Ref.TH.2135-CERN

DOUBLY RADIATIVE np CAPTURE : A PRECISION EVALUATION

J. Blomqvist *) and T. Ericson
CERN -- Geneva

A B S T R A C T

Our previous prediction $\sigma_{2\gamma} = 0.12\ \mu b$ is sharpened by model-independent considerations to $\sigma_{2\gamma} = (0.1176 \pm 0.0003)\ \mu b$. The contributions from tensor forces, inner region, retardation, P state interactions, M1 - M1 transitions and exchange currents are all demonstrated to be of order $1{:}10^3$ or smaller.

(To appear in Physics Letters)

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*) Now at Research Institute for Physics, Stockholm 50

Ref.TH.2135-CERN
11 February 1976

About a year ago the doubly radiative process $n+p \to D+2\gamma$ for thermal neutrons was reported observed [1] with a cross-section of $350\ \mu b$, i.e., a branching ratio of 10^{-3} to the singly radiative process $n+p \to D+\gamma$. Subsequent reinvestigations by a Chalk River group [2] and a Jülich group [3] instead gave upper limits of $\sigma_{2\gamma} < 33\ \mu b$ and $\sigma_{2\gamma} < 21\ \mu b$, respectively. Earle has recently reported [4] an improved upper limit of $\sigma_{2\gamma} < 8\ \mu b$. In addition, he indicates that further substantial improvement is possible, even to the region of the theoretically expected value [5] $\sigma_{2\gamma} \simeq 0.12\ \mu b$. The original experiment has been quantitatively criticized as due to "cross talk" between the detectors from positron annihilation in flight associated with the single photon process [6],[7].

The initial experimental anomaly [1] provoked a series of theoretical investigations [5],[8)-11], which demonstrated on the one hand that the discrepancy with expectations was over three orders of magnitude, and on the other hand that there was no plausible mechanism to explain the anomaly. Since the purpose was to establish magnitudes, minor differences exist between the various estimates. In view of the greatly improved experiments and future prospects we felt it justified to establish a more accurate prediction as well as its sensitivity to perturbing effects. We will below derive a generalization of our previous result [5], expected to be accurate to a level of a few ‰, independent of details of both the deuteron structure and the intermediate state interactions in the two-step process.

The dominant transition is the $E1-E1$ transition from the $^3S_1$ np state at threshold. The matrix element for this process is

$$M(E1,E1) =$$

$$= -\frac{1}{(2\pi)^6\sqrt{4\omega_1\omega_2}} \int d^3k \, \langle D| e\, \underset{\sim}{v_p}\cdot \underset{\sim}{\epsilon_2} |\underset{\sim}{k}\rangle \frac{1}{\omega_1+E_k} \langle \underset{\sim}{k}| e\, \underset{\sim}{v_p}\cdot \underset{\sim}{\epsilon_1} | np\rangle$$

$$+ (1 \leftrightarrow 2) + \text{gauge term}$$

For a local interaction with smooth currents the gauge term is

$$\frac{1}{(2\pi)^3\sqrt{4\omega_1\omega_2}} \frac{e^2}{M} \underset{\sim}{\epsilon_1}\cdot\underset{\sim}{\epsilon_2} \langle D | np\rangle$$

and vanishes in this limit (retardation is neglected).

The triplet wave functions contain S wave and D wave parts

$$Y_0 \chi_1 \frac{u(r)}{r} + [Y_2, \chi_1]_1 \frac{w(r)}{r}$$

The asymptotic $(r \to \infty)$ deuteron wave functions are

$$u_D \to \frac{N}{\sqrt{2\kappa}} e^{-\kappa r}$$
$$w_D \to \frac{N'}{\sqrt{2\kappa}} e^{-\kappa r} \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2}\right)$$

with $\kappa = \sqrt{MB} \cong (4.32\ \mathrm{fm})^{-1}$, and the asymptotic scattering wave functions

$$u_{np} \to r - a_t$$
$$w_{np} \to \frac{b}{r^2}$$

with $a_t \cong 5.42$ fm, and $b = -3\ N/N'\ a/\kappa^2$.

