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The Compact Muon Solenoid Experiment

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About the electromagnetic shower lateral profile in the lead tungstate

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Abstract

A comparison between test beam data and simulations provides an indirect measurement of the electron shower width in PbWO4 crystals. The result is compared with the Moliere radius approximation.

1. Introduction

In a previous note[1] we showed from GEANT Monte Carlo[2] simulations that 74.5% of the total energy deposited in a lead tungstate matrix (made of crystals with the standard barrel dimensions, 2.05*2.05cm^2 at the front face and 23 cm long) is contained in the central crystal. Since this study, a new release of the GEANT program was issued (3.21/05 which gives an higher value of 75.6%).These high containment properties of the lead tungstate makes a direct comparison between data and Monte Carlo difficult. Furthermore, the shower is not perfectly contained in depth, and at the time of our measurements (1995) the crystals had not exactly the same lateral dimensions.

2. The method and the experimental conditions

For all these reasons and also to avoid crystal intercalibration uncertainties, we consider only the crystal where the electron beam enters (at the centre of the front face) and we measure the deposited energy in this crystal as a function of the electron beam width. The data were recorded during the 1995 runs, when the 50 GeV electron beam was normal to the entry face of this central crystal.

The beam is defined by a 20*20mm^2 scintillator trigger (S2). The X(horizontal) and Y(vertical) beam profiles measured by two drift chambers are shown in Figs 1 and 2 respectively. The X profile shows a rather flat shape in the [-1,+1] cm range defined by S2, while the Y distribution is strongly asymmetric, reflecting a misalignment of the crystal matrix w. r. to the beam axis.

3. Data analysis

With the informations given by the chambers, we defined 4*4, 6*6. 8*8, 10*10,15*15 and 20*20 mm^2, beam dimensions. Taking into account the available statistics, the 4*4mm^2 is the smallest beam size we can consider (we are then left with 1380 events whilst the 20*20mm^2 cuts corresponds to 17500 events).

For each beam selection the peak of the energy deposited in the central crystal is determined by a gaussian fit.

4. Monte Carlo simulations

The test beam set up was described inside the GEANT program. The X and Y beam profiles are generated according to the experimental distributions of Figs 1 and 2. For each beam definition, 5000 to 7000 events are generated. As for the data the beam energy is 50 GeV. The tracking cuts are 10KeV both for electrons and photons. The event generation is performed on the Saclay HP9000 J200 computer, the computing time is about 40 seconds per event.These generated events are analyzed exactly in the same way as the test beam events.

5. Results

The deposited energy peak for 6*6, 8*8, 10*10, 15*15, and 20*20mm^2 beam dimensions normalized to the peak value obtained with the 4*4mm^2 beam is given in Table 1, both for data and Monte Carlo events.

A striking agreement between the two sets of values is observed. We notice no systematics in the difference between the Monte Carlo and the data in Table 1. Let us note however that the difference in the deposited energy in the central crystal between a point-like beam and the 20*20mm^2 beam is only 8%.

This result gives us a reasonable confidence in the GEANT program ability to describe the transverse profile of the electron shower. It can be shown that the shower transverse profile does not change with the incoming energy. Fig 3 shows this distribution for a 50 GeV electron. About 97% of the shower is contained in a cylinder of a 4 cm radius and 95% in a cylinder of 3.5 cm radius.

6. Parametrizations: the Moliere radius, the critical energy

The lateral extension of the electromagnetic showers is mainly due to the multiple scattering of the electrons and the smaller is their energy, the larger is the angle of scattering. This extension is limited by the range of these low energy electrons. The lateral spread of showers is often given in Moliere radius units [3]:

$$R_M = \frac{E_S}{E_c} X_0$$

With:

$$E_S = \sqrt{4\frac{\pi}{\alpha}} m_e c^2 = 21.2 MeV$$

where α is the fine structure constant, m_e the electron mass and X_0 the radiation length (0.89 cm for lead tungstate). E_c is called the critical energy. The number of particles in the shower increases until energy starts to dissipate, first by ionization and the Compton effect rather than radiation and pair production. At this point, the particles have reached the critical energy. In other words, the critical energy is the energy at which the loss by radiation $(dE/dX)_{rad}$ equals the loss by ionization $(dE/dX)_{col}$. Depending on the way the radiation loss is calculated, several estimations of Ec can be found, as illustrated for a copper absorber in ref. [4].

