Chapter 1

INTRODUCTION

M. Battaglia, A.J. Buras, J. Flynn, R. Forty, P. Gambino, P. Kluit, P. Roudeau, O. Schneider, A. Stocchi

1. Setting the scene

The understanding of flavour dynamics, and of the related origin of quark and lepton masses and mixings, is among the most important goals in elementary particle physics. In this context, weak decays of hadrons, and in particular the CP violating and rare decay processes, play an important role as they are sensitive to short distance phenomena. Therefore the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] that parametrizes the weak charged current interactions of quarks is currently a central theme in particle physics. Indeed, the four parameters of this matrix govern all flavour changing transitions involving quarks in the Standard Model (SM). These include tree level decays mediated by W bosons, which are essentially unaffected by new physics contributions, as well as a vast number of one-loop induced flavour changing neutral current (FCNC) transitions responsible for rare and CP violating decays in the SM, which involve gluons, photons, $W^{\pm}$, Z^0 and H^0, and are sensitive probes of new physics. This role of the CKM matrix is preserved in most extensions of the SM, even if they contain new sources of flavour and CP violation.

An important goal is then to find out whether the SM is able to describe the flavour and CP violation observed in nature. All the existing data on weak decays of hadrons, including rare and CP violating decays, can at present be described by the SM within the theoretical and experimental uncertainties. On the other hand, the SM is an incomplete theory: some kind of new physics is required in order to understand the patterns of quark and lepton masses and mixings, and generally to understand flavour dynamics. There are also strong theoretical arguments suggesting that new physics cannot be far from the electroweak scale, and new sources of flavour and CP violation appear in most extensions of the SM, such as supersymmetry. Consequently, for several reasons, it is likely that the CKM picture of flavour physics is modified at accessible energy scales. In addition, the studies of dynamical generation of the baryon asymmetry in the universe show that the size of CP violation in the SM is too small to generate a matter-antimatter asymmetry as large as that observed in the universe today. Whether the physics responsible for the baryon asymmetry involves only very short distance scales like the GUT or the Planck scales, or it is related to CP violation observed in experiments performed by humans, is an important question that still has to be answered.

To shed light on these questions the CKM matrix has to be determined with high accuracy and well understood errors. Tests of its structure, conveniently represented by the unitarity triangle, have to be performed; they will allow a precision determination of the SM contributions to various observables and possibly reveal the onset of new physics contributions.

The major theoretical problem in this program is the presence of strong interactions. Although the gluonic contributions at scales $\mathcal{O}(M_{\rm W}, M_{\rm Z}, m_{\rm t})$ can be calculated within the perturbative framework, owing to the smallness of the effective QCD coupling at short distances, the fact that hadrons are bound states of quarks and antiquarks forces us to consider QCD at long distances as well. Here we have to rely on the existing non-perturbative methods, which are not yet fully satisfactory. On the experimental side, the basic problem in extracting CKM parameters from the relevant rare and CP violating transitions is the smallness of the branching ratios, which are often very difficult to measure. As always in the presence of large theoretical and systematic uncertainties, their treatment in the context of global fits is a problematic and divisive issue.

In the last decade considerable progress in the determination of the unitarity triangle and the CKM matrix has been achieved through more accurate experiments, short distance higher order QCD calculations, novel theoretical methods like Heavy Quark Effective Theory (HQET) and Heavy Quark Expansion (HQE), and progress in non-perturbative methods such as lattice gauge simulation and QCD sum rules. It is the purpose of these proceedings to summarize the present status of these efforts, to identify the most important remaining challenges, and to offer an outlook for the future.

While many decays used in the determination of the CKM matrix are subject to significant hadronic uncertainties, there are a handful of quantities that allow the determination of the CKM parameters with reduced or no hadronic uncertainty. The prime examples are the CP asymmetry $a_{\psi K_S}$, certain strategies in B decays relevant for the angle γ in the unitarity triangle, the branching ratios for $\rm K^+ \to \pi^+\nu\bar\nu$ and $\rm K_L \to \pi^0\nu\bar\nu$, and suitable ratios of the branching ratios for rare decays $\rm B_{\mathit{d,s}} \to \mu^+\mu^-$ and $\rm B \to \mathit{X_{d,s}}\nu\overline{\nu}$ relevant for the determination of $|V_{td}|/|V_{ts}|$. Also the ratio $\Delta M_d/\Delta M_s$ is important in this respect.

The year 2001 opened a new era of theoretically clean measurements of the CKM matrix through the discovery of CP violation in the B system $(a_{\psi K_S} \neq 0)$ and further evidence for the decay $\rm K^+ \to \pi^+\nu\bar\nu$. In 2002 the measurement of the angle β in the unitarity triangle by means of $a_{\psi K_S}$ has been considerably improved. It is an exciting prospect that new data on CP violation and rare decays as well as $\rm B^0_{\mathit s} - \overline{B}^0_{\mathit s}$ mixing coming from a number of laboratories in Europe, USA and Japan will further improve the determination of the CKM matrix, possibly modifying the SM description of flavour physics.

Recently, there have been several workshops on B physics [3–5] that concentrated on studies at e^+e^- machines, at the Tevatron or at LHC, separately. Here we focus instead on the discussion of the CKM matrix and its determination from all available data at different machines.

2. CKM matrix and the Unitarity Triangle

2.1. General remarks

The unitary CKM matrix [1,2] connects the *weak eigenstates* (d', s', b') and the corresponding *mass eigenstates* d, s, b:

$$\left(\begin{array}{c} d' \\ s' \\ b' \end{array}\right) = \left(\begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array}\right) \left(\begin{array}{c} d \\ s \\ b \end{array}\right) \equiv \hat V_{\rm CKM} \left(\begin{array}{c} d \\ s \\ b \end{array}\right). \qquad (1)$$

Several parametrizations of the CKM matrix have been proposed in the literature; they are classified in [6]. We will use two in these proceedings: the standard parametrization [7] recommended by the Particle Data Group [8] and a generalization of the Wolfenstein parametrization [9] as presented in [10].

2.2. Standard parametrization

With $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ ($i, j = 1, 2, 3$), the standard parametrization is given by:

$$\hat{V}_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad (2)$$

where δ is the phase necessary for CP violation. c_{ij} and s_{ij} can all be chosen to be positive and δ may vary in the range $0 \le \delta \le 2\pi$. However, measurements of CP violation in K decays force δ to be in the range $0 < \delta < \pi$.

From phenomenological studies we know that s_{13} and s_{23} are small numbers: $\mathcal{O}(10^{-3})$ and $\mathcal{O}(10^{-2})$, respectively. Consequently to an excellent accuracy the four independent parameters can be chosen as

$$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}| \quad \text{and} \ \ \delta. \qquad (3)$$

As discussed in detail in Chapters 2 and 3, the first three parameters can be extracted from tree level decays mediated by the transitions $s \to u$, $b \to u$ and $b \to c$, respectively. The phase δ can be extracted from CP violating transitions or loop processes sensitive to $|V_{td}|$. We will analyse this in detail in Chapters 4–6.

2.3. Wolfenstein parametrization and its generalization

The absolute values of the elements of the CKM matrix show a hierarchical pattern with the diagonal elements being close to unity, the elements $|V_{us}|$ and $|V_{cd}|$ being of order 0.2, the elements $|V_{cb}|$ and $|V_{ts}|$ of order $4 \cdot 10^{-2}$ whereas $|V_{ub}|$ and $|V_{td}|$ are of order $5 \cdot 10^{-3}$. The Wolfenstein parametrization [9] exhibits this hierarchy in a transparent manner. It is an approximate parametrization of the CKM matrix in which each element is expanded as a power series in the small parameter $\lambda = |V_{us}| \approx 0.22$,

$$\hat{V} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,, \qquad (4)$$

and the set (3) is replaced by

$$\lambda, \qquad A, \qquad \varrho, \qquad \text{and} \ \ \eta\,. \qquad (5)$$

Because of the smallness of λ and the fact that for each element the expansion parameter is actually λ^2, it is sufficient to keep only the first few terms in this expansion.

The Wolfenstein parametrization is certainly more transparent than the standard parametrization. However, if one requires sufficient level of accuracy, the terms of $\mathcal{O}(\lambda^4)$ and $\mathcal{O}(\lambda^5)$ have to be included in phenomenological applications. This can be done in many ways [10]. The point is that since (4) is only an approximation the *exact* definition of the parameters in (5) is not unique in terms of the neglected order $\mathcal{O}(\lambda^4)$. This situation is familiar from any perturbative expansion, where different definitions of expansion parameters (coupling constants) are possible. This is also the reason why in different papers in the literature different $\mathcal{O}(\lambda^4)$ terms in (4) can be found. They simply correspond to different definitions of the parameters in (5). Since the physics does not depend on a particular definition, it is useful to make a choice for which the transparency of the original Wolfenstein parametrization is not lost.

In this respect a useful definition adopted by most authors in the literature is to go back to the standard parametrization (2) and to *define* the parameters $(\lambda, A, \varrho, \eta)$ through [10,11]

$$s_{12} = \lambda\,, \qquad s_{23} = A\lambda^2\,, \qquad s_{13}e^{-i\delta} = A\lambda^3(\varrho - i\eta) \qquad (6)$$

to *all orders* in λ. It follows that

$$\varrho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta. \tag{7}$$

The expressions (6) and (7) represent simply the change of variables from (3) to (5). Making this change of variables in the standard parametrization (2) we find the CKM matrix as a function of $(\lambda, A, \varrho, \eta)$ which satisfies unitarity exactly. Expanding next each element in powers of λ we recover the matrix in (4) and in addition find explicit corrections of $\mathcal{O}(\lambda^4)$ and higher order terms. Including $\mathcal{O}(\lambda^4)$ and $\mathcal{O}(\lambda^5)$ terms we find

$$\hat{V} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda+\mathcal{O}(\lambda^7) & A\lambda^3(\varrho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\varrho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2+\mathcal{O}(\lambda^8) \\ A\lambda^3(1-\overline{\varrho}-i\overline{\eta}) & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\varrho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \tag{8}$$

where [10]

$$\overline{\varrho} = \varrho(1-\frac{\lambda^2}{2}), \qquad \overline{\eta} = \eta(1-\frac{\lambda^2}{2}). \tag{9}$$

We emphasize that by definition the expression for V_{ub} remains unchanged relative to the original Wolfenstein parametrization and the corrections to V_{us} and V_{cb} appear only at $\mathcal{O}(\lambda^7)$ and $\mathcal{O}(\lambda^8)$, respectively. The advantage of this generalization of the Wolfenstein parametrization over other generalizations found in the literature is the absence of relevant corrections to V_{us}, V_{cd}, V_{ub} and V_{cb} and an elegant change in V_{td} which allows a simple generalization of the so-called unitarity triangle to higher orders in λ [10] as discussed below.

