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PION EXCHANGE AND THE UNIVERSAL IMPACT PARAMETER HYPOTHESIS

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ABSTRACT

In order to describe pion exchange reactions a modification to the "b-universality" hypothesis is proposed which simply relates the impact parameter profiles of helicity amplitudes for different values of net helicity-flip. This proposal is shown to give an excellent quantitative fit to the $\pi^- p \to \rho^o n$ and $\pi^- p \to f^o n$ data at 17.2 GeV/c. It also provides a natural explanation for the presence and size of the necessary absorptive cut corrections.

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In a recent series of papers [1-6] it has been suggested that the phenomenology of few body reactions, which is apparently rather complicated in terms of t-channel exchanges for the various helicity amplitudes, may look very simple when viewed in terms of s-channel dynamics.

For instance if one considers the impact parameter profile of each s-channel helicity amplitude $M_n(s,t)$ with net helicity-flip n, such that

$$\tilde{M}_n(s,b) = 1/q^2 \int_0^\infty d(-t')\, J_n(b\sqrt{-t'})\, M_n(s,t') \qquad (1)$$

it has been postulated that there exists a "universal impact parameter" criterion [2,5] or "b-universality" [4,6] which trivially relates these profiles for different values of n. Taking into account the kinematical constraint that

$$\tilde{M}_n(s,b) = O(b^n) \text{ near } b = 0$$

it was suggested in ref [4] that the profiles for all n may be given by

$$\tilde{M}_n(s,b) \propto b^n f(s,b). \qquad (2)$$

Here f(s,b) is assumed to be the same for all helicity amplitudes corresponding to the exchange of a particular set of quantum numbers. Therefore if one knows the structure of <u>one</u> helicity amplitude one can easily determine the profile for each n using eqs. (1), (2) and hence determine <u>all</u> the helicity amplitudes from the inverse transformation

$$\tilde{M}_n(s,t') = q^2 \int_0^\infty 2b\, db\, J_n(b\sqrt{-t'})\, \tilde{M}_n(s,b) \qquad (3)$$

In fact the great advantage of assuming the form of "b-universality" given by eq. (2) is that, together with eq. (3) it leads to simple derivative relationships between the different helicity amplitudes: i.e.

$$M_m(s,t') = \frac{\lambda_n(s)}{\lambda_m(s)} (-t')^{m/2} \left[\frac{d}{\sqrt{-t}\,d\sqrt{-t}}\right]^{m-n} \left\{\frac{M_n(s,t')}{(-t')^{n/2}}\right\} \qquad (4)$$

where $\lambda_m(s)$, $\lambda_n(s)$ may be some (complex) functionsof s.

These simple derivative relations, with [4] and without [7] the further assumption that there is one helicity amplitude that may be well approximated by a pure Regge pole exchange, have been found to give a good description of much of the current few-body data.

However, it is clear that there are certain very peripheral processes, i.e. pion exchange reactions, for which this form of b-universality and the corresponding derivative relations are likely to be invalid. For example if we assume eq. (2) is applicable even out to large values of b then this implies the peripheral dynamics (large b-behaviour) of the various helicity amplitudes will be significantly different for different values of the net helicity flip n. In terms of the derivative relations of eq. (4) this means that a simple pion pole propagator $1/(t-\mu^2)$ in one amplitude with $n=n_o$ say, will become a double and treble pole in the n_o+1, n_o+2 amplitudes respectively.

In other words the simple form of b-universility implied by eq. (2) destroys the t-channel analytic structure of the helicity amplitudes which in some cases, particularly pion exchange, has an important effect on the s-channel dynamics.

Here, therefore, we should like to propose a modification to the naive b-universality assumption of eq. (2) so that we can also incorporate pion exchange into this appealingly simple phenomenology of few-body reactions.

