

Fundamental physics in space: a Quantum-Gravity perspective¹

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ABSTRACT

I consider the possibility that space experiments be used to search for quantum properties of spacetime. On the basis of recent quantum-gravity results, I argue that insight on some quantum properties of spacetime can be obtained with experiments planned for the International Space Station, such as AMS and EUSO, and with satellite gamma-ray telescopes, such as GLAST.

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1 Introduction

Any attempt to test new ideas about the fundamental laws of Nature requires remarkable accuracy and/or the study of particles with very high energies. Since these objectives are often beyond the reach of conventional laboratories, there has been growing interest in the analysis of the implications of fundamental-physics theories in astrophysics and cosmology. More and more examples are being found in which a given picture of the fundamental laws of physics is established to have important consequences for certain classes of observations in astrophysics and cosmology. At first these studies focused on the implications of particle-physics theories, and the field of “astroparticle physics” is now one of the leading areas of fundamental-physics research. Over the last few years it has been realized that even the study of certain quantum-gravity models (which typically pertain to energy scales even beyond the ones of interest in particle physics) can benefit from suitable analyses of astrophysics and cosmology contexts.

Space experiments play an important role in these studies. Especially for certain observations in astrophysics, it is important to be able to observe the process before the disturbance introduced by the atmosphere. Moreover, in addition to its importance in astrophysics and cosmology, the opportunity to conduct experiments in space is also sometimes exploited in setting up high-precision experiments. The quieter environment of a space laboratory is in fact very valuable for achieving high precision in certain experiments.

Most of the experimental studies that are being considered in “quantum-gravity phenomenology” [1] rely on space experiments. In quantum-gravity phenomenology one is looking for the small effects predicted by quantum-gravity theories, effects with magnitude set by the ratio between the energy of the particles involved and the huge Planck energy scale ($E_p \simeq 10^{28} eV$). The overlap of interests between astrophysics and quantum-gravity phenomenology comes primarily from the availability in astrophysics of particles with energies significantly higher than the energies that can be achieved at particle colliders. And in quantum-gravity phenomenology there is of course strong interest in high-precision experiments, since the Planck-scale effects are always very small.

I here want to discuss some of the key aspects of this quantum-gravity phenomenology in space, with emphasis on some studies which can be conducted on the “ISS”, the International Space Station. In the first part of this paper (Sections 2-7) I review some results obtained in the quantum-gravity literature that are relevant for space experiments. I will stress that a key indication which is emerging from quantum-gravity research is that it is natural to question whether in quantum spacetime it is possible for ordinary (classical) Lorentz symmetry to preserve the role it holds in current (pre-quantum-gravity) theories. In light of these results, certain types of Lorentz-symmetry tests acquire interest from a quantum-gravity perspective. In addition to discussing the fate of Lorentz symmetry in quantum spacetime, I will also (more briefly) comment on another type of effect which has been considered in the quantum-gravity literature, the possibility that in monitoring a given length one might encounter an irreducible level of length fluctuations (connected with a sort of uncertainty principle for length measurements). In the second part of the paper (Section 8) I will discuss some characteristic signature of these quantum-gravity effects and comment on some space experiments that represent opportunities for searches of these quantum-gravity effects.

2 Lorentz symmetry and the three perspectives on the Quantum Gravity problem

It is probably fair to state that each quantum-gravity research line can be connected with one of three perspectives on the problem: the particle-physics perspective, the general-relativity perspective and the condensed-matter perspective.

From a particle-physics perspective it is natural to attempt to reproduce as much as possible the successes of the Standard Model of particle physics. One is tempted to see gravity simply as one more gauge interaction. From this particle-physics perspective a natural solution of the quantum-gravity problem should have its core features described in terms of graviton-like exchange in a background classical spacetime. Indeed this structure is found in String Theory, the most developed among the quantum-gravity approaches that originate from a particle-physics perspective.

The general-relativity perspective naturally leads to reject the use of a background spacetime [2, 3]. According to General Relativity the evolution of particles and the structure of spacetime are selfconsistently connected: rather than specify a spacetime arena (a spacetime background) beforehand, the dynamical equations determine at once both the spacetime structure and the evolution of particles. Although less publicized, there is also growing awareness of the fact that, in addition to the concept of background independence, the development of general relativity relied heavily on the careful consideration of the in-principle limitations that measurement procedures can encounter^a. In light of the various arguments suggesting that, whenever both quantum mechanics and general relativity are taken into account, there should be an in-principle Planck-scale limitation to the localization of a spacetime point (an event), the general-relativity perspective invites one to renounce to any direct reference to a classical spacetime [4, 5, 6, 7, 8]. Indeed this requirement that spacetime be described as fundamentally nonclassical (“fundamentally quantum”), the requirement that the in-principle measurability limitations be reflected by the adoption of a corresponding measurability-limited description of spacetime, is another element of intuition which is guiding quantum-gravity research from the general-relativity perspective. This naturally leads one to consider discretized spacetimes, as in the Loop Quantum Gravity approach, or noncommutative spacetimes.

The third possibility is a condensed-matter perspective on the quantum-gravity problem (see, *e.g.*, Refs. [9, 10]), in which spacetime itself is naturally seen as an emerging critical-point entity. Condensed-matter theorists are used to describe the degrees of freedom that are measured in the laboratory as collective excitations within a theoretical framework whose primary description is given in terms of much different, and often practically unaccessible, fundamental degrees of freedom. Close to a critical point some symmetries arise for the collective-excitations theory, which do not carry the significance of fundamental symmetries, and are in fact lost as soon as the theory is probed somewhat away from the critical point. Notably, some familiar systems are known to exhibit special-relativistic invariance in certain limits, even though, at a more fundamental level, they are described in terms of a nonrelativistic theory.

^aThink for example of the limitations that the speed-of-light limit imposes on certain setups for clock synchronization and of the contexts in which it is impossible to distinguish between a constant acceleration and the presence of a gravitational field.