2.4. Unitarity Triangle

The unitarity of the CKM-matrix implies various relations between its elements. In particular, we have

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{10}$$

Phenomenologically this relation is very interesting as it involves simultaneously the elements V_{ub}, V_{cb} and V_{td} which are under extensive discussion at present. Other relevant unitarity relations will be presented as we proceed.

The relation (10) can be represented as a *unitarity triangle* in the complex $(\overline{\varrho},\overline{\eta})$ plane. The invariance of (10) under any phase-transformations implies that the corresponding triangle is rotated in the $(\overline{\varrho},\overline{\eta})$ plane under such transformations. Since the angles and the sides (given by the moduli of the elements of the mixing matrix) in this triangle remain unchanged, they are phase convention independent and are physical observables. Consequently they can be measured directly in suitable experiments. One can construct five additional unitarity triangles [12] corresponding to other orthogonality relations, like the one in (10). Some of them should be useful when the data on rare and CP violating decays improve. The areas (A_Δ) of all unitarity triangles are equal and related to the measure of CP violation $J_{\rm CP}$ [13]: $\mid J_{\rm CP} \mid = 2\cdot A_\Delta$.

Noting that to an excellent accuracy $V_{cd}V_{cb}^*$ in the parametrization (2) is real with $|V_{cd}V_{cb}^*| = A\lambda^3 + \mathcal{O}(\lambda^7)$ and rescaling all terms in (10) by $A\lambda^3$ we indeed find that the relation (10) can be represented as the triangle in the complex $(\overline{\varrho},\overline{\eta})$ plane as shown in Fig. 1.1.

Let us collect useful formulae related to this triangle:

- We can express $\sin(2\alpha_i)$, $\alpha_i = \alpha, \beta, \gamma$, in terms of $(\overline{\varrho},\overline{\eta})$ as follows:

$$\sin(2\alpha) = \frac{2\overline{\eta}(\overline{\eta}^2+\overline{\varrho}^2-\overline{\varrho})}{(\overline{\varrho}^2+\overline{\eta}^2)((1-\overline{\varrho})^2+\overline{\eta}^2)}, \tag{11}$$

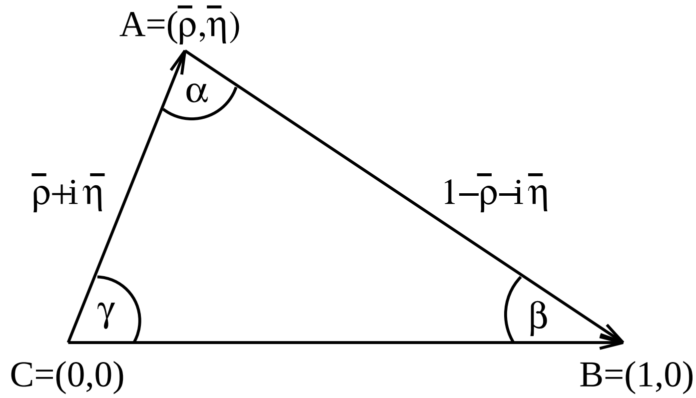

Fig. 1.1: *Unitarity Triangle.*

$$\sin(2\beta) = \frac{2\overline{\eta}(1-\overline{\varrho})}{(1-\overline{\varrho})^2+\overline{\eta}^2}, \tag{12}$$

$$\sin(2\gamma) = \frac{2\overline{\varrho}\overline{\eta}}{\overline{\varrho}^2+\overline{\eta}^2} = \frac{2\varrho\eta}{\varrho^2+\eta^2}. \tag{13}$$

- The lengths CA and BA to be denoted by R_b and R_t, respectively, are given by

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\overline{\varrho}^2+\overline{\eta}^2} = (1-\frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \tag{14}$$

$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1-\overline{\varrho})^2+\overline{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{15}$$

- The angles β and $\gamma=\delta$ of the unitarity triangle are related directly to the complex phases of the CKM-elements V_{td} and V_{ub}, respectively, through

$$V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}. \tag{16}$$

- The unitarity relation (10) can be rewritten as

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1\ . \tag{17}$$

- The angle α can be obtained through the relation

$$\alpha+\beta+\gamma = 180^\circ \tag{18}$$

  expressing the unitarity of the CKM-matrix.

Formula (17) shows transparently that the knowledge of (R_t,β) allows to determine (R_b,γ) through [14]

$$R_b = \sqrt{1+R_t^2-2R_t\cos\beta}, \qquad \cot\gamma = \frac{1-R_t\cos\beta}{R_t\sin\beta}. \tag{19}$$

Similarly, (R_t,β) can be expressed through (R_b,γ):

$$R_t = \sqrt{1+R_b^2-2R_b\cos\gamma}, \qquad \cot\beta = \frac{1-R_b\cos\gamma}{R_b\sin\gamma}. \tag{20}$$

These formulae relate strategies (R_t,β) and (R_b,γ) for the determination of the unitarity triangle that we will discuss in Chapter 6.

The triangle depicted in Fig. 1.1, together with $|V_{us}|$ and $|V_{cb}|$, gives the full description of the CKM matrix. Looking at the expressions for R_b and R_t, we observe that within the SM the measurements of four CP *conserving* decays sensitive to $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$ and $|V_{td}|$ can tell us whether CP violation ($\overline{\eta} \neq 0$) is predicted in the SM. This fact is often used to determine the angles of the unitarity triangle without the study of CP violating quantities.

Indeed, R_b and R_t determined in tree-level B decays and through $B_d^0 - \overline{B}_d^0$ mixing respectively, satisfy (see Chapters 3 and 4)

$$1 - R_b < R_t < 1 + R_b, \tag{21}$$

and $\overline{\eta}$ is predicted to be non-zero on the basis of CP conserving transitions in the B-system alone without any reference to CP violation discovered in $K_L \to \pi^+\pi^-$ in 1964 [15]. Moreover one finds

$$\overline{\eta} = \pm\sqrt{R_b^2 - \overline{\varrho}^2}\,, \qquad \overline{\varrho} = \frac{1 + R_b^2 - R_t^2}{2}. \tag{22}$$

Several expressions for $\overline{\varrho}$ and $\overline{\eta}$ in terms of R_b, R_t, α, β and γ are collected in Chapter 6.

2.5. The special role of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ elements

What do we know about the CKM matrix and the unitarity triangle on the basis of *tree level* decays? Here the semi-leptonic K and B decays play the decisive role. As discussed in detail in Chapters 2 and 3 the present situation can be summarized by

$$|V_{us}| = \lambda = 0.2240 \pm 0.0036 \qquad |V_{cb}| = (41.5 \pm 0.8) \cdot 10^{-3}, \tag{23}$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.086 \pm 0.008, \qquad |V_{ub}| = (35.7 \pm 3.1) \cdot 10^{-4} \tag{24}$$

implying

$$A = 0.83 \pm 0.02, \qquad R_b = 0.37 \pm 0.04\,. \tag{25}$$

This tells us only that the apex A of the unitarity triangle lies in the band shown in Fig. 1.2. While this

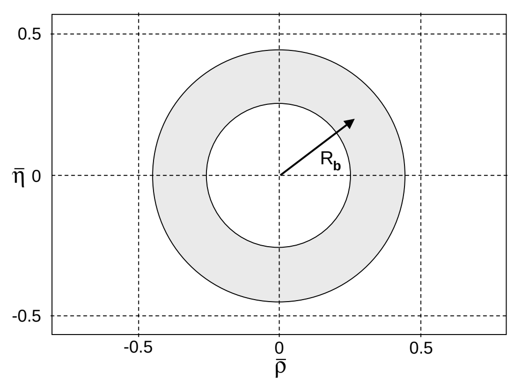

Fig. 1.2: *"Unitarity Clock"*

information appears at first sight to be rather limited, it is very important for the following reason. As $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and R_b are determined here from tree level decays, their values given above are to an excellent accuracy independent of any new physics contributions. That is, they are universal fundamental constants valid in any extension of the SM. Therefore their precise determination is of utmost importance. To find where the apex A lies on the *unitarity clock* in Fig. 1.2 we have to look at other decays. Most promising in this respect are the so-called *loop induced* decays and transitions and CP violating B decays which will be discussed in Chapters 4–6. They should allow us to answer the important question of whether the Cabibbo-Kobayashi-Maskawa picture of CP violation is correct and more generally whether the Standard Model offers a correct description of weak decays of hadrons. In the language of the unitarity triangle the question is whether the various curves in the $(\bar{\varrho},\bar{\eta})$ plane extracted from different decays and transitions using the SM formulae cross each other at a single point, as shown in Fig. 1.3, and whether the angles (α, β, γ) in the resulting triangle agree with those extracted from CP asymmetries in B decays and from CP conserving B decays.

