV})^2$ and so we fix its residue $\beta_\rho = 0.75$ to agree with duality and FESR expectations ^{*}) [see, for example, Ref. 14] rather than let it go free.

- E) The contribution from the circular contour in the fixed u dispersion relations is calculated using a Regge parametrization for some effective "exotic" exchange

$$F(\nu, u) = \gamma (1 + \gamma'u) \xi_+^x (\alpha'\nu)^{\alpha_x(u)} \tag{2}$$

with $\nu = (s-t)/2$. We choose the exotic trajectory to have intercept in the range $(-1.5, -0.5)$ and canonical slope, α' . The residue parameters γ and γ' are to be determined by the fit. Our results depend only weakly on the exact choice of the exotic trajectory, though of course the fitted values of γ and γ' are strongly α_x dependent.

We minimize the difference between the output real part obtained from the dispersion relations, by integrating over the data with a free phase ^{**)} $\varphi_0(s)$ for $1 < \sqrt{s} < 1.8 \text{ GeV}$, and the real part calculated directly from the data for $|F|$ using the same phase. This minimization is performed for values

^{*}) The factor $\alpha_\rho(t)/\alpha_\rho(0)$ in Eq. (1) approximates the desired t dependence of the ρ Regge residue ¹⁴⁾ in the near forward region we consider.

^{**)} First we took φ_0 as a free parameter constant in each 0.1 GeV interval in \sqrt{s} . The minimization was then repeated parametrizing this additional phase φ_0 as a quadratic in \sqrt{s} . The results of the two methods are compared below.

of t and u in the range $(0, -0.3)\text{GeV}^2$ and for values of \sqrt{s} in the range $(0.68, 1.75)\text{GeV}$ with a particularly fine grid of points in the region $1.25 < \sqrt{s} < 1.5 \text{ GeV}$ and a coarse grid at higher \sqrt{s} . The results for the real parts are shown in Fig. 2 for both solutions A and B at $t=0$ and $u=0$. We see rather dramatically how solution B, which has a sizeable ρ' to $\pi\pi$ coupling, satisfies analyticity very well with its over-all phase determined and displayed in Fig. 3, whereas solution A, which has no ρ' signal, fails badly for $1.3 < \sqrt{s} < 1.5 \text{ GeV}$. Indeed the fits for solution B are still better away from the physical region boundary values of Fig. 2, while the misfit for solution A in the region $M_{\pi\pi} = 1.4 \text{ GeV}$ persists for other fixed u values. As hoped analyticity discriminates between solutions just as Fig. 1 led us to expect. Moreover, most importantly our results are, within limits, not sensitive to the forward Regge parameters; for example doubling $\sigma^{\pi\pi}(\infty)$ does not affect the ability to distinguish between solutions, but only slightly worsens the fit.

We conclude that analyticity overwhelmingly favours ^{*)} the $\pi\pi$ partial wave solutions (B and D) with a sizeable ρ' coupling to $\pi\pi$ and determines the over-all phase of these solutions (see Fig. 3 for solution B). Detailed work on the remaining ambiguity between the B and D solutions is continuing. This depends on the Barrelet zero, z_2 , which, unlike z_1 , is near the middle of the physical region for $\sqrt{s} > 1.45 \text{ GeV}$ (where the bifurcation of $\text{Im } z_2$ occurs) and so just outside the range of validity of fixed momentum transfer dispersion relations. This makes matters more complicated. Moreover the existence of the spin-4 resonance ¹⁶⁾, the h , near $\sqrt{s} = 2.025 \text{ GeV}$ may play a more important rôle.

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*) Here we have assumed that the truncation of the partial wave series at $l=3$ (and moment series at $J=6$) is exact, so that the unknown phase φ_0 depends only on s and is independent of t . This is essentially correct for the energy region which is crucial here, that is $1.25 < \sqrt{s} < 1.5 \text{ GeV}$. However, when we allow a free phase at each value of u , as well as in the forward direction, each solution, A, B, C and D, rotates, on imposing analyticity, to basically the same resultant amplitude. This amplitude is essentially the same as solution B displayed here in Figs. 2 and 3.

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FIGURE CAPTIONS

Figure 1 : The modulus and phase of the $\pi^+\pi^-$ amplitude at $\sqrt{s} \equiv M_{\pi\pi} = 1.5$ GeV reconstructed from the partial wave analysis ⁴⁾ of the CERN-Munich data ⁶⁾. The Barrelet zeros, z_i , are also given and their positions ($\cos\theta \equiv z = \text{Re } z_i$) indicated. Solutions A and B correspond to $\text{Im } z_1 < 0$ and $\text{Im } z_1 > 0$ respectively. The $t=0$ phase is set arbitrarily to zero.

