

RADIATION DAMPING

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ABSTRACT

Energy and betatron oscillations in particle accelerators and storage rings are damped, if the particle energy is so high that a significant amount of synchrotron radiation is emitted. The basic mechanism leading to this damping is discussed. Examples are given and means commonly used to influence the amount of damping are described.

1. INTRODUCTION

The basic formulae concerning damping are derived with emphasis on the physics. More complete accounts of this topic can be found in articles 1,2,3) and textbooks 4,5). The influence of the radiation damping on the distribution of particles in the bunches is treated elsewhere together with quantum excitation 6).

2. ENERGY OSCILLATIONS

Consider a beam which is neither accelerated nor decelerated. Averaged over many turns, the energy lost per turn U_0 due to the synchrotron radiation 7) is compensated by the energy gained per turn in the RF accelerating cavities. The energy loss is given by

$$U_0 = \frac{2}{3} r_0 m_0 c^2 \gamma^4 \oint \frac{ds}{\rho^2} \quad (1)$$

where the integral is taken once around the closed orbit having ρ as local radius of curvature. Generally, U_0 is much smaller than the particle energy, typically by a factor 10^3 or more. Thus we can neglect the change of energy per revolution but we must consider effects accumulating over many turns.

Imagine only one short RF cavity in the ring. Particles may gain or lose energy $eV(t)$ depending upon their arrival time t .

$$eV(t) = eV_{RF} \sin(\Omega_{RF} t + \varphi_s) \quad (2)$$

The particle which always arrives in the right moment gaining exactly U_0 , as shown in Fig. 1, is called the synchronous particle. Its energy when leaving the cavity is E_s . Assume a particle injected with an energy deviation $\epsilon = E - E_s$ which is also ahead of the synchronous particle at the cavity by the distance cT as sketched in Fig. 2. Due to its higher energy it will travel on an orbit with a larger average radius and, therefore, it will have lost some of its advance when the two come around the next time. The change in time T is 8)

$$\Delta T = -\left(\alpha - \frac{1}{\gamma^2}\right) \frac{\epsilon}{E_s} T_0 \quad (3)$$

where T_0 is the revolution time and α is a positive constant given by the properties of the magnetic guide field. Since $\alpha \gg 1/\gamma^2$ for electrons, we can neglect $1/\gamma^2$. On average over the turn

$$\frac{dT}{dt} = -\alpha \frac{\epsilon}{E_s} \quad (4)$$

Establishing the energy balance for one turn yields the net energy gain per turn ΔE

$$\frac{\Delta E}{T_0} = \frac{eV(t) - U(E)}{T_0}$$

where t is the arrival time of the particle at the RF cavity. The energy loss U depends on E as shown later. It is this dependence which gives rise to the damping of the oscillations. Note: $U(E_s) = U_0$. Keeping only first order terms the average rate of energy change becomes

$$\frac{d\epsilon}{dt} = \frac{(U_0 + e \frac{dV}{dT}) - (U_0 + \frac{dU}{dE} d\epsilon)}{T_0} \quad (5)$$

After combination with (4) we obtain

$$\frac{d^2\epsilon}{dt^2} + \frac{1}{T_0} \frac{dU}{dE} \frac{d\epsilon}{dt} + \frac{e}{T_0} \frac{dV}{dT} \frac{\alpha}{E_s} \epsilon = 0 \quad (6)$$

Since $\alpha \ll \Omega_s$ the solution becomes

$$\epsilon(t) = A e^{-\alpha_\epsilon t} \cos(\Omega_s t - \vartheta_0) \quad (7)$$

The circular synchrotron frequency Ω_s is given by

$$\Omega_s^2 = \frac{e}{T_0} \frac{dV}{dT} \frac{\alpha}{E_s}$$

A and θ_0 are determined by the initial conditions. Equation (6) is the usual differential equation of the energy oscillations 8) apart from the second term which determines the damping rate

$$\alpha_\epsilon = \frac{1}{2T_0} \frac{dU}{dE} \quad (8)$$

due to synchrotron radiation. Using (7) in (4) gives the equation complementary to (7)

$$T(t) = -\frac{\alpha}{E_s \Omega_s} A e^{-\alpha_\epsilon t} \sin(\Omega_s t - \vartheta_0) \quad (9)$$

Assume that an observer plots ϵ and T of our particles in phase space after each revolution. Drawing a curve through the points would give a pattern as shown in Fig. 3. The centre of this spiralling motion is the synchronous particle. Since $dU/dE > 0$ as shown later, the particle loses energy in the upper part of its path in phase space relative to the synchronous particle, while it gains energy in the lower part. This leads to a continuous damping of the motion. The damping is governed by dU/dE , a quantity which can be derived entirely from the classical theory of radiation without invoking quantum effects.

