

## $D_s$ Lifetime, $m_b$ , $m_c$ and $|V_{cb}|$ in the Heavy Quark Expansion

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### Abstract

We present some straightforward applications of the QCD heavy quark expansion, stated in previous papers [1-3], to the inclusive widths of heavy flavour hadrons. We address the question of the  $D_s$  lifetime and argue that – barring Weak Annihilation (WA) –  $\tau(D_s)$  is expected to exceed  $\tau(D^0)$  by several percent; on the other hand WA could provide a difference of up to 10 ÷ 20% of *any* sign. We extract  $m_c$ ,  $m_b$  and  $|V_{cb}|$  from  $\Gamma_{SL}(D^+)$  and  $\Gamma_{SL}(B)$ . The values of the quark masses are somewhat higher, but compatible with estimates from QCD sum rules; we obtain  $|V_{cb}| \simeq 0.043$  for  $\tau(B) = 1.4$  psec and  $BR_{SL}(B) = 10.5\%$ . We discuss the associated uncertainties in the  $1/m_Q$  expansion as well as some consequences for other electroweak decays.

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Our theoretical understanding of both exclusive and inclusive heavy flavour decays has improved considerably over the last few years. This progress has been driven largely by treatments that involve expanding weak transition amplitudes in terms of  $1/m_Q$ ,  $m_Q$  being the mass of the heavy flavour quark, i.e.  $m_Q = m_{\text{beauty}}, m_{\text{charm}}$ . In this note we will analyze some phenomenological consequences that such a general treatment has for inclusive decays of beauty and charm hadrons.

The paper will be organized as follows : in Sect.1 we recapitulate briefly the salient features of our method which in Sect.2 is then applied to the lifetime of  $D_s$  mesons; in Sect.3 we extract the size of  $m_c$  from the semileptonic  $D$  width and infer the value of  $m_b$ ; the value of  $|V_{cb}|$  is then determined from the semileptonic  $B$  width. In Sect.4 we present our conclusions.

## 1 General Method

On very general grounds one expects that the spectator ansatz correctly describes inclusive heavy flavour decays for asymptotically large values of  $m_Q$ ; yet at finite  $m_Q$  there arise nonperturbative corrections to transition rates that among other things generate different lifetimes for the various types of hadrons  $H_Q$  carrying the flavour  $Q$ . It was shown in refs. [1-5] how these preasymptotic effects can be incorporated in a self-consistent and systematic way; this will be exemplified now for the case of charm lifetimes.

The width for the weak decay of a charmed hadron into an inclusive final state  $f$  is obtained from the transition operator that has been expanded into a series of local operators <sup>1</sup>:

$$\hat{\Gamma} = \frac{G_F^2 m_c^5}{192\pi^3} |KM|^2 \left\{ c_0(f) \bar{c}c + \frac{c_2(f)}{m_c^2} \bar{c}i\sigma Gc + \Sigma_i \frac{c_3(f)}{m_c^3} (\bar{c}\Gamma_i q)(\bar{q}\Gamma_i c) + \mathcal{O}(1/m_c^4) \right\} \quad (1)$$

where the dimensionless coefficients  $c_i(f)$  depend on the parton level characteristics of  $f$  and on the ratios of masses of the final state quarks to the mass of the  $c$  quark;  $KM$  denotes the appropriate product of the weak mixing angles.

The operators that appear on the right hand side of eq.(1) represent a rather universal cast, e.g. they control semileptonic as well as nonleptonic transitions and also distributions like semileptonic spectra:

- the scalar density  $\bar{c}c$  describing the ‘quasifree’ decay of the  $c$  quark;
- the chromomagnetic dipole operator  $\bar{c}i\sigma Gc$ ;
- the four-fermion operator  $(\bar{c}\Gamma q)(\bar{q}\Gamma c)$ . It actually contains terms differing in both the flavour of the light quark  $q$  and the particular Lorentz structure of the  $\gamma$ -matrices  $\Gamma_i$ .

The widths for  $D_s$ ,  $D^0$ ,  $D^+$  and  $\Lambda_c$  decays are then given by the diagonal matrix elements of the operator  $\hat{\Gamma}$  from eq.(1) between the corresponding states  $D_s$ ,  $D^0$ ,  $D^+$  and  $\Lambda_c$ . These matrix elements depend on long distance bound state dynamics; thus they cannot be evaluated perturbatively. Yet this aspect of the strong interaction

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<sup>1</sup>This expansion is based on a sufficiently large energy release. It is not applicable for beauty decays into  $\tau$  leptons, whose partial width is to be calculated explicitly. It is, however, the relevant approach to other annihilation-induced semileptonic widths [5].

cannot be ignored, for even the matrix element of the ‘quasifree’ operator  $\bar{c}c$  contains nonperturbative effects, giving rise to some nontrivial width splitting:

$$\langle H_c | \bar{c}c | H_c \rangle = 2M_{H_c} + \frac{1}{4m_c^2} \langle H_c | \bar{c}i\sigma Gc | H_c \rangle - \frac{1}{2m_c^2} \langle H_c | \bar{c}(i\vec{D})^2c | H_c \rangle + \mathcal{O}(1/m_c^3) \quad (2)$$

with  $D_\mu$  denoting the covariant derivative; a relativistic normalization has been employed here. In addition to the chromomagnetic operator another operator appears now, namely  $\bar{c}(i\vec{D})^2c$  which describes the kinetic energy of the charm quark in the gluon background field.

The size of the matrix element of the chromomagnetic operator between the heavy flavour mesons can be extracted from the hyperfine spin splitting between the pseudoscalar and vector states:

$$\frac{1}{2M_c} \langle D | \bar{c}i\sigma Gc | D \rangle \simeq \frac{3}{2} \cdot (M_{D^*}^2 - M_D^2) \quad . \quad (3)$$

For baryons it vanishes:

$$\langle \Lambda_c | \bar{c}i\sigma Gc | \Lambda_c \rangle \simeq 0 \quad . \quad (4)$$

The following general conclusions can be drawn from this approach:

- The leading nonperturbative corrections arise on the  $1/m_c^2$  level.
- There are two distinct sources of such  $1/m_c^2$  corrections: (i) The matrix elements of the  $d = 5$  operator  $\bar{c}i\sigma Gc$  that appear in eq.(1); its weight depends on which particular channel is considered. (ii) The expansion of the matrix elements of the ‘quasifree’ operator  $\bar{c}c$ , see eq.(2) that affects uniformly all decay modes of a particular hadron.
- These corrections are flavour independent: they do not depend explicitly on the flavour of the spectator antiquark in the meson; they do, however, differentiate between mesons and baryons.
- Order  $1/m_c^3$  corrections are produced by the four-fermion operators  $(\bar{c}\Gamma_i q)(\bar{q}\Gamma_i c)$  describing the nonspectator effects of ‘Pauli Interference’ (PI) and ‘Weak Annihilation’ (WA) or ‘Weak Scattering’ (WS) in baryons. These flavour-dependant operators generate different lifetimes even among the members of the same isomultiplet<sup>2</sup>.

