Born-Infeld strings in brane worlds

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Abstract

We study Born-Infeld strings in a six dimensional brane world scenario recently suggested by Giovannini, Meyer and Shaposhnikov (GMS). In the limit of the Einstein-Abelian-Higgs model, we classify the solutions found by GMS. Especially, we point out that the warped solutions, which lead to localisation of gravity, are the - by the presence of the cosmological constant - deformed inverted string solutions. Further, we construct the Born-Infeld analogues of the anti-warped solutions and determine the domain of existence of these solutions, while a analytic argument leads us to a "no-go" hypothesis: solutions which localise gravity do **not** exist in a 6 dimensional Einstein-Born-Infeld-Abelian-Higgs (EBIAH) brane world scenario. This latter hypothesis is confirmed by our numerical results.

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I. INTRODUCTION

The idea that we live in more than the observed 4 dimensions has been of huge interest ever since it was first suggested by Kaluza and Klein in the 1920s [1]. They studied a five-dimensional gravitational theory in a model with an extra *compact* dimension. The effective four dimensional theory then contains 4 dimensional gravity as well as the electromagnetic fields and a scalar field. In a similar way, (super)string theories contain extra *compact* dimensions [2] (superstring theories have 6 extra compact dimensions) with size of the order of the Planck length $= 1.6 \cdot 10^{-33} cm$. An example is the Calabi-Yau space in heterotic string theory whose properties determine the low energy effective field theory.

Other models, which were discussed extensively in recent years are so-called brane world scenarios [3, 4, 5, 6, 7, 8] which assume that the Standard model (SM) fields are confined to a 3-brane (a $3+1$ dimensional submanifold) which is embedded in a higher dimensional space-time. The extra dimensions now are *non-compact*. Since gravity is a property of space-time itself, a model which describes appropriately the well-tested Newton's law should localise gravity well enough to the 3-brane. This was achieved in [8] by placing a 3-brane into 5 dimensions with the 5th dimension being infinite. For the localisation of gravity in this model, the brane tension has to be fine-tuned to the negative bulk cosmological constant.

Recently, the localisation of gravity on different topological defects has been discussed [9]. This includes domain walls [10], Nielsen-Olesen strings [11, 12], and magnetic monopoles [13] in 5, 6 and 7 space-time dimensions, respectively. It was found [9, 11] that gravity-localising (so-called "warped") solutions are possible if certain relations between the defect's tensions hold. While in the case of domain walls and strings, gravity can only be localised when the bulk cosmological constant is negative, for magnetic monopoles the gravity-localisation is possible for both signs of the cosmological constant.

Originally introduced to remove singularities associated with point-like charges in electrodynamics [14], the generalisation of the Born-Infeld (BI) action to non-abelian gauge fields has gained a lot of interest in topics related to string theory [15, 16]. It became apparent that when studying low energy effective actions of string theory, the part of the Lagrangian containing the abelian Maxwell field strength tensor and its non-abelian counterpart in Yang-Mills theories has to be replaced by a corresponding (resp. abelian and non-abelian) BI term. That's why it seems interesting to generalise the brane world sce-

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nario for 6-dimensional Nielsen-Olesen strings recently proposed by Giovannini, Meyer and Shaposhnikov (GMS) [12] to Born-Infeld actions.

Our paper is organised as follows : in section II, we present the 6 dimensional Einstein-Born-Infeld-Abelian-Higgs (EBIAH) model, we give the equations of motion and especially present our analytic argument that no solutions, which localise gravity exist in the EBIAH model. In Section III, we present our numerical results for both the EBIAH model , namely, we present so-called "anti-warped" solutions, as well as for the limit of the Einstein-Abelian-Higgs (EAH) model. In this latter case, we put the emphasis on the connection between the GMS solutions and the four types of solutions available in the EAH model, namely the string branch, the inverted string branch, the Melvin branch and the Kasner branch [17, 18]. Especially, we show that the "warped" solutions found in [11, 12] are the - by the cosmological constant - deformed inverted string solutions. We give our conclusions in Section IV.

II.   THE MODEL

We have the following 6-dimensional action [9]:

$$S_d = S_{gravity} + S_{brane} \tag{1}$$

where the standard gravity action reads

$$S_{gravity} = -\int d^6x \sqrt{-g} \frac{1}{16\pi G_6} \left(R + 2\hat{\Lambda}_6 \right) \ . \tag{2}$$

$\hat{\Lambda}_6$ is the bulk cosmological constant, G_6 is the fundamental gravity scale with $G_6 = 1/M_{pl(6)}^4$ and g the determinant of the 6-dimensional metric.

The action S_{brane} for the Einstein-Born-Infeld-Abelian-Higgs (EBIAH) string is given by [17, 19]:

$$S_{brane} = \int d^6x \sqrt{-g_6} \left(\hat{\beta}^2(1 - \mathcal{R}) + \frac{1}{2} D_M\phi D^M\phi^* - \frac{\lambda}{4}(\phi^*\phi - v^2)^2 \right) \tag{3}$$

where

$$\mathcal{R} = \sqrt{1 + \frac{F_{MN}F^{MN}}{2\hat{\beta}^2} - \frac{(F_{MN}\tilde{F}^{MN})^2}{16\hat{\beta}^4}} \tag{4}$$

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with the covariant derivative $D_M = \nabla_M - ieA_M$ and the field strength $F_{MN} = \partial_M A_N - \partial_N A_M$ of the U(1) gauge potential A_M. v is the vacuum expectation value of the complex valued Higgs field ϕ and λ is the self-coupling constant of the Higgs field.