Clearly for the condensed-matter perspective on the quantum-gravity problem it is natural to see the familiar classical continuous Lorentz symmetry only as an approximate symmetry.

Results obtained over the last few years (which are partly reviewed later in these notes) allow us to formulate a similar expectation from the general-relativity perspective. Loop quantum gravity and other discretized-spacetime quantum-gravity approaches appear to require a description of the familiar (continuous) Lorentz symmetry as an approximate symmetry, with departures governed by the Planck scale. And in the study of noncommutative spacetimes some Planck-scale departures from Lorentz symmetry appear to be inevitable.

From the particle-physics perspective there is instead no obvious reason to renounce to exact Lorentz symmetry, since Minkowski classical spacetime is an admissible background spacetime, and in classical Minkowski there cannot be any a priori obstruction for classical Lorentz symmetry. Still, a break up of Lorentz symmetry, in the sense of spontaneous symmetry breaking, is of course possible, and this possibility has been studied extensively over the last few years, especially in String Theory (see, *e.g.*, Ref. [11] and references therein).

3 Quantum Gravity Phenomenology

The key challenge for the search of experimental hints relevant for the quantum-gravity problem is the smallness of the effects that one might expect to be induced by a quantum gravity. A key point for this “Quantum Gravity Phenomenology” [1] is that we are familiar with ways to gain sensitivity to very small effects. For example, our understanding of brownian motion is based on the fact that the collective result of a large number of tiny microscopic effects eventually leads to an observably large macroscopic effect. The prediction of proton decay within certain grandunified theories (theories providing a unified description of electroweak and strong particle-physics interactions) is really a small effect, suppressed by the fourth power of the ratio between the mass of the proton and the grandunification scale, which is only three orders of magnitude smaller than the Planck scale. In spite of this horrifying suppression, of order $[m_{proton}/E_{gut}]^4 \sim 10^{-64}$, with a simple idea we have managed to acquire a remarkable sensitivity to the possible new effect: the proton lifetime predicted by grandunified theories is of order $10^{39}s$ and “quite a few” generations of physicists should invest their entire lifetimes staring at a single proton before its decay, but by managing to keep under observation a large number of protons our sensitivity to proton decay is significantly increased. In proton-decay searches the number of protons being monitored is the (ordinary-physics) dimensionless quantity that works as “amplifier” of the new-physics effect.

In close analogy with these familiar strategies for the study of very small effects, in quantum-gravity phenomenology one must focus [1] on experiments which have something to do with spacetime structure and that host an ordinary-physics dimensionless quantity large enough that it could amplify the extremely small effects we are hoping to discover. The amplifier will often be the number of small effects contributing to the observed signal (as in brownian motion and in proton-stability studies).

Using these general guidelines, a few quantum-gravity research lines have matured over these past few years. I will later discuss studies of the fate of Lorentz symmetry

in quantum spacetime, emphasizing the relevance for observations of gamma rays in astrophysics [12, 13] and the relevance for the analysis of the cosmic-ray spectrum [14, 15, 16, 17, 18, 19, 20]. I will also discuss laser-interferometric tests of Planck-scale effects, both the ones [21] that are relevant for the study of the fate of Lorentz symmetry in quantum spacetime and the ones which explore the possibility of quantum-gravity-induced distance fluctuations [22, 23, 24].

I will not discuss here matter-interferometric limits on Planck-scale effects and limits on Planck-scale effects obtained using the sensitivity to new physics that one finds naturally in the neutral-kaon system (see, *e.g.*, Refs. [25, 26, 27, 28, 30, 29, 30]). In fact, for these Planck-scale effects it is not obvious that space experiments should be preferred to conventional on-ground laboratory experiments.

I should also stress that I intend to focus here on the possibility of a “genuinely quantum” spacetime, a spacetime which will typically have intrinsic noncommutativity and/or discretization at the Planck scale. Interesting ideas about the interplay between gravity and quantum mechanics which do not require a genuinely quantum spacetime can be found in Refs. [31, 32, 33, 34, 35, 36, 37].

4 A key issue: should we adopt a fundamentally quantum spacetime?

As I already stressed, it is rather obvious that from the particle-physics perspective one would not expect any modification of Lorentz symmetry, and on the contrary from the condensed-matter perspective Lorentz symmetry is naturally seen only as an approximate symmetry. It is instead less obvious what one should expect for the fate of Lorentz symmetry in quantum-gravity approaches based on the general-relativity perspective, and in fact some key insight, leading to the expectation that departures from Lorentz symmetry are usually present, has been gained only very recently, mostly in the study of loop quantum gravity and certain noncommutative spacetimes.

A key point that needs to be clarified when approaching the quantum-gravity problem from the general-relativity perspective is whether or not one should adopt a “genuinely quantum” spacetime. A genuinely quantum spacetime is essentially a spacetime in which an event (a spacetime point) cannot be sharply localized. Loop Quantum Gravity (the present understanding of Loop Quantum Gravity) and certain noncommutative spacetimes provide examples of genuinely quantum spacetimes. In the case in which one might be able to introduce coordinates for the event, in a quantum spacetime it must be impossible to determine sharply (in the sense of measurement) all of the coordinates of an event.

String Theory provides, in my opinion, an example of theory in which the status of spacetime is still not fully clarified. One introduces in fact in String Theory a background spacetime which can be (and usually is) classical, but then the theory itself tells us that the position of a particle cannot be sharply determined [38, 39, 40, 41]. This apparent logical inconsistency might suggest that the classical spacetime background should only be seen as a formal tool, void of operative meaning, which should be conveniently replaced by a physically meaningful spacetime picture in which no classical-spacetime idealization is assumed.

Various arguments suggest [4, 5, 6, 7, 8] that a theory that truly admits both general relativity and quantum mechanics as appropriate limits must renounce to any reference

to a classical spacetime, because such a theory is automatically incompatible with the possibility of localizing sharply a spacetime point.