The analysis of  $1/m_c^2$  [1-4] and  $1/m_c^3$  [6, 7, 5] corrections shows that their size is in general quite large in charm decays. For example the chromomagnetic operator produces corrections  $\sim [3(M_{D^*}^2 - M_D^2)]/(2m_c^2) \simeq 0.4$ . Higher order corrections can thus be expected to be still significant and we have to be satisfied with a typically semiquantitative analysis. For example the theoretical predictions  $\tau(D^+)/\tau(D^0) \sim 2$ ,  $\text{BR}_{\text{SL}}(D^+) \sim 16\%$  and  $\text{BR}_{\text{SL}}(D^0) \sim 8\%$  have to be seen as in agreement with the experimental findings within the uncertainties of such a treatment.

## 2 $D_s$ vs. $D^0$ Lifetimes

The lifetimes of  $D^0$  and  $D_s$  mesons could *a priori* differ substantially from each other due to  $SU(3)_{\text{F1}}$  breaking and in particular due to a different weight of WA in

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<sup>2</sup>Similar effects leading to the width splitting of the two  $D^0$  eigenstates are described by the corresponding four fermion operator of the form  $(\bar{c}u)^2$ . Its effect, however, is strongly suppressed and does not shift the central value of the width; we ignore it in our analysis.

$\Gamma(D^0)$  and  $\Gamma(D_s)$ . A previous analysis [7] suggested WA to be smallish already in charm meson decays; furthermore  $1/m_Q$  expansions naturally lead to the prediction of  $SU(3)_{\text{Fl}}$  breaking to be small in heavy flavour decays, namely of the order of several percent only [1]. Thus one predicts

$$\tau(D_s) \simeq \tau(D^0) \quad (5)$$

to first approximation, in agreement with recent E687 data:

$$\frac{\tau(D_s)}{\tau(D^0)} = 1.13 \pm 0.05 \quad (6)$$

A rather delicate analysis is required to go beyond the semi-quantitative prediction of eq.(5) to see whether, indeed, the  $D_s$  lifetime should be slightly longer than the  $D^0$  lifetime, as suggested by present data, and by how much.

There are two main sources for lifetime differences among charm mesons, namely explicitly flavour-dependent  $1/m_c^3$  terms and corrections of order  $m_s \mu_{\text{hadr}}/m_c^2$  due to  $SU(3)_{\text{Fl}}$  breaking in the ‘flavour-independent’  $1/m_c^2$  contributions.

Attempting to estimate relative corrections of a few percent in nonleptonic  $D$  decays is a bold enough enterprise and we will ignore processes contributing less than 1% to the total width. In that category are *e.g.* doubly Cabibbo suppressed transitions  $c \rightarrow d\bar{s}u$  with a relative rate  $\propto \tan^4 \theta_c \simeq 3 \cdot 10^{-3}$  as well as Penguin driven processes; the latter are suppressed by either tiny mixing angles with the b quark or by the small mass of the s quark.

Numerically large effects can arise only from Cabibbo-allowed channels: WA is present in nonleptonic  $D^0$  decays, and in semileptonic as well as nonleptonic  $D_s$  decays; it also drives  $D_s \rightarrow \tau\nu$ . Cabibbo-suppressed modes  $c \rightarrow s\bar{s}u$ ,  $d\bar{d}u$  can produce corrections to the total width of a few percent: WA affects  $D^0$ ,  $D^+$  and  $D_s$  decays; in addition to  $D^+$  PI intervenes also in  $D_s$  modes.

Since the impact of WA as compared to PI is reduced in meson decays [5, 6, 7, 8], it is natural to compare  $\tau(D_s)$  with  $\tau(D^0)$  rather than with  $\tau(D^+)$ . There are four distinct sources for a difference in  $\Gamma(D_s)$  vs.  $\Gamma(D^0)$  exceeding the 1% level:

**(a)** The decay  $D_s \rightarrow \tau\nu$ . **(b)** PI in those Cabibbo suppressed  $D_s$  decays that are driven by the quark level transition  $c \rightarrow s\bar{s}d$ . **(c)** The effects of  $SU(3)$  breaking on the matrix elements of the chromomagnetic and kinetic energy operators. **(d)** WA in nonleptonic  $D^0$  and in nonleptonic as well as semileptonic  $D_s$  decays.

While the corrections listed under (a) and (d) have been discussed extensively in the literature, the corrections referred to under (b) and especially (c) on the other hand are much less familiar.

**(a)** The width for the decay  $D_s \rightarrow \tau\nu_\tau$  is completely determined in terms of the axial decay constant for the  $D_s$  meson:

$$\Gamma(D_s \rightarrow \tau\nu_\tau) \simeq \frac{G_F^2 m_\tau^2 f_{D_s}^2 M_{D_s}}{8\pi} |V_{cs}|^2 (1 - m_\tau^2/M_{D_s}^2)^2 \quad . \quad (7)$$

For  $f_{D_s} \simeq 210$  MeV one gets numerically

$$\Gamma(D_s \rightarrow \tau\nu_\tau) \simeq .03 \Gamma(D^0) \quad . \quad (8)$$

This effect necessarily reduces  $\tau(D_s)$  relative to  $\tau(D^0)$ .

(b) PI in  $D_s$  appears in the  $c \rightarrow s\bar{s}u$  channel. Its weight is expressed in terms of the matrix elements of the four-fermion operators

$$\langle B_s | (\bar{c}_L \gamma_\mu s_L) (\bar{s}_L \gamma_\mu c_L) | B_s \rangle \quad , \quad \langle B_s | (\bar{c}_L \gamma_\mu \frac{\lambda^a}{2} s_L) (\bar{s}_L \gamma_\mu \frac{\lambda^a}{2} c_L) | B_s \rangle \quad (9)$$

with known coefficients that are computed perturbatively, see refs.[6, 7, 5]; the hybrid renormalization [6] of these operators down from the scale  $m_c^2$  has to be included. The most reliable way to estimate the effect of PI, we believe, is to relate it to the observed difference in the  $D^+$  and  $D^0$  widths. Both PI and WA contribute in reality to this difference; yet according to a detailed analysis [7, 8, 5] PI is the dominant effect in mesons (see also the discussion below). It is also worth noting that the size of the PI correction in  $D^+$  as estimated theoretically reproduces the observed width splitting for reasonable values of  $f_D$  – provided the hybrid renormalization of the operators involved is taken into account.

It is easy to see that the structure of the operators responsible for PI in  $D_s$  is exactly the same as in  $D^+$  if one replaces the  $d$  quark by the  $s$  quark and adds the extra factor  $\tan^2 \theta_c$ ; it is then *destructive* as well. From the assumed dominance of PI in the  $D^+ - D^0$  lifetime difference one thus arrives at:

$$\delta\Gamma_{\text{int}}(D_s) \simeq S \cdot \tan^2(\theta_c) (\Gamma(D^+) - \Gamma(D^0)) \simeq -S \cdot 0.03\Gamma(D^0) \quad (10)$$

where the factor  $S$  has been introduced to allow for  $SU(3)$  violation in the relevant matrix elements of the four-fermion operators <sup>3</sup>.

