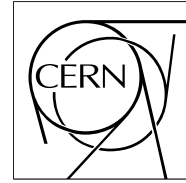


The Compact Muon Solenoid Experiment

CMS Note

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Digital filter optimisation for CMS ECAL

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Abstract

In this note the first results of M-C simulation of the pile-up noise in CMS ECAL including effects of digital filtering (DF) are presented. It is shown, that with the proper choice of the DF coefficients the pile-up noise could be reduced to the contribution from a single bunch-crossing. The performance of DF with respect to the other possible sources of noise is also discussed.

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1 Introduction

The noise conditions in CMS ECAL will be changing significantly over a wide range of accumulated neutron fluxes and luminosity. Therefore the usage of digital filtering as part of off-line data analysis is an important tool to improve the final physics performance of electromagnetic calorimeter. Considering the processes in the ECAL read-out chain resulting in errors of a physical signal measurement, one can combine them in three groups:

- sources of noise independent on the signal to be measured: such as the electronic noise in the amplifier, the leakage current in the APD, the external sources of noise (i.e. 50 Hz)
- pile-up noise is produced by other particles from the same bunch-crossing (BX) as the signal to be measured or from one of the previous bunch-crossings close enough in time.
- fluctuation processes depending on the signal to be measured: i.e. photo-statistical fluctuations of the light in the crystal, the avalanche multiplication fluctuations in the APD, “jitter” - the fluctuations in time synchronisation of the ADC clock with respect to the time of the signal being measured, a noise of “digitisation” defined by the width of the ADC count.

The amplitude of the signal at the output of the amplifier is measured in time domain with the 25 *nsec* pitch, and the final result is calculated as a combination of measured samples. In habitude, the output of such digital filter (*DF*) is a sum of samples taken with coefficients optimal for given noise conditions. The contribution of processes listed above to the error of the measurement depends on the choice of the DF coefficients.

In the present paper an approach to the DF optimisation in the CMS ECAL pile-up conditions is considered. The main content can be presented as following:

- the main expressions for noise calculation are given in Sec.2 and 3. This includes the noise of amplifier and APD, pile-up, photo-statistical fluctuations at the output of the digital filter. It is also shown that the contribution of photo-statistical fluctuations in pile-up noise is small, if the pile-up signal is larger than a few MeV.
- Taking into account the results of Sec.3 the DF for the minimal pile-up noise (DF_{mPU}) is built minimising the “pile-up integral” in Sec.4. This DF allows to decrease the pile-up noise to a contribution from single bunch-crossing. This conclusion is confirmed further by the direct MC simulation of the pile-up from minimum-bias events in CMS ECAL using PYTHIA+CMSIM code and simulating the signals and noise sources over 40 consequent BXs in time domain (Sec. 5 and 6).
- the comparison of noise performance of some given variants of DF is done in Sec.7. For instance, as mentioned above, the DF for minimal pile-up (DF_{mPU}) is not different very much from the DF for minimal parallel noise (DF_{mPN}), but it decreases the parallel noise by the factor of ~ 1.3 better than the “peak detector” (DF_{PD}), which is a single amplitude measurement at the maximum (peak) of the signal.

2 Digital filtering method and sources of noise

2.1 Main definitions

Further everywhere in text we will use the following definitions:

- $l(t)$ - the timing response of light in crystals. For simulations the following parametrisation was used:

$$l(t) = \sum_i (a_i/\tau_i) e^{(-t/\tau_i)} \quad (1)$$

where $a_i = 0.47, 0.43, 0.10$ and $\tau_i = 4.7, 20.7, 113 \text{ nsec}$. $\int_0^\infty l(\tau) d\tau = 1$.

- $h(t)$ - the amplifier impulse response, the shaping $RC - CR^2$ with peaking time $\tau_p = 40 \text{ nsec}$

$$h(t) = (t/\tau_p)^2 e^{2(1-t/\tau_p)}, \quad h(\tau_p) = 1$$

- $g(t)$ - the physical particle response after the amplifier:

$$g(t) = \int_0^\infty h(\tau) l(t - \tau) d\tau$$

- responses after digital filter

$$g_{DF}(t) = \sum_i g_i(t) d_i \quad h_{DF}(t) = \sum_i h_i(t) d_i$$

where d_i - DF coefficients, $g_i(t) = g(t + t_i)$ - samples and $t_i = i \times \Delta t$ - time of samples with period $\Delta t = 25 \text{ nsec}$ Thus $g_{DF}(t)$ - the physical particle response after DF, and $h_{DF}(t)$ - amplifier impulse response after DF.

