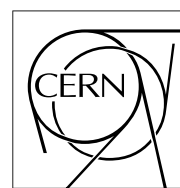


The Compact Muon Solenoid Experiment

CMS Note

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Evaluation of minimum number of signal events needed to measure the x_s mixing parameter

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Abstract

The evaluation of the minimum number of signal events needed to measure certain value of the mixing parameter x_s in the decay channel $B_s^0 \rightarrow D_s^- \pi^+ \rightarrow \phi \pi^- \pi^+ \rightarrow K^+ K^- \pi^- \pi^+$ is presented. In the study momentum and flight path resolutions are taken into account. Different signal to background ratios are tested. The range of the x_s values considered is from 20 to 50.

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1 Introduction

The investigation of $B^0 - \bar{B}^0$ mixing is important to constrain the parameter space of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], which is one of the main goals of B-physics. The measurement of the $B_d - \bar{B}_d$ mixing parameter helps to precisely determinate the $|V_{td}|$ matrix element. The mixing parameter x_d is measured already quite precise $x_d = 0.75 \pm 0.04$ [2] and further improvement does not help much because of theoretical uncertainties [3] in:

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 200 \pm 40 MeV \quad (1)$$

where f_{B_d} is the B_d decay constant and \hat{B}_{B_d} the hadron parameter. The measurement of the x_s value could improve remarkably the precision of the CKM matrix elements ratio $|V_{td}/V_{ts}|$, which can be extracted from the following relation:

$$\frac{x_s}{x_d} = \frac{\tau_s \hat{\eta}_{B_s} M_{B_s} (f_{B_s}^2 \hat{B}_{B_s}) |V_{ts}|^2}{\tau_d \hat{\eta}_{B_d} M_{B_d} (f_{B_d}^2 \hat{B}_{B_d}) |V_{td}|^2} \quad (2)$$

here τ_i are the life times, M_i are the masses, $\hat{\eta}_i$ are the QCD correction factors. Ratios in this equation are determined with much higher precision than each element alone. For example, the ratio $f_{B_s} \sqrt{\hat{B}_{B_s}} / f_{B_d} \sqrt{\hat{B}_{B_d}}$ is equal to 1.15 ± 0.05 which means $\sim 4\%$ accuracy instead of $\sim 20\%$ in the expression (1).

The measurement of the x_s value is a difficult task as the Standard Model (SM) predicts much higher frequency for the B_s oscillations than for B_d ones. The current experimental limit for the $B_s - \bar{B}_s$ mixing parameter is $x_s \geq 10.4$ [2]. The range allowed by the SM [3] is

$$12.9 \leq x_s \leq 26.1 \quad (3)$$

A sizable deviation from the SM prediction of the x_s value can indicate a presence of new physics beyond the SM.

In coming years several experiments, like HERA-B, CDF and D0 and later LHC experiments, will start serious investigation of the B_s meson oscillation. To evaluate the potential of these experiments it would be important to estimate the minimum number of signal events needed to observe oscillations and measure the x_s value. This paper is devoted to the questions: what values of the mixing parameter x_s can be measured and what is the required number of events taking into account background as well as proper time and momentum resolutions.

We restrict ourselves to one channel only:

$$B_s^0 \rightarrow D_s^- \pi^+ \rightarrow \phi \pi^- \pi^+ \rightarrow K^+ K^- \pi^- \pi^+ \quad (4)$$

This channel is interesting because of the fully reconstructible final states, reasonable branching ratio (about $\sim 6 \times 10^{-5}$) and potentially good separation from background. The result of the study depends on many parameters like number of signal events, signal to background ratio, proper time and momentum resolutions of the tracker system, tagging technique and trigger requirements. Almost all of these parameters are investigated in the paper.

A common method to observe oscillations is to split a sample of selected events into two samples: one contains events with B_s mesons having the same flavours and the other with B_s mesons having opposite flavours at the production and decay time. If there is enough signal statistics (statistical fluctuations are relatively small) and the dilution is not too big (i.e. the dilution factor is not too small) both distributions should show oscillations. Even if oscillations are not so evident, there are mathematical methods which can help to extract dominant oscillation harmonic which is directly connected with x_s value. The following methods to extract x_s value have been considered:

- log-likelihood analysis of proper time distributions using fitting functions:

$$P^+(\frac{t}{\tau_B}) = a^+ e^{-\frac{t}{\tau_B}} (1 + D^+ \cos(x_s \frac{t}{\tau_B})) \quad (5)$$

$$P^-(\frac{t}{\tau_B}) = a^- e^{-\frac{t}{\tau_B}} (1 - D^- \cos(x_s \frac{t}{\tau_B})) \quad (6)$$

Here, P^\pm are the probabilities for B_s to decay at the same/opposite flavour as it was produced, a^\pm are amplitudes, D^\pm are dilution factors, t is the proper time and τ_B is the life time of B_s ;

- Fourier transform of the proper time distributions.

The Note is organised as follows. In Section 2 the sample of signal events used in the study is described. With a restricted number of events the quality of the fit depends on the bin size. To perform the best fit of the proper time distribution one has to optimise bin size taking into account the proper time resolution. The binning of the proper time distributions and the proper time resolution is discussed in Section 3. Different algorithms of x_s value determination will be illustrated in Section 4. Dilution factor as a crucial element of the study will be analysed and the procedure to evaluate the minimum number of signal events needed to measure different x_s values will be introduced in Section 4 also. In the following section different scenarios will be considered. In Section 5.1 the analysis with the fixed value of the proper time resolution will be investigated. The parametrised proper time resolution as a function of the transverse momentum of B_s meson for the CMS tracker system will be used for more realistic analysis in Section 5.2. Using the tagging muon as a trigger will bias the sample of B_s mesons. The bias depends on the muon trigger threshold. The influence of various single muon trigger thresholds is discussed in Section 5.3. Results for x_s varying from 20 to 50 and signal to background ratios of 2:1, 1:1 and 1:2 will be presented in Section 6. Finally, in Section 7 conclusions of the study will be presented.