The factor  $S$  is expected to exceed unity somewhat; in the factorization approximation it is given by the ratio  $(f_{D_s}/f_D)^2$ . Various estimates yield the range  $S = 1 \div 1.7$ ; to be more specific we adopt  $S = 1.4$ . Then we conclude that

$$\delta\Gamma_{\text{int}}(D_s) \sim -0.04\Gamma(D^0) \quad . \quad (11)$$

(c) As already stated in Sect.1 the  $1/m_c^2$  corrections are given by the appropriate expectation values of two dimension five operators. As far as the chromomagnetic operator is concerned one has the general expressions:

$$\frac{1}{2M_c} \langle D^0 | \bar{c} i \sigma G c | D^0 \rangle \simeq \frac{3}{2} \cdot (M_{D^{0*}}^2 - M_{D^0}^2) \quad . \quad (12a)$$

$$\frac{1}{2M_c} \langle D_s | \bar{c} i \sigma G c | D_s \rangle \simeq \frac{3}{2} \cdot (M_{D_s^*}^2 - M_{D_s}^2) \quad . \quad (12b)$$

Since the measured values for  $D - D^*$  and for the  $D_s - D_s^*$  mass splittings are almost identical, the chromomagnetic operator cannot be expected to induce an appreciable difference between  $\tau(D_s)$  and  $\tau(D^0)$ .

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<sup>3</sup> $SU(3)_{Fl}$  violation also affects the coefficients of the operators that depend on the mass of the quarks in the final state; those corrections, however, are of the order of  $(m_s/m_c)^2$  whereas the factor  $S$  is linear in  $m_s/\mu_{\text{had}}$ , and we neglect the former.

The observation that the hyperfine splitting is largely independent of the flavour of the spectator can be understood in the following intuitive picture (see ref. [9]): using a simple constituent description one finds that the chromomagnetic field strength is proportional both to the chromomagnetic dipole moment of the spectator (or more generally, of the light degrees of freedom in the  $D$  meson) and to the wavefunction density at the origin,  $1/\langle r \rangle^3$ . The former is most naturally expected to decrease when the (current) mass of the spectator quark increases, whereas the latter is always assumed to increase when going from non-strange to strange particles. Such a behaviour can explicitly be demonstrated at least in the limit when the spectator becomes heavy enough as well <sup>4</sup>. These two effects may thus offset each other. The conclusion about the equal strength of the hyperfine splitting can and of course must be tested in  $B$  mesons where the mass formulae are more reliable due to the larger  $b$  quark mass.

The second operator that generates  $1/m_c^2$  corrections is the kinetic operator  $\bar{c}(i\vec{D})^2c$  which describes a nonrelativistic (“Fermi”) motion of the charm quark. As mentioned above one expects the spatial wavefunction to be more concentrated around the origin for  $D_s$  than for  $D$  mesons. This in turn implies that the mean value of  $\vec{p}^2$  is expected to be larger for  $D_s$  than for  $D$  mesons; in other words the charm quark undergoes more Fermi motion as a constituent of  $D_s$  than of  $D$  mesons. Correspondingly the lifetime of the charm quark is prolonged by time dilation to a higher degree inside  $D_s$  than inside  $D$  mesons. Eq.(2) makes this connection quite explicit [3, 4]: the factor  $1 - \langle \vec{p}^2 \rangle / 2m_c^2$  appearing in the expression for the transition operator is actually nothing but the mean value of the Lorentz factor  $\sqrt{1 - v^2}$  that suppresses the decay probability of a particle in a moving frame.

The qualitative trend of this effect is thus quite transparent. However its numerical size is not, at least not yet. One can expect future progress in QCD sum rules and/or simulations of QCD on the lattice to determine the appropriate matrix elements  $\langle H_c | \bar{c}(i\vec{D})^2c | H_c \rangle$ . Yet for the problem at hand, namely the  $D_s$ - $D^0$  lifetime difference one can in principle extract the relevant matrix element from the measured values of meson masses in the charm and beauty sector according to the following prescription [10]:

$$\begin{aligned} & \frac{1}{2M_D} (\langle D_s | \bar{c}(i\vec{D})^2c | D_s \rangle - \langle D | \bar{c}(i\vec{D})^2c | D \rangle) \simeq \\ & \simeq \frac{2m_b m_c}{m_b - m_c} \{ [\langle M_{D_s} \rangle - \langle M_D \rangle] - [\langle M_{B_s} \rangle - \langle M_B \rangle] \} \end{aligned} \quad (13)$$

where  $\langle M_{D,D_s,B,B_s} \rangle$  denote the ‘spin averaged’ meson masses: e.g. for  $D$  mesons

$$\langle M \rangle_D = \frac{M_D + 3M_{D^*}}{4}$$

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<sup>4</sup>The situation with the chromomagnetic dipole moment could in principle have been more complicated if the nonspectator (gluon) light degrees of freedom tended to form a ground state with a nonzero spin inside a meson; then for an arbitrarily weak interaction of the spectator’s spin the chromomagnetic term would be still finite. This abstract possibility however contradicts the spectrum of states in heavy quarkonia – it would produce an additional hyperfine splitting pattern due to interaction of the heavy spins with the spin of the light degrees of freedom.

and likewise for the other mesons. Accordingly one finds

$$\frac{\Delta\Gamma_{\text{Fermi}}(D_s)}{\bar{\Gamma}} \simeq -\frac{m_b}{m_c(m_b - m_c)} \{[\langle M_{D_s} \rangle - \langle M_D \rangle] - [\langle M_{B_s} \rangle - \langle M_B \rangle]\} \quad . \quad (14)$$

A 10 MeV shift in any of the ‘spin averaged’ mass terms  $\langle M \rangle$  in eq.(14) corresponds numerically to the kinetic energy operator generating approximately a 1% change in the ratio  $\tau(D_s)/\tau(D^0)$ . The meson masses have been measured with an accuracy of 2 MeV or better, which is sufficient for our analysis – with the exception of the  $B_s$ - $B_s^*$  sector. Very recent LEP/CDF data [11] indicate that the  $SU(3)$  mass splitting in the beauty sector is practically the same as in the charm sector, namely  $M_{B_s} - M_{B^0} \simeq (94.5 \pm 4.6 \text{ MeV})$  vs.  $M_{D_s} - M_D \simeq (99.5 \pm 0.7 \text{ MeV})$ . Nothing is known experimentally about the  $B_s^*$  mass; on the other hand retaining only the leading non-trivial term in a heavy quark expansion, one would conclude that the hyperfine splitting in the  $B_s$ - $B_s^*$  system be the same as in the  $B_d$ - $B_d^*$  – simply because the analogous equality holds for charm mesons. Combining the preliminary measurement of the  $B_s$  mass with these *theoretical* expectations about the  $B_s^*$  mass would lead to the rather surprising result that the mean momentum of the heavy quark is practically the same in strange and non-strange heavy-flavour mesons; the Fermi motion of the charm quark could then cause a difference in  $\tau(D_s)$  vs.  $\tau(D^0)$  of at most 1%.

This conclusion can be confronted with a general expectation: it is natural to expect the value of  $\langle \vec{p}^2 \rangle$  to be at least of order of  $(400 \text{ MeV})^2$  even in non-strange heavy mesons<sup>5</sup>; this by itself would suppress the width in charm by about 3% and increase the mass of the meson by 50 MeV. Assuming that  $SU(3)$  violation increases  $\langle \vec{p}^2 \rangle$  by a moderate factor 1.5 in the strange system one would then expect at least a 2% decrease in the width of  $D_s$ .