2.2 Optimal digital filter

If a DF is built as a linear combination of n samples with coefficients d_i , and a noise of each sample measurement is independent on the measured signal $g(t)$, than, as it is well known, the optimal DF coefficients can be obtained by minimization of the variance of the signal on the output of DF as following:

$$d_i = \sum_j M_{i,j}^{-1} g_j$$

where $M_{i,j} = \overline{N_i N_j}$ is a covariance matrix with $N_i(t)$ - noise at the i -th sample, which has to be precisely measured. Also it implies the knowledge of the response shape g_j . For calorimetry applications, this algorithm should be modified to account the photo-statistical and jitter fluctuations, which depend on the measured signal amplitude. Also a pile-up noise could be very much non-Gaussian, and then it needs an even-by-event reconstruction by deconvolution. Taking into account, that pile-up and leakage current noise conditions in CMS ECAL are very different for different η -regions and will change with time, it is worthwhile to study separately the performance of the DF with respect to various possible sources of noise.

3 Sources of noise.

3.1 Electronic noise.

The electronic noise in terms of equivalent noise charge can be expressed as following[3]:

$$ENC^2 = Ie \int_{-\infty}^{\infty} h_{DF}^2(\tau) d\tau + 2kTR_s C_d^2 \int_{-\infty}^{\infty} [h'_{DF}(\tau)]^2 d\tau \quad (2)$$

where I is the leakage current producing parallel shot noise and $R_s = r_s + \frac{0.7}{g_m}$ is the equivalent series resistance. C_d - the APD capacitance with the input capacitance of the amplifier. Integrals in (2) express the ‘‘parallel’’ and ‘‘series’’ integrals:

$$I_{parallel} = \int_{-\infty}^{\infty} h_{DF}^2(\tau) d\tau \quad (3)$$

$$I_{series} = \int_{-\infty}^{\infty} [h'_{DF}(\tau)]^2 d\tau \quad (4)$$

Here $h_{DF}(t)$ is the normalised amplifier impulse response after the DF.

3.2 Leakage in APD

A leakage current in APD is a source of the shot parallel noise at the input of the amplifier:

$$ENC_{APD}^2 = e I_{bulk} M^2 F \int_{-\infty}^{\infty} h_{DF}^2(\tau) d\tau \quad (5)$$

where I_{bulk} is a bulk leakage current which undergoes the multiplication with the gain coefficient $M \sim 50$ and with the excess noise factor $F \sim 2$. Here a possible contribution of a surface leakage is considered to be small.

3.3 Ballistic deficit.

As soon as a crystal light response is not a $\delta(t)$ -pulse, not all the light produced in the crystal is integrated by the filter. One has to see the difference between the fraction of the collected charge ε_c and the fraction of the collected photo-statistics ε_{st} . Provided that $h(t)$ is a normalised impulse response of the amplifier ($h_{max} = 1$), and $l(t)$ is a distribution function of the light ($\int l(\tau) d\tau = 1$), one obtains

$$\varepsilon_c = \int_0^{\infty} l(\tau) h_{DF}(t_{max} - \tau) d\tau \quad (6)$$

and

$$\varepsilon_{st} = \frac{\varepsilon_c^2}{\int_0^{\infty} l(\tau) h_{DF}^2(t_{max} - \tau) d\tau} \quad (7)$$

Here t_{max} is a peaking time of the response $g_{DF}(t)$. Evidently, that, if N_{LY} is a number of photo-electrons in a light pulse integrated over the infinite time for 1 MeV energy deposit, then the equivalent noise energy ENE can be expressed through the equivalent noise charge ENC at the amplifier input as following:

$$ENE = \frac{ENC}{N_{LY} \varepsilon_c M} \quad (8)$$

where M is APD gain. For the photo-statistical fluctuation contribution to the stochastic term in ECAL energy resolution the 'effective' number of photo-electrons is to be calculated as

$$N_{ph.el.} = N_{LY} \varepsilon_{st}.$$

3.4 Time jitter.

A value of the time jitter (the fluctuation of the time of particle hit with respect to the sampling ADC clock) is expected to be ~ 200 psec of the TTC system and ~ 200 psec defined by the length of the beam bunch at the interaction region. To minimise fluctuations of the signal after the DF it must reach a maximum at the ADC clock position:

$$S_{DF}(t_{max} + \delta t) \simeq 1 + \frac{S''_{DF}}{2} (\delta t)^2$$

In the table 1 the values of the signal decrement corresponding to the clock shift from the position of the signal maximum by $\delta t = t - t_{max} = 1$ nsec are presented.