Given an ensemble of particles, initially distributed in an arbitrary fashion in phase

space around the synchronous particle, one might wonder whether all these particles will finally end up clustered densely around the synchronous particle. Since they would get closer and closer to each other, the radiation would be emitted more and more coherently. As a consequence, the energy loss would finally exceed the available RF voltage and the cluster would be lost. However, this cannot happen because the excitation of the oscillations by quantum fluctuations eventually stops the contraction of the ensemble. The final particle distribution turns out to be governed by the equilibrium between damping and excitation 6).

In order to calculate dU/dE we recall 7)

$$U(E) = \oint P_{\gamma} dt = \oint P_{\gamma} \frac{dt}{ds} ds \quad (10)$$

where P_{γ} is the instantaneous power radiated by the particle. Note the first integral is taken along an orbit with $E \neq E_s$ while the second integral is taken along the orbit of the synchronous particle. Assume a sector magnet with endfaces perpendicular to the orbit and a particle displaced by x from the reference orbit as shown in Fig. 4.

Over a length ds the displaced particle has spent dt more time than the synchronous particle in the magnetic field. We get

$$\frac{dt}{ds} \approx \frac{1}{c} \left(1 + \frac{x(s)}{\rho_s} \right) \quad (11)$$

For vanishing betatron oscillation amplitude

$$x(s) = D_x(s) \cdot \varepsilon / E_s \quad (12)$$

Using (11) and (12) in equation (10) and differentiating yields

$$\frac{dU}{dE} = \frac{1}{c} \oint \left(\frac{dP_{\gamma}}{dE} + \frac{D_x}{\rho} \frac{P_{\gamma}}{E} \right) ds$$

The radiated power is given by 7)

$$P_{\gamma} = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho^2} \sim E^2 B^2(x(E)) \quad (13)$$

Hence

$$\frac{dU}{dE} = \frac{U_0}{E_s} \left[2 + \frac{1}{cU_0} \oint D_x P_{\gamma} \left(\frac{2}{B} \frac{dB}{dx} + \frac{1}{\rho} \right) ds \right] \quad (14)$$

and (8) becomes

$$\alpha_{\varepsilon} = \frac{1}{\tau_{\varepsilon}} = \frac{1}{2T_0} \frac{U_0}{E_s} (2 + \tilde{D}) \quad (15)$$

with

$$\tilde{D} = \frac{\oint \frac{D_x}{\rho^2} \left(\frac{2}{B} \frac{dB}{dx} + \frac{1}{\rho} \right) ds}{\oint \frac{ds}{\rho^2}} \quad (16)$$

The first term in \tilde{D} is due to magnets at $D_x \neq 0$ which have a dipole and quadrupole component in their field (combined-function magnets ⁹⁾). In separated-function rings ⁹⁾, where dipoles and quadrupoles are separated, this term is extremely small. A non-zero product $B \cdot dB/dx$ only occurs in the fringe field of dipole magnets which have endfaces not perpendicular to the orbit. However, this term becomes important if the closed orbit is accidentally or intentionally displaced away from the quadrupole centres.

The second term in the numerator of (16) comes from the fact that particles on an off-energy orbit spend a longer time radiating in the magnet than the synchronous particle. In separated-function machines, its contribution is

$$\Delta \tilde{D} = \frac{\alpha}{2\pi} \frac{1}{\rho_s} \oint ds \quad (17)$$

which can be neglected. If the endfaces of each dipole are parallel to each other ("straight magnets"), this term vanishes completely.

Therefore, we conclude $\tilde{D} \ll 1$ for a separated-function guide field which is the field configuration adopted for the recent large storage rings and synchrotrons. Consequently from (15)

$$\tau_E \approx T_0 \frac{E_s}{U_0} \quad (18)$$

indicating that the damping time is the time it takes a particle to radiate away its own energy. Note that τ_E scales like γ^{-3} for a given machine. For illustration, the numerical values for LEP at 55 GeV are ¹⁰⁾

$U_0 = 260$ MeV	energy loss per turn
$T_0 = 89$ μ s	revolution time
$T_S = 2\pi/\Omega_S \approx 10 \times T_0 = 1$ ms	synchrotron oscillation period
$\tau_E = 18$ ms	energy damping time

It can be seen from this example that the energy loss per turn is indeed very small compared to the particle energy. The damping is a rather slow process, the particle makes 18 revolutions in phase space (cf. Fig. 3) and 200 turns per damping time. Thus, it is admissible to use differential equations instead of difference equations.

