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Large cutoff effects of dynamical Wilson fermions *

R. Sommer[a]Zeu]DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany[b]Cern]CERN–TH, CH–1211 Geneva 23, Switzerland, S. Aoki[c]Utsu]Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan, M. Della Morte[d][Zeu], R. Hoffmann[e]HU]Institut für Physik, Humboldt–Universität zu Berlin, Newtonstr. 15, 12489 Berlin, Germany, T. Kaneko[f]KEK]High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan, F. Knechtli[g][HU], J. Rolf[h][HU], I. Wetzorke[i]NIC]NIC/DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany and U. Wolff[j][HU]
(ALPHA, CP-PACS and JLQCD Collaborations)

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We present and discuss results for cutoff effects in the PCAC masses and the mass dependence of r_0 for full QCD and various fermion actions. Our discussion of how one computes mass dependences – here of r_0 – is also relevant for comparisons with chiral perturbation theory.

1. INTRODUCTION

In the quenched approximation, large cutoff-effects were observed with the original Wilson action. In particular, they appeared as a strong dependence of the PCAC-mass, $m = \frac{1}{2}\langle\beta|\partial_\mu A_\mu|\alpha\rangle \,/\, \langle\beta|P|\alpha\rangle$, on the external states, $|\alpha\rangle, |\beta\rangle$, a dependence which has to be absent in the continuum limit [1]. These a-effects could be reduced to a tolerable level by non-perturbative O(a)-improvement [2]. The necessary improvement coefficient $c_{\rm sw}$ is now known also for the Wilson gauge action with $N_{\rm f} = 2$ flavors of dynamical Wilson fermions (action "W/SW") [3] and with the Iwasaki gauge action with $N_{\rm f} = 3$ (I/SW) [4]. It was pointed out already in [3] that O(a^2) effects may be sizeable at the lowest value of β considered, which roughly corresponds to $a \approx 0.1\,$fm. Here we report on O(a^2) effects with dynamical fermions, for W/SW and I/SW actions and for comparison we consider also the original Wilson (W/W) and Kogut Susskind (W/KS) actions.

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2. PCAC MASSES

A family of external states, $|\alpha\rangle, |\beta\rangle$, can be realized in the Schrödinger Functional (SF) [1, 2]. The (bare) PCAC-mass,

$$m = \left.\frac{\tilde\partial_0 f_{\rm A}(x_0) + c_{\rm A} a \partial_0^* \partial_0 f_{\rm P}(x_0)}{2 f_{\rm P}(x_0)}\right|_{x_0=L/2}, \qquad (1)$$

then depends on the resolution L/a of the $L^3 \times L$ space-time, on the spatial periodicity angle θ of the fermion fields and the Dirichlet boundary conditions for the gauge fields. In eq. (1) the standard SF correlation functions of the axial vector and the pseudoscalar density $f_{\rm A}, f_{\rm P}$ enter. Chiral symmetry predicts that m is (apart from cutoff ef-

action	β	N_f	$\Delta m[\mathrm{MeV}] \times \frac{a^{-1}}{2\,\mathrm{GeV}}$
W/SW	6.0	0	12(1)
W/SW	5.2	2	28(1)
W/W	5.5	2	96(4)
I/SW	2.2	2	−2(4)

Table 1
Δm at $a^{-1} \approx 2$GeV, $L/a = 8$. Values for m_∞ have been taken from [6–8]. Except for $N_\mathrm{f} = 0$, the 1-loop expressions for the improvement coefficient c_A are used. For the first three lines we have $\theta = 1/2$ and no background field in the SF, while for the last line, $\theta = 0$ and the background field of [2] was chosen.

fects) independent of L/a as well as of the other parameters, as long as the gauge coupling and the bare quark mass are kept constant. Denoting by m_∞ the PCAC mass computed in large volume with $|\alpha\rangle$ the $\mathbf{p} = 0$ one π state and $|\beta\rangle = |0\rangle$, one expects in particular

$$\Delta m \equiv m - m_\infty = \mathrm{O}(a^2) \tag{2}$$

in the O(a) improved cases. For W/W this difference is instead O(a). Indeed, after O(a) improvement Δm is not too large for $a \approx (2\mathrm{GeV})^{-1} \approx 0.1$fm in the quenched approximation. However for $N_\mathrm{f} = 2$ it is more than a factor two larger (Table 1) and, although not shown here, this roughly holds also for other such mass differences. Considering several other differences, we convinced ourselves that the only perturbatively known value for c_A is not the origin of the large cutoff effects seen for W/SW and $N_\mathrm{f} = 2$.