2 Signal sample

Proton-proton collisions at the centre of mass energy of 14 TeV have been simulated using PYTHIA5.7 event generator [4] and $b\bar{b}$ events were selected in such a way that both gluon splitting and fusion heavy quark production mechanisms have been taken into account. Simulation of each event was stopped at the parton level. About 10^6 of such events were stored (for details of simulation procedure see [5]).

For the analysis each $b\bar{b}$ event has been hadronised, fragmented and decayed 5 times to get reasonable statistics of selected B_s mesons. Both B mesons have been forced to decay: one B decays into $\mu + X$ (in this study the muon tagging technique is used) and B_s decays according to equation (4). The collected statistics corresponds to one month of the LHC operation assuming the low luminosity operation at $L = 10^{33} \text{cm}^{-2} \text{s}^{-1}$ and the total $b\bar{b}$ cross-section $\sigma_{b\bar{b}} = 500 \mu\text{b}$. For the final selection the following kinematic cuts have been applied: $p_t \geq 1 \text{GeV}$ and $|\eta| \leq 2.5$ for all final state particles (four hadrons and tagging muon).

Fig. 1 shows p_t distributions of B_s mesons and final state hadrons. This figure illustrates the fact that no significant bias was introduced by repeating 5 times hadronisation and decay of each event. Also, the plots show that the hardest particle in the final state is the pion which comes from B_s decay directly and softest particles are kaons which come from ϕ decay.

All results obtained with this sample of events and presented in this paper take into account momentum resolution. We use the momentum resolution of the CMS tracker. The following formula gives the parameterisation of the transverse momentum of a charged particle:

$$\sigma(p_t)/p_t = (1.5 \times 10^{-2} \times p_t \oplus 0.5) \times 10^{-2} \quad (7)$$

where, p_t is the transverse momentum in GeV and $\sigma(p_t)$ is the transverse momentum resolution.

3 Proper time resolution and binning

3.1 Bin size

The method to determine the x_s value used in the study is to fit the proper time distributions with functions like (5) and (6). To make such distributions one has to choose a bin size. As the goal of the study is to find a minimum number of signal events, the bin size should be as large as possible to minimise the influence of statistical fluctuations. On the other hand, the bin size can not be equal or larger than the period of oscillations. The bin size should be also such that an even number of bins coincides with a period of oscillations as the fitting function is proportional to the cosine. Two bins seems the extreme limit, hence for the study four bins per period have been chosen. The size of the bins have been optimised for each x_s value (keeping 4 bins per period) what is illustrated in the Table 1.

3.2 Example

Assuming $x_s = 20$, the number of signal events $N_s = 1000$, the signal to background ratio 1:1 and the proper time resolution $\sigma_t = 5 \%$, and following the standard procedure, by fitting the proper time distributions with functions (5) and (6) leaving the dilution factor as a free parameter. The resulting dilution factor and χ^2 of the fit as a function

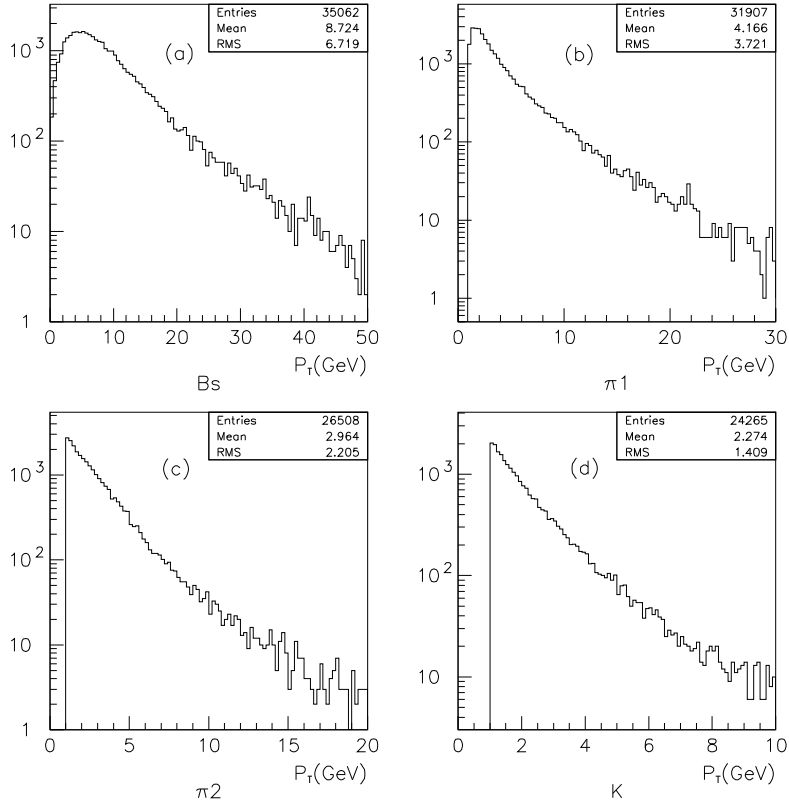


Figure 1: p_t distribution of B_s (a) and final state hadrons: $\pi 1$ (b) is the pion from B_s decay, $\pi 2$ (c) is the pion from D_s decay, K (d) comes from the ϕ decay.

x_s	T(ps)	Δt (ps)	$\Delta t/\tau_{B_s}$
10	1.01	0.25	0.16
15	0.67	0.17	0.11
20	0.51	0.13	0.08
25	0.40	0.10	0.06
30	0.34	0.09	0.06
35	0.29	0.07	0.04
40	0.25	0.06	0.04

Table 1: Periods of oscillation (T) and the corresponding bin size Δt for different x_s values

of the bin size of the proper time distributions is shown on Fig. 2. From Table 1 one can learn that a bin size of 0.08 ($\Delta t/\tau_{B_s}$) is the optimal one for $x_s = 20$. Fig. 2 justifies this fact: the dilution factor is maximum and χ^2 is minimum around 0.08 for both - like sign and unlike sign proper time distributions. A smaller binning is worse because of increased statistical fluctuations in each bin, a bigger bin size is also worse because of not an integer or even number of bins per period.