A. The Ansatz

The Ansatz for the 6-dimensional metric reads [9]:

$$ds^2 = M^2(r)\left(dx_1^2 - dx_2^2 - dx_3^2 - dx_4^2\right) - dr^2 - l^2(r)d\theta^2 \ . \tag{5}$$

For the gauge and Higgs field, we have [17]:

$$\phi(r,\theta) = vf(r)e^{in\theta} \ , \quad A_\theta(r,\theta) = \frac{1}{e}(n - P(r)) \tag{6}$$

where n is the vorticity of the string.

B. Equations of Motion

Introducing the following dimensionless coordinate x and the dimensionless function L:

$$x = \sqrt{\lambda}vr \ , \quad L(x) = \sqrt{\lambda}vl(r) \tag{7}$$

the set of equations depends only on the following dimensionless coupling constants:

$$\alpha = \frac{e^2}{\lambda} \ , \quad \gamma^2 = 8\pi G_6 v^2 \ , \quad \Lambda = \frac{\hat{\Lambda}_6}{\lambda v^2} \ , \quad \beta^2 = \frac{\hat{\beta}^2}{\lambda v^4} \ , \tag{8}$$

The gravitational equations then read:

$$3\frac{M''}{M} + \frac{L''}{L} + 3\frac{L'}{L}\frac{M'}{M} + 3\frac{M'^2}{M^2} + \Lambda = -\gamma^2 T_0^0 \tag{9}$$

$$6\frac{M'^2}{M^2} + 4\frac{L'}{L}\frac{M'}{M} + \Lambda = -\gamma^2 T_x^x \ , \tag{10}$$

$$6\frac{M'^2}{M^2} + 4\frac{M''}{M} + \Lambda = -\gamma^2 T_\theta^\theta \ , \tag{11}$$

where

$$T_0^0 = \left(\frac{(f')^2}{2} + \frac{(1-f^2)^2}{4} + \frac{f^2 P^2}{2L^2} + \beta^2\left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} - 1\right)\right) \ , \tag{12}$$

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$$T_x^x = \left(-\frac{(f')^2}{2} + \frac{(1-f^2)^2}{4} + \frac{f^2 P^2}{2L^2} + \beta^2 \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} - 1 \right) - \frac{P'^2}{\alpha L^2} \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} \right)^{-1} \right)$$
(13)

$$T_\theta^\theta = \left(\frac{(f')^2}{2} + \frac{(1-f^2)^2}{4} - \frac{f^2 P^2}{2L^2} + \beta^2 \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} - 1 \right) - \frac{P'^2}{\alpha L^2} \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} \right)^{-1} \right).$$
(14)

The Euler-Lagrange equations for the matter fields read:

$$\frac{(M^4 L f')'}{M^4 L} + (1 - f^2)f - \frac{P^2}{L^2} f = 0$$
(15)

and

$$\frac{L}{M^4} \left(\frac{M^4 P'}{L\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}}} \right)' - \alpha f^2 P = 0 .$$
(16)

The prime denotes the derivative with respect to x. For $\beta^2 = \infty$ these equations reduce to those studied in [9].

The equations (9)-(11) can be combined to obtain the following two differential equations for the two unknown metric functions:

$$\frac{(M^4 L')'}{M^4 L} + \frac{\Lambda}{2} = -\gamma^2 \left(T_0^0 + \frac{1}{4}T_x^x - \frac{3}{4}T_\theta^\theta \right)$$
(17)

which then gives:

$$\frac{(M^4 L')'}{M^4 L} + \frac{\Lambda}{2} = -\gamma^2 \left(\frac{1}{8}(1-f^2)^2 + \frac{f^2 P^2}{L^2} + \frac{\beta^2}{2} \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} - 1 \right) + \frac{P'^2}{2\alpha L^2} \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} \right)^{-1} \right)$$
(18)

and

$$\frac{(LM^3 M')'}{M^4 L} + \frac{\Lambda}{2} = -\frac{\gamma^2}{4} \left(T_x^x + T_\theta^\theta \right)$$
(19)

which then reads:

$$\frac{(LM^3 M')'}{M^4 L} + \frac{\Lambda}{2} = -\frac{\gamma^2}{4} \left(\frac{(1-f^2)^2}{2} + 2\beta^2 \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} - 1 \right) - \frac{2P'^2}{\alpha L^2} \left(\sqrt{1 + \frac{P'^2}{\alpha\beta^2 L^2}} \right)^{-1} \right)$$
(20)

C. Boundary conditions

We require regularity at the origin $x = 0$ which leads to the boundary conditions:

$$f(0) = 0 , \quad P(0) = n , \quad M(0) = 1 , \quad M'|_{x=0} = 0 , \quad L(0) = 0 , \quad L'|_{x=0} = 1 . \quad (21)$$

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The requirement for finiteness of the energy leads to:

$$f(\infty) = 1 , \quad P(\infty) = 0 .$$ (22)

D. String tensions

The string tensions are defined as follows:

$$\rho_i = \int_0^\infty dx \sqrt{-g} T_i^i .$$ (23)