From our above remarks it is clear that if we are not to introduce multiple poles into some helicity amplitudes we cannot use eq. (2) directly to define the impact parameter profiles at large values of b. Moreover, it can be derived using the properties of t-channel analyticity, that the impact parameter profiles, $\tilde{M}_n^{pole}(s,b)$, corresponding to amplitudes containing a single simple peripheral pole of the form $1/(t-\mu^2)$ all have the same asymptotic behaviour which is independent of n, i.e.

$$\tilde{M}_n^{pole}(s,b) \propto e^{-\mu b} \text{ for large } b$$

Hence we must modify our "b-universality" assumption so that while the profiles behave like b^n near $b = 0$, they now all have same n-independent behaviour at large b values. Clearly there are, in principle, an infinite number of ways in which this could be achieved. However, the most straight forward way to ensure that all amplitudes have the same leading t-channel singularity and yet are simply computed from one another is given by the following prescription:

(i) if there is an important near-by t-channel singularity (pole) we first define new singularity-free amplitudes; for example

$$\hat{M}_n(s,t') = (t-\mu^2)M_n(s,t') \tag{5}$$

(ii) assume a suitable parametrisation for one particular amplitude; e.g., we shall take a simple Regge pole parametrisation for the $\pi N \to \rho N$ amplitude $\hat{M}_{n=1}(s,t')$

(iii) now use the naive b-universality relations, eqs. (2),(4) to calculate the other singularity-free amplitudes.

(iv) finally determine the true helicity amplitudes $M_n(s,t')$ from eq. (5).

Thus all amplitudes explicitly display the single pole singularity and yet we still retain the advantage of being able to use the derivative relations, but now for the singularity free amplitudes rather than for the full amplitudes.

It is important to notice that where there is no prominant near-by t-channel singularity it makes only a small difference whether we use the derivative relations for the full amplitudes or the singularity-free amplitudes. For example in the case of vector or tensor exchanged trajectories the only effect of explicity taking into account the ρ or f^o pole would be to alter the exponential slope of the Regge residue function by less than $1(\mathrm{GeV/c})^{-2}$.

In the rest of this note therefore we shall concentrate on the pion exchange reactions $\pi N \to \rho N$ and $\pi N \to f^o N$ where such considerations will make a considerable difference.

Using the high statistics data [8] for the process $\pi^- p \to \pi^+ \pi^- n$ at 17.2 GeV/c it has been possible to determine fairly reliably the following combination of helicity amplitudes:

$$L_o = H^{L,o}$$
$$L_\lambda \pm = 1/\sqrt{2}\,(H^{L,\lambda} \mp (-1)^\lambda\, H^{L,-\lambda})$$

where L and λ refer to the spin and helicity of the produced dipion system; and (to leading order in s), $L_\lambda +$ and $L_\lambda -$ correspond to natural and unnatural parity exchanges. Here we shall only discuss the unnatural parity exchanges corresponding to possible pion exchange. Also in these equations it is understood that $L_\lambda \pm$ are incoherent sums of amplitudes with and without helicity flip at the nucleon vertex. Hence in terms of our basic s-channel helicity amplitudes $M_n^{(L)}(s,t)$ we have for

a) $\pi N \to \rho N$:
$$|P_o|^2 = m_o b_o |M_o^{(1)}|^2 + m_o b_1 |M_1^{(1)}|^2$$
$$|P_{1-}|^2 = \tfrac{1}{2}\, m_1 b_1 |M_o^{(1)} - M_2^{(1)}|^2$$

and for

b) $\pi N \to f^o N$:
$$|D_o|^2 = m_o b_o |M_o^{(2)}|^2 + m_o b_1 |M_1^{(2)}|^2 \qquad (6)$$
$$|D_{1-}|^2 = \tfrac{1}{2}\, m_1 b_1 |M_o^{(2)} - M_2^{(2)}|^2$$
$$|D_{2-}|^2 = \tfrac{1}{2}\, m_2 b_1 |M_1^{(2)} - M_3^{(2)}|^2$$

where we have used the parity invariance relation

$$M_n^{(L)} = (-1)^{n+1}\, M_{-n}^{(L)}.$$

The parameters $m_{|\Delta\lambda|}$, $b_{|\Delta\mu|}$ refer to a possible dependence of the normalisation of the amplitudes on the amount of helicity-flip $\Delta\lambda$, $\Delta\mu$ at the meson and nucleon vertex respectively *) since the data for P_o and D_o, see figs. 1,2 show the strong turn-over in the forward direction which is typical of a dominant n = 1 amplitude we may deduce that b_o and hence the contribution of the n = 0 amplitude to P_o and D_o must be very small.