Table 1 also indicates the proper time resolution needed to measure different x_s values. The resolution should not be much worse than the bin size chosen, otherwise the dilution will be too large to see any oscillations.

The proper time is defined as follows :

$$t = m \times L_{xy}/p_t \quad (8)$$

where t is the proper time, m is the B_s mass, p_t is the transverse momentum and L_{xy} is the flight path of B_s in the transverse plane. The main contribution in the proper time error comes from the flight path resolution which

depends on the quality of the vertex detector. To measure a certain x_s value the vertex detector resolution should not be worse than listed in Table 1. For example, to measure $x_s = 30$ the flight path resolution should be $0.09ps$ or better.

4 Discussion of algorithms

The algorithms used are based on the fit of the B_s proper time distributions. B_s/\bar{B}_s states can be tagged at the production time using the sign of μ which comes from the second B meson in the event and at the decay time using the sign of reconstructed D_s . The proper time distributions of 'like' sign and 'unlike' sign samples are diluted because of the presence of background, miss-tagging (wrong charge determination of μ , for example), momentum, mass and flight path resolutions. Table 2 shows contributions to the dilutions from the histogram binning, miss-tagging (only due to the second B_d or B_s oscillation), momentum resolution and background.

Contributions to Dilution	like sign	unlike sign
binning	0.9554	0.9395
miss-tagging	0.7604	0.7557
resolution	0.7037	0.6719
background($N_{sig}/N_{bkg} = 1$)	0.5220	0.5178

Table 2: Different contributions to the dilution for $x_s=20$ using 1000 signal events, momentum smearing according to equation (7) and fixed proper resolution of 5%.

Usually, the dilution factor due to miss-tagging is calculated as follows:

$$D_{mis} = 1 - 2W \quad (9)$$

where, W is the miss-tagging probability. A value of $W = 0.12$ is used in Table 2. For further analysis the

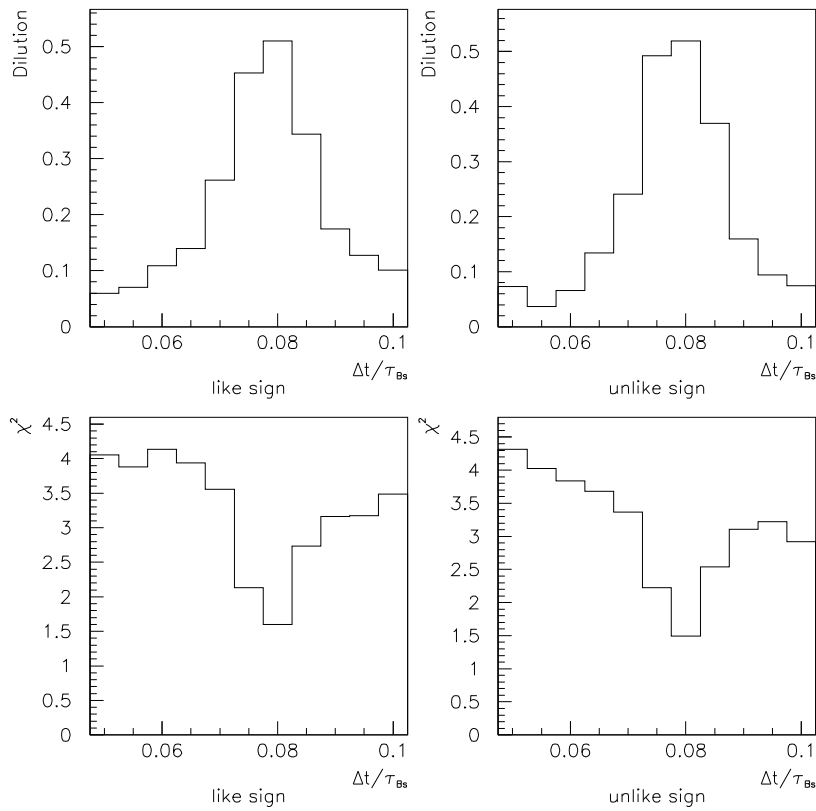


Figure 2: Dilution factor for different binning.

miss-tagging probability due to cascade decay ($b \rightarrow c \rightarrow \mu$) is added so that the total miss-tagging probability is equal to $W = 0.22$.

The classical maximum likelihood fit and Fourier analysis are applied. An improved method (1000 'experiments'), which is based on Fourier transform, is introduced to evaluate the minimum number of events needed to extract certain x_s value, as discussed in the following.

4.1 Maximum log-likelihood analysis

In this method, the frequency x_s^{min} which minimises $-\ln(L)$ is searched for, where $-\ln(L)$ can be either the fit of 'like' sign and 'unlike' sign proper time distributions respectively,

$$-\ln[L^{like}(x_s)] = \sum N_i^{like} \ln[P_i^{like}(x_s)] \quad (10)$$

$$-\ln[L^{unlike}(x_s)] = \sum N_i^{unlike} \ln[P_i^{unlike}(x_s)] \quad (11)$$

or the subtraction of 'like' sign and 'unlike' sign distributions:

$$-\ln[L^{like-unlike}(x_s)] = \sum N_i^{like-unlike} \ln[P_i^{like-unlike}(x_s)] \quad (12)$$

where $N_i^{like,unlike,like-unlike}$ is the 'like' and 'unlike' sign number of events or their subtraction in the time bin i and $P_i^{like,unlike,like-unlike}(x_s)$ is the proper time distribution probability expected for a mixing at a given frequency x_s . Fitting the subtraction instead of fitting respectively the 'like' sign and 'unlike' sign distributions is expected to suppress the influence of background which should have the same behaviour in both samples. The maximum value of log-likelihood corresponds to the real value of x_s . The precision of the x_s measurement can also be evaluated with this method. For instance, comparing Fig. 3(a ÷ c) with Fig. 3(d ÷ f), one can see that the error in x_s measurement increases immediately after introducing the momentum and secondary vertex resolutions.