We have calculated the collision and radiation stopping powers in the lead tungstate from the values obtained for the atomic constituents (Pb,W,and O) by BERGER and SELTZER [5] [6] [7], using the BRAGG-KLEEMAN rule which states that the stopping power of a compound can be approximated by a weighted sum of the atomic constituents stopping powers. Good agreement between measurements and this approximation based on this additivity rule was observed[8].

The total energy dependence of the collision and radiation stopping powers is plotted in Fig.4, together with the radiation loss as a function of the total energy according to the

ROSSI ‘approximation B’ [9] asymptotic formula:

$$\frac{1}{\rho}\bullet\left(\frac{dE}{dX}\right)_{rad} = \frac{E_{tot}}{X_0}$$

where ρ is the density. In this approximation E_c is then defined as the energy dissipated by collision in one radiation length by an electron of energy E_c.

This gives E_c = 9.9 MeV for the lead tungstate whilst the crossing point between the calculated radiation and collision stopping powers corresponds to E_c = 13 MeV. For comparison, ref [4] gives for these two estimations of E_c,19.6 and 24.8 MeV for a copper absorber. We obtained 8.1 and 11.3 MeV respectively, for a tungsten absorber.

One usually admits that 95% of the shower is contained in a cylinder with a $2R_M$ radius. From Fig 3 we compute R_M = 1.75 cm for the lead tungstate, assuming that the shower lateral spread is well described in terms of Moliere radius units. This gives E_c = 10.8 MeV. The approximate formula of [5]:

$$E_c = \frac{800MeV}{Z + 1.2}$$

gives E_c = 11.5 MeV using an “effective” atomic number Z=68.35 for the lead tungstate. The uncertainties in the critical energy estimations show that the measurement of the lateral spread of the showers in Moliere radius units is accurate to 10% at most.

The knowledge of the critical energy can also give a first estimation of the longitudinal shower development shape. At the point where particles have reached the critical energy, one can assume that the average energy of electrons, positrons and photons is the same, namely E_c[10]. Their number is then:

$$N_{max} = \frac{E_0}{3 \cdot E_c}$$

where E_0 is the incident energy. This occurs at a distance:

$$X_c = a \cdot X_0 \cdot Log\left(\frac{E_0}{E_c} - b\right)$$

from the incidence point. Ref. [10] gives a = 1.01 and b = 1 for electrons: with Ec = 9.9 MeV (ROSSI definition) we then find the maximum of the shower at 6.8, 7.33, and 9.48 cm from the front face of the crystal for 50, 100, and 1000 GeV electrons. The GEANT MonteCarlo gives 7.16, 7.53 and 8.77 cm respectively.

7. Conclusion

The GEANT program is in very good agreement with our test beam measurement. This observation gives us a good confidence in the description of the shower shape by the program which predicts that 95% of the shower is contained in a cylinder of a 3.5 cm radius. The Moliere radius approximation gives a description of the shower lateral spread with about 10% accuracy, depending on the assumptions made in the calculation of the critical energy. In the lead tungstate the Moliere radius is about 1.75 cm.