In order to discuss the qualitative features of our s-channel approach let us now parametrise the $\pi N \to \rho N$ n = 1 amplitude by the simple form **)

$$M_1(s,t) = \lambda_1 \sqrt{-t}\, e^{at}/(t-\mu^2). \tag{7}$$

Then using the above b-universality prescription we obtain

$$\begin{aligned} M_o(s,t) &= \lambda_o/a\ e^{at}/(t-\mu^2) \\ M_2(s,t) &= \lambda_2 at e^{at}/(t-\mu^2) \end{aligned} \tag{8}$$

and hence $P_{1-} = \dfrac{\lambda_o}{a}\ \dfrac{e^{at}}{t-\mu^2}\ (1-\dfrac{\lambda_2}{\lambda_o}\ a^2t)$ (9)

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*) This is in addition to the dependence on the total net helicity n given by the parameters λ_n in eq. (4).

**) For simplicity we shall assume in this discussion that the energy is sufficiently high so that t_{min} is negligible and hence we may equate t and t'.

It will be noticed immediately that now the n = 0 amplitude does not have the form $t\, e^{at}/(t-\mu^2)$ as would be the case for elementary pion exchange (for which parity conservation requires the equality of the n = 0 and n = 2 amplitudes). Instead we have

$$M_o = \frac{\lambda_o}{a} \frac{e^{at}}{t-\mu^2} = \frac{\lambda_o\, e^{at}}{a\mu^2} \left(\frac{t}{t-\mu^2} - 1\right)$$

That is to say the b-universality hypothesis predicts an absorptive cut correction of O(1) as required by the data and as prescribed by the William's "poor man's absorption model" [9]. Furtherore we see from eq. (9) that P_{1-} has a zero at

$$t = t_o = \frac{\lambda_o}{\lambda_2 a^2} \tag{10}$$

In terms of the poor man's absorption model, if we put

$$M_o = e^{at}\left(\frac{t}{t-\mu^2} - C\right)$$

then

$$t_o = -\frac{C\mu^2}{2-C} \tag{11}$$

This will also be the case for D_1. The data shown in figs. 1,2 indicate that the zero in P_1 is at $t_o \approx -\mu^2$ corresponding to $C \approx 1$, while in D_{1-}, $t_o \approx -1/2\ \mu^2$ corresponding to $C \approx 2/3$ [10-12].

Hence, for some unexplained reason in the Williams model, the absorptive cut effect has to decrease in strength in going from ρ to f^o production. In the present b-universality scheme, on the other hand, we see from eq. (10) that this shift in the value of t_o can be accomplished either by an increase in the relative size of the n = 2 amplitude, or by an increase in the slope, a, of the t-distribution.

In fact it has recently been suggested by Michael [13] that the Regge pole contributions to the production of specific spin states such as ρ and f^o resonances, may be simply connected by dual boosts. Specifically it was calculated that

$$M^{Regge}_{\pi N \to f^o N} = M^{Regge}_{\pi N \to \rho N} \left[m_f^2 + 2\mu^2 + (t-\mu^2)(2-1/\alpha' m_f^2)\right] / \left[m_f^2 - 4\mu^2\right]$$

$$\approx e^{0.9t}\, M^{Regge}_{\pi N \to \rho N} \quad \text{for } -t < 0.5\,(\text{GeV/c})^2$$

i.e., the slope parameter, a, in eq. (7) should be increased by 0.9 units from its value of 4.51 for ρ production to 5.41 for f^o production. Inserting these values in eq.(10) we find that if $t_o = -\mu^2$ for ρ production then $t_o \approx -.69\ \mu^2$ for f^o production with the same values of λ_o and λ_2.

Hence the decrease in the strength of the absorptive cut may be interpreted as a simple consequence of the dual-boosts describing the Regge pole production of Regge recurrences.

In previous studies [11,12] of these pion exchange reactions it has been pointed out that whereas the n = 1 amplitude for ρ production may be a pure Regge pole, it must develop a non-evasive absorptive cut correction to describe f^o production. That is to say it behaves like $\sqrt{-t}\ e^{a_f t}/(t-\mu^2)$ instead of having the form $(-t)^{3/2}\ e^{a_f t}/(t-\mu^2)$ which is required by parity conservation and the resulting equivalence of the $M^{\Delta\lambda\ =\ 2}_{\Delta\mu\ =\ 1}$ and $M^{\Delta\lambda\ =\ -2}_{\Delta\mu\ =\ 1}$ pole amplitudes. However, in the present s-channel approach this onset of an absorptive correction is given very naturally by assuming the structure of the n = 1 amplitude for f^o production is the same as for ρ production except for the slight change in the slope as explained above.