3. BETATRON OSCILLATIONS

3.1 Vertical oscillations

Ignoring the variation of β with s we can describe the motion by ⁹⁾

$$z = A \cos\left(\frac{s}{\beta} + \varphi\right)$$

$$z' = \frac{dz}{ds} = -\frac{A}{\beta} \sin\left(\frac{s}{\beta} + \varphi\right)$$

The amplitude

$$A^2 = z^2 + (\beta z')^2 \quad (19)$$

is invariant in the absence of radiation.

Consider first the emission of one photon of energy δE . During the emission the momentum \vec{p} of the particle is lowered by $|\delta\vec{p}| = \delta E/c$ due to the recoil as shown in Fig. 5a. Since the angle of photon emission θ_z relative to \vec{p} is very small ($\theta_{zrms} = 1/2\gamma$), we neglect it for the moment. Thus, neither z nor z' are changed during this process and the oscillation pattern is not disturbed provided that the vertical orbit position does not depend on energy, which is the case in most accelerators.

The energy loss will be compensated by the RF cavity adding $\delta p = \delta E/c$. Only the longitudinal component of the momentum is affected not the perpendicular component p_t . Hence, the slope z' after the RF acceleration has changed (see Fig. 5b) and becomes in the limit of $p_t \ll p$

$$z' + \delta z' = \frac{p_t}{p + \delta p} \approx z' \left(1 - \frac{\delta p}{p}\right)$$

From this

$$\delta z' = -z' \frac{\delta p}{p} \approx -z' \frac{\delta E}{E_s}$$

and we get for the concomitant change in amplitude

$$A \delta A = \beta^2 z' \delta z' = -(\beta z')^2 \frac{\delta E}{E_s}$$

Since the photon emission is a quantum effect, it does not occur in a predictable way. It may occur at some arbitrary distance before the RF cavity, and the phase of the oscillation at the cavity can take any value between 0 and 2π . Averaging over all possible phases yields

$$A \langle \delta A \rangle = -\frac{A^2}{2} \frac{\delta E}{E_s}$$

The total number of emissions per turn is very large ⁷⁾. For example, 7×10^3 emissions occur on average per turn in LEP at 55 GeV. Considering one revolution it is evident that the sum of all δE must give U_0 . The change of the amplitude due to all emissions per revolution time T_0 becomes

$$\frac{\Delta A}{A} = -\frac{U_0}{2E_s} \quad (20)$$

leading to

$$\frac{1}{A} \frac{dA}{dt} = \frac{\Delta A}{A T_0} = -\frac{U_0}{2E_s T_0}$$

and to the damping rate

$$\alpha_z = \frac{U_0}{2E_s T_0} \quad (21)$$

This is one half of the damping rate we found for the energy oscillations (15), provided \tilde{D} vanishes. Note that the emission itself has no damping effect on the oscillation. Only the compensation of the energy loss by the RF system causes the actual damping. A similar mechanism provides the damping of the betatron motion during acceleration to higher energy and is called adiabatic damping (11).

We have assumed that the emission of the photon is in the direction of the motion. In reality $\delta\vec{p}$ and \vec{p} are not quite collinear. This can give rise to some quantum excitation of the vertical motion. However, the effect is small and, in practical cases, masked by the much stronger energy transfer between the vertical and horizontal motion due to coupling.

3.2 Horizontal oscillations

In this case we have to take into account that the reference orbit changes in the moment a quantum is emitted because D_x is usually not zero in the dipole magnets. Although exceptions exist as for example DCI(12), the systematic D_z is zero in most machines, which is the reason why we ignored this effect for the vertical motion. Due to the energy drop δE by the photon emission the particle has suddenly a different reference orbit as sketched in Fig. 6 for the synchronous particle. Before emission the synchronous particle oscillates around the nominal orbit ($x = 0$), after emission the oscillation is around the orbit $x_\epsilon(s) = D_x(s) \delta E$. The particle position and the trajectory angle are unaffected by the emission. Hence

$$\delta x = \delta x_\beta + \delta x_\epsilon = 0$$

and with

$$\delta x_\epsilon = D_x \frac{\delta E}{E_s}$$

we get

$$\delta x_\beta = -D_x \frac{\delta E}{E_s}$$

Neglecting again the angle between \vec{p} and $\delta\vec{p}$ leads to

$$\delta x' = \delta x'_\beta + \delta x'_\epsilon = 0$$

and with

$$\delta x'_\epsilon = D'_x \frac{\delta E}{E_s}$$

to

$$\delta x'_\beta = -D'_x \frac{\delta E}{E_s}$$

Since $\delta E < 0$ the invariant A will have increased according to

$$A \delta A = x_\beta \delta x_\beta + \beta^2 x'_\beta \delta x'_\beta$$

$$A \delta A = - (D_x x_\beta + D'_x \beta^2 x'_\beta) \frac{\delta E}{E_s} \quad (22)$$

after the emission giving rise to some antidamping.