Figure 2 : The real part of the forward (and backward) $\pi^+\pi^-$ amplitude as obtained from the $t=0$ (and $u=0$) dispersion relations (the curve) and from the input data (the points). The values shown are the result of minimization at several values of t , u in the range $(0, -0.3)\text{GeV}^2$. For solution B we find $\sigma^{\pi\pi}(\omega) = 8.5$ mb and $\beta_f = 1.0$ (as compared to the exchange degeneracy value $\beta_f = \frac{3}{2}\beta_\rho = 1.1$). These Regge parameters are in complete agreement with a recent FNAL experiment ¹⁵⁾ that gives $\sigma_{\frac{1}{2}}(\pi^+\pi^-) = 15 \pm 4$ mb at $s = 20 \text{ GeV}^2$ and 13.5 ± 2.5 mb at $s = 32 \text{ GeV}^2$. The dispersion relation prediction near 1 GeV is not shown as it depends sensitively on the properties of the S^* resonance.

Figure 3 : Argand plot of the forward $\pi^+\pi^-$ scattering amplitude. The phase is determined by dispersion relations, i) by taking ϕ_0 as a free parameter in each 100 MeV interval (the points are the rotated data); ii) by parametrizing ϕ_0 as a quadratic in \sqrt{s} (the curve is the dispersion relation result). The normalization is $\text{Im } F(s, t=0) = \sqrt{s} q \sigma_{\frac{1}{2}}(\pi^+\pi^-) / 16\pi$.