We think that this oversimplified yet transparent line of reasoning cannot be ruled out yet; it would seem quite premature to conclude that the Fermi motion plays a negligible role in the  $D_s$ - $D^0$  lifetime difference. Namely the chromomagnetic field in  $D_s$  and  $D$  may not coincide so closely to predict the  $B_s^*$  mass with the necessary precision. For one has to allow for sizeable corrections to the mass formulae stated above where only the leading terms in  $1/m_b$  and  $1/m_c$  have been retained since  $m_c$  is not much larger than typical hadronic scales. There is no general reason to expect that the mass splitting formulae hold to better than 30% accuracy. Taking into account that the size of the hyperfine splitting in charm mesons constitutes about 140 MeV, such an uncertainty translates into a 3-4% shift in the lifetime ratio, see eq.(14). Therefore we conclude: ( $\alpha$ ) The Fermi motion of the charm quark inside the meson may well prolong  $\tau(D_s)$  by typically a few percent relative to  $\tau(D)$ . ( $\beta$ ) A better determination of the chromomagnetic field can be obtained from the  $B$  system. Therefore a more definite predictions can be made once the mass of the vector meson  $B_s^*$  has been measured and the experimental uncertainty on  $M(B_s)$  has been decreased.

One further comment is in order here. The discussion given above was concerned with nonleading corrections to the mass formulae for charm particles that could be sizeable because of the moderate value of  $m_c$ . Could there be analogous corrections

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<sup>5</sup>A recent QCD sum rule estimate yielded about  $0.5 \div 0.6 \text{ GeV}^2$  [12].

to the expansion of the *widths* of charm particles? In particular there could be a sizeable deviation from the relation  $\langle D_s | \bar{c} i \sigma G c | D_s \rangle \simeq \langle D | \bar{c} i \sigma G c | D \rangle$ ; this would show up as a violation of the prediction  $M_{B_s^*} - M_{B_s} \simeq M_{B^*} - M_B$  that is suggested by a simple extrapolation from charm. This deviation would reflect the weight of non-leading corrections in the charm system and could *a priori* have a significant impact on the width; for the chromomagnetic interaction seems to be more important numerically than the kinetic energy [4]. However we do not expect this to amount to an important effect. For the chromomagnetic operator appears twice, namely in the expansion of the transition operator  $\hat{\Gamma}$ , eq.(1), and in the ‘quasi-free’ operator  $\bar{c}c$ ; there is an almost complete cancellation between the chromomagnetic contributions from the nonleptonic and the two semileptonic channels if one uses for the coefficient  $c_2$  in eq.(1) the expression obtained in ref. [1]. Even allowing for a 30% uncertainty in the coefficient  $c_2$  the shift cannot exceed 1% for each 30 MeV in the hyperfine splitting of  $D$  or  $D_s$ . (A more accurate prediction had to include the next to leading *perturbative* corrections in the coefficient  $c_2$ ; computing them represents a task that is straightforward in principle, although tedious in practice.)

(d) The numerical impact of WA on charm meson lifetimes is the most obscure theoretical item in the analysis. The task becomes even harder for our present analysis addressing a difference in the WA contribution to  $\tau(D_s)$  and to  $\tau(D^0)$ . The uncertainty centers mainly on the question of how much the WA amplitude suffers from helicity suppression. In the valence quark description the answer is easily given to lowest order: the WA rate is suppressed by the ratio  $m_q^2/m_c^2$  where  $m_q$  denotes the largest quark mass in the final state. For a proper QCD treatment one has to use current rather than the larger constituent quark masses, at least for the  $1/m_c^3$  corrections. That would mean that WA is negligible in  $D^0$  decays where the appropriate factor reads  $(m_s/m_c)^2 \lesssim .01$  and *a fortiori* in  $D_s$  decays where only non-strange quarks are present in the final state. (Semi-)Hard gluon radiation cannot circumvent this suppression [7]. For such gluon corrections – when properly accounted for – drive the hybrid renormalization of the corresponding four fermion operators which however preserves their Lorentz structure and therefore does not eliminate the helicity suppression, at least in the leading *log* approximation. A helicity allowed amplitude can be induced only at the subleading  $\alpha_s(m_c^2)/\pi$  level and is thus expected to be numerically insignificant.

On the other hand nonperturbative dynamics can quite naturally vitiate helicity suppression and thus provide the dominant source of WA. These nonperturbative effects enter through nonfactorizable contributions to the hadronic matrix elements. This has been analyzed in considerable technical detail in ref.[5]. The expression for the weak annihilation operator in  $D_s$  as well as its renormalization can be easily obtained from the general expression for the case of  $D^0$  or  $B^0$  (see refs. [7, 5]) by interchanging the colour factors  $c_1$  and  $c_2$ :

$$\hat{\Gamma}_{\text{ann}} \simeq -\frac{G_F^2 |V_{cb}|^2}{6\pi} (p^2 \delta_{\mu\nu} - p_\mu p_\nu) (a_{\text{sing}} (\bar{c}_L \gamma_\mu q_L) (\bar{q}_L \gamma_\mu c_L) + a_{\text{oct}} (\bar{c}_L \gamma_\mu \frac{\lambda^a}{2} q_L) (\bar{q}_L \gamma_\mu \frac{\lambda^a}{2} c_L)) ;$$

$$a_{\text{sing}} = (3c_2^2 + 2c_1 c_2) \mathfrak{x}^{9/2} + \frac{1}{3} c_1^2 - \frac{1}{9} (\mathfrak{x}^{9/2} - 1) (3c_2^2 + 2c_1 c_2) \quad ,$$



$$\begin{aligned}
a_{\text{oct}} &= 2c_1^2 - (\mathfrak{x}^{9/2} - 1)(2c_2^2 + \frac{4}{3}c_1c_2) \quad , \\
c_1 &\simeq \frac{c_+ + c_-}{2} \quad , \quad c_2 \simeq \frac{c_+ - c_-}{2} \quad , \quad q = d \quad \text{for } D^0 \text{ decays} \quad , \\
c_1 &\simeq \frac{c_+ - c_-}{2} \quad , \quad c_2 \simeq \frac{c_+ + c_-}{2} \quad , \quad q = s \quad \text{for } D_s \text{ decays} \quad , \\
\mathfrak{x} &= \left[ \frac{\alpha_s(\mu^2)}{\alpha_s(m_c^2)} \right]^{1/b} \quad , \quad b = 11 - \frac{2}{3}n_f = 9 \quad . \quad (15)
\end{aligned}$$

For each of the two semileptonic channels in  $D_s$  one additionally has

$$a_{\text{sing}} = \frac{8}{9}\mathfrak{x}^{9/2} + \frac{1}{9} \quad , \quad a_{\text{oct}} = -\frac{2}{3}(\mathfrak{x}^{9/2} - 1) \quad . \quad (15a)$$

Numerically in  $D_s$  the overall singlet coefficient  $a_s$  is about 9 and the octet one is  $-2.3$  whereas in  $D^0$  they are  $-0.32$  and  $3.2$  respectively.

The reason behind  $a_{\text{sing}}$  being so much larger for  $D_s$  than for  $D^0$  mesons is that there is a colour-allowed WA contribution to  $D_s$  decays while WA is colour-suppressed in  $D^0$  decays. This colour-allowed contribution is obviously factorizable, however the factorizable part practically vanishes due to helicity suppression. Appreciable effects can then come only from nonfactorizable contributions or from  $\mathcal{O}(\alpha_s(m_c^2)/\pi)$  corrections to the leading log approximation. Naive colour counting rules suggest that nonfactorizable parts in the matrix elements of colour singlet operators are  $1/N_c$  suppressed as compared to those of colour octet operators. This line of reasoning is at best semi-quantitative, but if one adopts it one would conclude that the weight of WA is similar in inclusive  $D_s$  and  $D^0$  decays. As already stated nonleading perturbative corrections are capable of producing helicity unsuppressed contributions even to factorizable matrix elements; yet also they are colour-suppressed.