In figs. 1,2 we see that our modified b-universality prescription plus the simple form, eq.(7) for $M_1(s;t)$ not only explains the qualitative features of the data, but also provides an extremely good quantitative description. The parameters corresponding to these fits are given in table 1.

We also show in figs 1,2 the description of the data obtained using eq. (7) and the "old" b-universality hypothesis, i.e., using the derivative relations of eq. (4) for the whole amplitude such that

$$M_o = e^{a\mu^2}\ \mathrm{Ei}\ (a(t-\mu^2))$$
$$M_2 = ate^{at}/(t-\mu^2) - te^{at}/(t-\mu^2)^2, \text{ etc.} \tag{12}$$

where Ei(x) is the exponential integral function. The quantities P_o and D_o are still well described, of course, since they are given predominantly by the n = 1 amplitude of eq. (7) which is used in both methods. The other quantities however, involving M_o, M_2 and M_3 are now found to be very poorly described *)

Finally since there is growing evidence to suggest that the pion lies on a Regge trajectory with a typical slope of $\sim 0.9\ (\mathrm{GeV}/c)^{-2}$ just like the vector and tensor mesons, we have repeated the fit using our modified b-universality hypothesis and the Regge pole parametrisation of M_1 suggested by ref [11] i.e.

$$M_1 = \lambda_1 \sqrt{-t'}\ \frac{e^{at}\, e^{-i\frac{1}{2}\pi\alpha_\pi(t)}}{(t-\mu^2)} \left(\frac{P_{Lab}}{17-2}\right)^{\alpha_\pi(t)-1} \tag{13}$$

and we adopted the "usual" [4], [6] couplex parametrisation for $\lambda_n(s)$:

$$\lambda_n(s) = \lambda_n [\log\left(\frac{s}{s_o m_R^2}\right) - i\pi/2]^{1-n} \tag{14}$$

where m_R is the ρ or f^o resonance mass. The resulting fits to the data in this case are practically indistinguishable from those given by eqs. (7),(8), so we do not show them here. The parameters for these fits are again given in table 1.

It is interesting to note that in both of the good fits to the data, λ_2/λ_o does not decrease,(it is in fact constant),as the dipion mass increases. This result is in agreement with the universal limiting strength hypothesis of refs. [2],[5],[6]. However, while the t-dependence of the amplitudes is determined only by the total net helicity flip n, it would seem an important result that the normalisation of the helicity amplitudes is found to depend also on $|\Delta\lambda|$, $|\Delta\mu|$ the amount of helicity flip at each vertex.

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*) Of course this result depends on the assumption that M_1 is given by eq. (7). Other choices for M_1 in principle might give rather different fits. However, these high statistics data on P_{1-} do not allow much freedom in the modulus of $|M_1|$. Moreover it seems unlikely that any reasonable t-dependent phase could significantly improve the fit- particularly for D_2 which includes a strong triple pole term.

In conclusion therefore let us stress again that this simple s-channel approach provides a very natural explanation for the most puzzling features of the vector and tensor production data, namely the presence and relative strengths of the absorptive Regge cut effects. Moreover, the modified form of the b-universality hypothesis used here is, in many ways simpler than the old hypothesis of ref [4]. It assumes that while the impact parameter profiles still behave like b^n near $b = 0$. they are independent of this kinematic dependence on n for very large values of b. To illustrate this we show in fig. 3 the b-profiles corresponding to the amplitudes of eqs. (7),(8). It is amusing to not that this (simpler) form of the universal impact parameter hypothesis is closely related to the form originally proposed in ref. [2] for inelastic diffraction dissociation.

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TABLE 1

fit	reaction	$m_o b_o \lambda_o$	$m_o b_1 \lambda_1$	$m_1 b_1 \lambda_o$	$m_1 b_1 \lambda_2$	$m_2 b_1 \lambda_1$	$m_2 b_1 \lambda_3$	s_o
modified b- universality of eqs.(7),(8)	$\pi N \to \rho N$	0.	6.3	0.6	-2.07	-	-	-
	$\pi N \to f^o N$	0.	9.4	1.0	-3.45	0.16	-0.59	-
modified b- universality with Regge phases of eqs.(13),(14)	$\pi N \to \rho N$	0.	6.3	0.18	-0.064	-	-	0.0067
	$\pi N \to f^o N$	0.	9.4	0.15	-0.053	0.84	-0.35	0.0067
unmodified b-universality of eqs.(7),(12)	$\pi N \to \rho N$	0.	6.3	23.6	-3.00	-	-	-
	$\pi N \to f^o N$	0.	9.4	15.9	-1.87	4.6	0.18	-

Normalisation parameters of the various fits described in the text. In all fits the slope parameter, a, was taken to be 4.51 and 5.41 $(GeV/c)^{-2}$ for ρ and f^o production respectively.