Next we must relate δE to the power radiated per unit length. Consider a betatron oscillation of the synchronous particle in a combined-function sector magnet. Obviously

$$\delta E = P_y(B) \Delta \ell / c$$

Since $P_y \sim B^2(x)$

$$P_y(x_\beta) = P_y + 2 \frac{P_y}{B} \frac{dB}{dx} x_\beta$$

and since in a sector magnet

$$\Delta \ell \approx (1 + \frac{x}{\rho_s}) \Delta s$$

we get

$$\delta E = -(1 + \frac{2}{B} \frac{dB}{dx} x_\beta + \frac{x_\beta}{\rho_s}) \frac{P_y}{c} \Delta s \quad (23)$$

Introducing (23) into (22) and averaging over all phases of the betatron motion eliminates all terms except those with x_β^2 . We obtain

$$\langle \delta A \rangle = \frac{1}{2} \frac{D_x P_y}{c E_s} \left(\frac{2}{B} \frac{dB}{dx} + \frac{1}{\rho_s} \right) \Delta s$$

Summing over one turn yields for this antidamping due to non-zero D_x

$$\frac{\Delta A}{A} = \frac{1}{2} \oint D_x \frac{P_y}{c E} \left(\frac{2}{B} \frac{dB}{dx} + \frac{1}{\rho} \right) ds$$

Adding the damping by the acceleration in the RF cavity given by (20) finally results in

$$\alpha_x = \frac{U_0}{2 E_s T_0} (1 - \tilde{D}) \quad (24)$$

where \tilde{D} is given by (16). Comparison with (15) shows that the additional term, due to the orbit passing in combined-function fields or in sectors magnets, produces in the horizontal plane an effect opposite to the one it has for the energy oscillations.

One may wonder what happens if $D_x \neq 0$ at the RF cavity where the reference orbit is also changed due to the sudden increase in particle energy. Since this change is predictable, occurring always in the same place and not at random, only the closed orbit is affected but not the damping.

4. DAMPING PARTITION NUMBERS

From equations (15), (21) and (24) follows for the damping times

$$\tau_i = \frac{2 E_s T_0}{J_i U_0} \quad i = x, z, \epsilon \quad (25)$$

with the damping partition numbers

$$J_x = 1 - \tilde{D}, \quad J_z = 1, \quad J_\epsilon = 2 + \tilde{D} \quad (26)$$

Obviously

$$J_x + J_z + J_\epsilon = 4 \quad (27)$$

will hold for any \tilde{D} . Manipulating the magnetic guide field can change the damping repartition through a variation of \tilde{D} but the theorem (27) will always be valid. It has been shown that this theorem is very general. It is also valid for a general radiofrequency field which exerts longitudinal and radial forces on the particle depending on its trajectory in the cavity. The only requirement is that all the fields acting on the particle are not influenced by its motion but are determined a priori ^{1,2}).

5. ADJUSTMENT OF DAMPING RATES

It is sometimes useful or even necessary to influence all the damping rates or to change the repartition of damping. A few examples will illustrate this point. We group the examples according to the effect on the individual factors in (25).

5.1 Increase of U_0

Dipole wiggler magnets are proposed for LEP ¹⁰) for emittance control, for enhancement of the polarization rate and for reduction of the damping times at low energy. This reduction is beneficial for the beam accumulation at injection and for beam stability at low energy. A dipole wiggler consists of a string of dipole magnets with polarity - + - excited such that the total deflection is zero ¹³). Since the magnetic field in the main dipoles of LEP is very low at injection energy (about 22 mT at 20 GeV), the addition of a few dipoles with 1T substantially increases the integral in (1) and therefore U_0 . The planned wigglers double U_0 at 20 GeV and, consequently, all τ_i are halved according to (25). For example, τ_x will be reduced from 0.4 to 0.8 s at 20 GeV. Since LEP has a separated-function magnet lattice, the numerator in (16) is extremely small so that the change of the denominator is irrelevant. Thus, adding the dipole wigglers, does not change \tilde{D} and the partition numbers.