The operator is given by the standard relation

$$e \underset{\sim}{v_p} \cdot \underset{\sim}{\epsilon} \equiv e \underset{\sim}{\dot{r}_p} \cdot \underset{\sim}{\epsilon} = i [H, e \underset{\sim}{r_p} \cdot \underset{\sim}{\epsilon}]$$

The expression above for the two-photon matrix element can be easily evaluated. It is technically advantageous to use the operator identity

$$[H, \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_2}] \frac{1}{H + \omega_1 - E_0} [H, \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_1}] + (1 \leftrightarrow 2) =$$
$$= \omega_1 \omega_2 \left\{ \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_2} \frac{1}{H + \omega_1 - E_0} \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_1} + (1 \leftrightarrow 2) \right\} + [[H, \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_2}], \underset{\sim}{D} \cdot \underset{\sim}{\epsilon_1}] \tag{1}$$

in which the last term vanishes if $[[V, D \cdot \epsilon_2], D \cdot \epsilon_1] = 0$. Replacing the initial and final wave functions by their asymptotic S wave form and using free wave intermediate states immediately gives

$$\sigma_{2\gamma}(total) = N^2 \frac{2}{9} \alpha^2 \frac{c}{v_n} \left(\frac{B}{Mc^2}\right)^{5/2} a_t^2 \left\{ 20 + \pi - 32 \ln 2 - \frac{12}{5}\left(1 - \frac{1}{\kappa a_t}\right) + \frac{3}{2}\left(1 - \frac{1}{\kappa a_t}\right)^2 \right\} \tag{2}$$

We will show below that this expression is extremely accurate, since nearly the entire contribution to the integral comes outside the range of nuclear forces.

Previous estimates of the cross-section have all been obtained by the same general reasoning. Grechukhin [12] calculated the matrix element using square well wave functions giving approximately correct binding, scattering length and range, but with no D waves. He obtained the value $\sigma_{2\gamma} = 0.096\ \mu b$ [we have corrected his result by a factor of 16 coming from from inversion of factors for relative and centre-of-mass co-ordinates in his Eq. (18) and by a factor of $\frac{1}{2}$ coming from the spectral integration from 0 to B instead of 0 to $B/2$ as appropriate for identical particles]. Our original estimate [5] is Eq. (2) above neglecting effective range, i.e., $\kappa a_t = 1$, $N = 1$, but using the cross-section proportional to a_t^2 rather than κ^{-2}, since this is the area effectively entering into the initial np cross-section. This gives $\sigma_{2\gamma} = 0.124\ \mu b$. Finally, Hyuga and Gari [8] made a rougher closure estimate, neglecting the intermediate np energies $H-E_0$ as compared to $\omega_{1,2}$ in the energy denominators, which in the asymptotic approximation for wave functions gave $\sigma_{2\gamma} = 0.087\ \mu b$. Hyuga and Gari passingly state that their result is an upper limit. This is not so since the operator is non-positive, the exact conversion factor to the non-closure approximation with free P states and asymptotic wave functions being $\frac{3}{2}(20 + \pi - 32\ \ln 2) = 1.44$. This corresponds then to a cross-section $\sigma_{2\gamma} = 0.125\ \mu b$ in agreement with our result [5]. All these results are roughly consistent with each other within approximations in the evaluation. We will now discuss the sensitivity of the result (2) to various effects. Our conclusion is that their contributions are all very small for the simple reason that the 2γ process occurs mainly at an exceptional distance, typically about 15 fm, with an additional discrimination against the region inside and close to the deuteron by the propagator for the intermediate P state.