8. References

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Tab. 1. The energy peak values normalized to the 4*4 beam size for different beam sizes for data and Monte Carlo events

Beam size	normalized(to4*4) peak value (data)	normalized(to4*4) peak value (Monte Carlo)
6*6 mm2	1.0034	1.004
8*8 mm2	1.009	1.011
10*10mm2	1.017	1.019
15*15 mm2	1.047	1.047
20*20 mm2	1.077	1.073

Fig. 1 The horizontal (X) beam profile for 50 GeV electrons

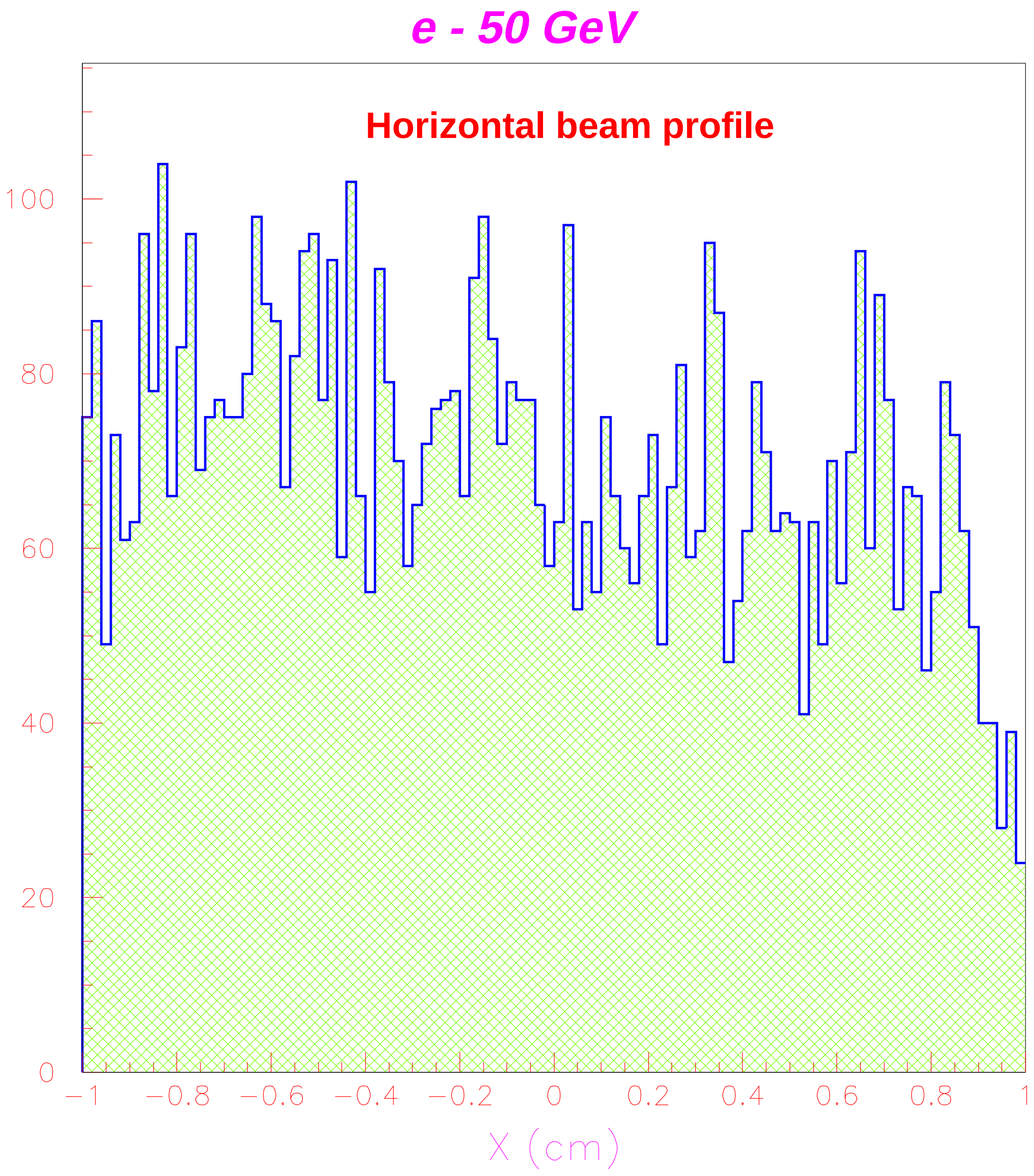

Fig. 2 The vertical (Y) beam profile for 50 GeV electrons

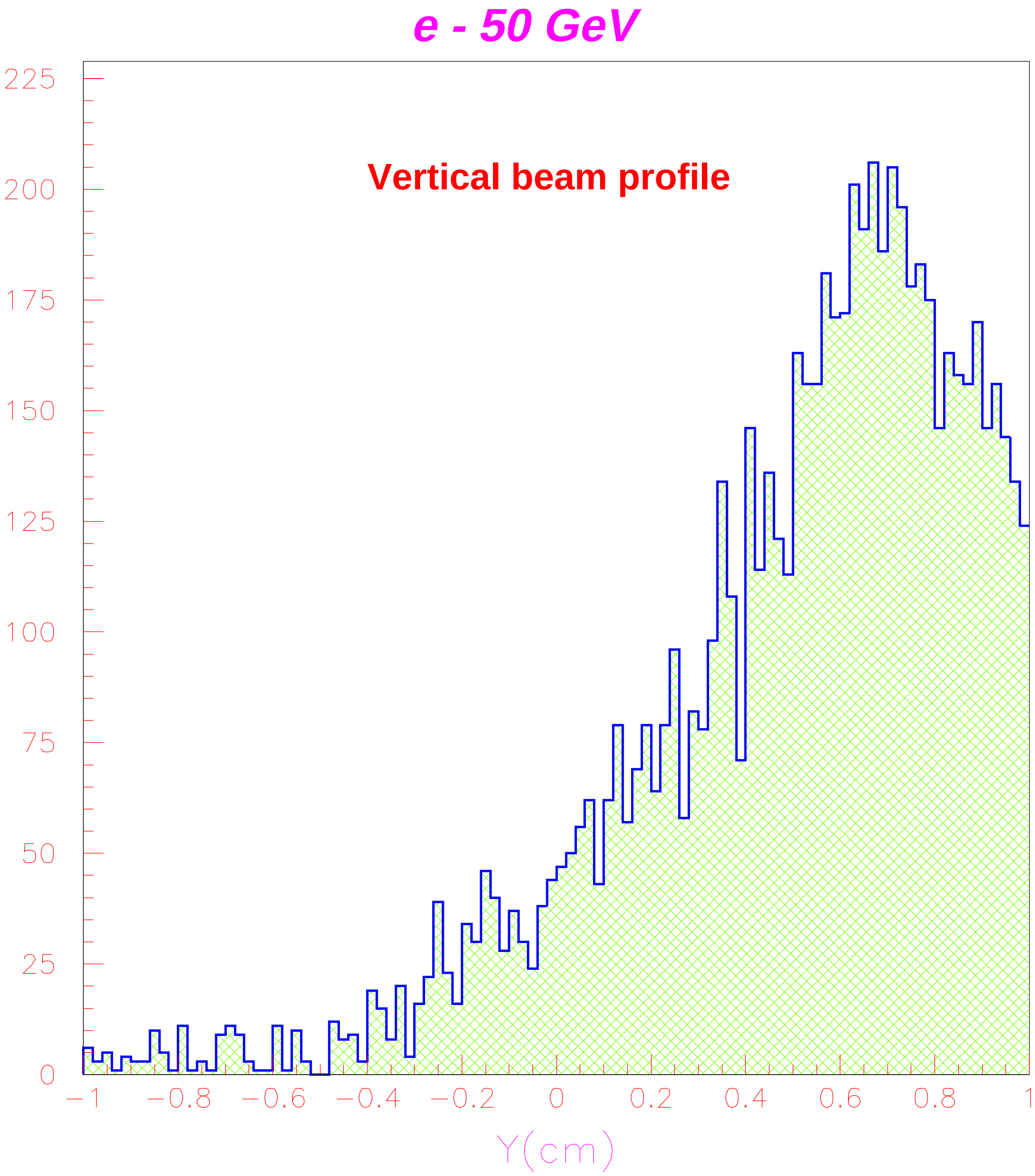

Fig. 3 The lateral shower profiles for 50 GeV electrons in lead tungstate from the GEANT Monte Carlo.

