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Quasi-particle Specific Heats for the Crystalline Color Superconducting Phase of QCD

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Abstract

We calculate the specific heats of quasi-particles of two-flavor QCD in its crystalline phases for low temperature. We show that for the different crystalline structures considered here there are gapless modes contributing linearly in temperature to the specific heat. We evaluate also the phonon contributions which are cubic in temperature. These features might be relevant for compact stars with an inner shell in a color superconducting crystalline phase.

1 Introduction

A number of theoretical studies have recently been devoted to QCD at low temperatures T and high densities. Besides the theoretical interest for the different QCD phases, the possibility of applications to compact stars, where dense quark matter might exist, has driven much of the recent interest.

At high density and small T, quarks at the Fermi surface are expected to condense, giving rise to color superconductivity, see [1, 2] and, for reviews,

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[3]. At the largest densities QCD with three flavors should be in the so called CFL (color-flavor locked) phase. For decreasing density one expects pairing between quarks of non vanishing total momentum, resulting in crystalline gap behaviors [4, 5, 6, 7] (for reviews see [8, 9]). Crystalline phases had already been discussed for electric superconductors [10, 11], so that one sometimes also refers to the crystalline phases as LOFF phases. A crystalline color superconductor shell might exist inside compact stars, between the external hadronic crust and the internal CFL core.

In the present letter we discuss dispersion law and specific heats of the quasi-particles present in the LOFF phase. These properties are relevant for the calculation of the thermal conductivity and neutrino emissivity and would affect the cooling of the compact stars.

Thus far crystalline color superconductivity has been studied only in a two-flavor model. Different crystal structures have been computed, with the conclusion, based on a Ginzburg-Landau discussion, that the most favored structure at zero temperature is a cubic structure very close to a face centered cube (FCC). However at non zero temperature the situation is far from being clear (see the discussion in Ref. [8]), therefore we will discuss other crystalline structures as well.

The plan of the paper is as follows. In section 2 we discuss the dispersion laws of the fermionic quasi-particles for different crystal structures of the LOFF phase of high density QCD. We present the one-plane wave structure, the strip, and the face centered cube. In section 3 we calculate the contribution of the fermionic quasi-particles to the specific heat for the three mentioned structures. In section 4 we use the effective phonon Lagrangian to calculate the phonon specific heats. Section 5 is devoted to the conclusions.

2 Fermi quasi-particle dispersion law

In this section we derive the Fermi quasi-particle dispersion law in the two-flavor QCD LOFF phase for a few different crystalline structures. We will work with an inhomogeneous condensate given by

$$\hat{\Delta}(\mathbf{r}) = \Delta(\mathbf{r})\,\epsilon_{\alpha\beta 3}\,\epsilon_{ij} \tag{1}$$

where α, β are color indices and $i, j = 1, 2$ are flavor indices. Notice that the gap term pairs together the color 1 and 2, whereas the fermions with color 3 are unpaired. We work in presence of a difference in the chemical potential of the u and the d quarks. We define

$$\mu_u = \mu + \delta\mu \ , \qquad \mu_d = \mu - \delta\mu \ . \tag{2}$$

As usual the quasi-particle dispersion law is obtained by looking at the zeros of the inverse propagator. For fermionic quasi-particles one has to solve the eigenvalue equation

$$(S^{-1})^{ij}_{\alpha\beta}\chi^\beta_j = 0\,, \tag{3}$$

where S^{-1} is the inverse propagator in the LOFF phase in the Nambu-Gorkov formalism and χ^β_j are the Green eigenfunctions. Following the notations in [8] we write S^{-1} as follows:

$$(S^{-1})^{ij}_{\alpha\beta} = \begin{pmatrix} \delta_{\alpha\beta}[\delta_{ij}(E+i\mathbf{v}\cdot\boldsymbol{\nabla})+\delta\mu(\sigma_3)_{ij}] & -\varepsilon_{\alpha\beta 3}\varepsilon_{ij}\Delta(\mathbf{r}) \\ -\varepsilon_{\alpha\beta 3}\varepsilon_{ij}\Delta(\mathbf{r})^* & \delta_{\alpha\beta}[\delta_{ij}(E-i\mathbf{v}\cdot\boldsymbol{\nabla})+\delta\mu(\sigma_3)_{ij}] \end{pmatrix} \tag{4}$$