It was shown in ref. [5] that a detailed experimental study of the semileptonic width and the lepton spectrum, in particular in the endpoint region, in  $D^0$  vs.  $D^+$  and/or in  $B^0$  vs.  $B^+$  decays would allow us to extract size of the matrix elements that control the weight of WA in all inclusive  $B$  and  $D$  decays. Since such data are not (yet) available we can at present draw only a qualitative conclusion: WA is not expected to affect the total lifetimes of  $D^0$  and  $D_s$  mesons by more than  $10 \div 20\%$ , see refs. [8, 13]. Furthermore WA does not necessarily enhance  $\tau(D_s)/\tau(D^0)$ : due to its interference with the spectator reaction it could even reduce it!

To summarize our findings on the  $D_s - D^0$  lifetime ratio:  $SU(3)_{\text{F1}}$  breaking in the leading nonperturbative corrections of order  $1/m_c^2$  can – due to ‘time dilatation’ – increase  $\tau(D_s)$  by  $3 \div 5\%$ . On the  $1/m_c^3$  level there arise three additional effects. Destructive interference in Cabibbo suppressed  $D_s$  decays increases  $\tau(D_s)$  again by  $3 \div 5\%$  whereas the mode  $D_s \rightarrow \tau\nu$  decreases it by  $3\%$ . These three phenomena together lead to  $\tau(D_s)/\tau(D^0) \simeq 1.0 \div 1.07$ . Any difference over and above that has to be attributed to WA. Taking these numbers at face value one can interpret the recent measurement [14] of the  $D_s$  lifetime in turn as more or less direct evidence for WA to contribute not more than 10-20 % of the lifetime ratio between charm mesons. As expected [6, 7], it does not constitute the major effect there. Finally the predictions just stated can be refined by future more accurate measurements, namely

- of the difference in the semileptonic spectra of charged and neutral mesons in the charm and in the beauty sector to extract the size of the matrix elements controlling the weight of WA;

- of  $M(\Lambda_b)$ ,  $M(B_s)$  and  $M(B_s^*)$  to better than 10 MeV to determine the expectation values of the kinetic energy operator.

### 3 Heavy Quark Masses and $|V_{cb}|$ – phenomenological approach

Conventional wisdom has it – based on considering the simplest perturbative diagrams – that semileptonic decays are easier to treat theoretically since they are less affected by the strong interactions. Our analysis of non-perturbative corrections in inclusive heavy flavour decays [1-5] offers additional evidence in support of this conviction. For there are smaller and fewer nonperturbative corrections in semileptonic than in nonleptonic decays of heavy flavour mesons. Even the first,  $\mathcal{O}(\alpha_s)$  perturbative corrections may appear to be quite large in nonleptonic  $b$  decays owing to a relatively large mass of the charm quark in the final state <sup>6</sup>. The semileptonic widths are then best suited to extract fundamental parameters like quark masses and KM angles.

The semileptonic width of the  $D$  meson is given by the expansion

$$\Gamma(D \rightarrow l\nu X) = \frac{G_F^2 m_c^5}{192\pi^3} |V_{cq}|^2 \cdot \left\{ z_0(x) - \frac{2\alpha_s(m_c^2)}{3\pi} (\pi^2 - 25/4) z_0^{(1)}(x) \right. \\ \left. \cdot (1 - K_D/m_c^2 + \frac{1}{4} G_D/m_c^2) - z_1(x) G_D/m_c^2 + \mathcal{O}(\alpha_s^2, \alpha_s/m_c^2, 1/m_c^3) \right\} \quad (16)$$

where the phase space factors  $z$  account for the mass of the quark  $q = s, d$  in the final state:

$$z_0(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x \quad , \quad z_1(x) = (1 - x)^4 \quad , \\ z_0^{(1)}(0) = 1 \quad , \quad z_0^{(1)}(1) = 3/(2\pi^2 - 25/2) \simeq 0.41 \quad , \quad x = (m_q/m_c)^2 \quad (17a)$$

(the function  $z_0^{(1)}$  can be found in ref.[15]), while  $K_D$  and  $G_D$  denote the kinetic energy and the chromomagnetic matrix elements respectively:

$$G = \frac{1}{2M_D} \langle D | \bar{c} i \sigma G c | D \rangle \simeq \frac{3}{2} \cdot (M_{D^*}^2 - M_D^2) \quad , \quad K = \frac{1}{2M_D} \langle D | \bar{c} \frac{(i\vec{D})^2}{2} c | D \rangle \equiv \frac{\langle \vec{p}^2 \rangle}{2} \quad . \quad (17b)$$

It should be noted that the explicit form of the order  $\alpha_s$  perturbative correction in eq.(16) refers to the on shell (pole) definition for the charm quark mass.

The expressions given above allow one to determine the mass of the  $c$  quark from the measured semileptonic width, provided the weak mixing angles  $|V_{cs}|$  and  $|V_{cd}|$  are known. We shall assume in our subsequent analysis that the weak mixing matrix is determined by the existence of only three generations; the quantities  $|V_{cs}|$  and  $|V_{cd}|$  are then known with an accuracy that is more than sufficient for our purposes<sup>7</sup>. Adopting

<sup>6</sup>We are grateful to V.Braun for presenting explicit arguments in favour of this option and to M.Shifman for a discussion of this problem.

<sup>7</sup>There are some direct experimental measurements of these two angles, however they suffer from relatively large uncertainties for  $V_{cs}$ .

for the semileptonic widths of  $D$  mesons the value of  $\Gamma_{\text{SL}} = \text{BR}_{\text{SL}}(D^+) \cdot \tau_{D^+}^{-1} \simeq 1.06 \cdot 10^{-13} \text{ GeV}$  and assuming  $\langle \vec{p}^2 \rangle \simeq .3 \text{ GeV}^2$  we then find

$$m_c = (1.57 \pm 0.03) \text{ GeV}$$

where we have included only the experimental error, coming mainly from  $\Gamma_{\text{SL}}$ . We used here the values  $\alpha_s(m_c^2) = .33$  and  $m_s = 140 \text{ MeV}$ .

The smallness of the error in  $m_c$  quoted above reflects the fact that the width depends on the fifth power of  $m_c$ . The value found for  $m_c$  is then also not very sensitive to details in the nonperturbative corrections to the spectator picture; these theoretical uncertainties will be discussed next.

The perturbative  $\mathcal{O}(\alpha_s)$  term reduces the width in eq.(16) by about 25%; the leading nonperturbative corrections  $\sim \mathcal{O}(1/m_c^2)$  are of comparable size: the chromomagnetic term and the kinetic term yield a reduction by  $\sim 25\%$  and by about 6%, respectively. However all these effects do not generate a prominent change in the value of  $m_c$ : the perturbative corrections increase  $m_c$  by 75 MeV; the chromomagnetic term and the kinetic energy term force  $m_c$  up by 85 MeV and by 20 MeV, respectively, for the stated value of  $\langle \vec{p}^2 \rangle$ . From these numbers we infer that the associated uncertainties in these corrections are rather insignificant. There is some uncertainty concerning the value of  $\alpha_s(M_Z^2)$  and the scale at which to evaluate  $\alpha_s$  in charm decays; yet those effects are quite unlikely to exceed 20% and can be addressed by including  $\mathcal{O}(\alpha_s^2)$  contributions. Potentially larger errors can be expected from the nonperturbative effects. As discussed in the previous section the uncertainty in the chromomagnetic field could conceivably be of order 30% in the charm system. Corrections of similar size can be expected from higher power terms in the  $1/m_c$  expansion. Finally the exact value of the kinetic term is not known. Yet its impact is generally somewhat suppressed as compared to the chromomagnetic interaction, and is dominated by the latter for reasonable sizes [12] of the mean Fermi momentum of the heavy quark; the dependence on the  $\langle \vec{p}^2 \rangle$  is illustrated in Table 1. It is worth adding that for obvious reasons there is no significant dependence of the extracted value for  $m_c$  on  $m_s$  when the latter is varied within reasonable limits for a current quark mass.