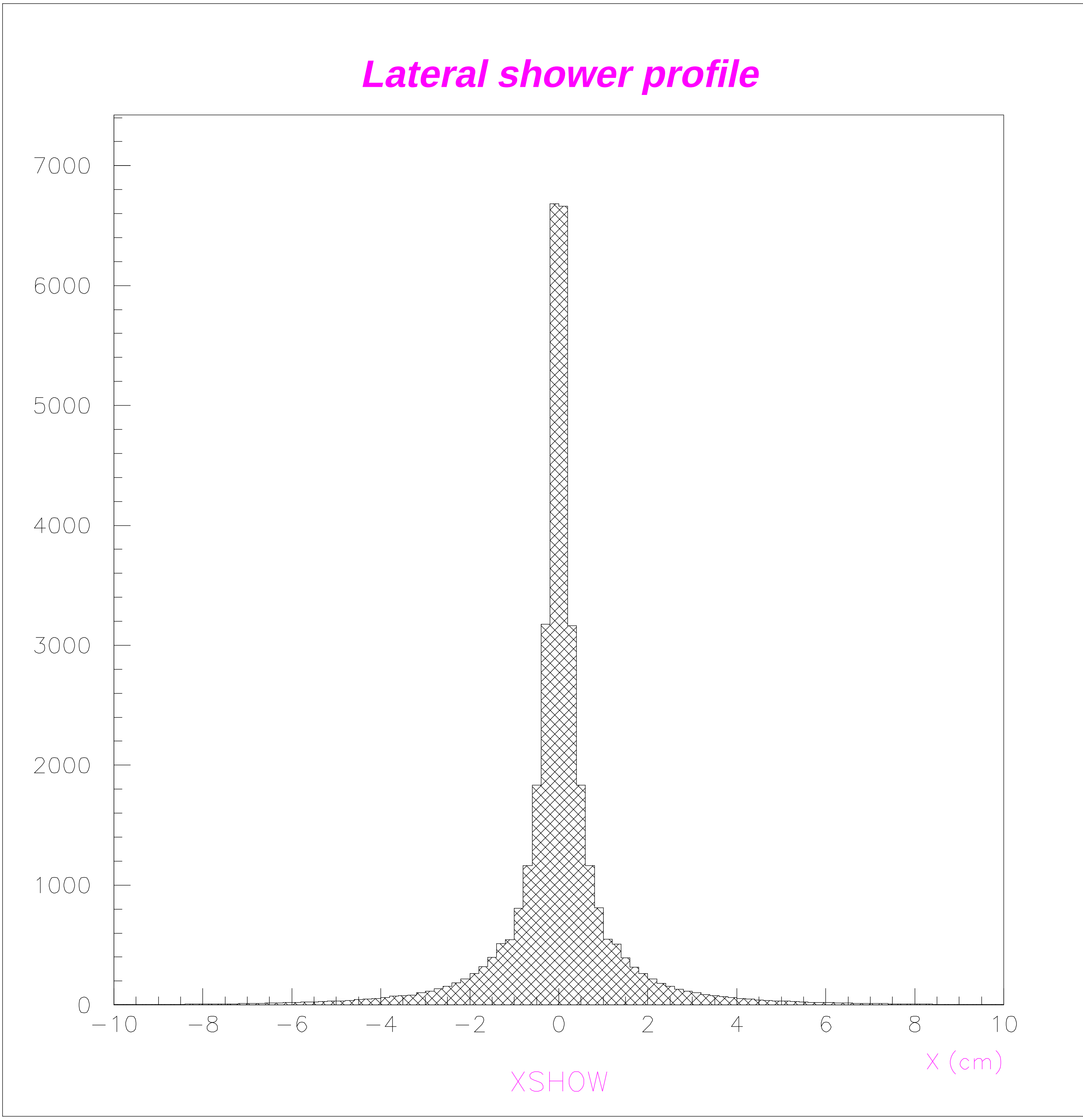

Fig. 4 The collision and radiation stopping powers as a function of the total energy in lead tungstate.