4 Pile-up calculation

As an approximation, one can consider the pile-up signal in the calorimeter read-out channel as a random Poisson's process, and then the variance can be evaluated by the Campbell's theorem[2]

$$D = \nu \overline{E^2} \int_0^{\infty} g^2(\tau) d\tau .$$

Here ν is the Poisson's process frequency (mean number of hits in the time unity), E is a single hit amplitude, and $g(\tau)$ is a system time response for a single hit. Indeed, the pile-up from minimum-bias events is a mixture of slow charged particles giving signals in the calorimeter with a significant delay and fast particles and gammas producing signals coherent with bunch-crossings. This fact can be easily taken into account with small modifications of the results, that do not change the main conclusions. In the case of the crystal light response, which has a slow decay time component [1], the distortion of the signal shape by the photo-statistics fluctuations has to be taken into account. Then the variance of the pile-up process at time t_0 is given by:

$$D = \overline{E^2} D_1 + \frac{F \overline{E}}{N_{LY}} D_2 \quad (9)$$

where

$$D_1 = \sum_{t_i \leq t_0} \left(\int l(t_i - \tau) h_{DF}(\tau) d\tau \right)^2 \quad D_2 = \sum_{t_i \leq t_0} \int l(t_i - \tau) h_{DF}^2(\tau) d\tau$$

$$h_{DF}(t) = \sum_i d_i h(t - i\Delta)$$

where E is the pile-up energy deposit from a single BX, F is the excess noise factor of an APD, N_{LY} is the light yield, expressed in photoelectrons/GeV, $l(t)$ is the light time response, $h(t)$ is an amplifier impulse response and d_i are the digital filter coefficients. A contribution of photo-statistical fluctuations is $\simeq \frac{FE}{N_{LY}E^2}$ and becomes small for a large pile-up energy.

4.1 Digital filter for minimal pile-up noise

Time response of the PWO crystal is considerably fast [1], and a contribution of the second term in (9) can be neglected. Then the pile-up noise can be expressed by the following:

$$\sigma_{pileup}^2 \simeq \overline{E^2} I_{pile-up} = \overline{E^2} \int g(t)_{DF}^2 dt \quad (10)$$

where $g_{DF}(t)$ is a normalised time response after an analog and digital filtering, and I_{pileup} is the so called "pile-up integral".

In a practical way, the signal can be expressed by a sum over n samples:

$$g_{DF}(t) = d_1 g(t) + d_2 g(t - \delta) + d_3 g(t - 2\delta) + d_4 g(t - 3\delta) + \dots$$

Here $g(t)$ is the amplifier ($CR - RC^2$) response on the crystal light signal, δ is the time between samples, and d_i are the digital filter coefficients, which in the case of the minimal pile-up noise must be determined by the minimization of the pile-up integral.

It was found, that in the case of the digital filter DF_{mPU} (see Tab.2) with the minimal pile-up integral (10) and in the case of DF_{mPN} with the minimal parallel noise integral (3) the value of the pile-up integral can be achieved less then 25 nsec (Tab.1), i.e. a noise contribution of the pile-up from charged particles slowly circulating in the magnetic field may be even less then from a single bunch-crossing. A comparison of noise performances for various digital filters is done in Tab.1. Filter coefficients are presented in Tab.2. Filter responses are shown on Fig.1. Often used in the beam tests the gated integrator with a gate of 150 nsec is presented for comparison. It can be seen, that in the beam test conditions the parallel noise is overestimated and the series noise is underestimated with respect to the peak detector.

Table 1: Digital filter performances: ε_c is a light signal collection coefficient; ε_{st} is a photo-statistics contribution in eq.(7); t_{max} is a peaking time after DF; I_{pileup} , $I_{parallel}$, ΔS_{max} are defined in the text in equations (3),(4) and in the section 3.4. The noise of the 200 nA bulk current in an APD and series noise for $C_d = 100$ pF are presented for a characterisation of the parallel and series noise performances. The filters coefficients are presented in Tab.2.

Filter	ε_c	ε_{st}	t_{max}	I_{pileup}	$I_{parallel}$	I_{series}^{-1}	$Noise$	$Noise$	ΔS_{max}
	charge %	ph.stat. %	nsec	nsec	nsec	nsec	$I_{bulk} = 200$ nA MeV	$C_d = 100$ pF MeV	$\Delta t = 1$ nsec %
DF_{mPN}	70	78	34	23.	20.	10.4	78.	43.	0.29
DF_{mPU}	69	78	33	21.	20.3	7.8	80.	50.	0.35
$\Sigma 1 - 1$	71	78	34	34.	28.7	10.7	93.	41.	0.25
$\Sigma 2 - 1$	79	86	58	38.	33.5	22.	90.	26.	0.14
$\Sigma 3 - 1$	85	91	81	50.	46.3	28.	97.	21.	0.10
$\Sigma 4 - 1$	89	93	103	63.	62.	31.	107.	19.5	0.07
$\Sigma 4 - 2$	89	93	103	66.	69.	21.	113.	24.	0.07
peak detector	82	87	52	60.	51.	24.	106.	24.	0.05
$\Sigma 2$	84	89	68	64.	55.	33.	108.	20.	0.04
$\Sigma 3$	87	92	86	71.	64.	40.	113.	18.	0.04
$\Sigma 4$	90	94	106	82.	77.	45.	120.	16.	0.03
Gate150ns	95	96	135	117.	117.	52.	140.	14.	0.016