The relation between the tensions ρ_0 and ρ_θ can then be evaluated. Using (12) and (14), we find:

$$\rho_0 - \rho_\theta = \int_0^\infty dx M^4 L \left(\frac{f^2 P^2}{L^2} + \frac{P'^2}{\alpha L^2 \sqrt{1 + \frac{P'^2}{\alpha \beta^2 L^2}}} \right)$$ (24)

and using (16), we find:

$$\rho_0 - \rho_\theta = \frac{1}{\alpha} \left(\frac{M^4 P P'}{\sqrt{L^2 + \frac{P'^2}{\alpha \beta^2}}} \right) \Bigg|_0^\infty .$$ (25)

For warped solutions, the infinity term should vanish, while for $x = 0$, we use the boundary conditions (21). Thus:

$$\rho_0 - \rho_\theta = -\frac{n|\beta|}{\sqrt{\alpha}}$$ (26)

Now, from [11], we know that $\rho_0 - \rho_\theta = \gamma^{-2}$ in order to have solutions which localise gravity. However, since the rhs of (26) is always negative, this relation can never be fulfilled, which leads us to the hypothesis that warped solutions don't exist in the Einstein-Born-Infeld model.

III. NUMERICAL SOLUTIONS

Before to discuss our numerical results for the EBIAH model, let us explain the pattern of solutions in the $\beta = \infty$ case i.e. in the Einstein-Abelian-Higgs (EAH) model. Since it was stated in [12] that no attempt to classify the solutions is done, the following discussion can be seen as such a classification.

A. Einstein-Abelian-Higgs (EAH) model ($\beta = \infty$ limit)

We first discuss the solutions in the case $\Lambda = 0$. The pattern of solution in the 6-dimensional EAH model is very similar to the one in the 4-dimensional EAH model [18] : for generic values of α, γ there are two branches of solutions: a branch for which $M(x \to \infty) = a$ and $L(x \to \infty) = bx + c$, where the parameters a, b depend on the coupling constants. For $\gamma = 0$, the Nielsen-Olesen string solutions [17] are recovered, this is why this branch of solutions is referred to as the *string branch*.

For $\gamma \ll 1$ the Nielsen-Olesen string is slightly deformed by gravity and $b \approx 1$, $a \approx 1$. When γ is increased (with α being fixed) the parameter a varies slowly while the parameter b decreases linearly with γ such that it becomes negative at some critical value of γ, say $\gamma = \gamma_{cr}^0(\alpha, \Lambda = 0)$. For $\gamma > \gamma_{cr}^0$ the solutions continue to exist but, since the function $L(x)$ possesses a zero , the solution is not regular on the full interval of the radial coordinate. The solutions for $\gamma > \gamma_{cr}^0$ are called *inverted string* solutions. We have constructed solutions for the self-dual limit ($\alpha = 2$) and for $\Lambda = 0$. We find that in complete agreement with the four dimensional case, the critical value $\gamma_{cr}^0(\alpha = 2) = 2$ and that in addition $M^{\alpha=2}(x) = 1$, while $b^{\alpha=2}(\gamma) = 1 - \frac{\gamma}{2}$ (see Fig. 1). Thus for $\gamma > \gamma_{cr}^0 = 2$, $L(x)$ is zero at some $x = x_0(\gamma)$, $L(x = x_0) = 0$. Note that the function $M(x)$ stays finite at $x = x_0$.

For the same values of the coupling constants a second branch of solutions, the so-called *Melvin branch*, exists. In $d = 6$, the solutions on this branch behave like $M(x \to \infty) = Ax^{2/5}$, $L(x \to \infty) = Bx^{-3/5}$. The constants A and B depend on the coupling constants of the model. There exist also solutions, the so-called *Kasner solutions*, for $\gamma > \gamma_{cr}^0$. For these strongly coupled gravitating solution the function $M(x)$ develops a node at some finite value of $x = \tilde{x}_0$ with $M(x = \tilde{x}_0) = 0$, while at the same time the function $L(x \to \tilde{x}_0) \to \infty$. For $\alpha = 2$, we present the dependence of the values A and B on the gravitational coupling γ in Fig. 1. We find that like in the case of the string branch $\gamma_{cr}^0(\alpha = 2) = 2$. For $\gamma \to 2$, A tends to zero, while B tends to infinity.

From the discussion of the solutions in the $\Lambda = 0$ limit, we note that the main feature is the existence of a critical value of γ, say $\gamma_{cr}^0(\alpha, \Lambda = 0)$, which clearly separates the classical solution into two distinguished domains; string branch and Melvin branch exist for $\gamma < \gamma_{cr}^0(\alpha, 0)$, inverted string branch and Kasner branch for $\gamma > \gamma_{cr}^0(\alpha, 0)$. However, as was noted in [18], the domain of existence of the Kasner solutions is limited. For the

4-dimensional Kasner solutions, it was found that for fixed $\gamma^{d=4} = 1.0$ and varying $\alpha^{d=4}$, the value $\tilde{x}_0^{d=4}$ tends to infinity at $\alpha^{d=4} \approx 0.15$. Thus, we expect that in analogy in $d = 6$, the Kasner solutions will exits only for small α when $\Lambda = 0$. Especially, for $\alpha = 2.0$ (which is the value of α mainly considered in this paper), we expect to find no Kasner solutions for $\Lambda = 0$, $\gamma > 2$. This is confirmed by our numerical results.

