

Transversity and single spin asymmetries

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The transverse spin of interacting light quarks is described by two independent non-perturbative inputs, twist-2 transversity and twist-3 quark-gluon correlators, while for free massive quarks they are related by the equations of motion. Similar relation emerges also in a model, where quark-hadron transition is treated in a probabilistic way. The single spin asymmetry in SIDIS appears due to the interplay of transversity with Collins fragmentation function, which appears to be of twist 3, when defined in a Lorentz invariant way.

1 Transverse spin for free and interacting quarks

The basic property of the quantum theory of angular momentum is its transformation for the rotation from the longitudinal to transverse direction, making the interference helicity-flip effects the probabilistic ones. The simplest way to see that is to observe that σ_z Pauli matrix is a diagonal one, contrary to σ_x and σ_y , and this is a core of the role played by the transversity basis. The important further distinction comes when free and interacting particles are considered. The state of free particle is completely described by its wave function, which determines the density matrix $\rho = 1/2(\hat{p} + m)(1 + \hat{s}\gamma_5)$, and the coefficients of all mutually orthogonal Dirac matrices are fixed. At the same time, the parametrization of transverse polarized nucleon matrix element ?? of light-cone quark operator leads to the similar expression, which may be therefore interpreted as a quark density matrix

$$q(x)\hat{P} + Mg_T(x)\hat{S}_T\gamma_5 + h_1(x)\hat{P}\hat{S}_T\gamma_5 \quad (1)$$

The mass of free particle is now substituted by a parameter of the order of hadron mass [1]. The coefficients of various Dirac matrices are now different parton distributions. The transverse spin may be therefore described by the

axial or pseudotensor terms. As the first one enters together with hadron mass, it corresponds to the subleading twist 3, while the second is just the twist 2 transversity.

The independence of these two functions is not a complete one. The operator equations of motions ??, which are crucial to guarantee both electromagnetic and colour gauge invariance lead to the integral relations between them, including also another important non-perturbative input, namely the quark-gluon correlator b :

$$Mg_T(x) + M \int dy b(x, y) = mh_1(x) \quad (2)$$

There are two extreme cases: when one may neglect these correlators, and the quark mass m is of order of the hadron one M , one is coming back to the free particle case and transversity happens to be equal to other spin distributions ??.

In the opposite case of very light quark, realized in Nature, transversity decouples, so that the very existence of such independent distribution is closely related to the spontaneous chiral symmetry breaking, being the mechanism of generation of the most of the observable mass.

The relation between various spin distributions is restored in some models. Let me briefly discuss the recent calculation [2] of transversity, which uses the elegant interpretation [3] as a DIS for the interfering vector and scalar currents. The obtained result is convoluted with the probabilistic momentum space distribution of quark in nucleon. This allows to perform the calculation in complete analogy with the earlier treatment [4] of unpolarized and polarized quark distribution. The obtained result looks especially simple when one neglects the quark mass. In the same approximation within this model [4] one reproduces the standard Wandzura-Wilczek (WW) relation, so it may be called the WW approximation for transversity:

$$g_T(x) + g_1(x) = h_1(x). \quad (3)$$

In particular, taking into account the Burkhardt-Cottingham sum rule,

$$2 \int dx g_1(x) = \int dx h_1(x), \quad (4)$$

which is not far from the model estimation [5]. Although model, in principle, may be applied to any quark flavour, one may have the violation of Soffer inequality if $g_1(x) < 0$, i.e. for d -quarks. This shows that the adopted probabilistic consideration of parton-hadron transition (while the interference effects are taken into account on the level of vector and scalar currents only) works worse for d -quarks. One may recall, that standard WW approximation for d -quarks may also violate the positivity bound [6].

2 T-odd fragmentation and distribution functions: twist and universality

The appearance of transversity in SIDIS may be only in the combination with chiral-odd fragmentation function, the most popular one being the T-odd Collins fragmentation function [5, 7],

As this quantity does not correspond to the standard one, being the intrinsic transverse momentum dependent, the notion of twist is not so trivial. The relevant tensor structure is $M^{-1}H(z, k_T)\epsilon_{\mu\nu P k_T}$, factor $1/M$ appearing due to the dimensional reasons. As soon as M and k_T are of the same order, any power suppression is absent which is usually considered as a manifestation of leading twist (two) contribution.

At the same time, the consideration of transverse momentum dependent functions did not yet receive the full field-theoretical justification beyond the Born approximation. The exception is provided by their expansion over the powers of k_T . In particular, linear in k_T terms constitute nothing else but WW approximation to inclusive [8] and exclusive [9] processes.

It is interesting, that WW approximation may be also obtained in the coordinate representation [10], when no reference on the transverse direction is needed. This suggests the possibility [11] to consider the Collins function in the coordinate space as well. The tensor structure is than $MI(z)\epsilon_{\mu\nu P z}$, so that the transverse momentum of the quark k_T is not present and the factor M appears now in the numerator, rather in the denominator, due to the dimensional reasons. However, when one is calculating the weighted (with p_T , being the transverse momentum of the produces *pion*) average, one is coming to the

same result, and $I(z)$ corresponds to the *moment* of the Collins function:

$$I(z) \sim \int dk_T^2 \frac{k_T^2}{M^2} H_1(z, k_T^2), \quad (5)$$

and the factor M^2 in the denominator of the r.h.s. is exactly the one resulting from the various appearance of M in the definitions of H and I . The proportionality of I to M makes this function the twist-3 one, according to the standard classification (like, say, the functions g_T and g_2). The trace of the twist-3 nature of I is the necessity of $1/M$ factor in the definition of the weighted cross-section,

$$\int dp_T^2 \frac{p_T}{M} \frac{d\Delta\sigma}{dp_T^2}, \quad (6)$$

so that the M should compensate the dimension of P_T , in order to get the unsuppressed expression. Consequently, if such a factor is not introduced by hand, the result should be suppressed like M , analogously to any other twist-3 observable. This suppression is absent, when P_T of order M (where factorization is, generally speaking, unapplicable beyond Born approximation) are considered. Let us also briefly comment on the complementary mechanism of generation of Single Spin Asymmetry in SIDIS, namely the Sivers function. As it was already discussed earlier [12], such a function can be only *effective*, (such a notion first suggested in [13]) or non-universal (like it is referred to now [7]), in the sense, that the imaginary phase emerges in the interaction, involving also the hard scattering and depending on its type. In other words, the respective cut, providing the imaginary phase, involves both hard and soft variables. Moreover [12], the model calculation [14] of Brodsky, Hwang and Schmidt, clearly exhibits this property, as well as elegant interpretation of Collins [15]. Let us also mention in this connection, that the similar calculation was performed earlier in twist-3 QCD [16] for the crossing related process of dilepton photoproduction. That result, when continued to the region $P_T \sim M$, will also not have any power suppression and looks formally as a twist 2 one.

At the same time, the imaginary phase in the Collins function should come from the cut with respect to the jet mass, and is therefore universal. The recent analysis [7] seems to confirm this picture.

3 Conclusions

The description of Collins function in coordinate space leads to its twist being equal to 3, while the absence of power suppression is related to the definition of observables. The Siverson function is expected to be an effective, or non-universal one, contrary to Collins function. The earlier calculation in twist 3 QCD [16], when continued to low P_T region, shows no power suppression and may be considered as an estimate of effective Siverson function.

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