We then considered the Iwasaki gauge action together with the values of c_sw used in [8]. We found a *much* smaller Δm, see Table 1.

3. MASS DEPENDENCE OF r_0

In lattice gauge theory computations, the dependence of r_0 on the mass(es) of the dynamical quarks has been neglected so far. However, in contrast to the $q\bar{q}$ force at very short distance r where only an effect of relative size $\mathrm{O}(\alpha_\mathrm{s} m^2 r^2)$ is present, for the force at $r \approx 0.5$ fm this dependence is not obviously that small. It should be computed by lattice gauge theory. Due to spontaneous chiral symmetry breaking, a linear term in m is expected for small quark masses.

In order to compute the mass dependence on the lattice, one must not introduce spurious mass dependences through renormalization. A mass independent renormalization scheme has to be chosen. In perturbation theory an example is provided by the lattice minimal subtraction scheme with renormalized coupling

$$\begin{aligned} \bar{g}^2_\mathrm{lat}(\mu) &= Z_g(\tilde{g}_0, a\mu)\, \tilde{g}_0^2 \\ &= \{1 - 2b_0 \ln(a\mu)\tilde{g}_0^2 + \ldots\}\, \tilde{g}_0^2\,. \end{aligned} \tag{3}$$

The above equation shows that the lattice spacing is fixed by $\tilde{g}_0$, independently of the quark mass. Apart from a small O(a)-correction in O(a)-improved QCD, which is due to [9]

$$\tilde{g}_0^2 = (1 + b_\mathrm{g} a m_\mathrm{q}) g_0^2\,, \quad b_\mathrm{g} = \mathrm{O}(g_0^2)\,, \tag{4}$$

the lattice spacing is determined by g_0 alone in a mass-independent renormalization scheme. We note in passing that it is also mandatory to choose such a scheme in a comparison of lattice QCD results with chiral perturbation theory predictions.

To define the mass-dependence, we first introduce a reference value m_ref of the quark mass via $r_0^2 m_\mathrm{PS}^2|_{m=m_\mathrm{ref}} = K$, which we later put to $K = 3$ in our numerical evaluation. Other choices are possible. The physical mass dependence of r_0 may then be described by the function $\rho_\mathrm{cont}(x) = r_0(m)/r_0(m_\mathrm{ref})$ where $x = m/m_\mathrm{ref}$ is finite since in the corresponding renormalized ratio the common Z-factor (defined in the massless scheme) cancels. A precise definition of ρ_cont is provided by ($\hat{r}_0 = r_0/a$)

$$\rho_\mathrm{cont}(x) = \lim_{1/\hat{r}_0(m_\mathrm{ref})\to 0} \rho(x, 1/\hat{r}_0(m_\mathrm{ref}))\,, \tag{5}$$

$$\rho(x, 1/\hat{r}_0(m_\mathrm{ref})) = \left.\frac{\hat{r}_0(m)}{\hat{r}_0(m_\mathrm{ref})}\right|_{\tilde{g}_0\ \mathrm{fixed}}\,. \tag{6}$$

As a first step we try to understand whether the mass dependence is a large effect and quantify scaling violations. Since we wanted to use results from the literature, it was easier to consider

$$R\left(x, \tfrac{1}{\hat{r}_0(m_\mathrm{ref})}\right) = \left.\frac{\hat{r}_0(m)}{\hat{r}_0(m_\mathrm{ref})}\right|_{\tilde{g}_0}\,,\ x = \frac{r_0^2 m_\mathrm{PS}^2|_m}{r_0^2 m_\mathrm{PS}^2|_{m_\mathrm{ref}}}$$

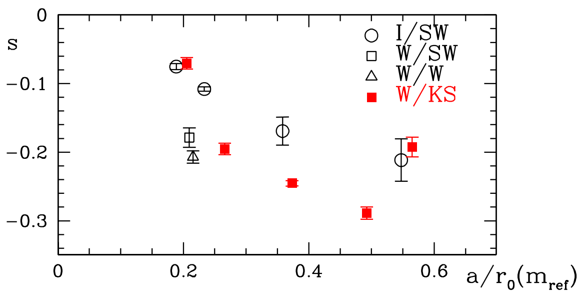

Figure 1. Lattice spacing dependence of slopes s.