Combining all of this we then estimate the present theoretical uncertainty in extracting  $m_c$  to be about 30 MeV; to be conservative one may increase it up to say 50 MeV:

$$m_c = (1.57 \pm 0.03 \pm 0.05) \text{ GeV} . \quad (18)$$

A more detailed understanding of the intrinsic limitations on the accuracy of such approach can be expected [1] in the future from explicit calculations of the higher order corrections, see e.g. refs. [16, 17].

Quite often another definition of the quark mass is used, namely the  $\overline{\text{MS}}$  one corresponding to a Euclidean renormalization point  $q^2 = -m_Q^2$ . At the one-loop level they are related by

$$m_Q^{\overline{\text{MS}}}(-m_Q^2) \simeq m_Q^{\text{pole}} \cdot \left(1 - \frac{4}{3} \frac{\alpha_s(m_Q^2)}{\pi}\right) ; \quad (19)$$

eq.(18) then yields 1.35 GeV for this definition of  $m_c$ .

It might appear at this point that the fifth power dependence of the decay width on the mass of the quark allows one to make a rather accurate extraction of  $m_c$  almost without any detailed information about the nature of nonperturbative corrections. Such a conclusion would however overstate the facts: for its validity rests on the absence of nonperturbative corrections of order  $1/m_Q$  in the total widths, as proven in refs. [1, 2]. This explained *a posteriori* why simple minded estimates made a long time ago that ignored nonperturbative corrections yielded a charm quark mass of around 1.5 GeV. The real shortcoming of these estimates was the following more subtle point: in these models one cannot distinguish between the mass of the charm *quark* and of the charm *hadron* in an unambiguous fashion; not surprisingly the estimates numerically fell somewhere in between. This alternative of course is a reformulation of the problem of nonperturbative  $1/m_Q$  corrections to widths. The QCD approach ensures that if  $m_c$  is understood as the (current) quark mass then these corrections are absent! This is a consequence of the conservation of the colour flow in QCD as can be seen by simple quantum mechanical arguments (see ref. [17] for details).

Having extracted a value for the charm quark mass one can then determine the mass of the beauty quark by employing an expansion of the heavy flavour hadron masses in terms of the heavy quark mass [10]:

$$m_b - m_c = \frac{M_B + 3M_{B^*}}{4} - \frac{M_D + 3M_{D^*}}{4} + \langle \vec{p}^2 \rangle \cdot \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) + \mathcal{O}(1/m_c^3, 1/m_b^3); \quad (20)$$

unfortunately the accuracy of this expansion is obviously controlled by the charm quark mass. Note that the absence of the perturbative corrections on the left-hand side of eq.(20) implies that pole masses are assumed. For the same value of the kinetic term the central value appears to be 5.0 GeV and 4.5 GeV for the pole and the  $\overline{\text{MS}}$  masses of the  $b$  quark, respectively. For such indirectly determined mass of the beauty quark the dependence on the size of  $\langle \vec{p}^2 \rangle$  is more significant as is illustrated by Table 1.

The same consideration fixes also the value of the scale  $\bar{\Lambda}$  that determines the asymptotic mass difference between the mass of the heavy flavour hadron and the mass of the constituent heavy quark. For the family of the lowest lying pseudoscalar and vector mesons one then has  $\bar{\Lambda} \approx 300$  MeV. This value is also correlated with the size of the kinetic term, and the latter seems to represent now the main uncertainty in the value of  $\bar{\Lambda}$ . Note that we have shown in a recent paper [18] that there is no lower bound on  $\bar{\Lambda}$  in contrast to a recent claim [19]; *a priori*  $\bar{\Lambda}$  could have been even negative (see also [17]).

Above we have discussed only the most obvious uncertainties that one encounters in the numerical evaluation of the heavy quark mass. There is a number of additional purely theoretical corrections that in principle can affect them. Those corrections were discussed in some detail in a recent paper [16]; a more comprehensive analysis will be given in a forthcoming publication [17]. We therefore do not dwell on them here and only comment that they are not expected to affect significantly the values extracted for the masses.

Turning to the extraction of the quark mixing parameter  $|V_{cb}|$  we first note that phenomenological studies have shown that the total widths and lepton spectra in

semileptonic  $B$  decays depend mostly on the difference  $m_b - m_c$  rather than on the absolute values of the heavy quark masses. In the framework of the heavy quark expansion these findings are the reflection of the fact that the  $c$  quark is rather heavy even as seen from the scale of the  $b$  quark mass; they can be understood as meaning that the error one makes in using the measured masses of the charm and beauty *hadrons* rather than those of the charm and beauty *quarks* is reduced relative to its natural scale  $1/m_Q$ . This error can actually be reduced even further by using values for the quark masses as they were extracted above from the widths of  $D$  mesons. A value for  $|V_{cb}|$  can then be obtained from the measured semileptonic width.

From a theoretical point of view such an inclusive method has some clear advantages over an extraction of  $|V_{cb}|$  from the exclusive decays  $B \rightarrow D, D^* l \nu$  suggested by the Heavy Quark Symmetry – even when taken at the “gold plated” point of zero recoil [22]. For the accuracy of the symmetry is governed by the *lightest* quark mass in the process, namely  $1/m_c^2$ ; on the other hand the expansion parameter for the inclusive widths is an inverse power of  $m_b$ , and this method can be applied even for light quarks in the final state.

The necessary expression for the width is given by eq.(16) where the obvious substitutions  $b$  for  $c$  and  $c$  for  $s$  are made. Using the values  $\tau_B \simeq 1.4\text{ps}$  and  $\text{BR}_{\text{SL}}(B) \simeq .105$ <sup>8</sup> we arrive at the results shown in Table 1. The main dependence is again due to the kinetic term, however now it is rather weak; the uncertainty associated with the value of  $\alpha_s$  is now smaller owing to the higher energy scale. It is tempting to conclude then that this method provides us at least three times better accuracy for the extraction of  $|V_{cb}|$  than other methods discussed so far.

The values for  $m_c$  and  $m_b$  that we have obtained in this Section are around the upper end of existing estimates. More conventional values emerge if one uses a smaller kinetic energy term of about  $(.4\text{GeV})^2$ . It is worth noting that the theoretical predictions of the QCD sum rules for  $\bar{\Lambda}$  prefer smaller values of about 400 MeV [21]; using this smaller value of  $\bar{\Lambda}$  decreases the predicted value of the kinetic term [12]. On the other hand it is most natural to expect [17] the mean value of the Fermi momentum not to exceed significantly  $\bar{\Lambda}$ . Therefore we think that values for the  $b$  quark mass around 4.95 GeV together with a relatively small scale for the Fermi momenta of the order of  $300 \div 500$  MeV are both selfconsistent and acceptable phenomenologically. This hypothesis can and will be cross-checked in detailed study of spectra in semileptonic and radiative  $B$  decays.