We will now discuss, how this pattern evolves when the cosmological constant is negative and construct the domain of existence of solutions in the γ-Λ-plane (with α fixed and $\gamma \geq 0$, $\Lambda \leq 0$). Our result are summarised in Fig. 2 for $\alpha = 2$, but we expect that the results are similar for generic values of α.

Considering first the part of the plane for which $\gamma < 2$, our numerical results indicate that both string branch and Melvin branch exist up to a critical value $\gamma < \tilde{\gamma}_{cr}(\alpha, \Lambda)$ (or equivalently $\Lambda > \tilde{\Lambda}_{cr}(\alpha, \gamma)$). In the limit $\gamma \rightarrow \tilde{\gamma}_{cr}(\alpha, \Lambda)$, the two solutions converge to a common solution indicated by the dashed line in Fig. 2. This phenomenon persists also for β finite and will thus be discussed in more detail in the next section. In order to make the connection with [12], let us mention that the so-called "anti-warped" solutions in Fig.s 15 and 17 of [12] belong, respectively, to the string and Melvin branch.

Let us now discuss the part of the γ-Λ-plane with $\gamma > 2$. As indicated in Fig. 2, below a critical line (solid), which we denote $\Lambda_{cr}(\alpha, \gamma)$, only inverted string solutions exist, while above this line only Kasner solutions exist. The critical phenomenon which limits the domain of existence of the inverted string solutions is related to the occurence of a zero $x = x_0$ of the function $L(x)$. Indeed, the numerical study of x_0 for α, γ fixed and varying Λ, reveals that x_0 is shifted to infinity for a finite value of Λ, which coincides with the critical value of Λ, $\Lambda_{cr}(\alpha, \gamma)$, mentioned above. This phenomenon is illustrated in Fig. 3 for two pairs of values (α, γ) such that $\gamma > \gamma_{cr}^0(\alpha, \Lambda = 0)$. We plot the quantity x_0 in dependence on the bulk cosmological constant Λ. Clearly at some $\Lambda = \Lambda_{cr}(\alpha, \gamma)$ x_0 tends to infinity and it can be checked that the functions $L(x)$ and $M(x)$ become exponentially decaying. We find that $\Lambda_{cr}(\alpha = 1, \gamma = 1.75) \approx -0.0014$ and $\Lambda_{cr}(\alpha = 2, \gamma = 2.157) \approx -0.0035$, which is in perfect agreement with the results of [11, 12]. Stated in another way, the crucial point about the construction in [12] is that the cosmological constant has to be fine tuned in such a way that the zero of the function $L(x)$ is pushed to infinity and consequently the metric functions

become exponentially decaying:

$$M(x \to \infty) = M_0 \exp(-\sqrt{-\Lambda/10}x) \quad , \quad L(x \to \infty) = L_0 \exp(-\sqrt{-\Lambda/10}x) \quad . \tag{27}$$

B. Einstein-Born-Infeld-Abelian-Higgs (EBIAH) model ($\beta < \infty$)

Classifying the solutions of the full EBIAH equations is a vast task because of the oc-curence of four parameters to be varied. First, we therefore limited our investigation to the case $\alpha = 1$ and $\beta^2 = 5$. Along with the case $\beta = \infty$ discussed above, we orientated first our numerical construction to small value of γ. In this case, the string and Melvin branches continue to exist and the phenomenon that these two branches converge to a common solu-tion for a Λ-depending critical value of γ is recovered. This is illustrated in Fig. 4, where we plot the metric functions $L(x)$ and $M(x)$ of the "anti-warped" solutions corresponding to the string and Melvin branches for $\Lambda = -0.1$. In this case, the critical value of γ is $\tilde{\gamma}_{cr}(\alpha = 1, \beta = \sqrt{5}, \Lambda = -0.1) \approx 1.0625$.

With this information, we constructed the domain of existence of solutions in the γ-Λ-plane for $\alpha = 2$ and $\beta = 1$. First, we determined the critical value of γ, $\gamma_{cr}^0(\alpha = 2, \beta = 1, \Lambda = 0)$, for which the string and Melvin solutions turn into closed solutions. We find that $\gamma_{cr}^0(\alpha = 2, \beta = 1, \Lambda = 0) \approx 2.045$ which differs only little from the value in the $\beta = \infty$ limit. For $\gamma < \gamma_{cr}^0$, we find -as mentioned before- that the domain of existence of the string and Melvin solutions is restricted by a curve $\tilde{\gamma}_{cr}(\alpha = 2, \beta = 1, \Lambda)$ (or $\tilde{\Lambda}_{cr}(\alpha = 2, \beta = 1, \gamma)$), at which the two solutions converge to a common solution (dashed line in Fig. 5). This is in complete analogy to the $\beta = \infty$ limit. In this part of the γ-Λ-plane, however, the domain of existence of the Melvin solutions is restricted by another phenomenon. At some $\tilde{\gamma}_{cr}^M(\alpha = 2, \beta = 2, \Lambda)$ (or $\tilde{\Lambda}_{cr}^M(\alpha = 2, \beta = 1, \gamma)$, dotted-dashed line in Fig. 5) with $\tilde{\gamma}_{cr}^M < \tilde{\gamma}_{cr}$, the value of the magnetic field at the origin tends to infinity. This phenomenon was previously noted for the $d = 4$, $\Lambda = 0$ case in [19]. Thus, Melvin solutions exist only in a small part of the γ-Λ-plane, namely for $\tilde{\gamma}_{cr}^M < \gamma < \tilde{\gamma}_{cr}$.