4.2 Fourier analysis

Fourier analysis is a powerful method to analyse periodic signals. The Fourier transform applied to binned distribution is defined [6] as follows:

$$y_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \exp\left(\frac{2\pi ijk}{N}\right) x_k, \quad (j = 0, 1, \dots, N) \quad (13)$$

where N has to be a power of 2. The results of the Fourier transformation are returned in the form:

$$y = Y_1 + iY_2 \quad (14)$$

The oscillation frequency x_s corresponds to the peak in the amplitude distribution:

$$A = \sqrt{Y_1^2 + Y_2^2} \quad (15)$$

The amplitude of the peak is damped due to the flight path, momentum and mass resolutions and background whereas the width of the peak increases (see Fig. 4). Statistical fluctuations of the sample generate noise in the Fourier transform. To study quantitatively the influence of these factors on the extracted x_s value, as well as to evaluate the minimum number of events needed to measure certain x_s value, the 1000 'experiments' method is introduced.

4.3 Repeat an 'experiment' 1000 times

This method is to repeat Fourier analysis with 1000 different samples under the same condition: number of events, secondary vertex resolution, signal to background ratio, input x_s value and the same binning which calculated according to the x_s value. Then the confidence level, defined as the fraction of Fourier transforms which have a

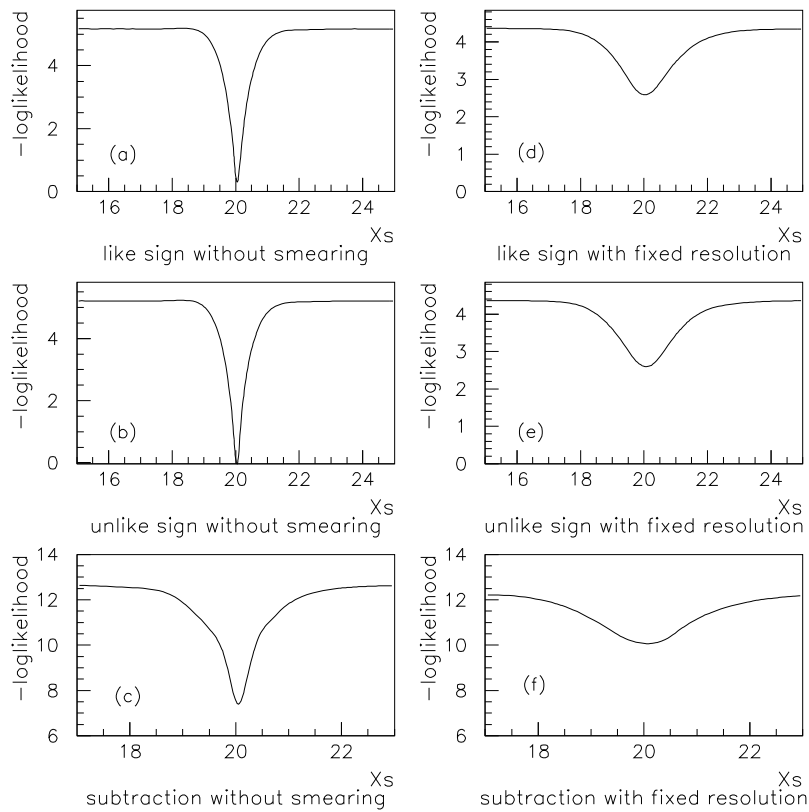


Figure 3: Log-likelihood of $x_s = 20$ with and without smearing.

peak at the expected x_s value, can be extracted. The minimum number of events to reach 95% confidence level is the minimum number of events we required to measure a certain x_s value under certain conditions.

The first problem of this method is that there is no such a large sample available with which 1000 independent

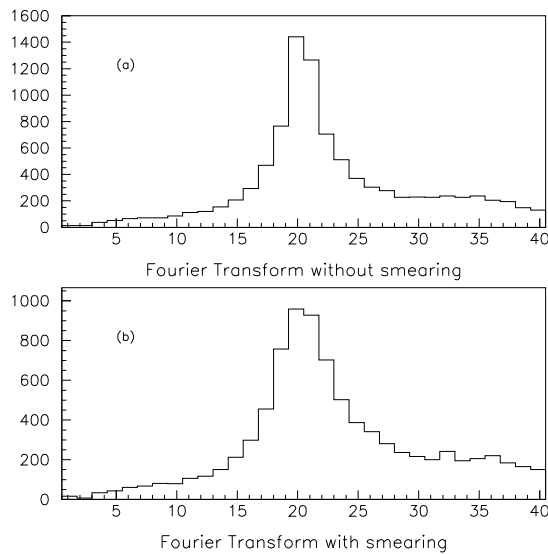


Figure 4: Fourier transform of $x_s = 20$ with and without smearing.

'experiments' can be performed. So, instead of really dividing a simulated sample into 1000 subgroups, oscillation functions such as eq. (5) and eq. (6) for 'like' and 'unlike' sign distributions respectively are used as 'seeds' to generate random distributions with the certain number of events which are used as signal distributions. Then certain number of background events are added to the 'like' and 'unlike' sign distributions respectively. This is one 'experiment'. 1000 such 'experiments' are repeated according to the same 'seed', but each time the signal and background distributions are randomly generated such that they are independent from each other.