where E is the quasi-particle energy and $\mathbf{v}$ is the Fermi velocity, that in QCD with massless quarks satisfies $v = |\mathbf{v}| = 1$. Let us define

$$\chi^\alpha_i = \begin{pmatrix} \bar{G}^\alpha_i \\ -i(\sigma_2)_{\alpha\beta}\bar{F}^\beta_i \end{pmatrix} . \tag{5}$$

Performing the unitary transformation

$$\bar{G}^\alpha_i = \left(e^{i\delta\mu\,\sigma_3\,\mathbf{v}\cdot\mathbf{r}/v^2}\right)_{ij} G^\alpha_j\,, \quad \bar{F}^\alpha_i = \left(e^{-i\delta\mu\,\sigma_3\,\mathbf{v}\cdot\mathbf{r}/v^2}\,\sigma_2\right)_{ij} F^\alpha_j \ \ , \tag{6}$$

one measures the energy of each flavor from its Fermi energy. The resulting equations for F^α_i and G^α_i are independent of color and flavor indices, that therefore will be omitted below:

$$\begin{aligned} (E+i\mathbf{v}\cdot\boldsymbol{\nabla})G - i\Delta(\mathbf{r})F &= 0\ , \\ (E-i\mathbf{v}\cdot\boldsymbol{\nabla})F + i\Delta(\mathbf{r})^*G &= 0\ . \end{aligned} \tag{7}$$

We solve these equations for three different crystalline structures, corresponding to different decompositions of (1) in plane waves:

1. One plane wave;

2. Two antipodal plane waves;

3. The face centered cubic (FCC) structure, formed by eight plane waves with momenta pointing to the vertices of a cube.

The first case is discussed for metals in [11] by Fulde and Ferrel (FF); for QCD is analyzed in [4]. The second case has been examined at $T = 0$ by [10]; we will refer to it as the *strip* below. The last case is the preferred solution at $T = 0$ in Ginzburg-Landau analysis of Ref. [7]. The reason why we examine these different structures is in the fact that the preferred structure at $T \neq 0$ can be different from that at $T = 0$, see e.g [6] and the discussion in [8].

2.1 One plane wave

For the FF condensate we take the direction of the Cooper pair total momentum $2\mathbf{q}$ along the $z-$axis. The gap

$$\Delta(\mathbf{r}) = \Delta e^{2iqz} \tag{8}$$

is therefore a complex number. If we take $G = \hat{G}(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}$, $F = \hat{F}(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}$, we get from Eqs.(7)

$$\begin{aligned} \left[E - qv_z + i\mathbf{v}\cdot\boldsymbol{\nabla}\right]\hat{G}(\mathbf{r}) &= +i\Delta\hat{F}(\mathbf{r}) \ , \\ \left[E - qv_z - i\mathbf{v}\cdot\boldsymbol{\nabla}\right]\hat{F}(\mathbf{r}) &= -i\Delta\hat{G}(\mathbf{r}) \ . \end{aligned} \tag{9}$$

These are the standard Gorkov equations for a uniform superconductor with energy $E - qv_z$. The eigenfunctions are simple plane waves

$$\hat{G}(\mathbf{r}) = ue^{i\mathbf{k}\cdot\mathbf{r}} \qquad\qquad \hat{F}(\mathbf{r}) = we^{i\mathbf{k}\cdot\mathbf{r}} \, , \tag{10}$$

and the quasi-particle spectrum is given by:

$$E_\pm = qv_z \pm \sqrt{\xi^2 + \Delta^2} \, , \tag{11}$$

where $\xi = \mathbf{k}\cdot\mathbf{v}$ is the residual longitudinal momentum, i.e. the longitudinal momentum measured from the Fermi surface. Because of the transformation (6), quasi-particle energies are computed from the corresponding Fermi energies $\mu_{u,d}$. Eq.(11) is the dispersion law of quasi-particle ($E_\pm \geq 0$) or hole states ($E_\pm < 0$). We notice that in this case, as in [12], [13] and [14], we are in presence of gapless superconductivity.