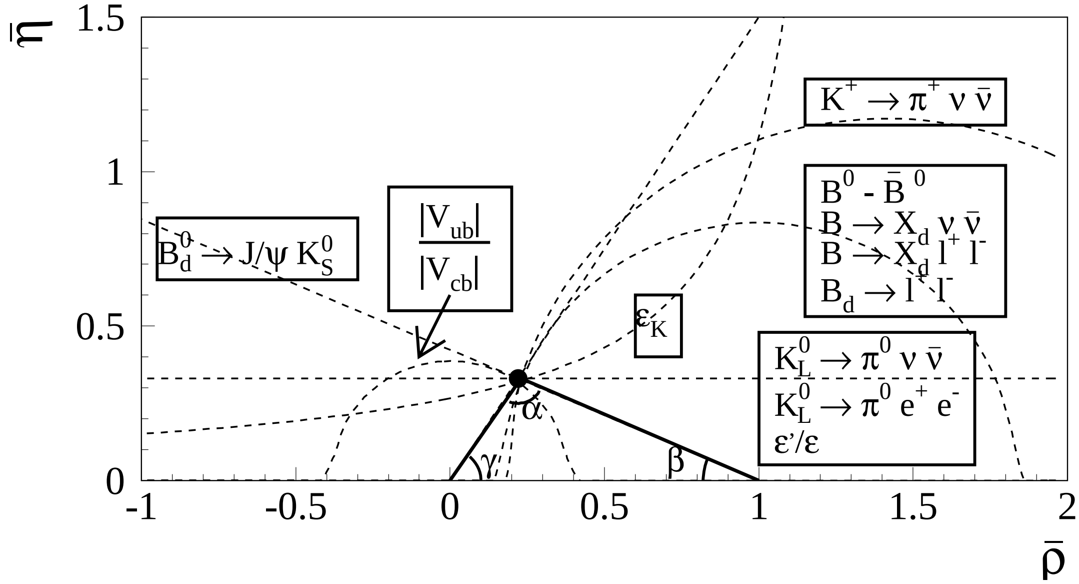

Fig. 1.3: *The ideal Unitarity Triangle*

Any inconsistencies in the $(\bar{\varrho},\bar{\eta})$ plane will then give us some hints about the physics beyond the SM. One obvious inconsistency would be the violation of the constraint (21). Another signal of new physics would be the inconsistency between the unitarity triangle constructed with the help of rare K decays alone and the corresponding one obtained by means of B decays. Also $(\bar{\varrho},\bar{\eta})$ extracted from loop induced processes and CP asymmetries lying outside the unitarity clock in Fig. 1.2 would be a clear signal of new physics.

In this context the importance of precise measurements of $|V_{ub}|$ and $|V_{cb}|$ should be again emphasised. Assuming that the SM with three generations and a unitary CKM matrix is a part of a bigger theory, the apex of the unitarity triangle has to lie on the unitarity clock obtained from tree level decays. That is, even if SM expressions for loop induced processes put $(\bar{\varrho},\bar{\eta})$ outside the unitarity clock, the corresponding expressions of the grander theory must include appropriate new contributions so that $(\bar{\varrho},\bar{\eta})$ is shifted back to the band in Fig. 1.2. In the case of CP asymmetries, this could be achieved by realizing that in the presence of new physics contributions the measured angles α, β and γ are not the true angles of the unitarity triangle but sums of the true angles and new complex phases present in extensions of the SM. Various possibilities will be discussed in the forthcoming CKM workshops. The better $|V_{ub}|$ and $|V_{cb}|$ are known, the thinner the band in Fig. 1.2 becomes, improving the selection of the correct theory. Because the branching ratios for rare and CP violating decays depend sensitively on the parameter A, precise knowledge of $|V_{cb}|$ is very important.

In order for us to draw such thin curves as in Fig. 1.3, we require both experiments and theory to be under control. Let us then briefly discuss the theoretical framework for weak decays.

3. Theoretical framework

3.1. Operator Product Expansion

The present framework describing weak decays is based on the operator product expansion (OPE) that allows short (μ_{SD}) and long distance (μ_{LD}) contributions to weak amplitudes to be separated, and on renormalization group (RG) methods that allow us to sum large logarithms $\log \mu_{SD}/\mu_{LD}$ to all orders in perturbation theory. A full exposition of these methods can be found in [16,17].

The OPE allows us to write the effective weak Hamiltonian for $\Delta F = 1$ transitions as an expansion in inverse powers of M_W. The leading term is simply

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\rm CKM} C_i(\mu) Q_i \tag{26}$$

with an analogous expression for $\Delta F = 2$ transitions. Here G_F is the Fermi constant and Q_i are the relevant local operators, built out of quark, gluon, photon and lepton fields, which govern the decays in question. The Cabibbo-Kobayashi-Maskawa factors $V^i_{\rm CKM}$ [1,2] and the Wilson coefficients $C_i(\mu)$ describe the strength with which a given operator enters the Hamiltonian. The latter coefficients can be considered as scale dependent *couplings* related to *vertices* Q_i and as discussed below can be calculated using perturbative methods, as long as μ is not too small. A well known example of Q_i is the $(V-A)\otimes(V-A)$ operator relevant for $\mathrm{K}^0-\overline{\mathrm{K}}^0$ mixing

$$Q(\Delta S = 2) = \overline{s}\gamma_\mu(1-\gamma_5)d \otimes \overline{s}\gamma^\mu(1-\gamma_5)d. \tag{27}$$

We will encounter other examples later on.

An amplitude for a decay of a given meson $M = \mathrm{K}, \mathrm{B}, ..$ into a final state $F = \pi\nu\overline{\nu},\ \pi\pi,\ \mathrm{D\,K}$ is then simply given by

$$A(M \to F) = \langle F|\mathcal{H}_{\rm eff}|M\rangle = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\rm CKM} C_i(\mu) \langle F|Q_i(\mu)|M\rangle, \tag{28}$$

where $\langle F|Q_i(\mu)|M\rangle$ are the matrix elements of Q_i between M and F, evaluated at the renormalization scale μ. An analogous formula exists for particle-antiparticle mixing.

The essential virtue of the OPE is that it allows the problem of calculating the amplitude $A(M \to F)$ to be separated into two distinct parts: the *short distance* (perturbative) calculation of the coefficients $C_i(\mu)$ and the *long-distance* (generally non-perturbative) calculation of the matrix elements $\langle Q(\mu)\rangle$. The scale μ separates, roughly speaking, the physics contributions into short distance contributions contained in $C_i(\mu)$ and the long distance contributions contained in $\langle Q_i(\mu)\rangle$. Thus C_i include the top quark contributions and those from other heavy particles such as W-, Z-bosons and charged Higgs or supersymmetric particles in the supersymmetric extensions of the SM. Consequently $C_i(\mu)$ depend generally on m_t and also on the masses of new particles if extensions of the SM are considered. This dependence can be found by evaluating so-called *box* and *penguin* diagrams with full W-, Z-, top- and new particles exchanges and properly including short distance QCD effects. The latter govern the μ-dependence of $C_i(\mu)$.

The value of μ can be chosen arbitrarily but the final result must be μ-independent. Therefore the μ-dependence of $C_i(\mu)$ has to cancel the μ-dependence of $\langle Q_i(\mu)\rangle$. In other words it is a matter of choice what exactly belongs to $C_i(\mu)$ and what to $\langle Q_i(\mu)\rangle$. This cancellation of the μ-dependence generally involves several terms in the expansion in (28). The coefficients $C_i(\mu)$ depend also on the renormalization scheme. This scheme dependence must also be cancelled by that of $\langle Q_i(\mu)\rangle$, so that physical amplitudes are renormalization scheme independent. Again, as in the case of the μ-dependence, cancellation of the renormalization scheme dependence generally involves several terms in the expansion (28).

Although μ is in principle arbitrary, it is customary to choose μ to be of the order of the mass of the decaying hadron. This is $\mathcal{O}(m_{\rm b})$ and $\mathcal{O}(m_{\rm c})$ for B decays and D decays respectively. For K decays the typical choice is $\mu = \mathcal{O}$(1-2 GeV) rather than $\mathcal{O}(m_K)$ that would be much too low for any perturbative calculation of the couplings C_i. Now since $\mu \ll M_{W,Z},\ m_t$, large logarithms $\ln M_{\rm W}/\mu$ compensate in the evaluation of $C_i(\mu)$ the smallness of the QCD coupling constant α_s, and terms $\alpha_s^n(\ln M_{\rm W}/\mu)^n$, $\alpha_s^n(\ln M_{\rm W}/\mu)^{n-1}$ etc. have to be resummed to all orders in α_s before a reliable result for C_i can be obtained. This can be done very efficiently by renormalization group methods. The resulting *renormalization group improved* perturbative expansion for $C_i(\mu)$ in terms of the effective coupling constant $\alpha_s(\mu)$ does not involve large logarithms. The related technical issues are discussed in detail in [16] and [17].

Clearly, in order to calculate the amplitude $A(M \to F)$ the matrix elements $\langle Q_i(\mu)\rangle$ have to be evaluated. Since they involve long distance contributions one is forced in this case to use non-perturbative methods such as lattice calculations, the 1/N expansion (where N is the number of colours), QCD sum rules, hadronic sum rules, chiral perturbation theory and so on. In the case of certain B-meson decays, the *Heavy Quark Effective Theory* (HQET) and *Heavy Quark Expansion* (HQE) also turn out to be useful tools. These approaches will be described in Chapter 3. Needless to say, all these non-perturbative methods have some limitations. Consequently the dominant theoretical uncertainties in the decay amplitudes reside in the matrix elements $\langle Q_i(\mu)\rangle$ and non-perturbative parameters present in HQET and HQE.

The fact that in many cases the matrix elements $\langle Q_i(\mu)\rangle$ cannot be reliably calculated at present is very unfortunate. The main goal of the experimental studies of weak decays is the determination of the CKM factors ($V_{\rm CKM}$) and the search for the physics beyond the SM. Without a reliable estimate of $\langle Q_i(\mu)\rangle$ these goals cannot be achieved unless these matrix elements can be determined experimentally or removed from the final measurable quantities by taking suitable ratios and combinations of decay amplitudes or branching ratios. Classic examples are the extraction of the angle β from the CP asymmetry in $\rm B \to \psi K_S$ and the determination of the unitarity triangle by means of $\rm K \to \pi\nu\bar{\nu}$ decays. Flavour symmetries like $SU(2)_{\rm F}$ and $SU(3)_{\rm F}$ relating various matrix elements can also be useful in this respect, provided flavour breaking effects can be reliably calculated. However, the elimination of hadronic uncertainties from measured quantities can be achieved rarely and often one has to face directly the calculation of $\langle Q_i(\mu)\rangle$.

One of the outstanding issues in the calculation of $\langle Q_i(\mu)\rangle$ is the compatibility (*matching*) of $\langle Q_i(\mu)\rangle$ with $C_i(\mu)$. $\langle Q_i(\mu)\rangle$ must have the correct μ and renormalization scheme dependence to ensure that physical results are μ- and scheme-independent. Non-perturbative methods often struggle with this problem, but lattice calculations using non-perturbative matching techniques can meet this requirement.