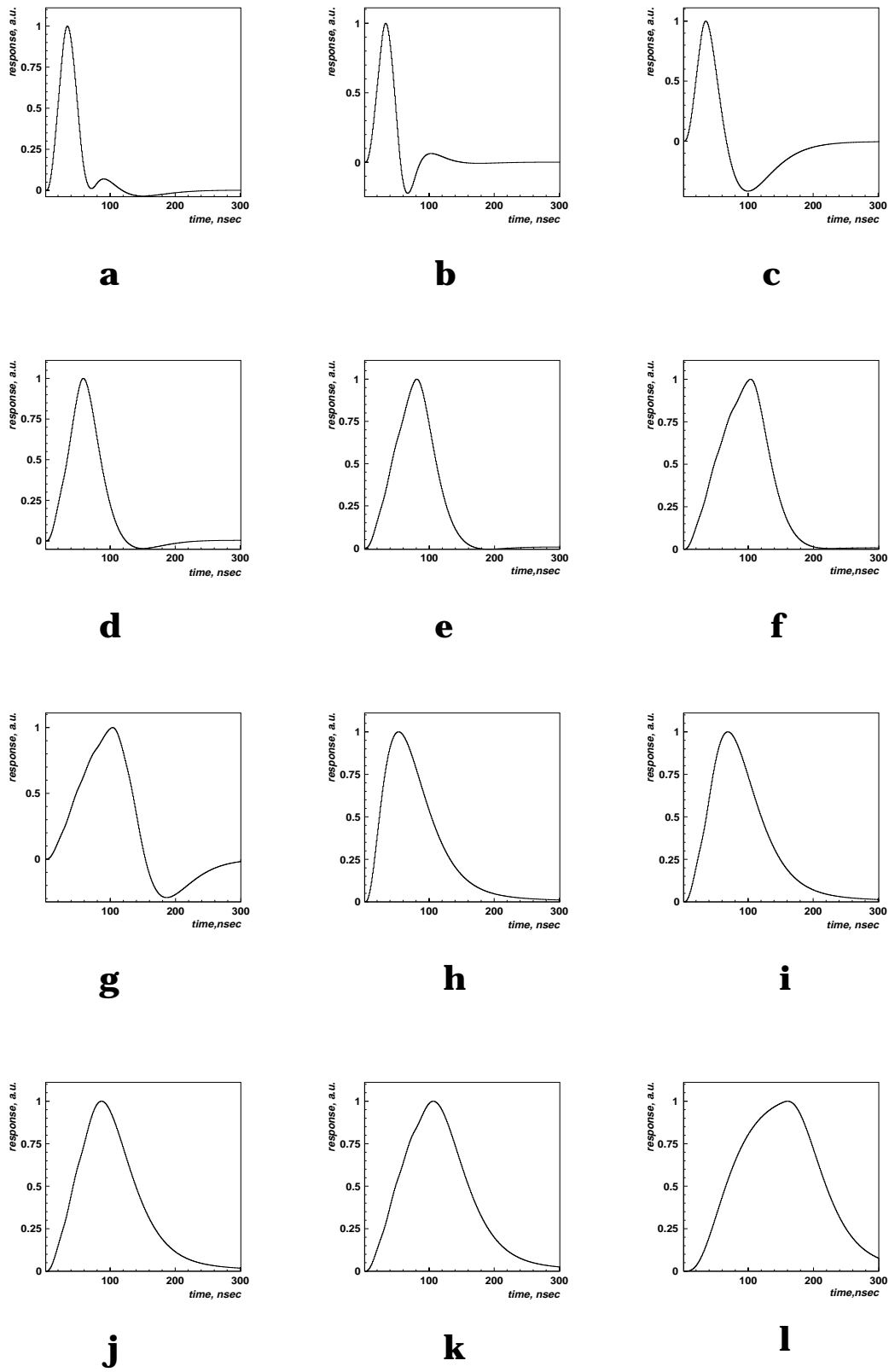


Figure 1: Time responses for each of the different digital filters (see Tab. 2): a) a filter with the minimal parallel noise integral, b) a filter with the minimal pile-up noise integral, c) one sample minus one presample, d) sum of 2 samples minus 1 presamples, e) sum of 3 samples minus 1 presamples, f) sum of 4 samples minus 1 presamples, g) sum of 4 samples minus 2 presamples, h) the peak detector, i) sum of 2 samples, j) sum of 3 samples, k) sum of 4 samples. l) The gated integrator ($gate = 150\text{ nsec}$), often used in beam tests, is shown for comparison.