An anisotropic dispersion law was also obtained in [4] by a different procedure (variational method). Their result reduces to (11) if one considers only the leading order in the asymptotic $\mu \to \infty$ limit.

2.2 Strip and FCC structures

Both the strip and the cubic structure have real $\Delta(\mathbf{r})$. The solutions of Eqs. (7) are Bloch functions

$$G = u(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}} \ , \qquad\qquad F = w(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}} \, , \tag{12}$$

with $u(\mathbf{r})$ and $w(\mathbf{r})$ periodic functions and $\mathbf{k}$ in the first Brillouin zone. They satisfy

$$\begin{aligned} [E-\xi+i\mathbf{v}\cdot\boldsymbol{\nabla}]\,u(\mathbf{r}) &= +i\,\Delta(\mathbf{r})w(\mathbf{r}) \ , \\ [E+\xi-i\mathbf{v}\cdot\boldsymbol{\nabla}]\,w(\mathbf{r}) &= -i\,\Delta(\mathbf{r})u(\mathbf{r}) \ . \end{aligned} \tag{13}$$

The corresponding quasi-particles are gapless [10]. In fact, for $E=0$ and $\xi=0$, the system (13) has two solutions. We have $w_\pm=\pm u_\pm$, with $u_\pm$ solutions of

$$\mathbf{v}\cdot\boldsymbol{\nabla}u_\pm(\mathbf{r}) = \pm\Delta(\mathbf{r})u_\pm(\mathbf{r})\, , \tag{14}$$

and given by

$$u_\pm(\mathbf{r}) = \exp\Big[\pm\int\Delta(\mathbf{r}')\frac{d(\mathbf{r}'\cdot\mathbf{v})}{v^2}\Big] \ . \tag{15}$$

Here the integration is over the path $\mathbf{v}=const.$

To find the dispersion law of quasi-particles for small values of ξ one uses degenerate perturbation theory and gets

$$E^2 = \frac{\xi^2}{A_+A_-} \, , \tag{16}$$

with

$$A_\pm = \frac{1}{V_c}\int_{\text{cell}} d\vec{r}\exp\left[\pm 2\int\Delta(\mathbf{r}')\frac{d(\mathbf{r}'\cdot\mathbf{v})}{v^2}\right] \, , \tag{17}$$

where V_c is the volume of a unit cell of the lattice.

Let us now specialize to the case of the strip, i.e. the crystalline structure formed by two plane waves with wave vectors $\pm 2\mathbf{q}$. The inhomogeneous gap is given by

$$\Delta(\mathbf{r}) = \Delta\,\cos\,2qz \tag{18}$$

and one gets

$$A_\pm^{(s)} \equiv A^{(s)} = I_0\left(\frac{\Delta}{qv\cos\theta}\right) \, , \tag{19}$$

where $I_0(z)$ is the modified Bessel function of the zeroth order.

Let us now turn to the FCC structure. Summing the eight plane waves, the condensate can be put in the form

$$\Delta(\mathbf{r}) = \Delta \cos 2qx \cos 2qy \cos 2qz\,. \tag{20}$$

With $|\mathbf{r}| = r$ and

$$\begin{aligned} B &= \frac{r}{4}\left(\frac{\sin 2q(x+y+z)}{x+y+z} + \frac{\sin 2q(x+y-z)}{x+y-z} + \frac{\sin 2q(x-y+z)}{x-y+z}\right. \\ &+ \left.\frac{\sin 2q(-x+y+z)}{-x+y+z}\right), \end{aligned} \tag{21}$$

one obtains for the cube

$$A_\pm^{(\rm fcc)} = \left(\frac{q}{\pi}\right)^3 \int_{cell} dV \exp\left\{\pm\frac{\Delta}{qv}B\right\}\,, \tag{22}$$

where the integration is over the elementary cell of volume $(\pi/q)^3$.