Finally, we would like to emphasize that in addition to the hadronic uncertainties, any analysis of weak decays, and in particular of rare decays, is sensitive to possible contributions from physics beyond the SM. Even if the latter are not discussed in this document and will be the subject of future workshops, it is instructive to describe how new physics would enter into the formula (28). This can be done efficiently by using the master formula for weak decay amplitudes given in [18]. It follows from the OPE and RG, in particular from (28), but is more useful for phenomenological applications than the formal expressions given above. This formula incorporates SM contributions but also applies to any extension of the SM:

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\rm QCD} V^i_{\rm CKM}[F^i_{\rm SM} + F^i_{\rm New}] + \sum_k B^{\rm New}_k [\eta^k_{\rm QCD}]^{\rm New} V^k_{\rm New}[G^k_{\rm New}]\,. \tag{29}$$

The non-perturbative parameters B_i represent the matrix elements $\langle Q_i(\mu)\rangle$ of local operators present in the SM. For instance in the case of $\rm K^0 - \overline{K}^0$ mixing, the matrix element of the operator $Q(\Delta S = 2)$ in (27) is represented by the parameter $\hat{B}_K$. An explicit expression is given in Chapter 4. There are other non-perturbative parameters in the SM that represent matrix elements of operators Q_i with different colour and Dirac structures. Explicit expressions for these operators and their matrix elements will be given in later chapters.

The objects $\eta^i_{\rm QCD}$ are the QCD factors resulting from RG-analysis of the corresponding operators. They summarise the contributions from scales $m_b \le \mu \le m_t$ and 1-2 GeV$\le \mu \le m_t$ in the case of B and K decays, respectively. Finally, $F^i_{\rm SM}$ stand for the so-called Inami-Lim functions [19] that result from the calculations of various box and penguin diagrams. They depend on the top-quark mass. $V^i_{\rm CKM}$ are the CKM-factors we want to determine.

New physics can contribute to our master formula in two ways. First, it can modify the importance of a given operator, already relevant in the SM, through a new short distance function $F^i_{\rm New}$ that depends on new parameters in extensions of the SM, such as the masses of charginos, squarks, and charged Higgs particles, or the value of $\tan\beta = v_2/v_1$, in the Minimal Supersymmetric Standard Model (MSSM). These new particles enter the new box and penguin diagrams. Second, in more complicated extensions of the SM new operators (Dirac structures) that are either absent or very strongly suppressed in the SM, can become important. Their contributions are described by the second sum in (29) with $B_k^{\rm New}, [\eta^k_{\rm QCD}]^{\rm New}, V^k_{\rm New}, G^k_{\rm New}$ the analogues of the corresponding objects in the first sum of the master formula. The $V^k_{\rm New}$ show explicitly that the second sum describes generally new sources of flavour and CP violation beyond the CKM matrix. This sum may, however, also include contributions governed by the CKM matrix that are strongly suppressed in the SM but become important in some extensions of the SM. A typical example is the enhancement of the operators with Dirac structures $(V-A)\otimes(V+A)$, $(S-P)\otimes(S\pm P)$ and $\sigma_{\mu\nu}(S-P)\otimes\sigma^{\mu\nu}(S-P)$ contributing to K^0-$\overline{\mathrm{K}}^0$ and $\mathrm{B}^0_{d,s}$-$\overline{\mathrm{B}}^0_{d,s}$ mixings in the MSSM with large $\tan\beta$ and in supersymmetric extensions with new flavour violation. The latter may arise from the misalignment of quark and squark mass matrices. The most recent compilation of references to existing next-to-leading (NLO) calculations of $\eta^i_{\rm QCD}$ and $[\eta^k_{\rm QCD}]^{\rm New}$ can be found in [20].

The new functions $F^i_{\rm New}$ and $G^k_{\rm New}$ as well as the factors $V^k_{\rm New}$ may depend on new CP violating phases, making the phenomenological analysis considerably more complicated. On the other hand, in the simplest class of the extensions of the SM where the flavour mixing is still entirely given by the CKM matrix and only the SM low energy operators are relevant [21] the formula (29) simplifies to

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\rm QCD} V^i_{\rm CKM}[F^i_{\rm SM} + F^i_{\rm New}] \tag{30}$$

with $F^i_{\rm SM}$ and $F^i_{\rm New}$ real. This scenario is often called *Minimal Flavour Violation* (MFV) [21], although one should be mindful that for some authors MFV means a more general framework in which also new operators can give significant contributions [22].

The simplicity of (30) allows to eliminate the new physics contributions by taking suitable ratios of various quantities, so that the CKM matrix can be determined in this class of models without any new physics uncertainties. This implies a universal unitarity triangle [21] and a number of relations between various quantities that are universal in this class of models [23]. Violation of these relations would indicate the relevance of new low energy operators and/or the presence of new sources of flavour violation. In order to see possible violations of these relations and consequently the signals of new sources of flavour violation it is essential to have a very precise determination of the CKM parameters. We hope that the material presented in this document is a relevant step towards this goal.

3.2. Importance of lattice QCD

Lattice calculations of the matrix elements $\langle Q_i(\mu)\rangle$ are based on a first-principles evaluation of the path integral for QCD on a discrete space-time lattice. They have the advantage of being systematically improvable to approach continuum QCD results with no additional parameters beyond those of QCD itself. Indeed, lattice QCD can be applied to determine these QCD parameters — the quark masses and the coupling constant. The most notable application of lattice QCD for CKM-fitting is to the mixing parameters for neutral kaons (B_K) and neutral B-mesons (F_B and B_B). Uncertainties in these quantities are now dominant in CKM fits. Lattice calculations are also important for determining form factors used to extract $|V_{ub}|$ from exclusive semileptonic B decays to light pseudoscalars or vectors, and for providing the endpoint form factor normalization needed to extract $|V_{cb}|$ from semileptonic B to $\mathrm{D}^{(*)}$ decays. With the advent of CLEO-c, the current round of lattice calculations for charm physics will be tested at the few-percent level. The charm calculations share several features with their analogues in the b sector, so a favourable outcome would bolster confidence in lattice techniques.

In recent years much effort has been devoted to non-perturbative techniques for improvement, to reduce discretization errors, and for renormalization and matching, to relate lattice results either directly to physical quantities or to quantities defined in some continuum renormalization scheme. With non-perturbative matching, the μ- and scheme-dependence of the matrix elements $\langle Q_i(\mu)\rangle$ is correctly matched with that of the $C_i(\mu)$.

The outstanding issue for the lattice is the inclusion of dynamical quark effects or *unquenching*. Many phenomenologically important quantities have been or are being calculated with dynamical quarks. However the dynamical quarks cannot be simulated with light enough masses to describe physical up and down quarks (the state-of-the-art is a mass of about $m_s/5$). Likewise, the *valence* quarks, whose propagators are used to evaluate matrix elements, also cannot be simulated with physical up and down masses. The combined extrapolations (chiral extrapolations) of both kinds of masses to realistic values are a major current focus of activity.

For heavy quarks the issue is to avoid discretization errors proportional to positive powers of $m_Q\, a$ where m_Q is the mass of the heavy quark and a the lattice spacing. Since present-day inverse lattice spacings are in the range $2\,\mathrm{GeV} < a^{-1} < 4\,\mathrm{GeV}$ or so, $m_b\, a$ is intolerably large for the b-quark. One approach is to restrict calculations to masses around that of charm and extrapolate to the b-quark regime guided by HQET, but the extrapolation can be significant and may amplify the $m_Q\, a$ errors, unless a continuum limit is taken first. In the last few years much has been learned about how to disentangle heavy quark mass-dependence and discretization effects using an effective theory approach where QCD is expanded in powers of μ/m_Q, where μ denotes other dimensionful scales in the problem, and discretization errors are proportional to powers of μa (so that μ should be smaller than m_Q and a^{-1}). This has been pioneered by the Fermilab group and implemented by them and others in numerical simulations for B-meson decay constants and semileptonic decay form factors. Lattice discretizations of HQET and NRQCD are also effective theory approaches which are used in simulations. In the effective theories one has to ensure that corrections in powers of $1/m_Q$ are calculated accurately, which involves issues of renormalization and the proliferation of terms as the power of $1/m_Q$ increases. By combining lattice HQET with direct simulations around the charm mass, the b-quark can be reached by interpolation, but this makes sense only if the continuum limit is taken for both calculations first. Currently, results obtained with different approaches to treating heavy quarks agree fairly well for b-physics.

An important theoretical advance in 1998 was the realization that full chiral symmetry could be achieved at finite lattice spacing, allowing the continuum and chiral limits to be separated. Lattice actions incorporating chiral symmetry are being used notably in calculations for kaon physics, including B_K, the $\Delta I = 1/2$ rule and ϵ'/ϵ, where the symmetry can be used to simplify the structure of the calculation. However, these calculations are currently quenched and have not yet had much impact on phenomenology.

4. Experimental aspects of B physics and the CKM matrix elements

In this report we will review B decay properties relevant for the measurement of the $|V_{ub}|$ and $|V_{cb}|$ CKM matrix elements, and $\mathrm{B}^0-\overline{\mathrm{B}}^0$ oscillations which constrain $|V_{td}|$ and $|V_{ts}|$, allowing to test the Standard Model through the CKM Unitarity Triangle. However, many additional measurements of B mesons properties (masses, branching fractions, lifetimes etc.) are necessary to constrain Heavy Quark theories to enable a precise extraction of the CKM parameters. These measurements are also important because they propagate to the CKM-related measurements as systematic errors.

4.1. B physics at colliders

In the last 15 years — before the start of asymmetric B-factories — the main contributors to B hadron studies have been symmetric e^+e^- colliders operating at the $\Upsilon(4S)$ and at the Z^0 resonance, and also the Tevatron $p\overline{p}$ collider (see Table 1.1).