Now turning to solutions for $\gamma > \gamma_{cr}^0$, again, we find no Kasner solutions for $\Lambda = 0$ since $\alpha = 2$ is too large. We were only able to construct inverted string solutions for $\alpha = 2$, $\beta = 1$ and $\Lambda = 0$. The inverted string solutions exist up to a critical value $\Lambda_{cr}(\alpha = 2, \beta = 1, \gamma)$ (or $\gamma_{cr}(\alpha = 2, \beta = 1, \Lambda)$, solid line in Fig. 5). For $\Lambda > \Lambda_{cr}(\alpha = 2, \beta = 1, \gamma)$ only Kasner solutions exist. Following our analytic argument in Section II.D. no "warped solutions" exist

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such that the solid line in Fig. 5 now represents a line where no solutions at all exist. Thus our numerical results confirm the analytic argument that no gravitation-localising solutions exist in a Born-Infeld brane world scenario.

IV. CONCLUSIONS

In this paper, we have studied a 6-dimensional Einstein-Born-Infeld-Abelian-Higgs model in a brane world scenario recently suggested by [12]. In the $\beta = \infty$ limit, our model reduces to that of [12]. As a cross-check of our model, we reconsidered the solutions found in this paper, and managed to determine the domain of the γ-Λ-plane in which solutions of this model exist. Especially, we identify the four different types of branches that exist for $\Lambda = 0$, namely the string, Melvin, inverted string and Kasner branches and put the emphasis on the extention of these branches in the $\Lambda < 0$ case. We identified the warped solution of [12] as the solution sitting on the limit of the domain of existence of the inverted string solution (decreasing Λ and keeping the other parameters fixed). We also considered the influence of the Born-Infeld interaction on the solutions and recovered the existence of the four types of solutions in this case. We construct the domain of existence of solutions in the γ-Λ-plane for $\alpha = 2$ and $\beta = 1$. Especially, we find that in agreement with our analytic argument, which uses the reasoning based on the string tensions of [12], no "warped solutions" exist in the Born-Infeld case.

Let us remark that we have only taken into account the Born-Infeld terms of the effective action of string theory and have neglected higher derivative corrections in this paper. These arise naturally in the effective action of open string theory and are related to the essential non-locality of string field theory. These terms could, of course, influence the "no-go" result of this paper. We plan to address this question in a future publication.

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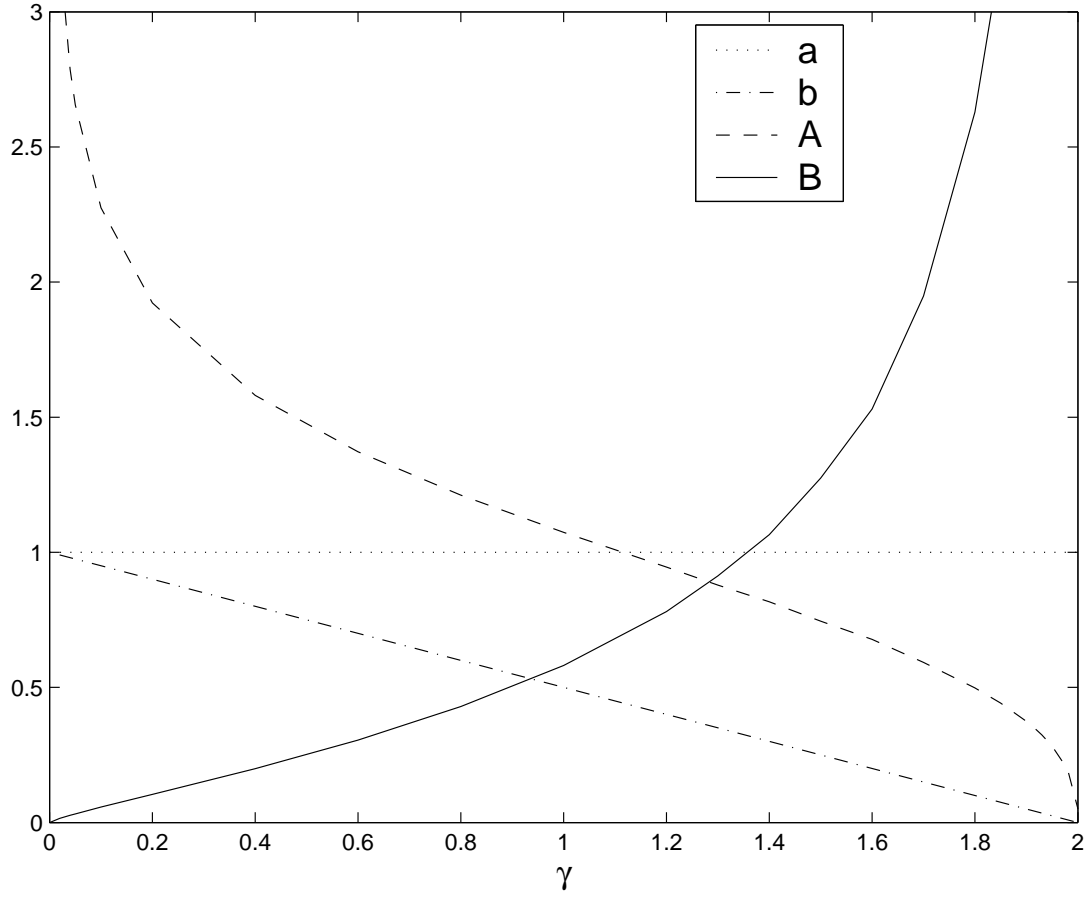[20] M. Giovannini and H. B. Meyer, Phys. Rev. **D64** (2001), 124025.