Let us notice that whereas for the strip in Eq. (19) one has a closed formula, for the FCC only numerical or approximate expressions can be given. Since the parameter $x = \Delta/qv$ is expected to be small, expanding $A_\pm^{(\rm fcc)}$ as a power series one obtains

$$\sqrt{A_+^{(\rm fcc)}(x)A_-^{(\rm fcc)}(x)} = 1 + 0.035\,x^2 + \mathcal{O}[x]^4\,. \tag{23}$$

In conclusion for both the strip and the FCC the quasi-fermion spectrum is gapless. This result remains valid also for massive quarks, since the effect of the quark mass can be accounted for by reducing the quark velocity from the value $v = 1$ valid for the massless case [15].

3 Specific heat of the Fermi quasi-particles

The contribution of the Fermi quasi-particles to the specific heat per unit volume is

$$c_v = \rho \int \frac{d\Omega}{4\pi} \int d\xi\, E\, \frac{dn(E,T)}{dT}\,, \tag{24}$$

where, for the two flavor case (and $|\mathbf{v}| = 1$),

$$\rho = \frac{4\mu^2}{\pi^2}\,, \tag{25}$$

while $n(E,T)$ is the Fermi distribution function and the angular integration is over the directions of $\mathbf{v}$.

We should also remind that, in the case of two flavors, the two quarks of color 3 are ungapped and as such each contributes to the specific heat as follows:

$$c_v = \frac{\mu^2}{3} T\,. \tag{26}$$

Let us now specialize Eq. (24) to the three crystalline structure under scrutiny. We limit the analysis to the small T range.

3.1 One plane wave

The dispersion law of quasi-particles is given by (11). Using (24) one gets in the small temperature limit ($T \ll \Delta$) and for $\Delta < q$

$$c_v^{(\mathrm{FF})} = \frac{\rho T \pi^2}{3}\sqrt{1-\frac{\Delta^2}{q^2}} \qquad \text{(quarks)}\ . \tag{27}$$

The specific heat depends linearly on temperature because the quasi-particle dispersion law (11) gives rise to gapless modes. There is also a contribution to the specific heat that comes from gapped modes, but this contribution is exponentially suppressed with the temperature.

3.2 Strip and FCC structures

From Eq. (24), using (16) that is valid for the strip and the FCC alike, one obtains, for $T \ll \Delta$:

$$c_v = \frac{\rho T \pi^2}{3}\int \frac{d\Omega}{4\pi}\sqrt{A_+ A_-}\ . \tag{28}$$

This expression can be evaluated in closed form for the strip, when $A_\pm = A^{(\mathrm{s})}$. One gets

$$c_v^{(\mathrm{s})} = \frac{\rho T \pi^2}{3}\ {}_1F_2\left(-1/2;\ 1/2,\ 1;\ (\Delta/(qv))^2\right) \qquad \text{(quarks)} \tag{29}$$

where ${}_1F_2$ denotes the generalized hypergeometric function [16]. Differently from the analysis of [10], here v is not small and we can take $z = \Delta/qv \to 0$ near the second order phase transition. Since for small z one has

${}_1F_2(-1/2;\,1/2,\,1;z^2)\simeq 1-z^2$, it is easily seen that the normal Fermi liquid result (26) is obtained for $z=0$. On the other hand, at finite Δ, the specific heat turns out to be smaller.

For the face centered cube, $A_\pm^{(\rm fcc)}$ are isotropic and the angular integration is trivial. One gets for this case

$$c_v^{(\rm fcc)}=\frac{\rho T\pi^2}{3}\sqrt{A_+^{(\rm fcc)}A_-^{(\rm fcc)}}\qquad\qquad (\text{quarks})\ . \tag{30}$$

Using the expansion (23) one can get an approximate formula giving a power series of Δ/qv. One recognizes that the specific heat for the cubic structure is larger than in the ungapped case (26).