In Section 6 I will summarize results obtained over the last 3 or 4 years which suggest that, if spacetime is “quantum” in the sense of noncommutativity or discreteness, the familiar (classical, continuous) Lorentz symmetry naturally ends up being only an approximate symmetry of the relevant “flat spacetime limit” of quantum gravity.

5 The three possibilities for the fate of Lorentz symmetry in quantum gravity

There are three possibilities for the fate of Lorentz symmetry in quantum gravity: unmodified (exact, ordinary) Lorentz symmetry, broken Lorentz symmetry, and deformed Lorentz symmetry.

It is easy to understand what it means to preserve Lorentz symmetry without modification. We are also all familiar with the concept of “broken Lorentz symmetry” that is being encountered and discussed in some quantum-gravity research lines. This is completely analogous to the familiar situation in which the presence of a background selects a preferred class of inertial observers. For example, there is a modification of the energy/momentum dispersion relation for light travelling in water, in certain crystals, and in other media. Of course, the existence of crystals is fully compatible with a theoretical framework that is fundamentally Lorentz invariant, but in presence of the crystal the Lorentz invariance is manifest only when different observers take into account the different form that the tensors characterizing the crystal (or other background/medium) take in their respective reference systems. If the observers only take into account the transformation rules for the energy-momentum of the particles involved in a process the results are not the ones predicted by Lorentz symmetry. In particular, the dispersion relation between energy and momentum of a particle depends on the background (and therefore takes different form in different frames since the background tensors take different form in different frames).

While the case in which Lorentz symmetry is not modified and the case in which Lorentz symmetry is broken are familiar, the third possibility recently explored in the quantum-gravity literature, the case of deformed Lorentz symmetry introduced in Ref. [42], is rather new and it might be useful to describe it here intuitively. It is the idea that in quantum gravity it might be appropriate to introduce a second observer-independent scale, a large-energy/small-length scale, possibly given by the Planck scale. It would amount to another step of the same type of the one that connects Galilei Relativity and Einstein’s Special Relativity: whereas in Galilei Relativity the (mathematical) description of boost transformations does not involve any invariant/observer-independent scale, the observer-independent speed-of-light scale “ c ” is encoded in the Lorentz boost transformations (which can be viewed as a c -deformation of the Galileo boost transformations), and similarly in the case of a deformed Lorentz symmetry of the type introduced in Ref. [42] there are two scales encoded in the boost transformations between inertial observers (observers which are still indistinguishable, there is no preferred observer). In addition to the familiar observer-independent velocity scale c , there is a second, length (or inverse-momentum), observer-independent scale λ .

In order to provide additional intuition for the concept of deformed Lorentz symmetry let me consider the particular case [42, 43, 44] in which the deformation involves

a new dispersion relation $m^2 = f(E, p; \lambda)$ with $f(E, p; \lambda) \rightarrow E^2 - p^2$ in the limit $\lambda \rightarrow 0$. A modified dispersion relation can also emerge (and commonly emerges) when Lorentz symmetry is broken, but of course the role of the modified dispersion relation in the formalism is very different in the two cases: when Lorentz symmetry is broken the modified dispersion relation reflects properties of a background/medium and the laws of boost/rotation transformation between inertial observers are not modified, while when Lorentz symmetry is deformed the modified dispersion relation reflects the properties of some new laws of boost/rotation transformation between inertial observers. This comparison provides an invitation to consider again the analogy with the transition from Galilei Relativity to Special Relativity. In Galilei Relativity, which does not have any relativistic-invariant scale, the dispersion relation is written as $E = p^2/(2m)$ (whose structure fulfills the requirements of dimensional analysis without the need for dimensionful coefficients). As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a special velocity scale appeared to require (since it was assumed that the validity of the Galilei transformations should not be questioned) the introduction of a preferred class of inertial observers, *i.e.* the “ether” background. Special Relativity introduces the first observer-independent scale, the velocity scale c , its dispersion relation takes the form $E^2 = c^2 p^2 + c^4 m^2$ (in which c plays a crucial role for what concerns dimensional analysis), and the presence of c in Maxwell’s equations is now understood not as a manifestation of the existence of a preferred class of inertial observers but rather as a manifestation of the necessity to deform the Galilei transformations (the Lorentz transformations are a dimensionful deformation of the Galilei transformations). Analogously in some recent quantum-gravity research there has been some interest in dispersion relations of the type $c^4 m^2 = E^2 - c^2 \vec{p}^2 + f(E, \vec{p}; E_p)$ and the fact that these dispersion relations involve an absolute energy scale, E_p , has led to the assumption that a preferred class of inertial observers should be introduced in the relevant quantum-gravity scenarios. But, as I stressed in the papers proposing physical theories with deformed Lorentz symmetry [42], this assumption is not necessarily correct: a modified dispersion relation involving two dimensionful scales might be a manifestation of new laws of transformation between inertial observers, rather than a manifestation of Lorentz-symmetry breaking.

6 Spacetime and Lorentz symmetry in popular quantum-gravity approaches

6.1 Spacetime and Lorentz symmetry in String Theory

String Theory is the most mature quantum-gravity approach from the particle-physics perspective. As such it of course attempts to reproduce as much as possible the successes of quantum field theory, with gravity seen (to a large extent) simply as one more gauge interaction. Although the introduction of extended objects (strings, branes, ...) leads to subtle elements of novelty, in String Theory the core features of quantum gravity are essentially described in terms of graviton-like exchange in a background classical spacetime.

Indeed String Theory does not lead to spacetime quantization, at least in the sense that its background spacetime has been so far described as completely classical. However, this point is not fully settled: it has been shown that String Theory eventually

leads to the emergence of a fundamental limitation on the localization of a spacetime event [38, 39, 40, 41] and this might be in conflict with the assumption of a physically-meaningful classical background spacetime.

If eventually there will be a formulation of String Theory in a background spacetime that is truly quantum, it is likely that Lorentz symmetry will then not be an exact symmetry of the theory. If instead somehow a classical spacetime background can be meaningfully adopted, of course then there is no *a priori* reason to contemplate departures from Lorentz symmetry: classical Minkowski spacetime would naturally be an acceptable background, and a theory in the Minkowski background can be easily formulated in Lorentz-invariant manner.