Experiments	Number of $b\bar{b}$ events ($\times 10^6$)	Environment	Characteristics
ALEPH, DELPHI OPAL, L3	$\sim$ 1 per expt.	Z^0 decays ($\sigma_{bb} \sim$ 6nb)	back-to-back 45 GeV b-jets all B hadrons produced
SLD	~ 0.1	Z^0 decays ($\sigma_{bb} \sim$ 6nb)	back-to-back 45 GeV b-jets all B hadrons produced beam polarized
ARGUS	~ 0.2	$\Upsilon(4S)$ decays ($\sigma_{bb} \sim$ 1.2nb)	mesons produced at rest B^0_d and B^+
CLEO	~ 9	$\Upsilon(4S)$ decays ($\sigma_{bb} \sim$ 1.2nb)	mesons produced at rest B^0_d and B^+
CDF	$\sim$ several	$p\overline{p}$ collisions $\sqrt{s}$ = 1.8 TeV	events triggered with leptons all B hadrons produced

Table 1.1: *Summary of the recorded statistics for experiments at different facilities and their main characteristics.*

At the $\Upsilon(4S)$ peak, B^+B^- and $B^0_d\,\overline{B}^0_d$ meson pairs are produced on top of a continuum background, with a cross section of about 1.2 nb. At the energy used, only $B^\pm$ and B^0_d mesons are produced, almost at rest, with no additional hadrons. The constraint that the energy taken by each B meson is equal to the beam energy is useful for several measurements which rely on kinematic reconstruction.

At the Z^0 resonance the production cross section is $\sim$ 6 nb, about five times larger than at the $\Upsilon(4S)$, and the fraction of $b\overline{b}$ in hadronic events, is $\sim 22\%$, very similar to that obtained at the $\Upsilon(4S)$. Further, at the Z^0 peak B^0_s mesons and B baryons are produced in addition to $B^\pm$ and B_d mesons. B hadrons carry, on average, about 70% of the available beam energy, resulting in a significant boost which confines their decay products within well-separated jets. The resulting flight distance of a B hadron, $L = \gamma\beta c\tau$, is on average about 3 mm at these energies. Since the mean charged multiplicity in B decays is about five, it is possible to tag B hadrons using a lifetime tag based on the track topology. Additional hadrons are created in the fragmentation process which can be distinguished from the heavy hadron decay products using similar procedures.

Finally, at $p\overline{p}$ colliders b quarks are produced predominantly through gluon-gluon fusion. At the Tevatron energy of $\sqrt{s}$ = 1.8 TeV the b-production cross section is observed to be around 100 μb, which is huge. As the B decay products are contained in events with a much greater multiplicity than at the Z^0 pole and as backgrounds are important, only specific channels, such as fully reconstructed final states, can be studied with a favourable signal-to-background ratio.

Most of the precision measurements in B physics performed since SLC/LEP startup have been made possible by the development of high resolution Vertex Detectors, based on Silicon sensors. As the average flight distance of the b quark is of the order of 3 mm at Z^0 energies and as the typical displacement of secondary charged particles from the event primary vertex is of the order of 200 μm, secondary particles can be identified and the decay topology of short-lived B hadrons can be measured. The typical resolution of silicon detectors varies between a few and a few tens of microns depending on particle momentum and on detector geometry. A typical LEP $b\bar{b}$ event is shown in Fig. 1.4. In spite of a smaller Z^0 data set, the SLD experiment has proven to be highly competitive, due to a superior CCD-based vertex detector, located very close to the interaction point.

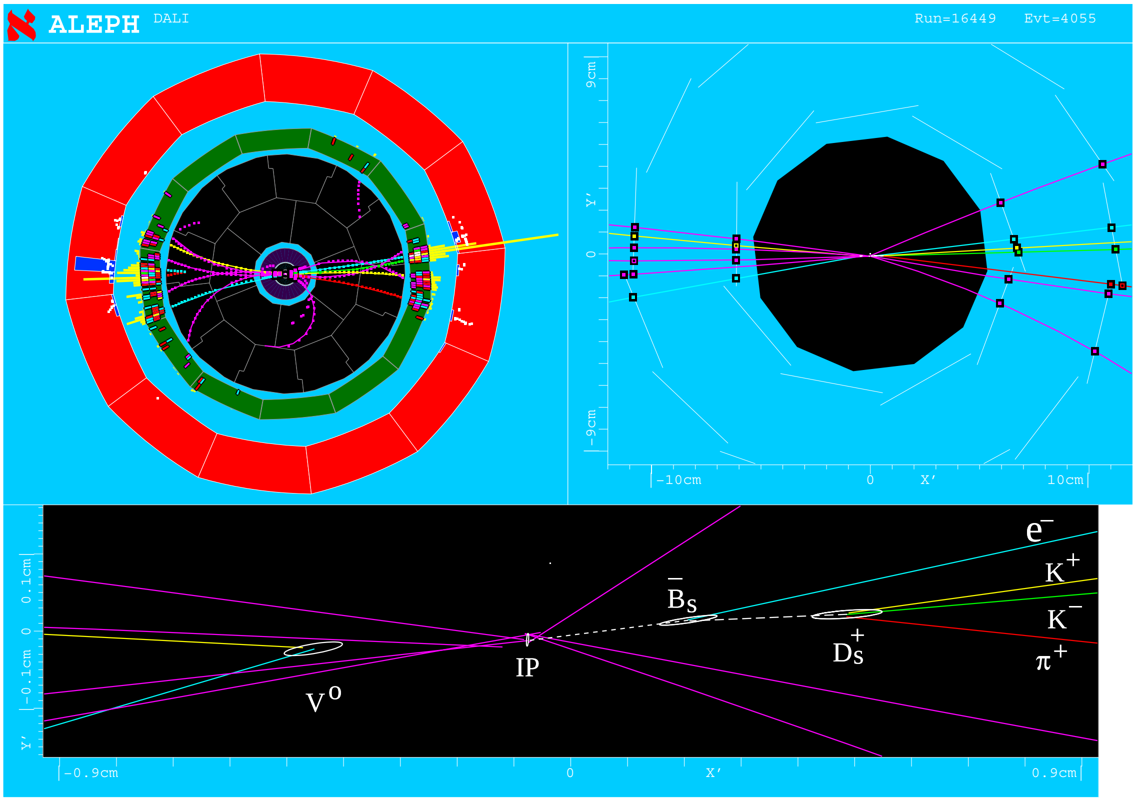

Fig. 1.4: *A $b\bar{b}$ event at LEP recorded by the ALEPH detector. The event consists of two jets containing the decay products of the two B hadrons and other particles. In one hemisphere a $\overline{\mathrm{B}}{}_s^0$ decays semileptonically : $\overline{\mathrm{B}}{}_s^0 \rightarrow D_s^+ e^- \overline{\nu}_e X$, $D_s^+ \rightarrow K^+ K^- \pi^+$ (tertiary vertex).*

The physics output from the data taken on the Z^0 resonance at LEP and SLC has continued to improve, with a better understanding of the detector response and new analysis techniques. Better-performing statistical treatments of the information have been developed. As a result, the accuracy of several measurements and the reach of other analyses have been considerably enhanced.

In 1984, six years after the discovery of the $b\bar{b}$ bound state Υ, the first experimental evidence for the existence of $\mathrm{B_d}$ and B^+ mesons was obtained by ARGUS at DORIS and CLEO at CESR and the B mesons joined the other known hadrons in the Review of Particle Physics listings. By the time LEP and SLC produced their first collisions in 1989, the inclusive b lifetime was known with about 20$\%$ accuracy from measurements at PEP and PETRA. The relatively long b lifetime provided a first indication for the smallness of the $|V_{cb}|$ matrix element. Branching fractions of $\mathrm{B_d}$ and B^+ meson decays with values larger than about few 10^{-3} had been measured.

In the early 90's the B sector landscape was enriched by the observation of new states at LEP. Evidence of the Λ_b baryon was obtained in the $\Lambda_b \rightarrow \Lambda \ell \nu X$ decay mode [24]. This was followed by the observations of the B_s^0 meson, in the decay $\overline{\mathrm{B}}{}_s^0 \rightarrow \mathrm{D}_s^+ \ell^- \overline{\nu}_\ell$, in 1992 and of the Ξ_b baryon in 1994. These analyses used semileptonic decays with a relatively large branching ratio of the order of a few $\%$ in combination with a clean exclusive final state (D_s , Λ or Ξ). Using right and wrong sign combinations, the background could be controlled and measured using the data. Selection of those signals is shown

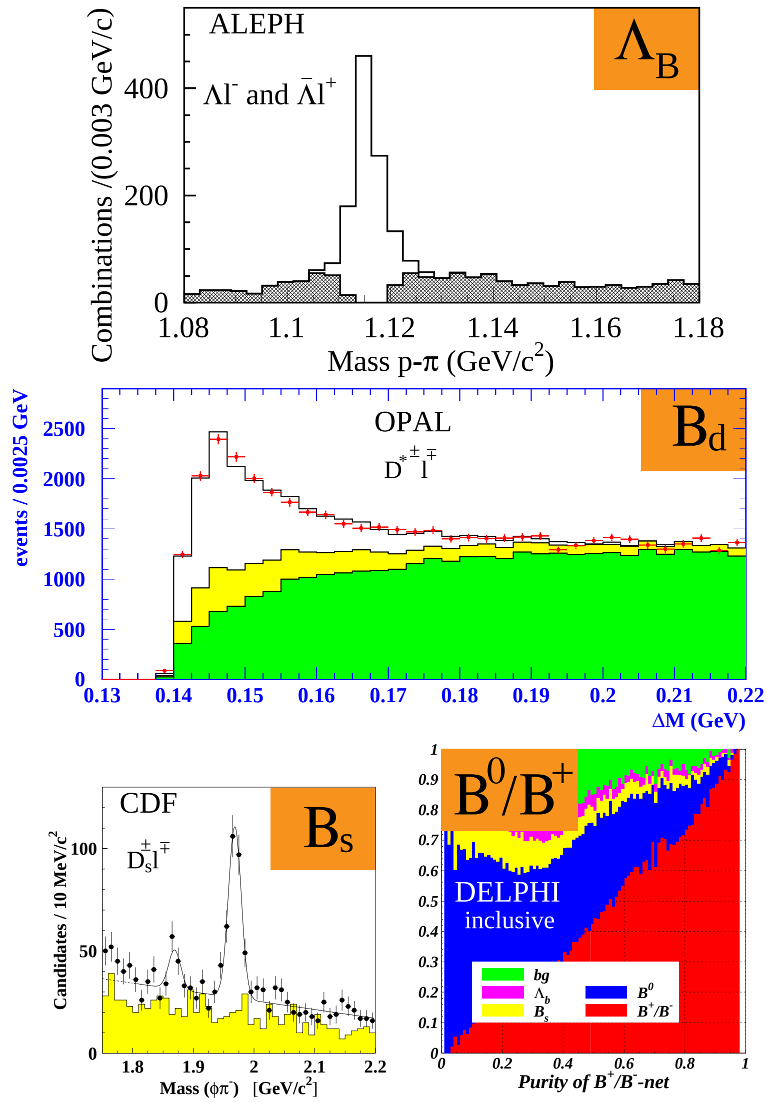

Fig. 1.5: *Signals from B hadrons. From top left to bottom left are the invariant mass spectra of* Λ, $((\mathrm{D}^0\pi)-\mathrm{D}^0)$, $\mathrm{D_s}$ *which are obtained in correlation with an opposite sign lepton. These events are attributed mainly to the semileptonic decays of* Λ_b, $\mathrm{B^0_d}$ *and* $\mathrm{B^0_s}$ *hadrons, respectively. The bottom right figure shows the possibility of distinguishing charged from neutral B mesons based on inclusive techniques.*

in Fig. 1.5. The orbitally excited B hadrons ($L = 1$) (B^{**}) [25] were also found and studied starting in 1994. These analyses were mostly based on partial reconstruction, profiting from the characteristic decay topology, and estimated the backgrounds relying to a large extent on the data themselves.