$\pi^+\pi^-$ AMPLITUDE AT
 $M_{\pi\pi} = 1.55 \text{ GeV}$

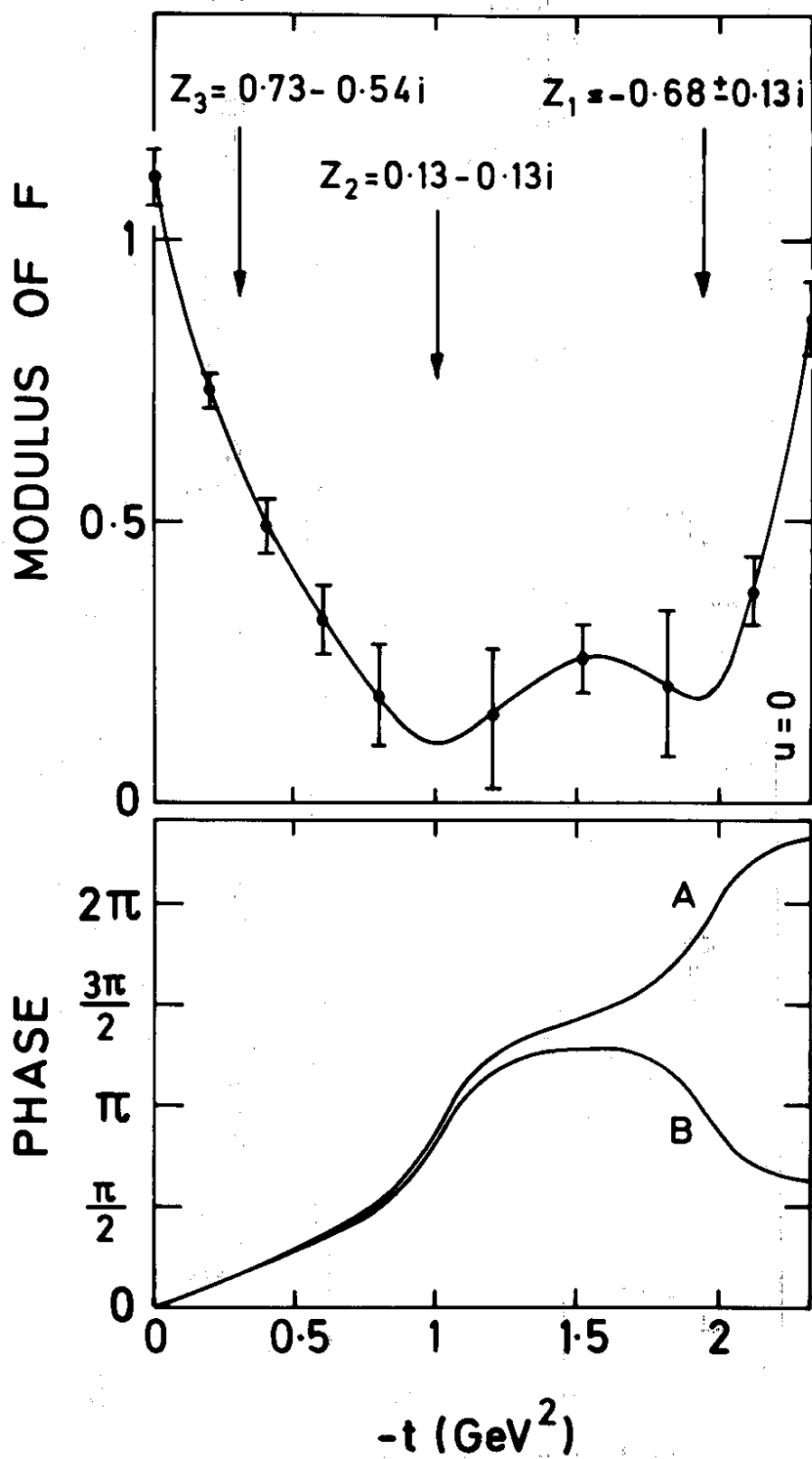


FIG. 1

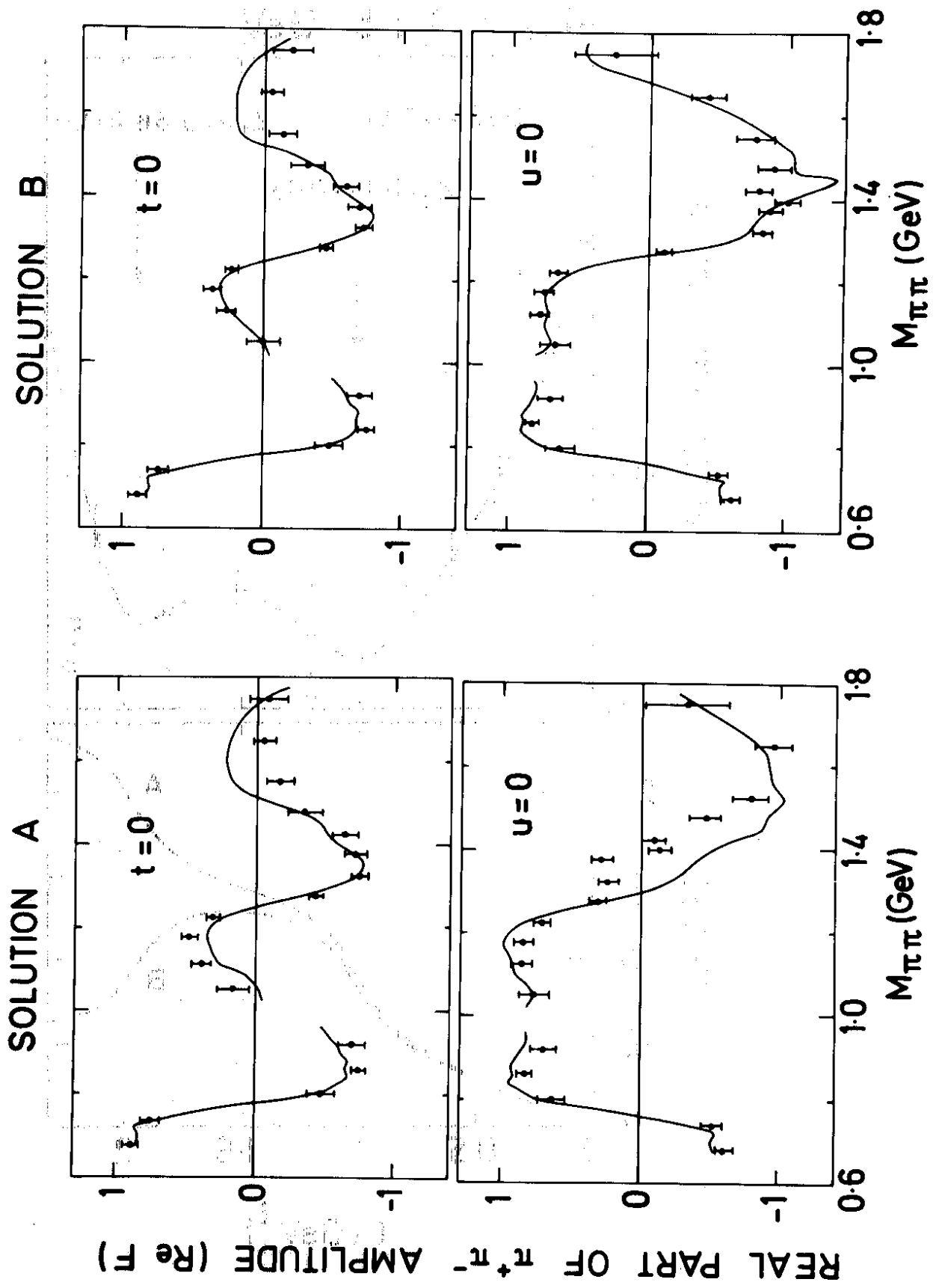