which agrees with ρ at small quark masses and lattice spacings due to $m \sim m_{\rm PS}^2$. In practice only g_0 could be kept constant in our evaluation of R, assuming that the $b_{\rm g} a m_{\rm q}$ term is negligible ($b_{\rm g} \approx 0.02 + {\rm O}(g_0^4)$ for W/SW [9]). It turns out that R is well represented by the Taylor expansion

$$R(x, 1/\hat{r}_0(m_{\rm ref})) = 1 + s(1/\hat{r}_0(m_{\rm ref}))\,(x-1) \quad (7)$$

for the data considered [8, 6, 7, 10], namely

I/SW	$x \in [0.4, 2.0]$,	$\hat{r}_0(m_{\rm ref}) \in [1.8, 5.3]$
W/W	$x \in [0.8, 1.6]$,	$\hat{r}_0(m_{\rm ref}) \approx 4.6$
W/SW	$x \in [0.6, 2.0]$,	$\hat{r}_0(m_{\rm ref}) \approx 4.8$
W/KS	$x \in [0.3, 2.0]$,	$\hat{r}_0(m_{\rm ref}) \in [1.8, 4.9]$

Our figure shows the available results for the slope s. Both the I/SW numbers and the W/KS ones point to a very small $|s|$ in the continuum limit with a rather strong a-dependence and unclear mutual consistency. The two other actions appear to show even larger cutoff effects at $a \approx 0.1$ fm.

4. DISCUSSION

Our results are not easy to interpret, apart from the general observation that dynamical fermions may introduce significant cutoff effects at $a \approx 0.1$ fm. We recall some other findings.

- UKQCD found a very small and strongly mass dependent 0^{++} glueball mass at $a \approx 0.1$ fm with $N_{\rm f} = 2$ W/SW [11].
- For the same action and parameters, the auto-correlation times increase as the quark mass is increased.
- The pure SU(3) gauge theory has a phase transition in the $\beta_{\rm F}, \beta_{\rm A} > 0$ half-plane not very far from $a \approx 0.1$ fm.
- The JLQCD collaboration found a phase transition with $N_{\rm f} = 3$ W/SW [12], preventing simulations unless a is quite small.

These hints lead us to the conjecture that for $N_{\rm f} = 2$, W/SW, $a \approx 0.1$ fm, one may be close to a phase transition and thus suffer from cutoff effects which are not necessarily described by the Symanzik expansion. Concerning the PCAC masses (see section 2), I/SW is much better. However, we are lacking an understanding why this is so and our figure shows significant a-effects in the mass dependence of r_0 for I/SW.

At the present time we draw two **conclusions**. 1) The issue of what is a good action for full lattice QCD deserves more attention and one should investigate scaling properties of actions which are used to compute physical observables.
2) The mass dependence of r_0 and in particular the lattice artifacts contained in this mass dependence has to be understood before drawing conclusions on the chiral behavior of quantities such as F_π and m_π^2/m. Section 3 outlines what needs to be done but our numerical investigation should be considered only as a first step.

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REFERENCES

1. K. Jansen et al., Phys. Lett. B372 (1996) 275.
2. M. Lüscher et al., Nucl. Phys. B491 (1997) 323, hep-lat/9609035.
3. ALPHA, K. Jansen and R. Sommer, Nucl. Phys. B530 (1998) 185, hep-lat/9803017.
4. CP-PACS and JLQCD collaborations, S. Aoki et al., Nucl. Phys. Proc. Suppl. 119 (2003) 433, hep-lat/0211034.
5. R. Sommer, Nucl. Phys. B411 (1994) 839.
6. JLQCD, S. Aoki *et al.*, Phys. Rev. D68 (2003) 054502, hep-lat/0212039.
7. TχL, N. Eicker et al., Phys. Rev. D59 (1999) 014509, hep-lat/9806027.

8. CP-PACS, A. Ali Khan et al., Phys. Rev. D65 (2002) 054505, hep-lat/0105015.
9. M. Lüscher et al., Nucl. Phys. B478 (1996) 365, hep-lat/9605038.
10. MILC, S. Tamhankar and S. Gottlieb, Nucl. Phys. Proc. Suppl., 83-84 (2000) 212 and private communication by S. Gottlieb.
11. UKQCD, A. Hart and M. Teper, Phys. Rev. D65 (2002) 034502, hep-lat/0108022.
12. JLQCD, S. Aoki et al., Nucl. Phys. Proc. Suppl. 106 (2002) 263, hep-lat/0110088.