1. - Contribution from the internal region

Straightforward integration over the region $0 < r < r_o \approx r_t$ (the effective range) using asymptotic S state wave functions gives a contribution of order

$$\frac{1}{40}(\kappa r_o)^5 < 10^{-3}.$$

The two additional powers of r_o as compared to the closure approximation reflect quenching by the propagator. Since asymptotic wave functions are considerably larger than actual ones in the inner region, our estimate should be considered a very generous upper limit. The internal region is thus of negligible importance.

2. - Retardation

The effect of the finite wavelength is of order $\omega^2 < r^2 >$; more exactly $-0.6\ \epsilon/M = -1.5\times10^{-3}$. It is thus small *).

3. - Contributions from D states

The tensor force gives a D state content to both the initial and final states ; it is closely related to the deuteron quadrupole moment which in dimensionless units is $\kappa^2 Q = 1.4.10^{-2}$. The SD cross-terms

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*) In a recently circulated preprint and the preceding paper [13], Lee and Khanna advocate that the dipole operator should be not $\frac{1}{2}\, e\omega(\underset{\sim}{r}\cdot\underset{\sim}{\epsilon})$ but

$$3e\, j_1\left(\frac{\omega r}{2}\right)(\hat{\underset{\sim}{r}}\cdot\underset{\sim}{\epsilon}) = \frac{1}{2}\, e\omega(\underset{\sim}{r}\cdot\underset{\sim}{\epsilon})\left[1 + O(\omega^2 r^2)\right]$$

and that the difference is of importance. However, this is only the retarded version of the normal operator, and their description is fully equivalent to the standard one. They further argue that $\frac{1}{2}e(\underset{\sim}{v}\cdot\underset{\sim}{\epsilon})$ gives anomalous results for non-local interactions. This, however, is based on a misunderstanding and follows from the fact that they neglect to include the gauge term in this case. Their value $\sigma_{2\gamma} = 6.9.10^{-2}\ \mu b$ results from a cruder approximation to asymptotic deuteron wave function normalization than implicit in previous work, but it is otherwise consistent with them.

average to zero in the total cross-section. The D state to D state contribution can be directly evaluated with asymptotic wave functions. Its contribution is of order

$$5(\kappa^2 Q)^2 = 10^{-3}.$$

It is interesting to note that in the closure approximation the corresponding term is cut-off dependent and using the effective range it is five times larger. Once more closure overemphasizes the region close to the origin, while the effective cut-off is $\sqrt{2}\,\kappa^{-1} = 6$ fm. The quadrupole contribution is thus model-independent and small.

4. - Interaction in the intermediate P states

Equation (2) was obtained neglecting np interactions in relative P states. In order to control this approximation we will assume the P state interaction to be of short range, described by a potential with the scattering volume a_1, the same for the three angular momentum states. It is then rather simple to analytically evaluate the interaction effect to linear terms in the scattering volume, using the same wave functions as previously. The effect is of order

$$\frac{1}{15}\kappa^3 a_1 = \frac{a_1\,(\mathrm{fm})^3}{1200}.$$

Typical values for the scattering volumes a_{1J} are [14]: $a_{10} = -3.0$ fm^3 ; $a_{11} = 1.8$ fm^3 ; $a_{12} = -0.3$ fm^3. The spin averaged value $\bar{a}_1 = 0.1$ fm^3, due to very strong cancellations, should be used above. In this case the P state interaction is of order 10^{-4} and completely negligible. In order to be very conservative we average the absolute values of a_{1J}, giving $\bar{a}_1 \approx 1$ fm^3. Even in this case the P wave enters only on the level $1:10^3$, and we conclude that this effect is small.