Table 2: Digital filters coefficients. For easier comparison, $d_1 = 1$.

Filter	d_1	d_2	d_3	d_4	d_5	d_6
DF_{mPN}	1.	-1.14	0.56	-0.17	0	0
DF_{mPU}	1.	-1.4	0.8	-0.2	0	0
$\Sigma 1 - 1$	1	-1	0	0	0	0
$\Sigma 2 - 1$	1	1	-1	0	0	0
$\Sigma 3 - 1$	1	1	1	-1	0	0
$\Sigma 4 - 1$	1	1	1	1	-1	0
$\Sigma 4 - 2$	1	1	1	1	-1	-1
peak detector	1	0	0	0	0	0
$\Sigma 2$	1	1	0	0	0	0
$\Sigma 3$	1	1	1	0	0	0
$\Sigma 4$	1	1	1	1	0	0

5 Signal processing simulation

In order to check the noise performance of a DF with realistic conditions, Fortran code was written for a signal processing simulation in time domain, including the following processes:

- Pile-up energy at a given bunch-crossing, distributed by spectrum, obtained with full PYTHIA-Geant(CMSIM) simulation of minimum-bias events in CMS geometry. At the present step there was no time-of-flight information available for the particle hits.
- Light time response including the photo-statistics.
- Electronic noise: series one for detector capacitance, parallel one in a preamplifier, and a leakage current in an APD.
- Noise of the multiplication in an APD. The time response of the APD was not simulated.
- Noise of digitisation.

Figures 2 and 3 show the time response $g_{DF}(t)$ (2.1) from a physical particle in the case of the DF for the minimal parallel noise integral (DF_{mPN}) and in the case, when the only one sample at the maximum of the response is taken (the peak detector).

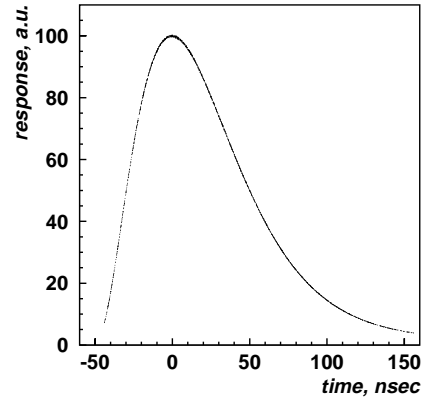
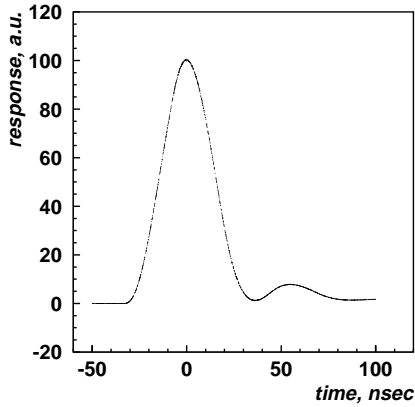


Figure 2: Time response $g_{DF}(t)$ of the DF for minimal parallel noise integral

Figure 3: Time response $g_{DF}(t)$ of the peak detector.

The figure 4 shows some typical minimum-bias events in time domain for 20 consequent bunch-crossings in 5×5 crystals at $\eta = 2.25$ for high luminosity. It proves, that even under one of the most severe conditions, when a pile-up energy deposited per a BX is large and an occupancy in CMS ECAL is high, the DF built for a minimal pile-up integral reconstructs the original energy well.