Still, it is noteworthy that, even assuming that it makes sense to consider a classical background spacetime, the fate of Lorentz symmetry in String Theory is somewhat uncertain: it has been found that under appropriate conditions (a vacuum expectation value for certain tensor fields) Lorentz symmetry is broken in the sense I described above. In these cases String Theory admits description (in the effective-theory sense) in terms of field theory in a noncommutative spacetime [11] with most of the studies focusing on the possibility that the emerging noncommutative spacetime is “canonical” (see Susection 6.3).

In summary in String Theory (as presently formulated, admitting classical backgrounds) it is natural to expect that Lorentz symmetry be preserved. In some cases (when certain suitable background/“external” fields are introduced) this fundamentally Lorentz-invariant theory can experience Lorentz-symmetry breaking. There has been so far no significant interest or results on deformation of Lorentz symmetry in String Theory (see, however, Ref. [45]).

6.2 Spacetime and Lorentz symmetry in Loop Quantum Gravity

Loop Quantum Gravity is the most mature approach to the quantum-gravity problem that originates from the general-relativity perspective. As for String Theory, it must be stressed that the understanding of this rich formalism is in progress. As presently understood, Loop Quantum Gravity predicts an inherently discretized spacetime [46]. There has been much discussion recently, prompted by the studies [47, 48], of the possibility that this discretization might lead to departures from ordinary Lorentz symmetry. Although there are cases in which a discretization is compatible with the presence of continuous classical symmetries [49, 50, 51], it is of course natural, when adopting a discretized spacetime, to put Lorentz symmetry under careful scrutiny. Arguments presented in Refs. [47, 48, 52], support the idea of broken Lorentz symmetry in Loop Quantum Gravity.

Moreover, very recently Smolin, Starodubtsev and I proposed [53] (also see the follow-up study in Ref. [54]) a mechanism such that Loop Quantum Gravity would be described at the most fundamental level as a theory that in the flat-spacetime limit admits deformed Lorentz symmetry. Our argument originates from the role that certain quantum symmetry groups have in the Loop-Quantum-Gravity description of spacetime with a cosmological constant, and observing that in the flat-spacetime limit (the limit of vanishing cosmological constant) these quantum groups might not contract to a classical Lie algebra, but rather contract to a quantum (Hopf) algebra.

In summary in Loop Quantum Gravity the study of the fate of Lorentz is still at a preliminary stage. All three possibilities (preserved, broken and deformed) are still being explored. It is noteworthy however that until 3 or 4 years ago there was a nearly general consensus that Loop Quantum Gravity would preserve Lorentz symmetry, whereas presently the intuition of a majority of experts has shifted toward the possibility that Lorentz symmetry be broken or deformed.

6.3 On the fate of Lorentz symmetry in noncommutative spacetime

There has been much recent interest in flat noncommutative spacetimes, as possible quantum versions of Minkowski spacetime. Most of the work has focused on various parts of the two-tensor parameter space

$$[x_\mu, x_\nu] = i \frac{1}{E_p^2} Q_{\mu\nu} + i \frac{1}{E_p} C_{\mu\nu}^\beta x_\beta, \quad (1)$$

The assumption that the commutators of spacetime coordinates would depend on the coordinates at most linearly is adopted both for simplicity and because it captures a very general intuition: assuming that the Planck scale governs noncommutativity (and therefore noncommutativity should disappear in the formal $E_p \rightarrow \infty$ limit) and assuming that the commutators do not involve singular, $1/x^n$, terms one cannot write anything more general than (1).

Most authors consider two particular limits [55]: the “canonical noncommutative spacetimes”, with $C_{\mu\nu}^\beta = 0$,

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} \quad (2)$$

and the “Lie-algebra noncommutative spacetimes”, with $Q_{\mu\nu} = 0$,

$$[x_\mu, x_\nu] = i\gamma_{\mu\nu}^\beta x_\beta \quad (3)$$

(I am adopting notation replacing $Q_{\mu\nu}/E_p^2 \rightarrow \theta_{\mu\nu}$ and $C_{\mu\nu}^\beta/E_p \rightarrow \gamma_{\mu\nu}^\beta$).

An intuitive characterization of the fate of Lorentz symmetry in canonical noncommutative spacetimes can be obtained by looking at wave exponentials. The Fourier theory in canonical noncommutative spacetime is based [55] on simple wave exponentials $e^{ip^\mu x_\mu}$ and from the $[x_\mu, x_\nu] = i\theta_{\mu\nu}$ noncommutativity relations one finds that

$$e^{ip^\mu x_\mu} e^{ik^\nu x_\nu} = e^{-\frac{i}{2} p^\mu \theta_{\mu\nu} k^\nu} e^{i(p+k)^\mu x_\mu}, \quad (4)$$

i.e. the Fourier parameters p_μ and k_μ combine just as usual, with the only new ingredient of the overall phase factor that depends on $\theta_{\mu\nu}$. The fact that momenta combine in the usual way reflects the fact that the transformation rules for energy-momentum from one (inertial) observer to another are still the usual, undeformed, Lorentz transformation rules. However, the product of wave exponentials depends on $p^\mu \theta_{\mu\nu} k^\nu$, it depends on the “orientation” of the energy-momentum vectors p^μ and k^ν with respect to the $\theta_{\mu\nu}$ tensor. This is a first indication that in these canonical noncommutative

spacetimes there is Lorentz symmetry breaking. The $\theta_{\mu\nu}$ tensor plays the role of a background that identifies a preferred class of inertial observers. Different particles are affected by the presence of this background in different ways, leading to the emergence of different dispersion relations, as shown by the results [56, 57, 58, 59] of the study of field theories in canonical noncommutative spacetimes.