In parallel with studies on B spectroscopy, inclusive and individual B^0_d, B^+, B^0_s and b-baryon exclusive lifetimes were measured at LEP, SLD and CDF with increasing accuracies (as shown in Fig. 1.6) down to the present final precision, of a few percent.

Rare decays have been traditionally a hunting ground for the CLEO experiment, which benefited from the large statistics recorded at CESR. With about 9M $\mathrm{B\overline{B}}$ meson pairs registered, B decay modes with branching fractions down to 10^{-5} could be observed. The first signal for the loop-mediated $\mathrm{B} \to K^*\gamma$ decay was obtained in 1993. Evidence for charmless decay of B mesons followed [26] (see Fig. 1.7). At LEP, where the data sets were smaller, topological decay reconstruction methods and the efficient separation of decay products from the two heavy mesons allowed access to some transitions having branching fractions of order 10^{-4}–10^{-5} [27].

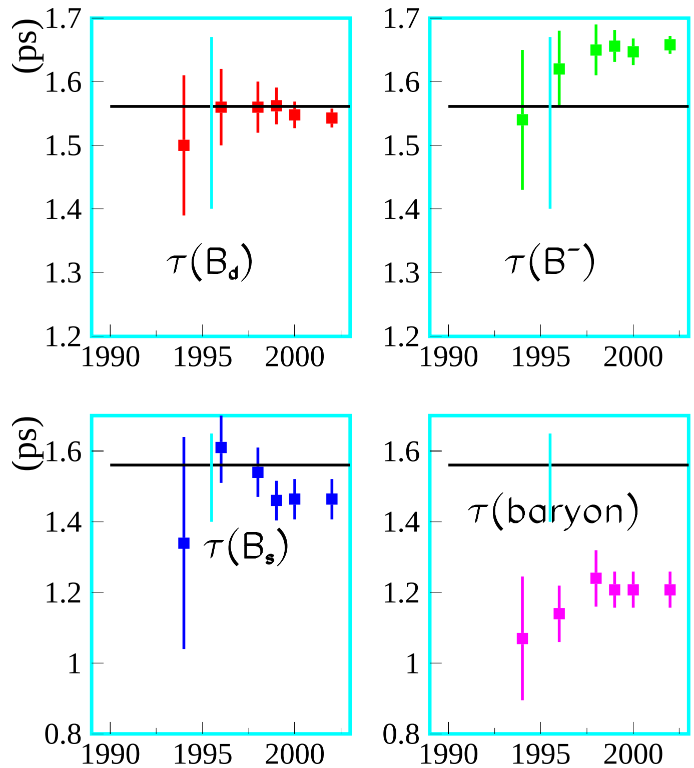

Fig. 1.6: *Evolution of the combined measurement of the different B hadron lifetimes over the years (the last point for* B_d^0 *and* B^- *meson lifetimes includes measurement obtained at* b*-factories). The horizontal lines indicate the values of the inclusive* b *lifetime, while the vertical lines indicate the end of LEP data taking at the* Z^0 *resonance.*

A value close to 10% for the semileptonic (s.l.) b branching fraction was not expected by theorists in the early 90's. More recent theoretical work suggests measuring both the s.l. branching fraction and the number of charmed particles in B decays. In fact, a s.l. branching ratio of 10% favours a low value of the charm mass and a value for the B branching ratio into double charm $b \to \bar{c}cs$ of about 20%. Much experimental effort has been made in recent years by the CLEO and LEP collaborations, allowing a coherent picture to emerge. The interplay among data analyses and phenomenology has promoted these studies to the domain of precision physics. The s.l. B branching fraction is presently known with about 2% accuracy and much data has become available for fully inclusive, semi-inclusive and exclusive decays. Inclusive and exclusive s.l. decays allow the extraction of $|V_{cb}|$ and $|V_{ub}|$ with largely independent sources of uncertainties and underlying assumptions. The inclusive method is based on the measured inclusive s.l. widths for $b \to X_{c,u}\ell\overline{\nu}_\ell$ interpreted on the basis of the Operator Product Expansion predictions. The exclusive method uses processes such as $\overline{\mathrm{B}}_d^0 \to \mathrm{D}^{*+}\ell^-\overline{\nu}$ and $\mathrm{B}^- \to \rho\ell\overline{\nu}$ and relies on Heavy Quark Effective Theory and form factor determinations. The requirements of precision tests of the unitarity triangle are now setting objectives for further improving our understanding of these decays and their application in the extraction of the CKM parameters.

The second major source of information on the magnitude of the relevant elements in the CKM matrix comes from oscillations of neutral B mesons. A B^0 meson is expected to oscillate into a $\overline{\mathrm{B}}^0$ with a probability given by: $P_{\mathrm{B_q^0}\to\mathrm{B_q^0}(\overline{\mathrm{B}}_\mathrm{q}^0)} = \frac{1}{2}e^{-t/\tau_q}(1 \pm \cos\Delta M_q t)$ where ΔM_q is proportional to the

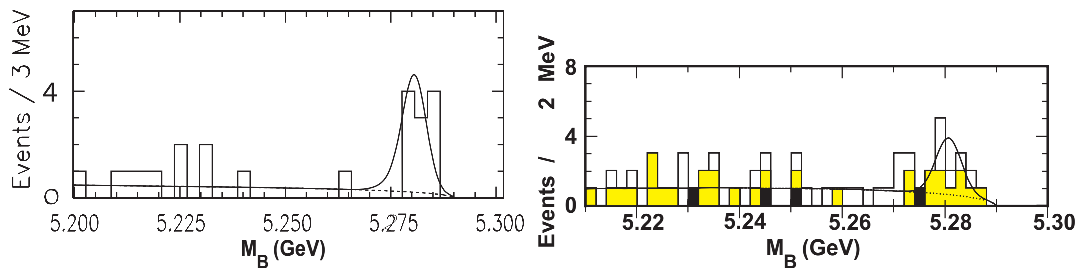

Fig. 1.7: *Left Plot: the* $\mathrm{K}^*\gamma$ *mass spectrum reconstructed by CLEO in the* $\mathrm{B} \rightarrow \mathrm{K}^*\gamma$ *decay modes. Right Plot : charmless B decays observed by CLEO :* $\pi\pi$ *and* $K\pi$ *mass spectrum in the* $\mathrm{B} \rightarrow \pi\pi(\mathrm{K}\pi)$ *decay modes. In dark the events with an identified pion. The plots show the updated signals in 1995.*

magnitude of the V_{tq} element squared. The first signals for B_d mixing were obtained in 1987 by the ARGUS [28] and CLEO [29] experiments. The UA1 experiment at the CERN S$p\overline{p}$S collider showed evidence for mixing due to combined contributions from both B^0_d and B^0_s mesons [30].

At energies around the Z^0 peak, where both B^0_d and B^0_s mesons are produced with fractions f_{B_d} and f_{B_s}, the mixing parameter χ is given by $\chi = f_{B_d}\chi_d + f_{B_s}\chi_s$ (where $\chi_{d(s)}$ is the probability to observe a $\overline{\mathrm{B}}^0_{d(s)}$ meson starting from a $\mathrm{B}^0_{d(s)}$ meson and $f_{B_{d(s)}}$ is the $\mathrm{B}^0_{d(s)}$ production fraction). Owing to the fast B^0_s oscillations, the χ_s value is close to 0.5 and becomes very insensitive to ΔM_s. Therefore even a very precise measurement of χ_s does not provide a determination of $|V_{ts}|$.

It became clear that only the observation of time evolution of $\mathrm{B}^0 - \overline{\mathrm{B}}^0$ oscillations, for B_d and B_s mesons separately, would allow measurement of ΔM_d and ΔM_s. Time dependent $\mathrm{B}^0_d - \overline{\mathrm{B}}^0_d$ oscillation was first observed [31] in 1993. The precision of the measurement of the B_d oscillation frequency has significantly improved in recent years. Results have been extracted from the combination of more than thirty-five analyses which use different event samples from the LEP/SLD/CDF experiments. At present, new results from the B-factories are also being included. The evolution of the combined results for the ΔM_d frequency measurement over the years is shown in Fig. 1.8, reaching, before the contribution from the B-factories, an accuracy of $\sim$ 2.5$\%$. New, precise measurements performed at the B-factories further improved this precision by a factor of 2.

As the B^0_s meson is expected to oscillate more than 20 times faster than the B^0_d meson ($\sim 1/\lambda^2$) and as B_s mesons are less abundantly produced, the search for $\mathrm{B}^0_s - \overline{\mathrm{B}}^0_s$ oscillations is much more difficult. To observe these fast oscillations, excellent resolution on the proper decay time is mandatory. Improvements in the ΔM_s sensitivity are depicted in Fig. 1.9. As no signal for $\mathrm{B}^0_s - \overline{\mathrm{B}}^0_s$ oscillations has been observed so far, the present limit implies that B^0_s mesons oscillate at least 30 times faster than B^0_d mesons. The impact of such a limit on the determination of the unitarity triangle parameters is already significant.