The second problem is the quality of the 'seed' functions which is of vital importance to the results. We discuss this problem in the next Section.

4.4 Dilution factor

The average values of the dilutions factors (D) are extracted by randomly taking and fitting 100 sets of signal events from the sample. Fig. 5 compares the dilution factor distribution of 100 samples randomly taken from the total sample with that of the 1000 'experiments'. The mean values and width of the distributions are quite similar, which proves the correctness of the chosen procedure. When the number of signal events used in one experiment increases to nearly total number of events in the simulated sample, the procedure described above can not be used, as soon as all 100 sets of signal events will contain almost the same events. Nevertheless, additional study shows (Fig. 6) that the dilution factors remain almost constant for small and large number of signal events. This result allows us to use the asymptotic value of the dilution factors for large number of signal events.

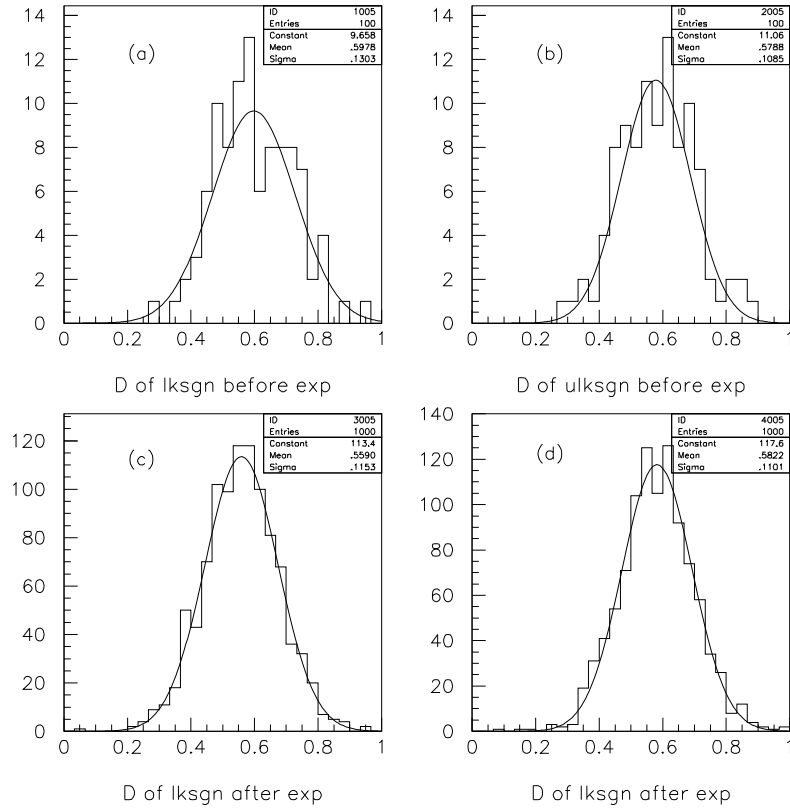


Figure 5: Dilution factor before and after generating signal distribution

5 Results

Proper time resolution is a key issue in measuring the x_s mixing parameter. Usually, a relative proper time resolution (resolution divided by proper time) has a Gaussian behaviour with a non-Gaussian tail. A Gaussian part

characterises the quality of the tracker system. Non-Gaussian tails depend on tracker design, selection cuts and vertex fitting algorithm. In this Section two cases are considered: fixed relative proper time resolution and proper time resolution as a function of the transverse momentum of B_s . Results of the study also depend on the bias introduced by the tagging technique. In CMS the tagging of the flavour of B_s at production time will be done using the sign of the trigger muon. How results depend on the muon trigger threshold is also considered in this Section.

5.1 Fixed proper time resolution

The relative proper time resolution is fixed at 5%. This value is chosen because it is close to the resolution declared by LHC-B: $\sim 3\%$ [7], ATLAS: $\sim 4.5\%$ [8] and CMS: $\sim 4\%$ [9]. We consider these results as an ideal case when non-Gaussian tails are vanished. This could be the case, for example, if the cut on flight path $L \geq n\sigma_L$ is used and n is quite large: $n \geq 5 \div 7$.

Fig. 7 shows the confidence level as a function of number of signal events for $x_s = 20$ and different values of signal to background ratio. One can derive from these plots that 250 to 700 signal events are needed to measure $x_s = 20$ for different background conditions.

5.2 Proper time resolution as a function of B_s momentum

The parameterisation of the secondary vertex resolution obtained for CMS [9] is used in this Section. The parameterisation of the Gaussian part is done with a formula:

$$\frac{\sigma_t}{L_{xy}} = \sqrt{\left(\frac{P_1}{p_t}\right)^2 + P_2^2} \quad (16)$$

Here, σ_t is the error of the flight path in the transverse plane, L_{xy} is the transverse flight path, p_t is the transverse momentum of B_s , P_1 and P_2 are parameters of the fit. Non-Gaussian tails have been parameterised also with a Gaussian of wider width. Parameters of the fit are $P_1 = 37\%$ and $P_2 = 4\%$. The width of the wide Gaussian is about 5 times bigger than the width of the narrow one. This parameterisation is done with a sample of B_s events triggered with a muon threshold of 6.5GeV which is a case for the low luminosity regime.

Fig. 8 shows the same distributions as Fig. 7 but for the parametrised proper time resolution. One can see that the number of signal events needed to measure x_s values is much bigger: from 1000 to 3000 depending on the signal to background ratio. The reason of that is a presence of non-Gaussian tails which degrade the secondary vertex resolution.