In canonical noncommutative spacetimes Lorentz symmetry is “broken” and there is growing evidence that Lorentz symmetry breaking occurs for most choices of the tensors θ and γ . It is at this point clear, in light of several recent results, that the only way to preserve Lorentz symmetry is the choice $\theta = 0 = \gamma$, *i.e.* the case in which there is no noncommutativity and one considers the familiar classical commutative Minkowski spacetime. Typically Lorentz symmetry is broken, but recent results suggest that for some special choices of the tensors θ and γ Lorentz symmetry might be deformed, rather than broken. In particular, this appears to be the case for the Lie-algebra κ -Minkowski [60, 61, 62, 63, 64, 65] noncommutative spacetime ($l, m = 1, 2, 3$)

$$[x_m, t] = \frac{i}{\kappa} x_m, \quad [x_m, x_l] = 0. \quad (5)$$

κ -Minkowski is a Lie-algebra spacetime that clearly enjoys classical space-rotation symmetry; moreover, κ -Minkowski is invariant under “noncommutative translations” [64]. Since I am focusing here on Lorentz symmetry, it is particularly noteworthy that in κ -Minkowski boost transformations are necessarily modified [64]. A first hint of this comes from the necessity of a deformed law of composition of momenta, encoded in the so-called “coproduct” [60, 61]. One can see this clearly by considering the Fourier transform. It turns out [62, 63, 66] that in the κ -Minkowski case the correct formulation of the Fourier theory requires a suitable ordering prescription for wave exponentials:

$$: e^{ik^\mu x_\mu} : \equiv e^{ik^m x_m} e^{ik^0 x_0}. \quad (6)$$

These wave exponentials are solutions of a κ -Minkowski wave equation [62]. While wave exponentials of the type $e^{ip^\mu x_\mu}$ would not combine in a simple way (as a result of the κ -Minkowski noncommutativity relations), for the ordered exponential one finds

$$(: e^{ip^\mu x_\mu} :)(: e^{ik^\nu x_\nu} :) =: e^{i(p\dot{+}k)^\mu x_\mu} :. \quad (7)$$

The notation “ $\dot{+}$ ” here introduced reflects the behaviour of the mentioned “coproduct” composition of momenta in κ -Minkowski spacetime:

$$p_\mu \dot{+} k_\mu \equiv \delta_{\mu,0}(p_0 + k_0) + (1 - \delta_{\mu,0})(p_\mu + e^{\lambda p_0} k_\mu). \quad (8)$$

As argued in Refs. [42] the nonlinearity of the law of composition of momenta should require an absolute (observer-independent) momentum scale, just like upon introducing a nonlinear law of composition of velocities one must introduce the absolute observer-independent scale of velocity c . The inverse of the noncommutativity scale λ should play the role of this absolute momentum scale. This invites one to consider the possibility [42] that the transformation laws for energy-momentum between different observers would have two invariants, c and λ , as required for a doubly-special-relativity (deformed-Lorentz-symmetry) framework [42].

In summary there is growing evidence that in the rather general class of noncommutative spacetimes described by Eq.(1) it is never possible to preserve Lorentz symmetry unmodified: in most cases Lorentz symmetry is broken and in a few (possibly only in κ -Minkowski) Lorentz symmetry is deformed.

7 Spacetime foam and distance fluctuations

I have so far focused on the fate of Lorentz symmetry in quantum spacetime. Space experiments that can look for effects possibly due to Planck-scale departures from ordinary Lorentz symmetry will be discussed in the next Section. In preparation for the discussion of space experiments, I must first here introduce another class of effects which has been discussed in the quantum-gravity literature and for which space experiments represent an important opportunity. This originates from the prediction of nearly all approaches to the unification of general relativity and quantum mechanics that at very short distances the sharp classical concept of space-time should give way to a somewhat “fuzzy” (or “foamy”) picture. In particular any given length/distance would be affected by an irreducible level of uncertainty/fluctuations.

One way to characterize this operatively makes direct reference to interferometers [22, 24]. Interferometers are the best tools for monitoring the distance between test masses, and an operative definition of quantum-gravity-induced distance fluctuations can be expressed directly in terms of strain noise in interferometers^b. In achieving their remarkable accuracy modern interferometers must deal with several classical-physics strain noise sources (*e.g.*, thermal and seismic effects induce fluctuations in the relative positions of the test masses). Importantly, strain noise sources associated with effects of ordinary quantum mechanics are also significant for modern interferometers: the combined minimization of *photon shot noise* and *radiation pressure noise* leads to a noise source which originates from ordinary quantum mechanics [67]. An operative definition of fuzzy distance can characterize the corresponding quantum-gravity effects as an additional source of strain noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that the read-out of an interferometer would still be noisy (because of quantum-gravity effects) even in the idealized limit in which all classical-physics and ordinary-quantum-mechanics noise sources are completely eliminated. Just like the quantum properties of the non-gravitational degrees of freedom of the apparatus induce noise (*e.g.* the mentioned combination of *photon shot noise* and *radiation pressure noise*) it is of course plausible that noise be induced by the quantum properties of the gravitational degrees of freedom of the apparatus (*e.g.* the distances between the test masses).

This operative definition of quantum-gravity-induced distance fuzziness immediately confronts us with a potentially serious challenge, which is the central challenge of all quantum-gravity-phenomenology research lines: if indeed this distance fuzziness is proportional to (some power of) the Planck length L_p , the smallness of $L_p \equiv 1/E_p \sim 10^{-35}m$ will automatically lead to very small effects. However, modern interferometers have a truly remarkable sensitivity to distance fluctuations and it is not inconceivable that this sensitivity would be sufficient for the detection of fluctuations occurring genuinely at the Planck scale. In order to support this observation with a simple intuitive argument let us consider the possibility that the distances L between the test masses of an interferometer be affected by Planck-length fluctuations of random-walk type occurring at a rate of one per Planck time ($t_p = L_p/c \sim 10^{-44}s$). It is easy to show [22] that such fluctuations would induce strain noise with power

^bSince modern interferometers were planned to look for classical gravity waves (gravity waves are their sought “signal”), it is reasonable to denominate as “noise” all test-mass-distance fluctuations that are not due to gravity waves.

spectrum given by $L_p c L^{-2} f^{-2}$. For $f \sim 100\text{Hz}$ and $L \sim 1\text{Km}$ (estimates relevant for some modern interferometers) this corresponds to strain noise at the level 10^{-37}Hz^{-1} , which is of course rather small because of the L_p suppression but is still well within the reach of the sensitivity of modern interferometers.