5. Heavy flavour averages

5.1. Motivation and history

Averaging activities have played an important role in the LEP community and several different working groups were formed to address the issue of combining LEP results. The first working group to appear was the LEP Electroweak WG with members from ALEPH, DELPHI, L3 and OPAL, soon followed in 1994

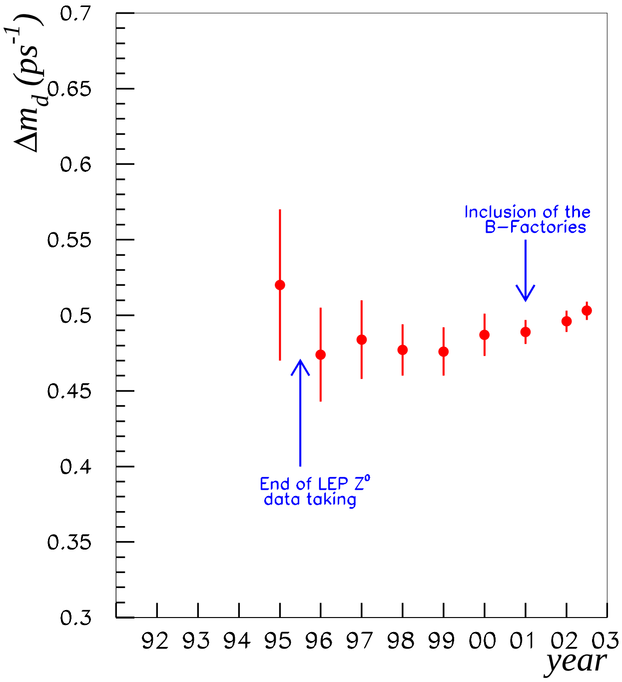

Fig. 1.8: *Evolution of the average of ΔM_d frequency measurements over the years.*

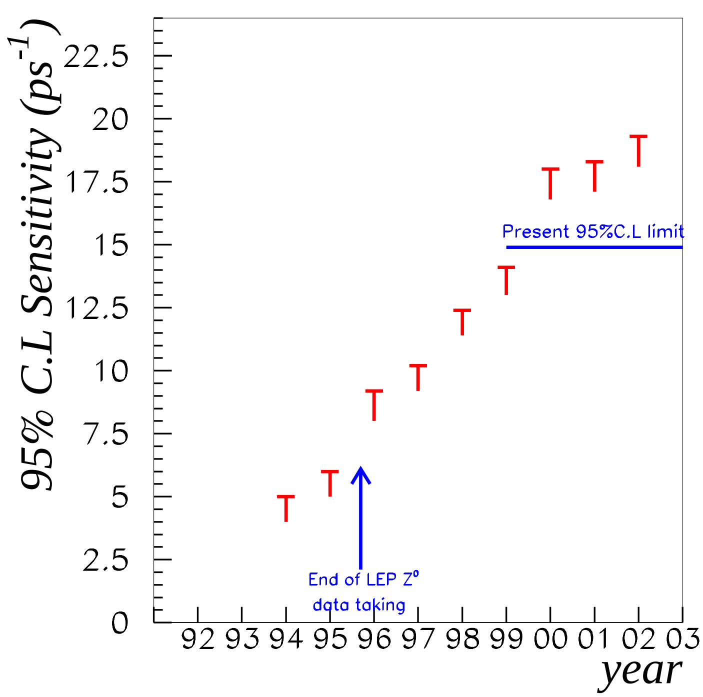

Fig. 1.9: *Evolution of the combined ΔM_s sensitivity over the last decade.*

by the b-hadron lifetime WG. They both rather quickly felt the need to enlarge their scope, and provide *world averages* rather than just LEP averages, so these groups have grown to include also representatives from the SLD collaboration, as well as from the CDF collaboration in the case of the lifetime WG. The B oscillations WG was formed in 1996 (once the need for combining B mixing results arose), and was also joined by SLD and CDF a year later.

In fall 1998, the four LEP collaborations decided to create the Heavy Flavour Steering Group (HFS), with members from the ALEPH, CDF, DELPHI, L3, OPAL and SLD collaborations. Within the scope of heavy flavour physics — in particular beauty physics — its mandate was to identify new areas where combined results are useful, and coordinate the averaging activities.

The HFS quickly spawned three new working groups on $\Delta\Gamma_s$, $|V_{cb}|$ and $|V_{ub}|$, and also supported or initiated activities in other areas like charm-counting in b-hadron decays, determination of the b-fragmentation function, and extraction of the CKM parameters. The coordination of all these activities resulted in better communication between experimenters and theorists and, as a product, a more coherent set of averages in b physics updated on a regular basis [32]. In order to provide world averages, contacts have also been established with representatives of other collaborations (CLEO, and more recently BABAR and BELLE).

The results of the b-lifetime WG were used by the Particle Data Group from 1996 onwards; later also averages from the B oscillation and b-fractions (1998), the $|V_{cb}|$ and the $|V_{ub}|$ Working Groups (2000) were also included. During this Workshop an Open Forum was organised for an orderly hand-over of the responsibility for heavy flavour physics world averages. This forum was chaired by HFS and PDG members. As a result, in the future, after the HFS group disbands, these averaging activities will be continued in the framework of a new Heavy Flavour Averaging Group [33], in which the Particle Data Group is also taking part.

In 2000 and 2001, the HFS group has produced reports [34] containing combined results on b-hadron production rates and decay properties from the ALEPH, CDF-1, DELPHI, L3, OPAL and SLD experiments. A final report is expected soon after all major results from these experiments have been published. In the remainder of this chapter, we will give some information on the combination procedures used for extracting averages for the b-hadron lifetimes, oscillations parameters and b-hadron fractions, $|V_{cb}|$ and $|V_{ub}|$. More details as well as technical aspects can be found in [34].

5.2. Averages of b-hadron lifetimes

Methods for combining b-hadron lifetime results were established in 1994, following a study [35] triggered by a rather puzzling fact: the world averages for the B_s^0 lifetime quoted by independent reviewers at the 1994 Winter Conferences differed significantly, although they were based essentially on the same data. Different combination methods have been developed [36] in the b-hadron lifetime WG to take into account the underlying exponential behaviour of the proper time distribution, as well as handling the resulting asymmetric uncertainties and biases in low statistics measurements.

The b-hadron lifetime WG provides the following averages: the B^+ lifetime, the mean B^0 lifetime, the $\mathrm{B}^+/\mathrm{B}^0$ lifetime ratio, the mean B_s^0 lifetime, the b-baryon lifetime (averaged over all b-baryon species), the Λ_b^0 lifetime, the Ξ_b lifetime (averaged over the two isospin states), and various average b-hadron lifetimes (e.g. for an unbiased mixture of weakly decaying b-hadrons). These averages take into account all known systematic correlations, which are most important for the inclusive and semi-inclusive analyses: physics backgrounds (e.g. $\mathrm{B} \to \mathrm{D}^{**}\ell\nu$ branching ratios), bias in momentum estimates (from b fragmentation, decay models and multiplicities, branching ratios of b- and c-hadrons, b-baryon polarization, etc.), and the detector resolution. For the B^+ and B^0 lifetimes, the fractions of weakly-decaying b-hadrons determined by the B oscillation WG (see Sec. 5.4. below) are used as an input to the averaging procedure. The b-lifetime averages are used as input by the other working groups for the determination of other b-physics averages.

5.3. Averages of B oscillation frequencies

The main motivation for the creation of the B oscillation WG was to combine the different lower limits obtained on ΔM_s. In 1995, the ALEPH collaboration proposed the so-called *amplitude method* [37], as a way to present the ΔM_s results in a form which allowed them to be combined in a straightforward manner. Each analysis provides the measured value of the B_s oscillation amplitude as a function of the oscillation frequency, normalized in such a way that a value of 1 is expected for a frequency equal to ΔM_s, and 0 for a frequency much below ΔM_s. A limit on ΔM_s can be set by excluding a value of 1 in a certain frequency range, and the results can be combined by averaging the measurements of this amplitude at each test frequency, using standard techniques.

The B oscillation working group played a major role in promoting this method, which was eventually adopted by each experiment studying B_s oscillations. As a result, all published papers on ΔM_s since 1997 give the *amplitude spectrum*, i.e. the B_s oscillation amplitude as a function of the oscillation frequency. As the individual ΔM_s results are limited by the available statistics (rather than by systematics), the overall sensitivity to ΔM_s is greatly increased by performing a combination of the results of the ALEPH, CDF, DELPHI, OPAL and SLD experiments.

It should be noted that the sensitivity of the inclusive analyses depends on the assumed value for the fraction of B_s mesons in a sample of weakly decaying b-hadrons. This is taken into account in the combination procedure, which is performed assuming the latest average value for this fraction (see Sec. 5.4. below).

The B oscillation working group also combines the many measurements of ΔM_d: in February 2002, 34 measurements were available from 8 different experiments. Several correlated systematic (and statistical) uncertainties are taken into account. Systematic uncertainties come from two main sources: experimental effects (which may be correlated amongst analyses from the same experiment), and imperfect knowledge of physics parameters like the b-hadron lifetimes and b-hadron production fractions which are common to all analyses. Since different individual results are assuming different values for the physics parameters, all measurements are re-adjusted to a common (and recent) set of average values for these parameters before being combined.

The average ΔM_d value is also combined with the B^0 lifetime to get a value for x_d, and with the time-integrated measurements of χ_d performed at ARGUS and CLEO, to get world averages of ΔM_d and χ_d.

5.4. Averages of *b*-hadron fractions in *b*-jets

Knowledge of the fractions of the different hadron species in an unbiased sample of weakly-decaying b hadrons produced in high-energy b jets is important for many b physics measurements. These fractions are rather poorly known from direct branching ratio measurements: for example the fraction of B_s mesons is only known with a $\sim 25\%$ uncertainty. However, mixing measurements allow this uncertainty to be reduced significantly, roughly by a factor 2.

Because these fractions play an important role in time-dependent mixing analyses, the B oscillation WG was also committed to provide b-hadron fractions (as well as a complete covariance matrix) that incorporate all the available information. A procedure was developed by this group, in which the determinations from direct measurements are combined with the world average of χ_d and the value of $\overline{\chi}$ (the mixing probability averaged over all b-hadron species) provided by the LEP electroweak WG, under the assumption that $\chi_s = 1/2$ (as is known from the limit on ΔM_s).