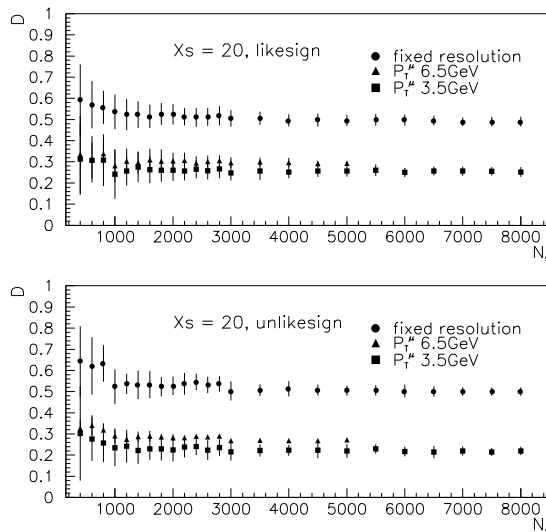


Figure 6: Dilution factor for $x_s = 20$, for like sign and unlike sign samples.

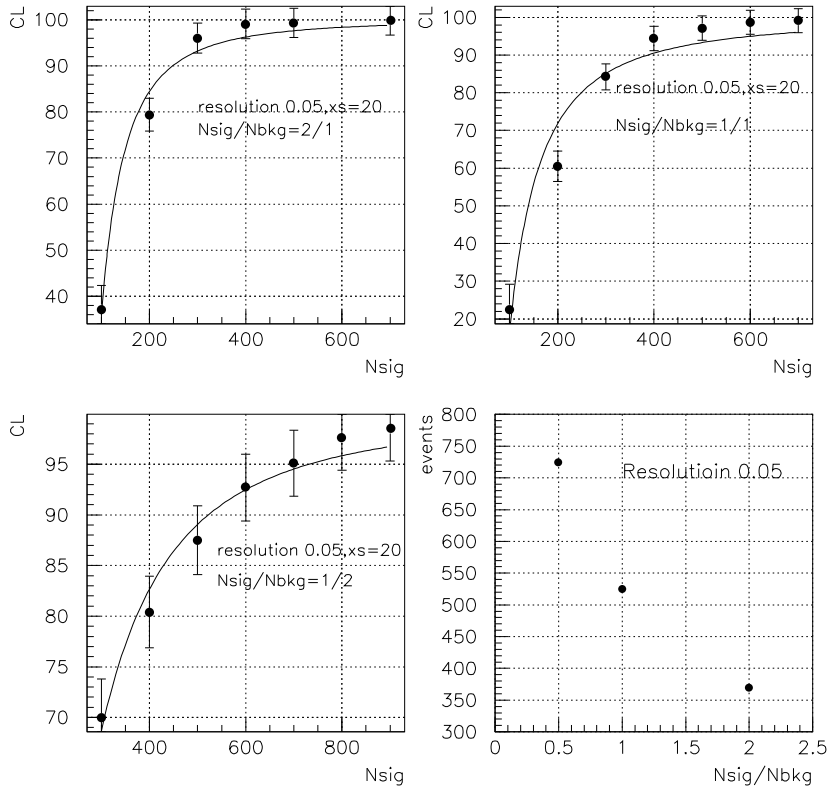


Figure 7: CL versus number of signal for different S:B ratio, fixed proper time resolution. Last plot: number of events needed to measure $x_s = 20$ vs S:B ratio at 95% CL.

5.3 Influence of the muon trigger threshold

The lower threshold of the trigger muon, could be the case at the LHC start up, when the integrated luminosity is about few times $10^3 pb^{-1}$. With such a luminosity and the muon trigger threshold of $3.5 GeV$ the rate of single muon trigger will be similar as with the threshold of $6.5 GeV$ at integrated luminosity of $10^4 pb^{-1}$ for CMS: the luminosity is decreased by factor 3-5, but the number of accepted events are increased due to the lower threshold by the roughly the same factor (see Fig. 9c).

The sample of B_s will be softer in this case. Fig. 9a shows the dependence of the mean transverse momentum of B_s on the muon trigger threshold. This also means that the flight path of B_s will be shorter (Fig. 9b). Hence, the flight path resolution will be worse. Using the same parameterisation of the flight path resolution as in the previous section but the muon trigger threshold of $3.5 GeV$ one needs more signal events than in the case with the threshold of $6 GeV$. Fig. 10 provides results for the muon trigger threshold of $3.5 GeV$. Now the same x_s values can be measured with 1400 to 4000 (instead of 1000 to 3000) signal events depending on the signal to background ratio. This can be easily understood keeping in mind the following dependence [8]:

$$x_s^{max} \sim \frac{L}{\sigma_L} \quad (17)$$

where L is the flight path and σ_L is the flight path resolution of B_s .

6 Discussion of results

In this study the range of x_s from 20 to 50 and signal to background ratios 2:1, 1:1 and 1:2 have been investigated. Results are summarised and presented in Fig. 11, where minimum number of signal events are plotted versus the x_s value for different signal to background ratios and different assumptions about flight path resolution.