FIG. 1: The dependence of the parameters A, B, a and b on γ is shown for the solutions in the $\beta = \infty$, $\alpha = 2$ limit on the string branch ($M(x \to \infty) = a$, $L(x \to \infty) = bx + c$) and on the Melvin branch ($M(x \to \infty) = Ax^{2/5}$, $L(x \to \infty) = Bx^{-3/5}$), respectively.
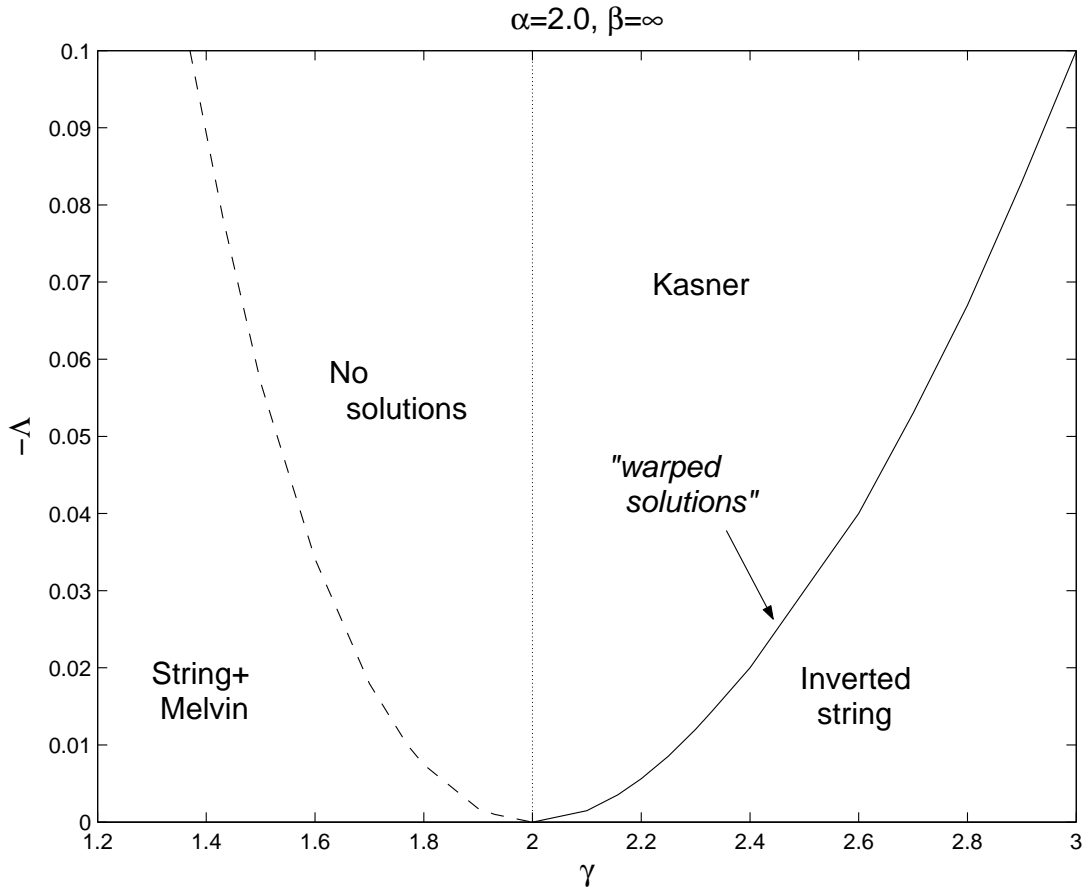
FIG. 2: The domain of existence of solutions in the γ-Λ-plane is presented for $\alpha = 2.0$, $\beta = \infty$. Note that the dashed line represents $\tilde{\gamma}_{cr}(2, \Lambda)$, while the solid line denotes $\Lambda_{cr}(2, \gamma)$.
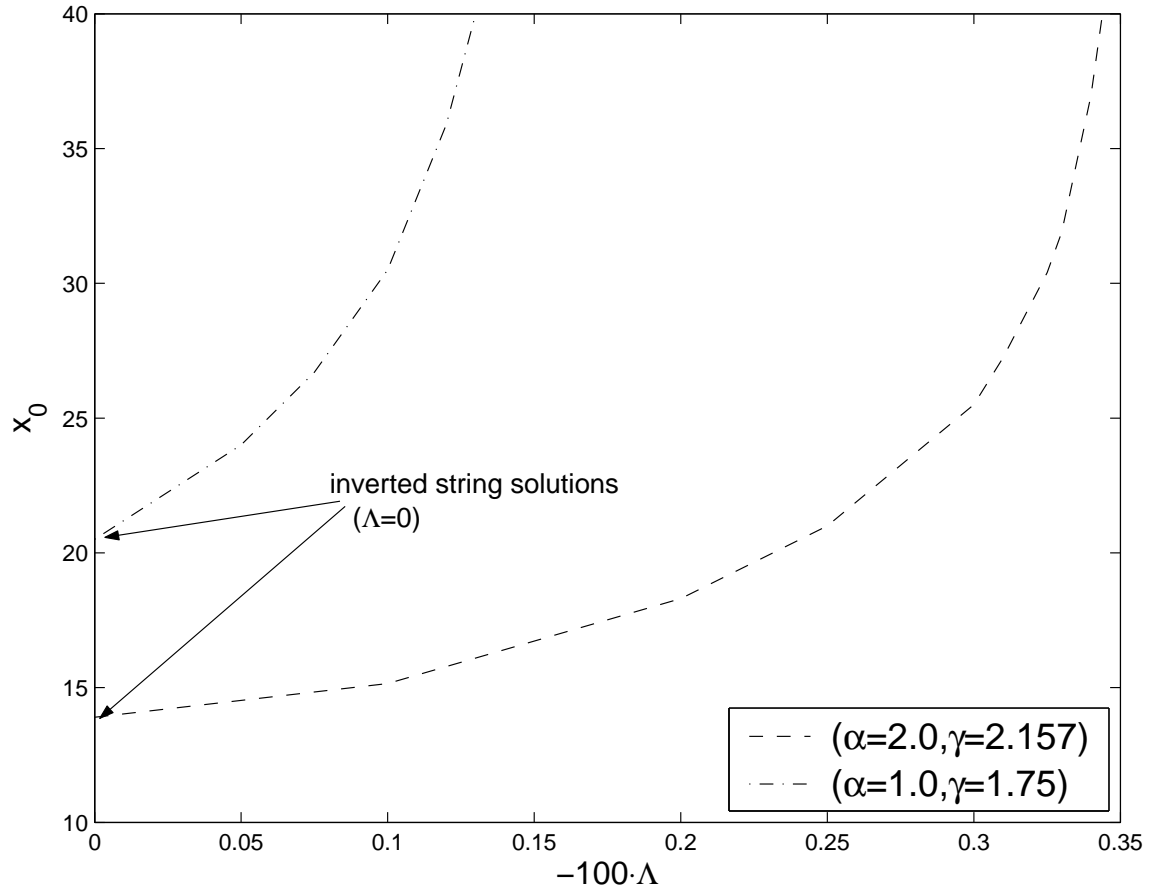