FIGURE CAPTIONS

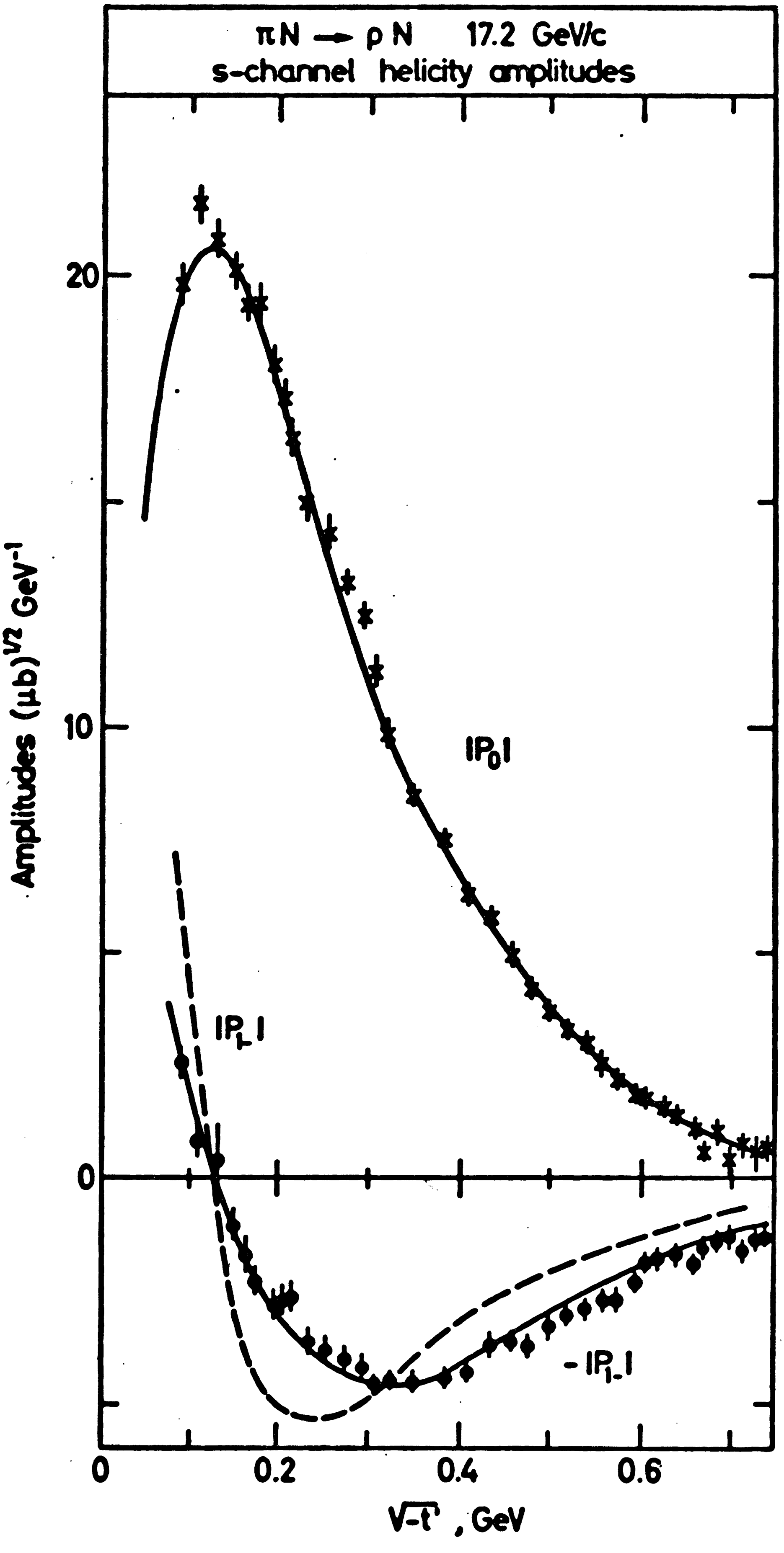
Fig. 1 $\pi^- p \to \rho n$ s-channel helicity amplitudes P_o and P_{1-} of refs [8],[11]. The solid and dashed curves represent the fit to these data using eq. (7) and the modified and unmodified b-universality hypothesis, eqs (8) or eqs (12) respectively. The description of P_o is identical in both methods.