The b-hadron fractions are used as input for the ΔM_d combination procedure. Because the final fractions can only be known once the average ΔM_d is computed (and vice versa), the calculation of the b-hadron fractions and the ΔM_d averaging are part of the same fitting procedure, in such a way that the final results form a consistent set. The fractions are also used as input for the ΔM_s combination, for the lifetime averages, and for the $|V_{cb}|$ average.

5.5. Averages of $|V_{cb}|$ and $|V_{ub}|$ elements

The $|V_{cb}|$ working group started to combine LEP results and has by now evolved in a worldwide effort including results from the collaborations BABAR, BELLE, CDF, and CLEO. Only the case of exclusive $b \to c$ transitions presents specific problems. To combine the different results, central values and uncertainties on $\mathcal{F}(1)|V_{cb}|$ and ρ^2 have been rescaled to a common set of input parameters and ranges of values. The $\mathcal{F}(1)|V_{cb}|$ central value has then been extracted using the parametrization of Ref. [38], which is based on the experimental determination of the R_1 and R_2 vector and axial form factors. LEP results have been rescaled accordingly. In the averaging, the correlations between the different measurements and that between $\mathcal{F}(1)|V_{cb}|$ and ρ^2 have been taken into account. The working group also provides the combination of inclusive and exclusive determinations.

In order to average the inclusive charmless semileptonic branching fraction results from the LEP experiments, uncorrelated and correlated systematic errors are carefully examined. The correlated systematical errors come from the description of background $b \to c$ and from the theoretical modelling of signal $b \to u$ transitions. They are assumed to be fully correlated between the different measurements. The four measurements of BR($b \to X_u \ell \overline{\nu}$) have been averaged using the Best Linear Unbiased Estimate technique [39].

From this average branching fraction, using as an input the average b lifetime value, the probability density function for $|V_{ub}|$ has been derived. To obtain this function all the errors have been convoluted assuming that they are Gaussian in BR($b \to X_u \ell \overline{\nu}$) with the exception of the HQE theory error which is assumed to be Gaussian in $|V_{ub}|$. The negligible part of this function in the negative unphysical $|V_{ub}|$ region is discarded and the probability density function renormalised accordingly. The median of this function has been chosen as the best estimate of the $|V_{ub}|$ value and the corresponding errors are obtained from the probability density function.

6. Outline

This document is organized as follows:

Chapters 2 and 3 are dedicated to the determination of the elements V_{ud}, V_{us}, V_{cb} and V_{ub} by means of tree level decays. In Chapter 2 we summarize the present status of the elements V_{ud} and V_{us}. In Chapter 3 we discuss in detail the experimental and theoretical issues related to the determination of V_{cb} and V_{ub} from semileptonic inclusive and exclusive B decays and we discuss status and perspectives for B^0-$\overline{\mathrm{B}}^0$ lifetime differences and for the ratios of the lifetime of B hadrons.

In Chapter 4 we consider the determination of the elements $|V_{ts}|$ and $|V_{td}|$, or equivalently of $\overline{\varrho}, \overline{\eta}$ by means of $\mathrm{K}^0 - \overline{\mathrm{K}}^0$ and $\mathrm{B}^0_{d,s} - \overline{\mathrm{B}}^0_{d,s}$ mixings. The first part of this chapter recalls the formalism for ε_K and the mass differences ΔM_d and ΔM_s. Subsequently, the present status of the non-perturbative calculations of $\hat{B}_K$, $\sqrt{\hat{B}_{B_d}} F_{B_d}$, $\sqrt{\hat{B}_{B_s}} F_{B_s}$, and ξ is reviewed. The final part of this chapter deals with the measurements of $\mathrm{B}^0_{d,s} - \overline{\mathrm{B}}^0_{d,s}$ oscillations, parameterized by the mass differences $\Delta M_{d,s}$.

In Chapter 5 we describe two different statistical methods for the analysis of the unitarity triangle: the Bayesian approach and the frequentist method. Subsequently, we compare the results obtained in the two approaches, using in both cases the same inputs from Chapters 2-4. We also investigate the impact of theoretical uncertainties on the CKM fits.

Chapter 6 deals with topics that will be the focus of future CKM workshops. In this respect it differs significantly from the previous chapters and consists of self-contained separate contributions by different authors. After a general discussion of future strategies for the determination of the Unitarity Triangle, a few possibilities for the determination of its angles α, β and γ in B decays are reviewed. The potential of radiative and rare leptonic B decays and of $\mathrm{K} \to \pi \nu \overline{\nu}$ for the CKM determination is also considered.

Finally, Chapter 7 has a summary of the main results of this workshop and the conclusion.

References

[1] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.

[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[3] The BaBar Physics Book, eds. P. Harrison and H. Quinn, (1998), SLAC report 504.

[4] B Decays at the LHC, eds. P. Ball, R. Fleischer, G.F. Tartarelli, P. Vikas and G. Wilkinson, hep-ph/0003238.

[5] B Physics at the Tevatron, Run II and Beyond, K. Anikeev et al., hep-ph/0201071.

[6] H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45** (2000) 1.

[7] L.L. Chau and W.-Y. Keung, Phys. Rev. Lett. **53** (1984) 1802.

[8] Particle Data Group, Euro. Phys. J. C **15** (2000) 1.

[9] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.

[10] A.J. Buras, M.E. Lautenbacher and G. Ostermaier, Phys. Rev. D **50** (1994) 3433.

[11] M. Schmidtler and K.R. Schubert, Z. Phys. C **53** (1992) 347.

[12] R. Aleksan, B. Kayser and D. London, Phys. Rev. Lett. **73** (1994) 18;
J.P. Silva and L. Wolfenstein, Phys. Rev. D **55** (1997) 5331;
I.I. Bigi and A.I. Sanda, hep-ph/9909479.

[13] C. Jarlskog, Phys. Rev. Lett. **55**, (1985) 1039; Z. Phys. C **29** (1985) 491.

[14] A.J. Buras, P.H. Chankowski, J. Rosiek and Ł. Sławianowska, Nucl. Phys. B **619** (2001) 434.

[15] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys. Rev. Lett. **13** (1964) 128.

[16] A.J. Buras, hep-ph/9806471, in *Probing the Standard Model of Particle Interactions*, eds. R. Gupta, A. Morel, E. de Rafael and F. David (Elsevier Science B.V., Amsterdam, 1998), p. 281.

[17] G. Buchalla, A.J. Buras and M. Lautenbacher, Rev. Mod. Phys **68** (1996) 1125.

[18] A.J. Buras, in *Kaon 2001*, eds. F. Constantini, G. Isidori and M. Sozzi, Frascati Physics Series, p. 15, hep-ph/0109197.

[19] T. Inami and C.S. Lim, Progr. Theor. Phys. **65** (1981) 297.

[20] A.J. Buras, in *Theory and Experiment Heading for New Physics*, ed. A. Zichichi, World Scientific, 2001, page 200, hep-ph/0101336.

[21] A.J. Buras, P. Gambino, M. Gorbahn, S. Jäger and L. Silvestrini, Phys. Lett. B **500** (2001) 161.

[22] C. Bobeth, T. Ewerth, F. Krüger and J. Urban, Phys. Rev. D: **64** (2001) 074014; **66** (2002) 074021;
G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155.

[23] S. Bergmann and G. Perez, Phys. Rev. D **64** (2001) 115009, JHEP **0008** (2000) 034;
A.J. Buras and R. Fleischer, Phys. Rev. D **64** (2001) 115010;

S. Laplace, Z. Ligeti, Y. Nir and G. Perez, Phys. Rev. D **65** (2002) 094040;
A.J. Buras, hep-ph/0303060.

[24] ALEPH Coll., Phys. Lett. B **297** (1992) 449; Phys. Lett. B **294** (1992) 145;
OPAL Coll., Phys. Lett. B **316** (1992) 435; Phys. Lett. B **281** (1992) 394;
DELPHI Coll., Phys. Lett. B **311** (1993) 379.

[25] OPAL Coll., Zeit. Phys. C **66** (1995) 19;
DELPHI Coll., Phys. Lett. B **345** (1995) 598;
ALEPH Coll., Zeit. Phys. C **69** (1996) 393.

[26] CLEO Coll., Phys. Rev. Lett. **71** (1993) 3922.

[27] P. Kluit, Nucl Instr. Meth. A **462** (2001) 108.

[28] ARGUS Coll., Phys. Lett. B **192** (1987) 245.

[29] CLEO Coll., Phys. Rev. Lett. **58** (1987) 18.

[30] UA1 Coll., Phys. Lett. B **186** (1987) 247, erratum ibid B **197** (1987) 565.

[31] ALEPH Coll., Phys. Lett. B **313** (1993) 498; DELPHI Coll., Phys. Lett. B **332** (1994) 488; OPAL Coll., Phys. Lett. B **336** (1994) 585.

[32] The LEP Heavy Flavour Steering group maintains a web page at *http://www.cern.ch/LEPHFS/* with links to the web sites of each of the different working groups, where the latest averages can be found.

[33] The HFAG Steering group maintains a web page at *http://www.slac.stanford.edu/xorg/hfag/* with links to the web sites of each of the different working groups, where the latest averages can be found.

[34] D. Abbaneo *et al*, LEP Heavy Flavour Steering group and the different Heavy Flavour working groups (for the ALEPH, CDF, DELPHI, L3 OPAL, and SLD collaborations), CERN-EP/2000-096 and update in *idem*, CERN-EP/2001-050.

[35] R. Forty, CERN-PPE/94-144.

[36] L. Di Ciaccio *et al.*, OUNP 96-05, ROM2F/96/09.

[37] ALEPH Coll., contributed paper EPS 0410 to Int. Europhysics Conf. on High Energy Physics, Brussels, July 1995; the amplitude method is described in H.-G. Moser and A. Roussarie, Nucl. Instrum. Meth. A **384** (1997) 491.

[38] R. A. Briere *et al.* [CLEO Coll.], Phys. Rev. Lett. **89** (2002) 081803, [hep-ex/0203032].

[39] L. Lyons, D. Gibaut and G. Burdman, Nucl. Instrum. Meth. A **270** (1988) 110. The fit program code blue.f (author: P. Checchia) has been employed by the COMBOS program (authors: O. Schneider and H. Seywerd) developed by the LEP B Oscillation Working Group (see *http://www.cern.ch/LEPOSC/combos/*).