FIG. 3: The value of the quantity x_0 $(L(x = x_0) = 0)$ is plotted as function of Λ for two different pairs of values (α, γ) and $\beta = \infty$.
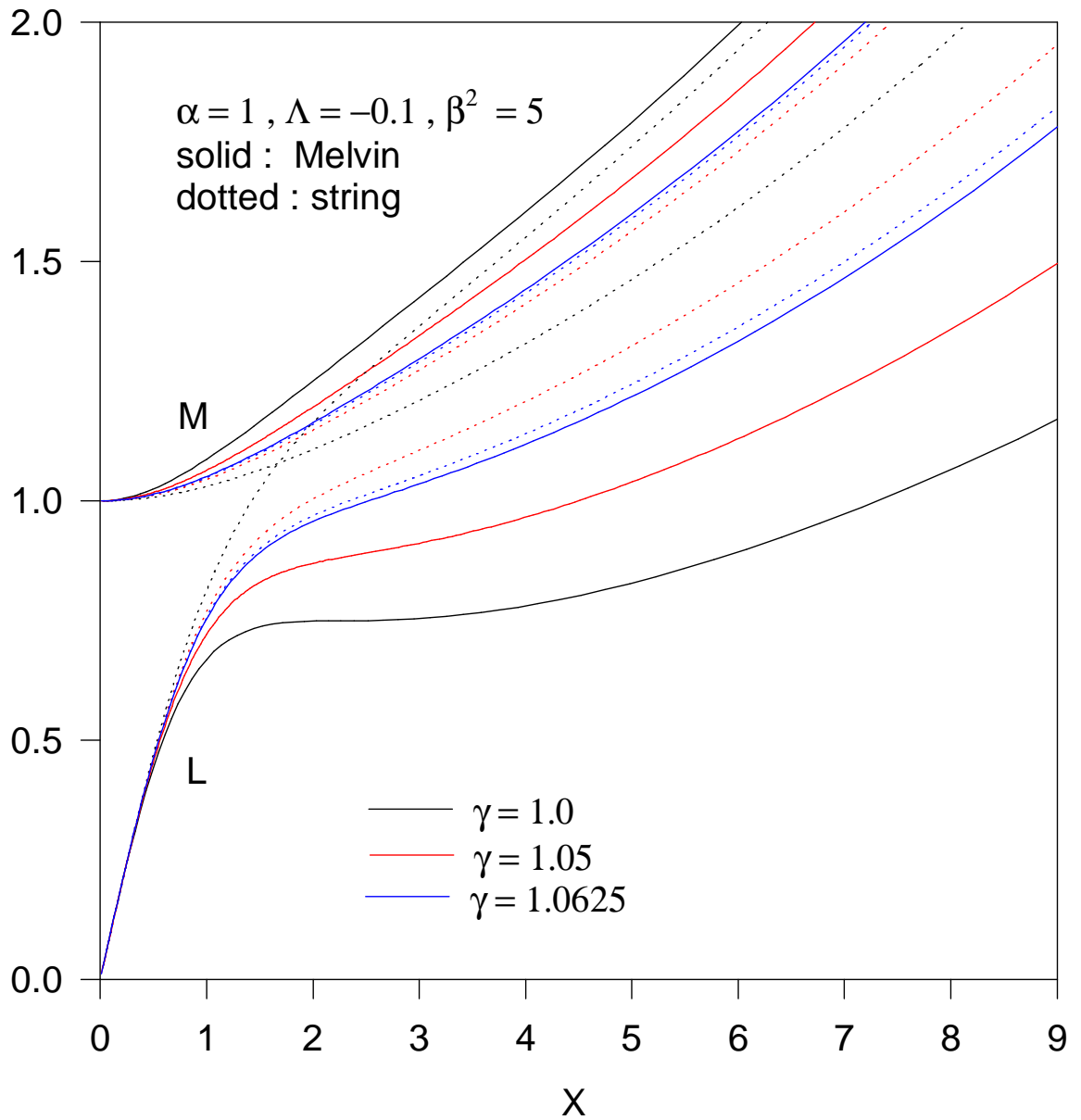
FIG. 4: *(Colour figure)* The "anti-warped" solutions of the Born-Infeld equations with $\alpha = 1.0$, $\Lambda = -0.1$, $\beta^2 = 5.0$ are presented for different values of γ ($\gamma = 1.0$ (black), $\gamma = 1.05$ (red) and $\gamma = 1.0625$ (blue)) approaching $\gamma_{cr}(\alpha = 1, \beta^2 = 5, \Lambda = -0.1) \approx 1.063$.
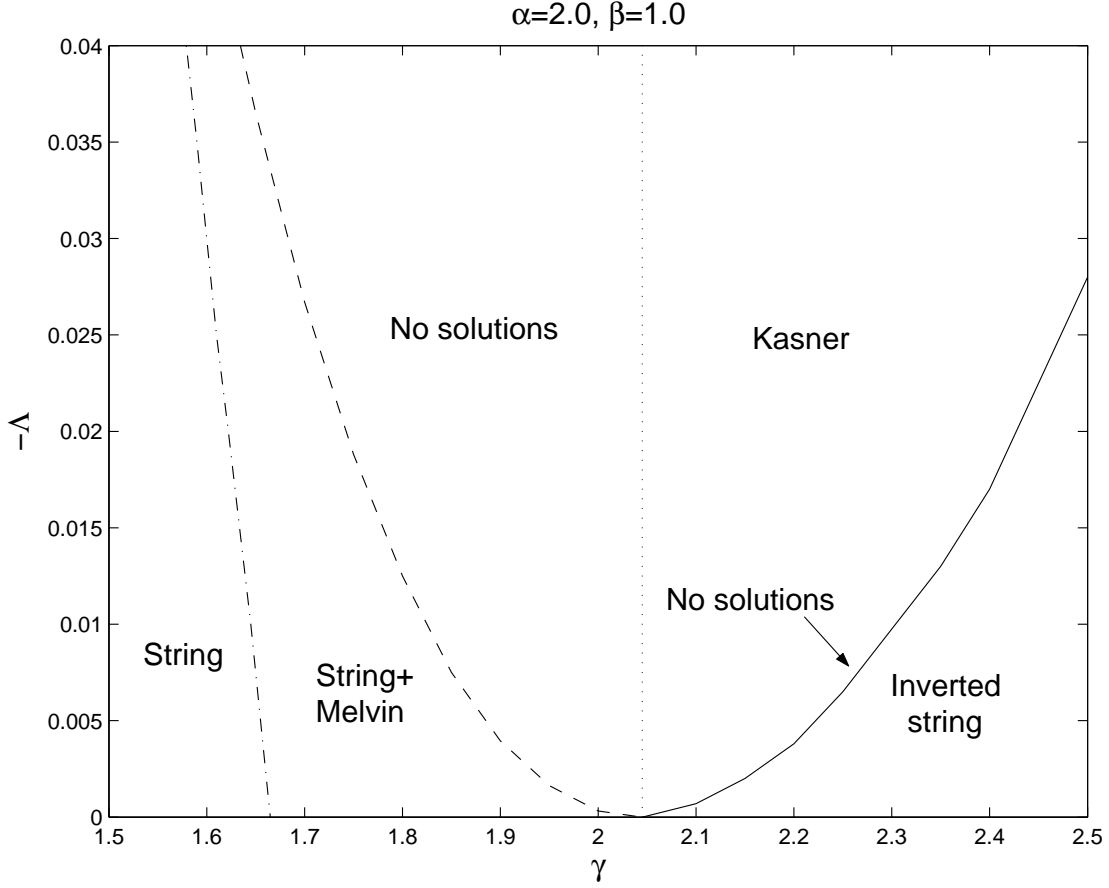
FIG. 5: The domain of existence of solutions in the γ-Λ-plane is presented for $\alpha = 2.0$, $\beta = 1.0$. Note that the dotted-dashed line corresponds to those values of γ and Λ for which the magnetic field of the Melvin solutions tends to infinity at the origin ($\tilde{\gamma}_{cr}^{M}(\alpha = 2, \beta = 1, \Lambda)$). The dashed line corresponds to $\tilde{\gamma}_{cr}(\alpha = 2, \beta = 1, \Lambda)$ at which the string and Melvin solutions converge to a common solution, while the solid line represents the limit of existence of inverted string solutions, $\Lambda_{cr}(\alpha = 2, \beta = 1, \gamma)$. The dotted line corresponds to $\gamma_{cr}^{0}(\alpha, \beta, \Lambda = 0)$.