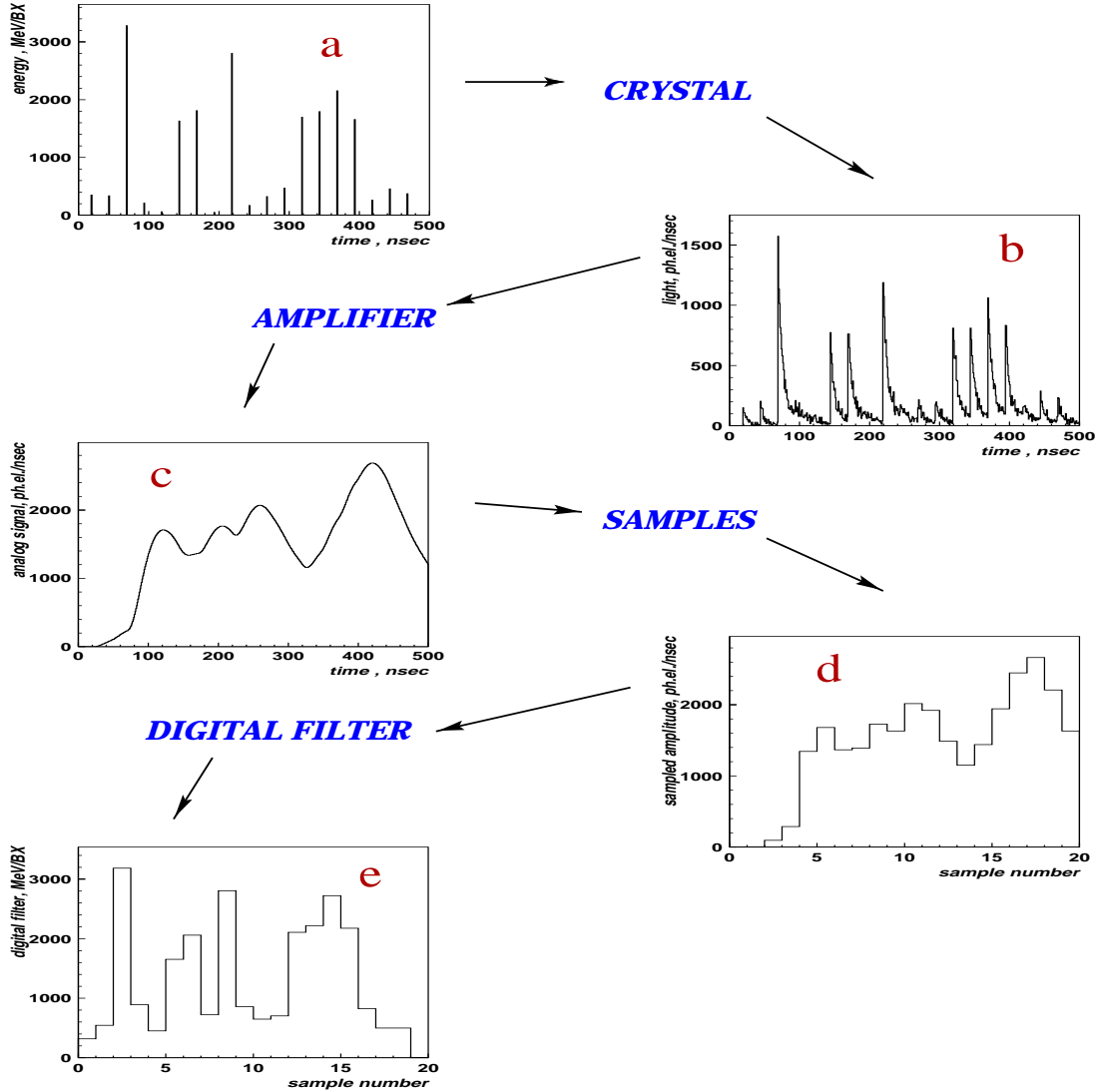


Figure 4: Minimum bias event display in 5×5 crystals at $\eta = 2.25$ for high luminosity. Following the arrows, the pictures of signals in time over 20 BX show: a) energy deposited in each BX; b) light signals from crystals; c) signal after amplifiers; d) signals measured by sampling ADC; e) reconstructed energies per BX after DF for minimal pile-up noise integral (DF_{mPI} in Tab. 2)

6 Pile-up simulation

To evaluate the pile-up noise in ECAL, the bank of 3000 minimum bias events was generated by PYTHIA with settings tuned to 55 mb of an inelastic cross-section of hard high- P_t interactions([1],p.306). Then the hits were obtained using full Geant simulation(CMSIM v.113). Next, the hits from event bank were mixed in a random way to obtain a multi-interaction event for a bunch-crossing. The number of minimum bias events per BX was distributed by Poisson's law with the mean value taken to be 17.3 for the full design luminosity ($10^{34} \text{ cm}^{-2} \text{ s}^{-1}$) case. To simulate the signals in time domain the hits from 40 random bunch-crossings were taken as consecutive. As the first approximation, the time of flight of particles from the interaction point to the hits in the calorimeter was not simulated and all the energy deposits were supposed to be synchronised with beam bunch-crossings. Thus obtained the impulse energy deposits in ECAL cells were used to simulate the time responses, as it was described above (Sec.5). To estimate the noise only from the pile-up there was no electronic noise simulated. For the light simulation the following parameters were used: light yield was 4 ph.el./MeV , time response was parametrised with three exponents (eq.1). The avalanche gain fluctuations in an APD were not simulated in detail and were taken into account by correspondent average increase of statistical fluctuations of the number of photo-electrons with the value of the excess noise factor $F = 2$.