Fig. 2 $\pi^- p \to f^o n$ s-channel helicity amplitudes of refs [8] [11]. The solid curve represents the fit to these data using the modified b-universality hypothesis of eqs (7), (8). The dashed curves are the description of the D_{1-}, D_{2-} amplitudes provided by the unmodified hypothesis eqs (7),(12).

Fig. 3 Impact parameter profiles for helicity amplitudes with quantum numbers appropriate for pion exchange:

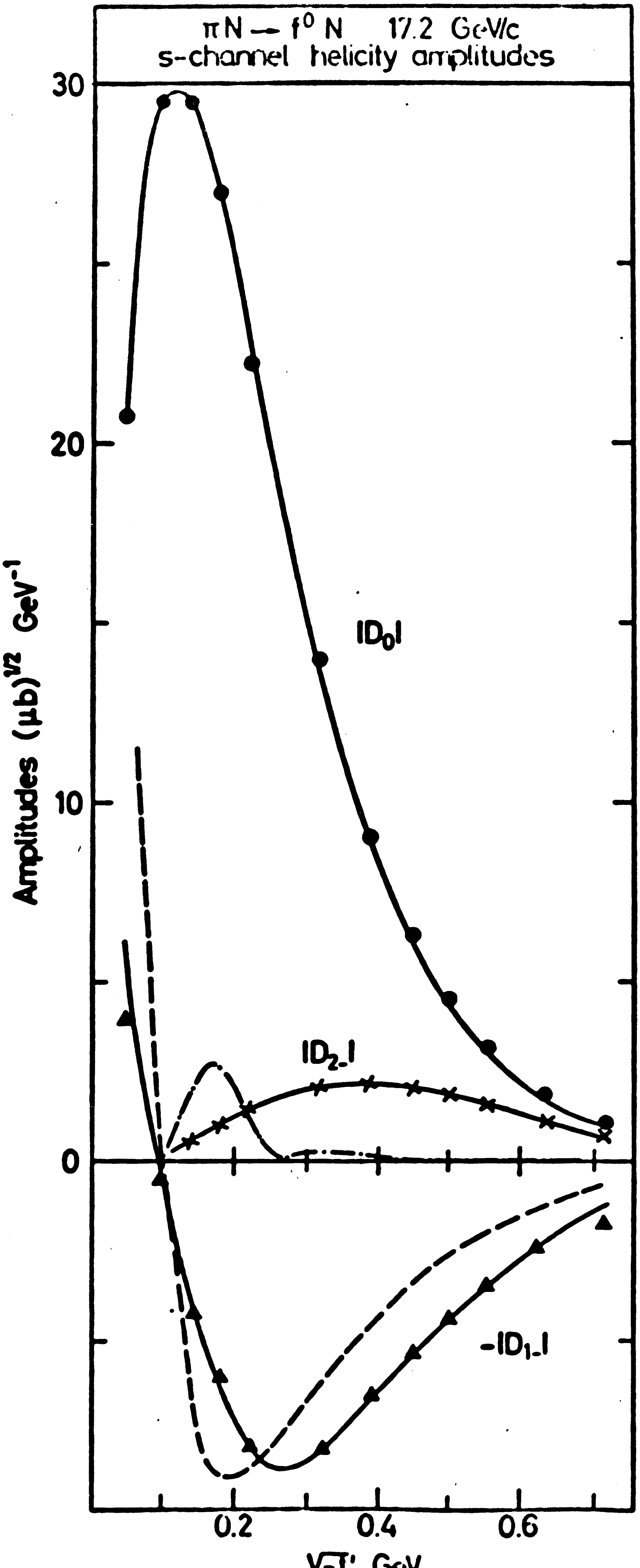
a) determined from eq. (7) and the modified b-universality hypothesis i.e., eq (8).

b) given by the old b-universality hypothesis of eq. (2). The normalisation is arbitrary.