FIG. 2

THE FORWARD $\pi^+\pi^-$ AMP., $F(s,t=0)$

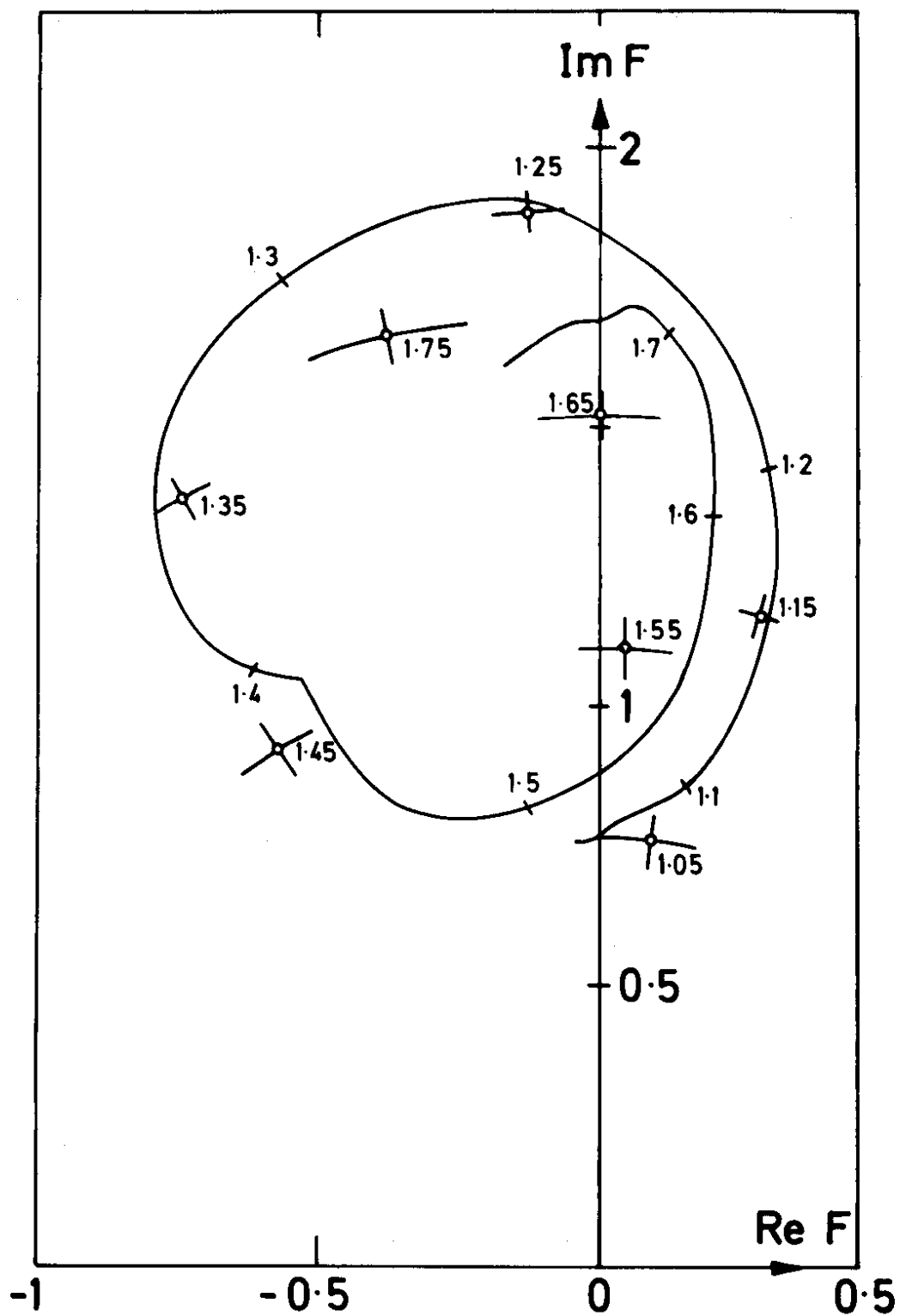


FIG. 3

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