## 4 Conclusion

In this note we have applied a systematic expansion in  $1/m_Q$  that exists for the inclusive widths, to a few phenomenologically interesting issues concerning the properties of charm and beauty mesons. The general method we use has been suggested earlier [1, 2, 6] and refined in subsequent papers [3, 4, 5, 16, 17]. The main object of our analysis was the pattern of the charm meson widths, including the  $D_s$  width.

Generally the size of preasymptotic nonperturbative effects is almost of order unity in charm; this makes it difficult to arrive at conclusions that are both definite and

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<sup>8</sup>For the sake of definiteness we used  $\Gamma_{\text{sl}}(b \rightarrow u) = .01\Gamma_{\text{sl}}(b \rightarrow c)$ .

reliable. The detailed classification of the corrections enable us to conclude nevertheless that the width splitting between  $D_s$  and  $D^0$  must be reasonably small. There are four sources for this difference that can produce effects on the few percent level: 1) A larger Lorentz reduction of the decay probability for the  $c$  quark inside the  $D_s$  meson due to a more rapid Fermi motion. 2) Destructive interference in the Cabibbo suppressed decays of  $D_s$ ; each of these effects are estimated to decrease the width of  $D_s$  by  $3 \div 5\%$ . 3) The mode  $D_s \rightarrow \tau \nu_\tau$  increases it by about 3%. 4) Nonleptonic decays of both mesons are affected by WA which is present also in semileptonic decays of  $D_s$ . According to the analysis of ref. [5] all hadronic parameters governing the strength of WA in different decays can be obtained by a careful comparison of the lepton spectra in semileptonic decays of charged and neutral  $B$  mesons. However such data do not exist for now; therefore the size of WA remains an unknown and – unfortunately – the potentially largest effect numerically in  $\tau(D_s)/\tau(D^0)$ .

Other lines of reasoning suggest that the impact of WA on inclusive charm decays can hardly be large: presumably it does not exceed  $10 \div 20\%$  of the total  $D^0$  width; furthermore it can be of either sign! We see therefore that the data on the lifetimes of charm mesons fit reasonably well the theoretically expected pattern. On the other hand the existing experimental determination of the  $D_s$  lifetime can be viewed as some phenomenological constraint on the non-factorizable matrix elements of the four-fermion operators in charm mesons:

$$\left| \frac{f_{D_s}^2}{f_D^2} (g_{\text{singl}}^{(D_s)} - 0.25g_{\text{oct}}^{(D_s)}) + 0.03g_{\text{singl}}^{(D)} - 0.3g_{\text{oct}}^{(D)} \right| \lesssim 0.01 \cdot \left( \frac{200 \text{ MeV}}{f_D} \right)^2 \quad (21)$$

where we have used the notations introduced in eq.(16) of ref. [5]. Of course the extremely small number in the *rhs* of eq.(21) should not be taken too literally.

The theoretical prediction for the  $D_s$  lifetime can be further clarified by accurate measurements of the masses of  $B_s$  and  $B_s^*$  masses on the one hand and on the other a better understanding of the scale of Fermi motion in heavy hadrons which can be obtained from an accurate analysis of spectra in semileptonic and/or radiative decays of beauty particles.

Similar preasymptotic corrections splitting the widths of beauty particles are expected to be essentially smaller [7]. A careful analysis of the QCD corrections [23, 7] has lead to the observation that the width difference between the two mass eigenstates in the  $B_s$ - $\bar{B}_s$  sector quite probably represents the largest numerical difference in the family of beauty mesons:

$$\frac{\Delta\Gamma_{B_s}}{\bar{\Gamma}} \simeq .15 \frac{f_{B_s}^2}{(200 \text{ MeV})^2} \quad (22)$$

This estimate is valid for  $f_{B_s}$  – which acts as an expansion parameter – not too large. To a very good approximation one can identify the two mass eigenstates as  $CP$  eigenstates. Obviously the upper bound for the width difference is reached when all final states in the decay channel  $b \rightarrow c\bar{c}s$  for the decays of  $B_s$  have the same  $CP$  parity and  $|\Delta\Gamma_{B_s}| \simeq 2\Gamma(b \rightarrow c\bar{c}s)$ . The rough estimate for this partial decay width is given by the parton expression and constitutes about 20%, therefore the estimate eq.(22) is sensible up to  $f_{B_s} \lesssim 300 \text{ MeV}$ .

A straightforward, in principle, application of a  $1/m_Q$  expansion of inclusive decay widths [1, 2] is the determination of the mass of charm quark from the semileptonic  $D$  width, then obtaining the  $b$  quark mass from the mass formulae and subsequently extracting  $|V_{cb}|$  from the  $B$  meson semileptonic width. In charm non-perturbative effects dominate the more familiar perturbative corrections: the former constitute about 35% whereas  $\mathcal{O}(\alpha_s)$  corrections yield approximately 25%. None of these however produce a significant change in the resulting value of  $m_c$  owing to the fifth power dependence of the decay width on the mass. In fact this conclusion is a consequence of the non-trivial fact that there are no corrections of order  $1/m_c$  to the heavy flavour widths, as shown in refs. [1, 2]; this is a peculiar feature of the gauge nature of the strong interaction producing the bound state in the initial state and driving hadronization dynamics in the final states, and it reflects the conservation of the colour current. A reasonable estimate for the theoretical uncertainty of the value of the  $c$  quark is about 50 MeV and one can count on it being decreased in the future.

The masses of  $b$  and  $c$  quarks obtained in such a way naively seem to be some 100 MeV higher than the conventional values inferred from the direct QCD analysis, and in particular from estimates based on charmonium sum rules. In fact a careful analysis<sup>9</sup> undertaken recently suggested a rational explanation of this difference in the framework of QCD. We will report on it in the forthcoming paper [17].

Using the observed semileptonic width of  $B$  mesons and including the leading corrections to the parton formulae one obtains the value of the KM mixing angle defining the strength of the  $b \rightarrow c$  transitions:

$$|V_{cb}| \simeq 0.043 \quad .$$

This method seems to be the most accurate and reliable way to obtain the value of  $|V_{cb}|$ . For its accuracy is governed by powers of  $1/m_b$  as confronted to the decays into exclusive final state  $D$  and  $D^*$  [22] where actually the  $c$  quark sets the mass scale for the corrections to the Heavy Spin-Flavour Symmetry. The corrections to the inclusive decays have been calculated explicitly through order  $1/m_b^3$  and in principle the expansion can be extended, whereas  $1/m_c^2$  effects for the exclusive decays [24] may already constitute the limiting factor for improving the estimates.

Our expressions for the semileptonic decay widths can be easily translated into the semi-phenomenological parameters  $z_c$  and  $z_u$  that were introduced long ago to account for the final quark mass suppression in inclusive semileptonic widths as well as for all other possible corrections to the parton formulae. Their usual definition was

$$\Gamma_{\text{SL}}(b \rightarrow q) = z_q \cdot \frac{G_F^2 (5 \text{ GeV})^5}{192\pi^3} |V_{qb}|^2 \quad , \quad q = u, c \quad . \quad (23)$$

For the thus defined factor  $z_c$  we get values varying from .36 at small  $\langle \vec{p}^2 \rangle$  to .43 for  $\langle \vec{p}^2 \rangle \simeq 0.8 \text{ GeV}^2$ ; these values seem to agree with the estimates obtained from experiment using fits based on some phenomenological models [25] of a heavy quark decay. The ratio  $z_u/z_c \simeq 2.08$  appears to be independent of the scale of the Fermi motion, which also reproduces the expectations that one inferred from those models.