Fluctuations genuinely at the Planck scale (the simple scheme I used to illustrate my point involves Planck-length fluctuations occurring at a rate of one per Planck time) can lead to an effect that, while being very small in absolute terms, compares well with the sensitivity of modern interferometers. This originates from the fact that random-walk fluctuations do not fully average out. They have zero mean (in this sense they do average out) but the associated standard deviation grows with the time of observation (with the random-walk-characteristic \sqrt{t} dependence which translates [22, 68] into the f^{-2} dependence of the power spectrum). A reasonable scale to characterize the time of observation in interferometry is provided by f^{-1} which, for $f \sim 100\text{Hz}$, is huge in Planck-time units.

8 Space experiments and quantum properties of spacetime

8.1 Preliminaries on a simple model for Planck-scale departures from Lorentz symmetry

Studies of the fate of Lorentz symmetry in quantum gravity provide a good prototype of quantum-gravity-phenomenology research line. As discussed in the previous Sections, in several (though, of course, not all) approaches to the quantum-gravity problem one finds some evidence of departures from ordinary Lorentz symmetry. Like other effects discussed in the quantum-gravity literature, the ones associated with departures from Lorentz symmetry are very striking from a conceptual perspective. While different intuitions for the quantum gravity problem may lead to favouring one or another of these effects, there is a general consensus that some strikingly new effects should be present in quantum gravity. It was however traditionally believed that even such strikingly new effects (certainly leading to very characteristic signatures) could not be tested because of their small magnitude, set by the small ratio between the energy of the particles involved and the Planck energy scale. Work on quantum-gravity phenomenology has proven that this old expectation is incorrect. And this point is very clearly illustrated in the context of tests of Planck-scale departures from Lorentz symmetry.

Rather than providing a more general discussion, for simplicity I focus here on the possible emergence of Planck-scale-modified dispersion relations,

$$E^2 = m^2 + \vec{p}^2 + f(\vec{p}^2, E, m; L_p) , \quad (9)$$

which are found in the large majority of quantum-gravity-motivated schemes for deviations from ordinary Lorentz invariance (see, *e.g.*, Refs. [12, 47, 52, 57, 61, 62, 69]).

If the function f is nontrivial^c and the energy-momentum transformation rules are unmodified (the familiar Lorentz transformations) then clearly f cannot have the exact

^cFor example, it would be pointless to introduce an $f = L_p^2[E^2 - \vec{p}^2 - m^2]^2$, since then the dispersion relation (9) would be equivalent to $E^2 = m^2 + \vec{p}^2$.

same structure for all inertial observers. In this case Lorentz symmetry is necessarily “broken”, in the sense clarified earlier. In that case it is then legitimate to assume that, in spite of the deformation of the dispersion relation, the rules for energy-momentum conservation would be undeformed.

If instead f does have the exact same structure for all inertial observers, then necessarily the laws of transformation between observers must be deformed (they cannot be the ordinary Lorentz transformation rules). In this case Lorentz symmetry must be deformed, in the sense of the doubly special relativity [42] discussed earlier. There is no preferred frame. The deformation of the laws of transformation between observers impose that one must also necessarily [42] deform the rules for energy-momentum conservation.

While the case of deformed Lorentz symmetry might exercise a stronger conceptual appeal (since it does not rely on a preferred class of inertial observers), for the purposes of this paper it is sufficient to consider the technically simpler context of broken Lorentz symmetry. Upon admitting a broken Lorentz symmetry it becomes legitimate, for example, to adopt a dispersion relation with leading-order-in- L_p form

$$E^2 \simeq \vec{p}^2 + m^2 - \eta(L_p E)^n \vec{p}^2, \quad (10)$$

without modifying the rules for energy-momentum conservation. In (10) η is a phenomenological parameter of order 1 (and actually, for simplicity, I will often implicitly take $\eta = 1$). n , the lowest power of L_p that leads to a nonvanishing contribution, is model dependent. In any given noncommutative geometry one finds a definite value of n , and it appears to be equally easy [42, 43, 70] to construct noncommutative geometries with $n = 1$ or with $n = 2$. In Loop Quantum Gravity one might typically expect [70] to find $n = 2$, but certain scenarios [47] have been shown^d to lead to $n = 1$.

I will use this popular Lorentz-symmetry breaking scenario, with dispersion relation (10) and unmodified rules for energy-momentum conservation, to illustrate how a tiny (Planck-length suppressed) effect can be observed in certain experimental contexts. The difference between the case $n = 1$ and the case $n = 2$ is very significant from a phenomenology perspective. Already with $n = 1$, which corresponds to effects that are linearly suppressed by the Planck length, the correction term in Eq. (10) is very small: assuming $\eta \simeq 1$, for particles with energy $E \sim 10^{12}eV$ (some of the highest-energy particles we produce in laboratory) it represents a correction of one part in 10^{16} . Of course, the case $n = 2$ pays the even higher price of quadratic suppression by the Planck length and for $E \sim 10^{12}eV$ its effects are at the 10^{-32} level.