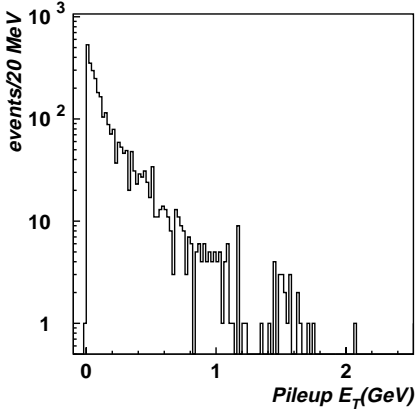


Figure 5: Pile-up transverse energy in 5x5 crystal array at full high luminosity at $\eta = 0.1$.

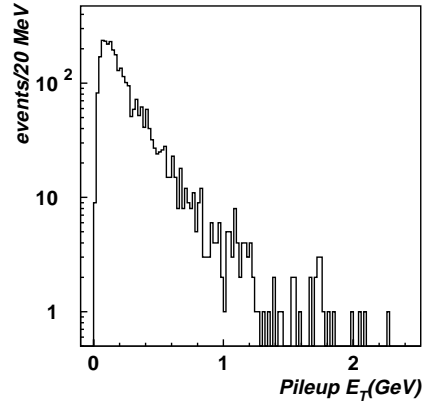


Figure 6: Pile-up transverse energy in 5x5 crystal array at full high luminosity at $\eta = 2.25$.

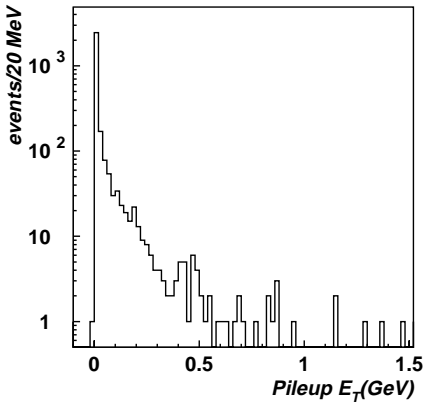


Figure 7: Pile-up transverse energy in 5x5 crystal array at full high luminosity at $\eta = 0.1$ after signal simulation.

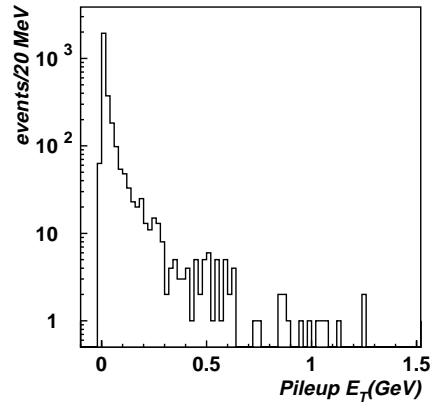


Figure 8: Pile-up transverse energy in 5x5 crystal array at full high luminosity at $\eta = 2.25$ after signal simulation.

Figures 5 and 6 show the pile-up transverse energy deposit in a single bunch-crossing in a 5×5 crystal array at full design (high) luminosity at $\eta = 0.1$ and 2.25 . The r.m.s. values of distributions were found to be 105 and 250 MeV . Both spectra have long high energy tails.

Figures 7 and 8 show the measured pile-up transverse energy for the same events after including effects of the light emission in crystals, of the shaping of an amplifier and of the digital filtering for a minimal pile-up integral. The r.m.s. values, in agreement with predictions, are found to be close to the original ones: 105 and 265 MeV .

But, because of the long high energy tail, simply taken, r.m.s. of the pile-up spectrums could be an exaggeration of the effect of the pile-up noise on the experimental study of physical processes. To obtain an adequate approach one has to apply the spectra in a situation close to experimental conditions. For e-m shower energy measurement in ECAL, the pile-up noise should be compared with a typical energy resolution. For instance, in case of $H \leftarrow \gamma\gamma$ decay study, ECAL energy resolution at 100 GeV is about 500 MeV . As a solution, the following procedure was used to approximate the effect of pile-up noise: a) the E_t spectra of pile-up events taken for each value of η were convoluted with a Gaussian of 500 MeV width; b) obtained distributions were fitted with Gaussian, and c) the value of the “pedestal” of 500 MeV was subtracted back quadratically from the resulting σ . The result appears weakly dependent on the initially chosen width of Gaussian.

One can see the obtained pile-up noise contribution as a function of pseudo-rapidity on Fig.9. It can be seen that in the most regions of ECAL, except only at the highest values of η , the pile-up noise has a rather small effect on the energy resolution. Moreover, it should be noted that the effect will be even smaller, because of the luminosity will not be constant, but rather decaying during the LHC fill.

One can see also in Fig.9, that the fast digital filter algorithm obtained for the minimal pile-up noise integral effectively rejects the pile-up from other bunch-crossings except the one, that gives a trigger. The upper points with triangular marks show the result, when one ADC sample taken at the maximum of signal (peak detector) was used. This results in ~ 1.5 times larger noise.