4 Specific heat of phonons

Besides Fermi quasi-particles we consider also the massless Nambu-Goldstone bosons (NGB). Even though we expect a parametrically smaller $\mathcal{O}(T^3)$ contribution in this case, the contribution of the NGB might be relevant for future applications, e.g. for the calculation of thermal conductivity. For two flavor QCD the only NGB are phonons [17, 18].

4.1 One or two plane waves

Let us begin with the phonon Lagrangian for the plane wave

$$\mathcal{L}=\frac{1}{2}\left(\dot\phi^2-v_\parallel^2(\nabla_\parallel\phi)^2-v_\perp^2|\nabla_\perp\phi|^2\right)\ , \tag{31}$$

where ϕ is the phonon field, $\nabla_\parallel=\mathbf{n}\cdot\boldsymbol{\nabla}$, $\boldsymbol{\nabla}_\perp=\boldsymbol{\nabla}-\mathbf{n}\nabla_\parallel$ and $\mathbf{n}$ along the $z-$axis, i.e. the direction of $\mathbf{q}$. In [17] we found

$$v_\parallel^2=\cos^2\theta_q\ ,\qquad\qquad v_\perp^2=\frac{1}{2}\sin^2\theta_q\ , \tag{32}$$

with $\cos\theta_q=\delta\mu_2/q=0.833$. Here $\delta\mu_2$ is the $\delta\mu$ value corresponding to the second order phase transition from the LOFF phase to the normal one. The dispersion law, relating the phonon quasi-momentum $\mathbf{k}$ and energy ω, therefore is

$$\omega(\mathbf{k})=\sqrt{v_\perp^2(k_x^2+k_y^2)+v_\parallel^2k_z^2}\ . \tag{33}$$

Let N be the total number of oscillatory modes. There is an oscillator for any quark pair and the total number of quark pairs is given by the available phase space, i.e. the pairing region. If V is the available volume, then

$$\frac{N}{V} = \frac{g}{2}\int \frac{d^3k}{(2\pi)^3}, \tag{34}$$

where $g = 4 \times 2$ and we divide by two due to the fact that there are two quarks in the pair. The integration over the pairing region gives

$$\frac{N}{V} \simeq \frac{g}{2}\,\frac{4\pi\mu^2\zeta}{(2\pi)^3}\,(2\delta) \ . \tag{35}$$

A factor 2δ arises from the integration over the longitudinal residual momentum ξ; δ is the ultraviolet cutoff, of the order of μ while ζ is the fraction of the phase space available for pairing in the LOFF phase. It is estimated of the order of Δ/q.

The ratio N/V can be expressed in terms of a cutoff frequency by a procedure analogous to the introduction of the Debye frequency for ordinary crystals. We write

$$\frac{N}{V} = \int_0^{\omega_D} f(\omega)d\omega \tag{36}$$

where $f(\omega)$ is given by

$$f(\omega) = \frac{g}{2}\int \frac{d^3k}{(2\pi)^3}\delta(\omega - \omega(\mathbf{k})) = \frac{g\omega^2}{4\pi^2 v_\perp^2 v_\parallel} \ . \tag{37}$$

Substituting in (34) we see that ω_D is of the order of $\mu(\Delta/q)^{1/3}$ therefore large (excluding the second order phase transition region). More precisely we get the formula

$$\frac{N}{V} = \frac{g\omega_D^3}{12\pi^2 v_\perp^2 v_\parallel} \ . \tag{38}$$

These results also hold in the case of the strip, i.e. two antipodal plane waves. As a matter of fact one can prove, following the same procedure as in [17], that the effective Lagrangian for the phonon field is still given by (31).

The specific heat per unit volume at small temperatures ($\omega_D \gg T$), is given by ($k_B = \hbar = 1$):

$$c_v = \frac{4N\pi^4}{5V}\left(\frac{T}{\omega_D}\right)^3 , \tag{39}$$

and therefore

$$c_v^{(\mathrm{FF})} = c_v^{(\mathrm{s})} = \frac{8\pi^2}{15 v_\perp^2 v_\parallel} T^3 \qquad \text{(phonons)} \ . \tag{40}$$

This result holds for the two considered structures (one or two plane waves).