8.2 Gamma-ray bursts and Planck-scale-induced in-vacuo dispersion

A deformation term of order $L_p^n E^n p^2$ in the dispersion relation, such as the one in (10), leads to a small energy dependence of the speed of photons of order $L_p^n E^n$, by applying the relation $v = dE/dp$. An energy dependence of the speed of photons

^dNote however that the Loop-Quantum-Gravity scenario of Ref. [47] does not exactly lead to the dispersion relation (10): for photons ($m = 0$) Ref. [47] describes a polarization-dependent effect (birefringence).

of order $L_p^n E^n$ is completely negligible (both for $n = 1$ and for $n = 2$) in nearly all physical contexts, but, at least for $n = 1$, it can be significant [12, 13] in the analysis of short-duration gamma-ray bursts that reach us from cosmological distances. For a gamma-ray burst a typical estimate of the time travelled before reaching our Earth detectors is $T \sim 10^{17}s$. Microbursts within a burst can have very short duration, as short as $10^{-4}s$. There is therefore one of the “amplifiers” mentioned in Section 3: the ratio between time travelled by the signal and time structure in the signal is a (conventional-physics) dimensionless quantity of order $\sim 10^{17}/10^{-4} = 10^{21}$. It turns out that this “amplifier” is sufficient to study energy dependence of the speed of photons of order $L_p E$ ($n = 1$). In fact, some of the photons in these bursts have energies in the $100MeV$ range and higher. For two photons with energy difference of order $100MeV$ an $L_p E$ speed difference over a time of travel of $10^{17}s$ leads to a relative time-delay on arrival that is of order $\Delta t \sim \eta T L_p \Delta E \sim 10^{-3}s$ (where ΔE is the difference between the energies of the two photons). Therefore such a quantum-gravity-induced time-of-arrival delay could be detected [12, 13] upon comparison of the structure of the signal in different energy channels.

A space telescope, the GLAST [71] gamma-ray satellite telescope (scheduled to start taking data in 2006), is presently considered to be the best opportunity to look for this effect. AMS [71], on the International Space Station, could also contribute to these studies.

While GLAST (and possibly AMS) should have sufficient sensitivity to look for the $n = 1$ case, *i.e.* $L_p E$ corrections, the much weaker energy dependence of the speed of light found in the case $n = 2$ ($L_p^2 E^2$ corrections) is clearly beyond the reach of GLAST. For $n = 2$ the time-of-arrival-difference (again in the illustrative example of two photons with energies in the $100MeV$ range) comes out to be of order $10^{-18}s$, which is not only beyond the sensitivities achievable with GLAST but also appears to be unreachable for all foreseeable gamma-ray observatories.

Some access to effects characterized by the $n = 2$ case could be gained by exploiting the fact that, according to current models [72], gamma-ray bursters should also emit a substantial amount of high-energy neutrinos. Models of gamma-ray bursters predict in particular a substantial flux of neutrinos with energies of about 10^{14} or 10^{15} eV. Comparing, for example, the times of arrival of these neutrinos emitted by gamma-ray bursters to the corresponding times of arrival of low-energy photons, the case $n = 1$ would predict a huge time-of-arrival difference ($\Delta t \sim 1year$) and even for the case $n = 2$ the time-of-arrival difference could be significant (*e.g.* $\Delta t \sim 10^{-6}s$).

Current models of gamma-ray bursters also predict some production of neutrinos with energies extending to the $10^{19}eV$ level. For such ultra-energetic neutrinos one would expect an even more significant signal, possibly at the level $\Delta t \sim 1s$ for $n = 2$. A $1s$ timing accuracy will surely be comfortably^e available to the EUSO [73] observatory, being planned for the International Space Station. EUSO is expected to be

^eFor this strategy relying on ultra-high-energy neutrinos the delicate point is clearly not timing, but rather the statistics (sufficient number of observed neutrinos) needed to establish a robust experimental result. Moreover, it appears necessary to understand gamma-ray bursters well enough to establish whether there are typical at-the-source time delays. For example, if the analysis is based on a time-of-arrival comparison between the first (triggering) photons detected from the burster and the first neutrinos detected from the burster it is necessary to establish that there is no significant at-the-source effect such that the relevant neutrinos and the relevant photons are emitted at significantly different

able to observe high-energy neutrinos (even with energies higher than 10^{19} eV) and one could, for example, attempt to correlate such detections of high-energy neutrinos with corresponding detections of lower-energy gamma-ray-burst particles (*e.g.* MeV photons).

8.3 Cosmic rays and Planck-scale-modified thresholds

Let us now consider another significant prediction that follows from the dispersion relation (10). While in-vacuo dispersion, discussed in the preceding Subsection, only depends on the deformation of the dispersion relation^f, the effects considered in this Subsection, the so-called “threshold anomalies” [17, 74], also depend on the rules for energy-momentum conservation, which are not modified in the Lorentz-symmetry-breaking scenario I am considering.

Certain types of energy thresholds for particle-production processes may be sensitive to the tiny $L_p^n E^n p^2$ modification of the dispersion relation I am considering. Of particular interest is the analysis of the photopion production process $p + \gamma \rightarrow p + \pi$ when the incoming proton has high energy E while the incoming photon has much smaller energy ϵ ($\epsilon \ll E$). In fact, adopting the modified dispersion relation (10) and imposing ordinary (unmodified) energy-momentum conservation one finds [17] that, for fixed photon energy ϵ , the process is allowed when $E > E_{th}$, with E_{th} given by the modified threshold relation

$$E_{th} \simeq \frac{(m_p + m_\pi)^2 - m_p^2}{4\epsilon} - \eta \frac{L_p^n E_{th}^{2+n}}{4\epsilon} \left(\frac{m_p^{1+n} + m_\pi^{1+n}}{(m_p + m_\pi)^{1+n}} - 1 \right). \quad (11)$$

Of course, the ordinary threshold relation for photopion production is obtained by taking the $L_p \rightarrow 0$ limit of (11), in which the correction of order $L_p^n E_{th}^{2+n}/\epsilon$ disappears. The correction of order $L_p^n E_{th}^{2+n}/\epsilon$ can be relevant for the analysis of ultra-high-energy cosmic rays. A characteristic feature of the expected cosmic-ray spectrum, the so-called “GZK limit”, depends on the evaluation of the minimum energy required of a cosmic ray in order to produce pions in collisions with cosmic-microwave-background photons. According to ordinary Lorentz symmetry this threshold energy is around $E_{th} \simeq 5 \cdot 10^{19} eV$, and cosmic rays emitted with energy in excess of this value should

times. The fact that this “time history” of the gamma-ray burst must be obtained only with precision of, say, 1s (which is a comfortably large time scale with respect to the short time scales present in most gamma-ray bursts) suggests that this understanding should be achievable in the not-so-distant future.