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<sup>9</sup>It was elaborated in joint discussions with M.Shifman and A.Vainshtein; we are grateful for their permission to mention it prior to publication of that result.

The phenomenological extraction of  $m_c$  and  $m_b$  enables one to determine the hadronic parameter  $\bar{\Lambda}$  which in the nonrelativistic description of a heavy hadron plays a role of the constituent mass of the spectator(s). As was stated in ref. [5]  $\bar{\Lambda}$  actually completely defines the leading,  $1/m_Q$ , non-perturbative shift in the average invariant mass of the final hadrons in semileptonic or radiative decays. In the quasi two particle decays like  $b \rightarrow s + \gamma$  the correction is given by  $\delta m_{\text{np}}^2 \simeq \bar{\Lambda} m_b + \mathcal{O}(\mu_{\text{hadr}}^2)$  if one neglects the mass of strange quark. We see that the average final state hadronic mass is increased by some  $1.5 \div 2 \text{ GeV}^2$  as compared to a perturbative estimate. We note that once again the leading nonperturbative effects seem to be at least comparable in size to the perturbative corrections [26] even in beauty decays. Accordingly they have to be included in a proper quantitative treatment; together with the effect of the Fermi motion the  $\bar{\Lambda}$  parameter describes the significant  $1/m_b$  corrections to the spectrum of photons, which in turn define the relative weight low lying exclusive final states can command in such decays.

*Note added:* While this paper was written up we become aware [27] of the work of ref.[28] which have a significant overlap with the part concerning the determination of quark masses and the mixing parameter. At that point we present a more detailed discussion of nonperturbative corrections as they are relevant for extracting  $m_c$ ,  $m_b$  and  $|V(cb)|$ . The two treatments thus complement each other. As was mentioned in Section 3 we also believe that using the upper bound of Guralnik and Manohar – that constitutes an important element of the analysis of paper [28] – is irrelevant [18] in this context.

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## References

- [1] I.I.Bigi, N.G.Uraltsev, A.Vainshtein, *Phys.Lett.* **293B** (1992) 430.
- [2] B.Blok, M.Shifman, *Nucl.Phys.* **B399** 441; 459.
- [3] I.I.Bigi, B.Blok, M.Shifman, N.G.Uraltsev, A.Vainshtein, *Proc. of the 1992 DPF meeting of APS*, Fermilab, November 1992; **preprint** UND-HEP-92-BIG07, TPI-MINN-92/67-T.
- [4] I.I.Bigi, M.Shifman, N.G.Uraltsev, A.Vainshtein, *Phys.Rev.Lett.* **71** (1993) 496.
- [5] I.I.Bigi, N.G.Uraltsev, **preprint** UND-HEP-93-BIG02, September 1993; to be published in *Nucl.Phys. B*.
- [6] M.A.Shifman, M.B.Voloshin, *Sov.Journ.Nucl.Phys.* **45** (1987) 292; *Sov.Phys.-ZhETF* **64** (1986) 698.
- [7] I.I.Bigi, N.G.Uraltsev, *Phys.Lett.* **280B** (1992) 120.
- [8] B.Blok, M.Shifman, *Yad.Phys.* **45** (1987) 841.
- [9] J.Rosner, M.B.Wise, **preprint** CALT-68-1807, EFI-92-40, 1992.
- [10] I.I.Bigi, N.G.Uraltsev, A.Vainshtein, **preprint** UND-HEP-92-BIG04, TPI-MINN-92/30-T, FNAL-PUB-92/158-T; ref.[1].
- [11] See W.Venus, Rapporteur's talk at the XVI International Symposium on Lepton-Photon Interactions, Cornell, August 1993.
- [12] P.Ball, V.M.Braun, **preprint** MPI-Ph/93-51, July 1993.
- [13] V.M.Braun, private communication; N.U. is gratefull to him for making this QCD sum rule estimate.
- [14] See M.Witherell, Rapporteur's talk at the XVI International Symposium on Lepton-Photon Interactions, Cornell, August 1993.
- [15] N.Cabbibo, L.Maiani, *Phys.Lett.* **79B** (1978) 109; original calculation was in: R.E.Behrends et al., *Phys.Rev.* **101** (1956) 866.
- [16] I.I.Bigi, B.Blok, M.Shifman, A.Vainshtein, **preprint** CERN-TH.7082/93, November 1993.
- [17] I.I.Bigi, M.Shifman, N.G.Uraltsev, A.I.Vainshtein, *in preparation*.
- [18] I.I.Bigi, N.G.Uraltsev, **preprint** UND-HEP-93-BIG07, CERN-TH.7091/93, November 1993; to be published in *Phys.Lett. B*.
- [19] Z.Guralnik, A.V.Manohar, *Phys.Lett.* **302B** (1993) 103.
- [20] I.I.Balitsky, *Phys.Lett.* **273B** (1991) 282; I.Balitsky, M.Beneke, V.Braun, **preprint** MPI-Ph/93-62, September 1993.

- [21] E.Bagan, P.Ball, V.M.Braun, H.G.Dosch, *Phys.Lett.* **278B** (1992) 457.
- [22] M.Neubert, *Phys.Lett.* **264B** (1991) 455.
- [23] M.A.Shifman, M.B.Voloshin, V.A.Khoze, N.G.Uraltsev *Sov.J.Nucl.Phys.* **46** (1987) 112.
- [24] A.F.Falk, M.Neubert, *Phys.Rev.* **D 47** (1993) 2965.
- [25] G.Altarelli et al., *Nucl.Phys.* **B208** (1982) 365.
- [26] A.Ali, C.Greub, *Zeit.f.Physik* **C49** (1991) 431; *Phys.Lett.* **259B** (1991) 182.
- [27] We thank T.Mannel for bringing the work of Luke and Savage to our attention.
- [28] M.Luke, M.Savage, preprint UCSD/PTH 93-25.

$\frac{\langle D \bar{c}(i\vec{D})^2c D\rangle}{2M_D}$	$m_c$ , GeV	$m_c(\overline{\text{MS}})$ , GeV	$m_b$ , GeV	$m_b(\overline{\text{MS}})$ , GeV	$\bar{\Lambda}$ , GeV	$ V_{cb} $
$(0.1 \text{ GeV})^2$	1.55	1.33	4.89	4.43	0.422	0.0442
$(0.3 \text{ GeV})^2$	1.56	1.34	4.91	4.45	0.390	0.0437
$(0.5 \text{ GeV})^2$	1.57	1.35	4.96	4.50	0.327	0.0427
$(0.7 \text{ GeV})^2$	1.59	1.36	5.03	4.56	0.234	0.0413
$(0.8 \text{ GeV})^2$	1.60	1.37	5.07	4.60	0.177	0.0404
$(0.9 \text{ GeV})^2$	1.61	1.39	5.12	4.64	0.112	0.0395

Table 1: Dependence of the extracted parameters on the size of the kinetic energy operator in nonstrange mesons. We used here the strength of the QCD running coupling corresponding to  $\Lambda_{\text{QCD}} = 180 \text{ MeV}$ .