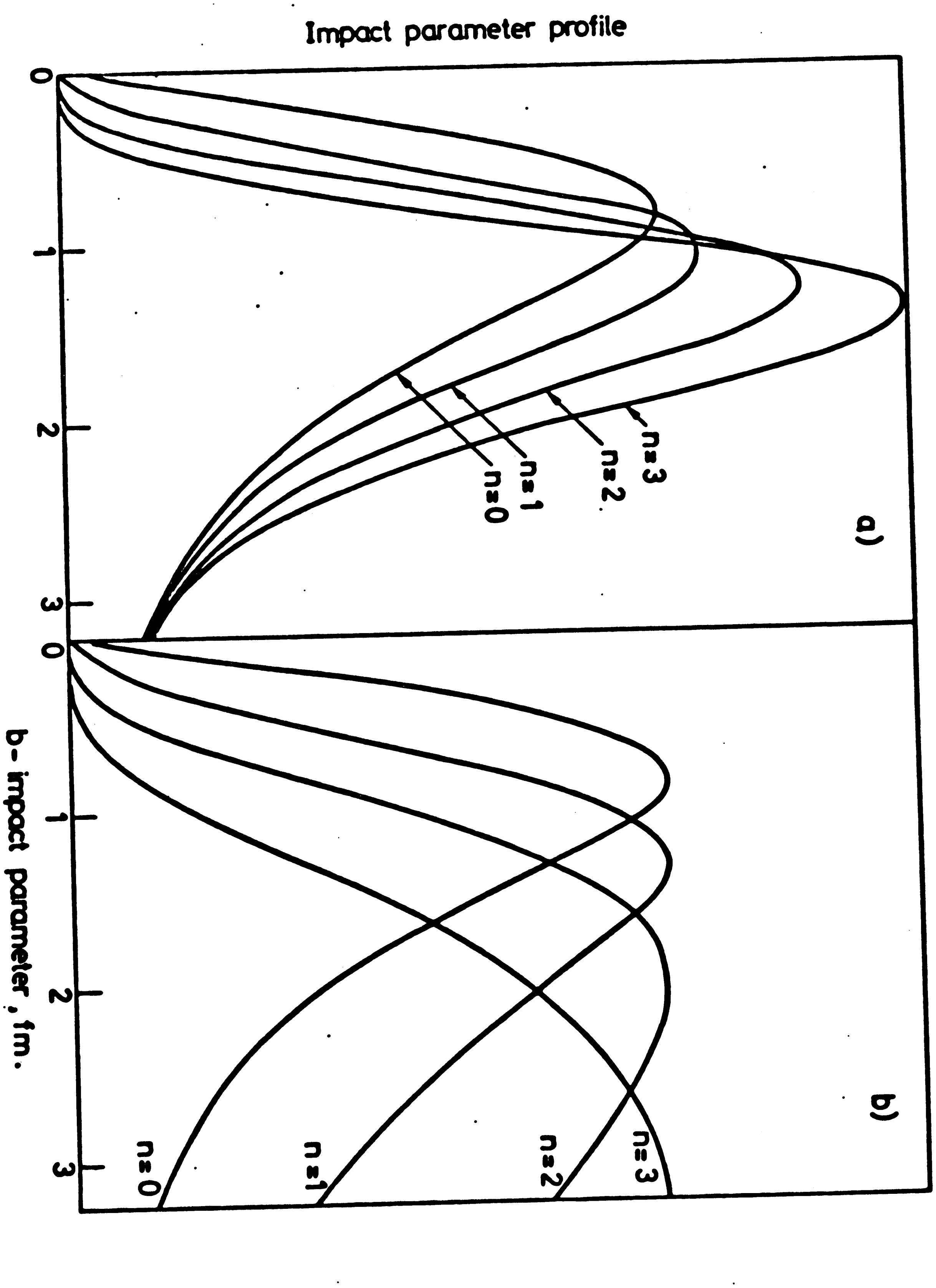


πN → ρN 17.2 GeV/c
s-channel helicity amplitudes
Amplitudes (μb)^1/2 GeV^-1
20
10
0
|P0|
|P1-|
-|P1-|
0
0.2
0.4
0.6
√-t' , GeV

fig.1



πN → f⁰N 17.2 GeV/c
s-channel helicity amplitudes
Amplitudes (μb)^1/2 GeV^-1
30
20
10
0
|D0|
|D2-|
-|D1-|
0.2
0.4
0.6
√-t', GeV



Impact parameter profile
a)
b)
n=0
n=1
n=2
n=3
0
1
2
3
b- impact parameter, fm.

fig.3