^fThe dispersion relation (10) can also be implemented in a doubly special relativity (deformed Lorentz symmetry) scenario [42]. The in-vacuo-dispersion analysis discussed in the preceding Subsection applies both to Lorentz-symmetry-breaking and Lorentz-symmetry-deformation scenarios adopting (10). When (10) is adopted in a Lorentz-symmetry deformation scenario it is necessary [42] to consistently modify the laws of energy-momentum conservation. Therefore the analysis of Planck-scale-modified thresholds discussed in this Subsection, which assumes unmodified laws of energy-momentum conservation, does not apply to the scenario in which (10) is adopted in a Lorentz-symmetry deformation scenario. Planck-scale-modified thresholds are present also in the case of Lorentz-symmetry deformation, but there are significant quantitative differences [42].

lose the excess energy through pion production before reaching the Earth. Strong interest was generated by the observation [14, 15, 16, 17, 18, 75] that the Planck-scale-modified threshold relation (11) leads, for positive η , to a higher estimate of the threshold energy, an upward shift of the GZK limit. This would provide a description of the observations of the high-energy cosmic-ray spectrum reported by AGASA [76], which can be interpreted as an indication of a sizeable upward shift of the GZK limit. Both for the case $n = 1$ and for the case $n = 2$ the Planck-scale-induced upward shift would be large enough [14, 15, 16, 17, 18, 70, 75] for quantitative agreement with the cosmic-ray observations reported by AGASA.

There are other plausible explanations for the AGASA “cosmic-ray puzzle”, and the experimental side must be further explored, since another cosmic-ray observatory, HIRES, has not confirmed the AGASA results. The situation will become clearer with planned more powerful cosmic-ray observatories, such as the Pierre Auger (ground) Observatory, which will soon start taking data, and EUSO (which, as mentioned, is planned for the International Space Station).

8.4 Interferometry and Planck-scale-induced in-vacuo dispersion

Planck-scale modifications of Lorentz symmetry could also affect certain laser-interferometric setups, as Lämmerzahl and I recently observed [21]. Our observation is based on the idea of operating a laser-light interferometer with two different frequencies^g, possibly obtained from a single laser beam by use of a “frequency doubler” (see *e.g.* [77]).

A description of some interferometric setups that could be used for this purpose is given in Ref. [21]. At least at a simple-minded level of comparison between the magnitude of the Planck-scale-induced phase difference and the phase-difference sensitivity of LIGO/VIRGO-type or LISA-type interferometers we find some encouragement for this proposal.

In addition to studies of Planck-scale modifications of Lorentz symmetry, laser-light interferometry could also be used to explore the possibility of quantum-gravity-induced distance fluctuations, in the sense here discussed in Section 7. The strategy that should be followed is rather clear since in Section 7 I characterized physically the distance fluctuations directly in terms of interferometry. One should look for excess strain noise, noise in excess with respect to the one expected from classical mechanics and ordinary quantum mechanics. Whereas for Planck-scale departures from Lorentz symmetry there is a natural phenomenology in terms of modified dispersion relations, and one manages to find evidence in support of such dispersion relations in certain quantum-gravity proposals (at least in noncommutative spacetimes and in Loop Quantum Gravity), it is much more difficult to develop a phenomenological model for distance fluctuations. The derivation of the much discussed “uncertainty principle for lengths/distances” in quantum gravity theories remains at a rather heuristic level, and it remains to be established which (if any) relation connects the

^gModern interferometers achieve remarkable accuracies also thanks to an optimization of all experimental devices for response to light of a single frequency. The requirement of operating with light at two different frequencies is certainly a challenge for the realization of interferometric setups of the type proposed in Ref. [21]. This and other practical concerns are not discussed here. The interested reader can find a preliminary discussion of these challenges in Ref. [21].

new uncertainty principle to the structure of distance fluctuations. The simple-minded description of random-walk distance fluctuations given in Section 7 does not necessarily provide a reliable estimate and a reliable formula for quantum-gravity-induced interferometric noise [1, 22], but it shows that effects genuinely at the Planck scale (Planck-length fluctuations with Planck-time frequency) could lead to observably large noise levels. Moreover, the random-walk distance-fluctuation picture also provides insight on another perhaps unexpected feature of quantum-gravity-induced noise: this noise might be peaking at low frequencies [1, 22], rather than at the characteristic quantum-gravity frequency scale (the Planck frequency, given by the inverse of the Planck time). Through the example of random-walk fluctuations one can see that the value of the frequency at which noise is highest is not simply set by the inverse of the characteristic time scale of the fluctuations: random-walk fluctuations will always lead to noise peaking at low frequencies, independently of the time scale of fluctuations. [The fact that it peaks in the low-frequency limit is a direct consequence [68, 22] of the fact that the standard deviation of random-walk fluctuations grows in time, with the characteristic \sqrt{t} behaviour.]

As long as we lack a satisfactory model of distance fluctuations we cannot do any better than look for excess noise at low frequencies. In this respect space interferometers can be important, since in space some of the low-frequency noise sources present in on-ground interferometers are absent. In particular, the LISA space interferometer is expected to achieve a remarkably low level of low-frequency noise, and one could perhaps hope that this might open a window of opportunity for the discovery of quantum-gravity-induced low-frequency excess noise. Moreover, in the development of this quantum-gravity-phenomenology research lines based on laser-light interferometry an important role could be played by the operation of other lasers and cavities in space laboratories [78, 79], such as the International Space Station. The (relatively) quiet environment of space could be ideally suited for the development of new laser-interferometric techniques, such as the ones needed for the proposal of Ref. [21].

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