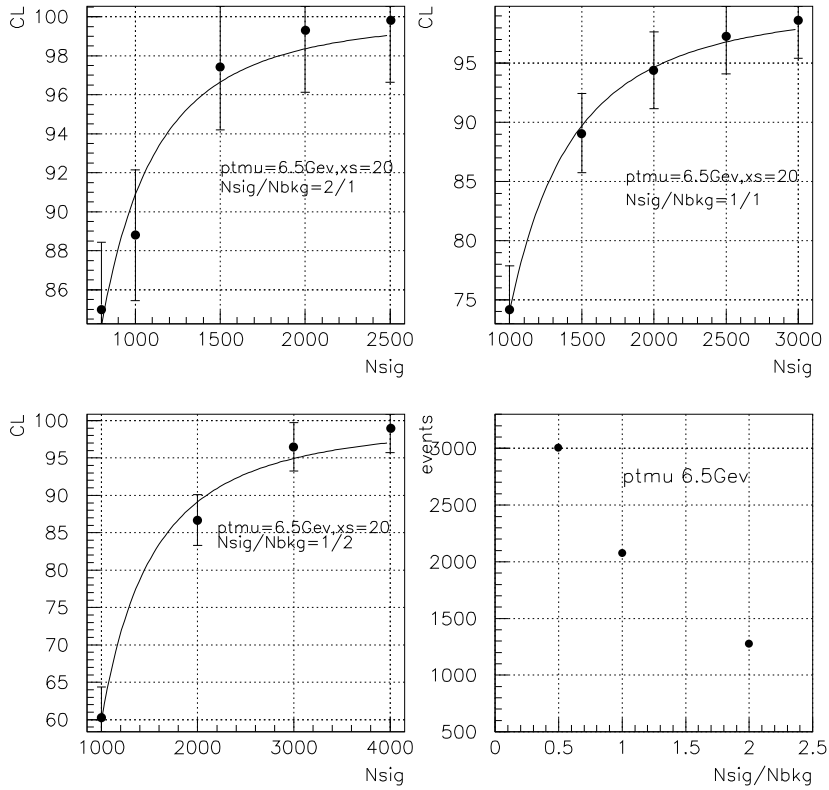


Figure 8: Confidence level for $x_s = 20$ and $p_t^\mu = 6.5 \text{ GeV}$, $\sigma_{sv} = f(p_t)$.

From these plots one can conclude that the flight path resolution plays a crucial role in the observation of B_s oscillations. The best results are obtained with the fixed flight path resolution of 5%. One needs less than 1000 signal events to measure x_s up to 40 even in the worst case of the signal to background ratio of 1:2. As soon as one uses parameterised flight path resolution (even with better asymptotic value of 4%) including 'non-Gaussian' tails, the number of signal events needed to measure the same x_s values with the same signal to background ratios will be increased by about a factor of 4. Using the same parameterisation but softer B_s sample (lower muon trigger threshold) and, hence, worse flight path resolution, 30%-50% more signal events are needed again.

There is an obvious dependence of the number of signal events on the x_s value. Increasing x_s value one need to use smaller bin size which leads to a decreasing of the number of events in bin and increases the influence of statistical fluctuations (the dilution factor due to fluctuations becomes smaller). And also, decreasing the signal to background ration one decreases the dilution factor due to background which results in increasing the number of signal events needed to measure certain x_s value.

Results obtained are in quite a good agreement with results expected by the ATLAS Collaboration [8]. In [8] $x_s = 42$ is mentioned as a reachable value for the following assumptions:

- the number of signal events is ~ 4800 : two of the B_s decay channels have been analysed: $B_s \rightarrow D_s^- \pi^+$ [10] and $B_s \rightarrow D_s^- a_1^+$ [11];
- the signal to background ratio is equal to 1;
- the muon trigger threshold of 6 GeV ;
- the mean flight path resolution is $\sigma_t = 0.07 \text{ ps}$.

This set of parameters are very close to the one discussed in Section 5.2. One can find a point in Fig. 11c which corresponds to this set of parameters. The number of signal events required to measure $x_s = 40$ is about 4400 in our study.

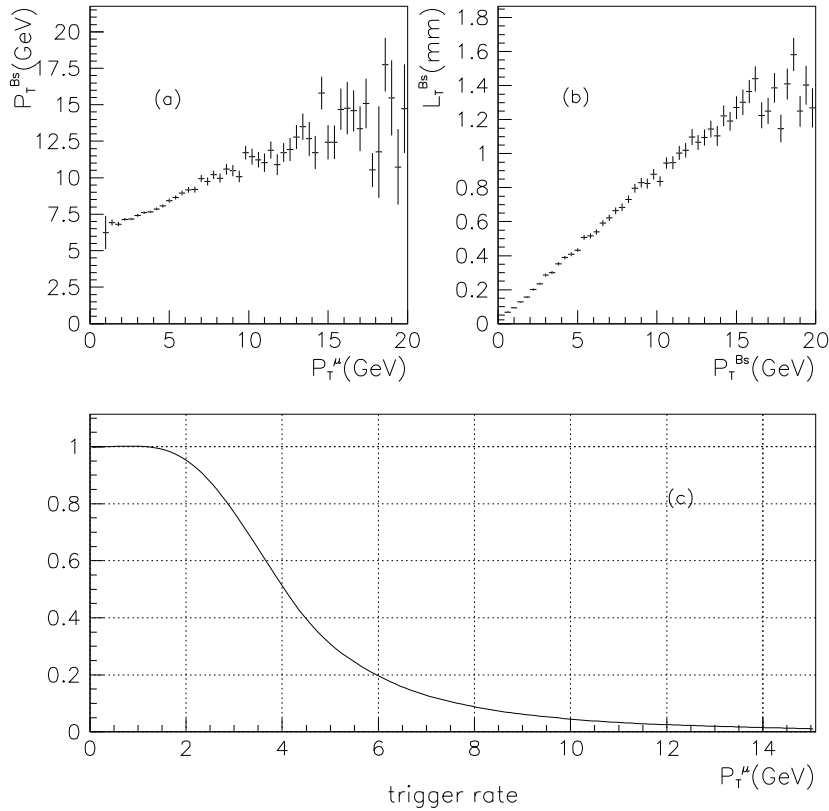


Figure 9: p_t of B_s versus transverse momentum of trigger muon (a); flight path in the transverse plane of B_s versus p_t of B_s (b) and cumulative muon trigger rate for the CMS detector (c).

7 Conclusion

The minimum numbers of signal events have been evaluated under different assumptions about flight path resolution, signal to background ratio and for $x_s = 20 \div 50$.

Using the expected flight path resolutions, number of signal events, signal to noise ratios of coming new experiments, one can conclude that the predicted SM range $\sim 13 \leq x_s \leq 30$ can be easily reached. Even if x_s is about $40 \div 50$, $B_s - \bar{B}_s$ oscillations will be observed.