4.2 Face-centered-cube

For the FCC crystal structure the phonon Lagrangian is given by [19]

$$\mathcal{L} = \frac{1}{2}\sum_{i=1,2,3}(\dot{\phi}^{(i)})^2 - \frac{a}{2}\sum_{i=1,2,3}|\boldsymbol{\nabla}\phi^{(i)}|^2 - \frac{b}{2}\sum_{i=1,2,3}(\partial_i\phi^{(i)})^2 - c\sum_{i<j=1,2,3}\partial_i\phi^{(i)}\partial_j\phi^{(j)} \, . \tag{41}$$

In [18] we found

$$a = 1/12 \qquad b = 0 \ , \qquad c = (3\cos^2\theta_q - 1)/12 \, . \tag{42}$$

Eq. (36) is still valid with

$$f(\omega) = \frac{1}{3}\sum_{r=1}^{3}\frac{g}{2}\int\frac{d^3k}{(2\pi)^3}\delta(\omega - \omega_r(\mathbf{k})) \ , \tag{43}$$

since $\omega_r(\vec{k}) = v_r(\hat{\mathbf{n}})k$ we get

$$f(\omega) = \frac{\omega^2 g}{48\pi^3}\sum_{r=1}^{3}\int\frac{d\hat{\mathbf{n}}}{v_r^3(\hat{\mathbf{n}})} = \frac{\omega^2 g}{24\pi^2} K \, . \tag{44}$$

Here the velocities $v_r^2 = v_r^2(\hat{\mathbf{n}})$ are the eigenvalues of the matrix

$$\begin{pmatrix} a + b n_1^2 & c\, n_1 n_2 & c\, n_1 n_3 \\ c\, n_1 n_2 & a + b n_2^2 & c\, n_2 n_3 \\ c\, n_1 n_3 & c\, n_2 n_3 & a + b n_3^2 \end{pmatrix} , \tag{45}$$

with $n_1 = \sin\theta\cos\varphi$, $n_2 = \sin\theta\sin\varphi$, $n_3 = \cos\theta$.

To get the Debye frequency and the specific heat we use the numerical result

$$K = \frac{1}{2\pi}\sum_{r=1}^{3}\int\frac{d\hat{\mathbf{n}}}{v_r^3(\hat{\mathbf{n}})} \approx 3.3 \times 10^2 \tag{46}$$

corresponding to the values (42) of the parameters. For the specific heat we get, similarly to Eq. (40), the result

$$c_v^{(\mathrm{fcc})} = \frac{4\pi^2}{15} K\, T^3 \qquad \text{(phonons)} \ . \tag{47}$$

One can note that numerically $c_v^{(\mathrm{fcc})} > c_v^{(\mathrm{s})}$.

5 Conclusions and Outlook

We have considered three different crystalline structures for high density QCD in the LOFF phase: one plane wave, the strip, and the favorite face-centered-cubic strucure. Gapless fermionic quasi-particles are found to exist even in the presence of quark mass terms. These gapless fermions provide for the dominant contributions to the thermal properties for low temperatures, such as of possible interest for compact stars. Typically they contribute to specific heats with terms linear in T which are dominant at low T. The massless phonons contribute instead as T^3 and gapped particles have an exponential suppression at low T. The previous calculations could be applied to the evaluation of transport properties of compact stars. At the present a reliable model for the LOFF phase in compact stars is still lacking, since the study of the inhomogeneous superconducting phase of QCD with three flavors has not yet been performed. Nevertheless one can make some conjecture by imagining the existence of the LOFF phase inside the star, but outside the inner core, where one might suppose the existence of quark matter in the CFL phase. An application of previous results might be the evaluation of the thermal conductivity in the LOFF phase for the existing model of 2-flavor QCD. We plan to come to this subject in the future.

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