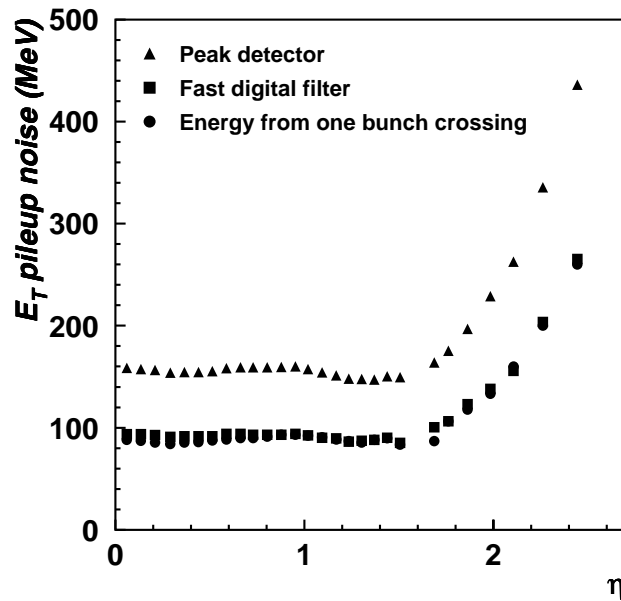


Figure 9: Effective pile-up (transverse energy) as function of η .

7 Leakage current in APD. DF for parallel and series noise.

The typical value of neutron flux in the barrel of ECAL is being estimated as $2 \times 10^{13} \text{ n/cm}^2$ for 10 years of LHC operation. It causes the leakage current generation in two APDs to be about 200 nA, which goes through the multiplication region and produces a shot-like noise. Following the equations (5) and (8), the equivalent noise energy at the output of the digital filter can be expressed as:

$$ENE_{leakage}^2 = \frac{FI_{bulk}}{(\varepsilon_c N_{LY})^2 e} \int_0^\infty h_{DF}^2(\tau) d\tau$$

$$\varepsilon_c = \int_0^\infty l(\tau) h_{DF}(t_{max} - \tau) d\tau$$

Here $ENE_{leakage}^2$ is a noise variance in MeV^2 , ε_c is a fraction of the light signal $l(t)$ eq.(1) integrated by the filter response $h_{DF}(\tau)$, N_{LY} is a light yield expressed in photo-electrons per MeV, and $h_{DF}(\tau)$ is a normalised single photo-electron time response after digital filtering. So the optimal digital filter performance for the leakage current noise depends on the parallel noise integral and on the time of light emission in crystals. For example, the digital filter DF_{mPN} for a minimal parallel noise integral, described above, accumulates 69% of the light signal, and the value of the parallel noise integral was found equal to 20 nsec. The peak detector collects 82% of the light and integrates the parallel noise over 52 nsec. So one obtains $\sqrt{51/20} \times 0.69/0.82 = 1.34$ times better a parallel noise with a fast digital filter DF_{mPN} , then with a peak detector. Thus the leakage current of 100 nA , expected after 5 years of the LHC run, with the excess noise factor $F = 2$ and the light yield of 4 photo-electrons/ MeV produces noise of 75 MeV with a peak detector in comparison with 56 MeV in case of a DF_{mPN} digital filter. With respect to a series noise the performance of DF is also very different. As it is shown in Tab.1, the peak detector gives 2 times better noise than the fast digital filters. In general, the filters $\Sigma 2 - 1$, $\Sigma 3 - 1$ and $\Sigma 4 - 1$ look very promising for the conditions of a moderate leakage current and pile-up noise.

8 Conclusion

A digital filter performance has been studied with respect to CMS ECAL conditions. It was found that

- In case of the fastest digital filter response, pile-up integral can be obtained $\sim 21 - 23 \text{ nsec}$, that is close to a single BX contribution.
- The pile-up and leakage current noise obtained with the proper choice of the digital filter coefficients improves by factor of 1.35-1.5 in comparison with the peak detector.
- At the peak of the signal after DF a jitter sensitivity is found to be acceptably low (less than 0.1% for jitter of 0.3 nsec). To minimise the jitter noise, the proper clock synchronisation is necessary. To change the digital filter, generally, one has to change the timing. The monitoring of signal shape is also obligatory. This, probably, needs some special investigations.
- A presence of the low frequency noise, like 50 Hz , will require the sum of DF coefficients $\Sigma d_i = 0$. Depending on the number of ADC samples used, this will restrict the performance of a digital filter with slow time response. The filters for minimal pile-up and parallel noise, described above, already provide good low frequency noise suppression and will suffer only a small modification.

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