Nevertheless, we would like to stress once again the importance of the flight path resolution. Potential problems with the primary and secondary vertices precision or unexpectedly big non-Gaussian tails in the flight path resolution could result in larger number of signal events needed. LHC-B experiment has quite a large safety margin as in this experiment is expected to reconstruct about 35500 signal events in the decay mode

$$B_s^0 \rightarrow D_s^- \pi^+ \rightarrow \phi \pi^- \pi^+ \rightarrow K^+ K^- \pi^- \pi^+$$

with a signal to background ration of 10:1 [7]. The upper limit of x_s reachable using this statistics is about of 90 [7]. Also ATLAS and CMS experiments have enough safety margin as it is reasonable to expect the increasing of the signal statistics from $3000 \div 5000$ events up to $2 \div 3$ times using additional decay modes of B_s and D_s . D0 and CDF have also good chance to measure the x_s mixing parameter. From [12], the CDFII experiment, for example, expects to collect about one to two thousand events in the decay mode investigated in this Note. According to our results, CDFII has a good chance to measure x_s about 20, provided that the flight path resolution will be not much worse than the one considered in this paper.

We would like to thank F.Pauss for the attention to this work, advises and corrections have been made during preparation of this Note.

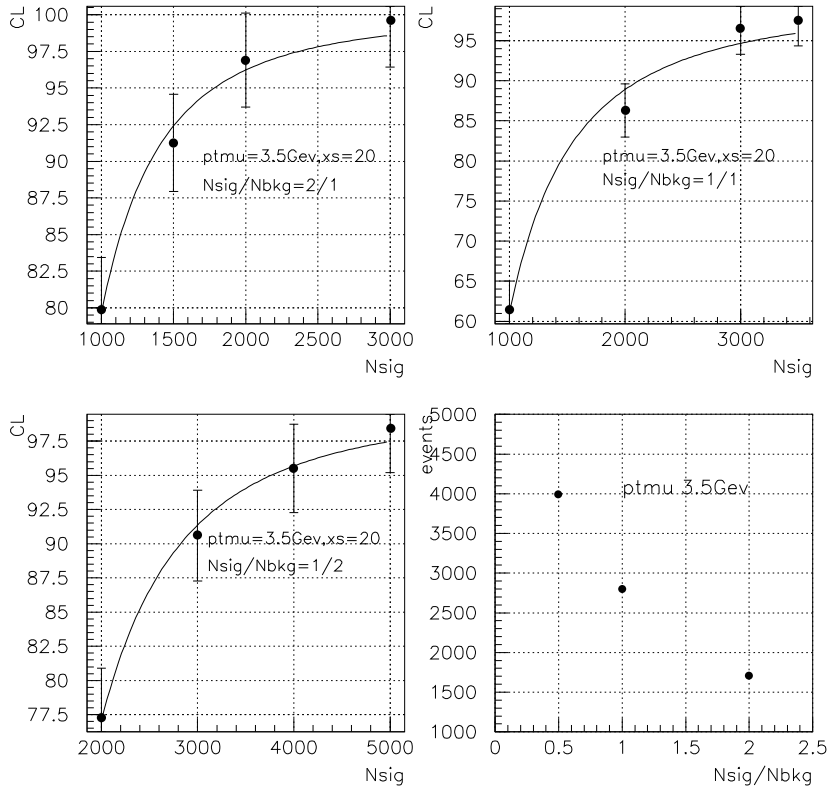


Figure 10: Confidence level for $x_s = 20$ and $p_t^\mu = 3.5 \text{ GeV}$, $\sigma_{sv} = f(p_t)$.

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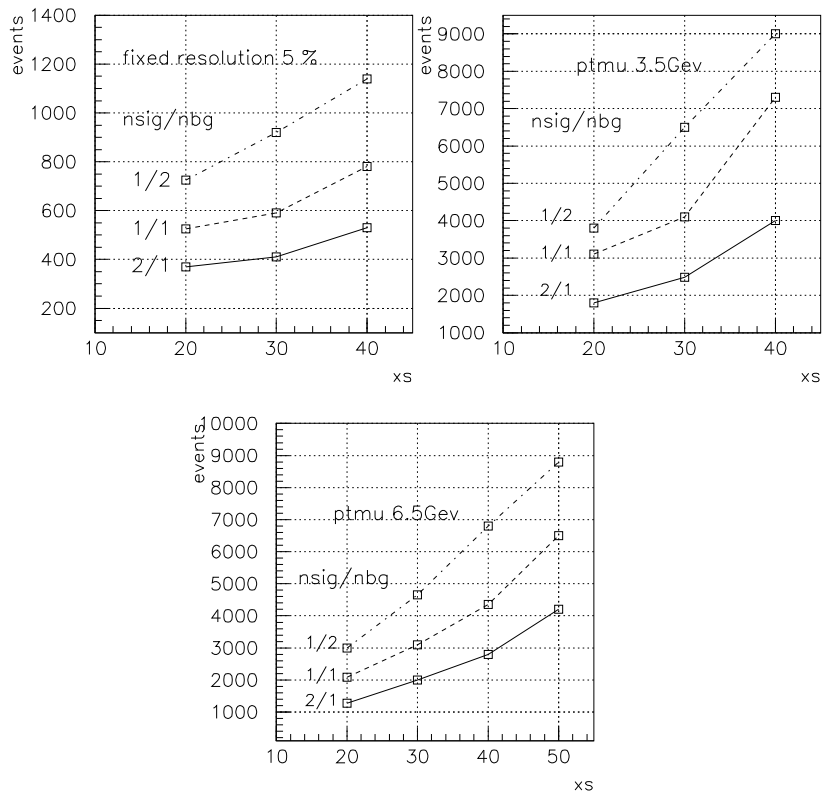


Figure 11: Minimum number of events versus x_s value for different signal to background ratios.