5. - Meson exchange currents

In principle exchange currents could play a non-negligible rôle in processes of this kind. Hyuga and Gari [8] show in their derivation of the result equivalent to the operator identity (1) that the gauge term contains exactly the double commutator

$$- [[H, \underset{\sim}{D} \cdot \underset{\sim}{\epsilon}_2], \underset{\sim}{D} \cdot \underset{\sim}{\epsilon}_1]$$

so that such terms cancel exactly, yielding a matrix element proportional to

$$\omega_1 \omega_2 \left\{ \langle f \,|\, \underset{\sim}{D} \cdot \underset{\sim}{\epsilon}_2 \frac{1}{H + \omega_1 - E_0} \underset{\sim}{D} \cdot \underset{\sim}{\epsilon}_1 \; + \; (1 \leftrightarrow 2) \,|\, i \rangle \right\} + \frac{(Ze)^2}{2AM} \underset{\sim}{\epsilon}_1 \cdot \underset{\sim}{\epsilon}_2 \langle f | i \rangle \tag{3}$$

This result can be interpreted as the non-diagonal generalization of the Thomson theorem ; it is well known that the Thomson amplitude depends only on the total charge and mass, and is independent of detailed structure. Equation (3) is an important general result for E1 - E1 transitions. At this point we further note that quite generally the 2γ matrix element is <u>exactly</u> the non-diagonal electric polarizability in the limit when $\omega_{1,2}$ can be neglected compared to nuclear intermediate excitation energies. The present case of the np system is, however, much closer to the opposite limit of degenerate intermediate states (closure approximation).

6. - The M1 - M1 transitions [5]

Both the singlet and triplet np states can decay by double M1 transitions at a rate

$$\left[\frac{\hbar (\mu_p - \mu_n)}{Mc\, a_t} \right]^4 \approx 10^{-3}$$

of the double E1 rate. They add incoherently to the total cross-section. Good expressions for these cross-sections were given by us previously [5] : $\sigma_t(\mathrm{M1\ M1}) = 9.10^{-5}\ \mu b$ and $\sigma_s(\mathrm{M1\ M1}) = 2.10^{-5}\ \mu b$. These are small and can be well calculated.

We are now ready to evaluate Eq. (2) with known precision. Using the best values for the np parameters [15] $\epsilon = 2.2246$ MeV, $\varkappa^{-1} = (4.3167 \pm 0.00009)$ fm, $a_t = (5.418 \pm 0.005)$ fm, $\rho(-\epsilon,-\epsilon) = (1.771 \pm 0.007)$ fm and $N^2 = (1-\varkappa\rho(-\epsilon,-\epsilon))^{-1} = (1.696 \pm 0.004)$ substitution into Eq. (2) gives the total cross-section

$$\sigma_{2\gamma} = (0.1176 \pm 0.0003)\,\mu b,$$

which agrees to 6% with our crude first estimate.

The uncertainty of 2.5 parts in 10^3 reflects the uncertainty in the experimental determination of the triplet range parameter $\rho(-\epsilon,-\epsilon)$. The principal corrections to this cross-section as discussed above are : a) the M1 - M1 contribution (+1.5 parts in 10^3) ; b) the D state contribution (+1 part in 10^3) and c) the P state interaction (1 part in 10^3, but probably smaller), and d) retardation (-1.5 parts in 10^3), while exchange currents and the inner region give negligible contributions. All the important corrections can be calculated to higher precision when required.

Finally, the spectral shape and the angular distribution of the photons are rather insensitive to detailed assumptions. They are to good approximation given by the simple expression

$$\frac{d\sigma}{d\omega_1\, d(\cos\theta_{12})} \propto \omega_1\omega_2 \left\{ \frac{1}{1+\sqrt{\frac{\omega_1}{B}}} + \frac{1}{1+\sqrt{\frac{\omega_2}{B}}} - \frac{3}{2}(\kappa a_t - 1) \right\}^2 (1+\cos^2\theta_{12})$$
$$\approx \text{constant} \times \omega_1\omega_2\,(1+\cos^2\theta_{12})$$

which is a slight improvement on our previous expression $(\varkappa a_t = 1)$ [5].

In conclusion, both the cross-section and the spectrum of the process $n + p \to D + 2\gamma$ for thermal neutrons are given to very high accuracy by the asymptotic properties of the wave functions ; they are insensitive to non-localities and exchange currents in the interaction.

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