5.2 Change of \tilde{D}

Electron synchrotrons

Most of the earlier accelerators as the 8 GeV synchrotron (DESY) in Hamburg or the 28 GeV synchrotron (PS) at CERN have a combined-function lattice. In this case the first term in the numerator of (16) is very big producing a sizeable \tilde{D} . If the bending radius is the same in all magnets (isomagnetic machine) $\tilde{D} = 2$ within a few percent. Thus, the energy oscillations are very strongly damped ($J_\epsilon = 4$) at the expense of the damping of the horizontal oscillations which become antidamped ($J_x = -1$).

If the acceleration is very rapid, the beam has not enough time to horizontally grow to a level which would make ejection impossible. This is the case in DESY. Fig. 7 shows the evolution of the horizontal emittance versus energy ¹⁴). It can be seen that at first the emittance is reduced due to adiabatic damping which dominates at low energy. At higher energy, the radiation antidamping becomes progressively stronger leading together

with the quantum excitation to a blow-up of the emittance. But since the emittance is still acceptable at 7 GeV, no harm is done.

The situation is different in the PS which has to accelerate electrons and positrons from 0.6 to 3.5 GeV as part of the LEP injector chain 15). The rise time of the horizontal amplitude growth would be 76 ms at 3.5 GeV ($J_x = -1$), which is much shorter than the cycling time of the synchrotron being about 1.2 s. This would lead to an unacceptably wide beam impossible to hold without losses or to eject.

Various means to change the damping repartition have been proposed¹⁶⁾. The simplest one is to decrease the average radius R of the orbit, which reduces the first term in the numerator of (16). This is done by increasing the radio-frequency by Δf according to

$$\Delta f / f = - \Delta R / R$$

Obviously, the energy and the revolution period are also affected by ΔR , which in turn influence τ_i by virtue of (25), but the change in τ_i is dominated by the variation of J_i (\tilde{D}). Fig. 8 shows J_x and J_e versus the ΔR . It can be seen that, in order to make $J_x = 1$, a fairly large ΔR would be required with a concurrent loss in useful aperture. It is preferable to use a dipole-quadrupole wiggler, also called Robinson wiggler 1,16). It consists of a string of four about identical magnet blocks having a dipole and quadrupole field. The variation of field polarity is such that the total deflection vanishes as shown in Fig. 9. Given the polarity of the field gradient indicated in Fig. 9 the wiggler decreases the numerator of (16). Since the wiggler increases the denominator in (16) very little, the overall effect is a strong decrease of \tilde{D} . For illustration, the new line J_x versus ΔR for a wiggler producing $\Delta J_x = +2$ is shown in Fig. 8. Obviously, the main effect of the wiggler is a shift of the whole pattern to the right.

Fig. 10 shows the cross-section of the prototype wiggler magnet which has been successfully tested 17). It is planned to install 3 wiggler magnets so that even $J_x \approx 3$ ($\tilde{D} = -4$) becomes possible. According to (26) this not only establishes horizontal damping, proving then a small transverse emittance, but also reduces the damping of energy oscillations. Hence, quantum excitation can lead to larger energy oscillation amplitudes producing in turn longer bunches. The concomitant reduction in line density is necessary for beam stability 15,18).

EPA

Another example for the manipulation of the repartition functions is the Electron or Positron Accumulation ring (EPA) in the LEP injector chain. This ring serves as a buffer between the linac providing 100 pulses per second and the slow-cycling PS. The pulses are accumulated in horizontal betatron phase space. Efficient injection requires that the large horizontal betatron amplitudes inherent in the injection process are quickly damped before the next pulse arrives 15).

Since the energy was fixed and since the magnetic field determining ρ in the rather short bending magnets was already as high as possible, U_0 could not be increased further. Hence, the repartition was changed in order to enhance the horizontal damping at

the expense of the longitudinal damping 19). This was done by introducing a gradient in the bending magnets of the originally separated-function guide field. A gradient

$$\frac{dB}{dx} = -1 \text{ T/m}$$

was added to the dipole field of $B = 1.4\text{T}$ resulting in $\Delta\tilde{D} = -1$ and $J_x = 2$. The concurrent reduction of J_z also has the advantage of reducing the line density in the bunch.

PETRA, LEP

While the change in J_z per 1 mm increase in average radius is very weak ($dJ_z/dR < 0.1 \text{ mm}^{-1}$) in machines like EPA or the PS, it is quite substantial

$$dJ_z/dR = 0.5 \text{ mm}^{-1}$$

in PETRA and LEP. Hence, the damping repartition can be conveniently changed by acceptably small deviations from the nominal radial beam position. The strong variation of \tilde{D} is caused by the first term in (16) which can become very significant when the beam circulates off-centre in the quadrupoles. This feature is used operationally in PETRA 20) and will be exploited in LEP 10) in order to influence the longitudinal line density in the bunch and to control the transverse emittance.

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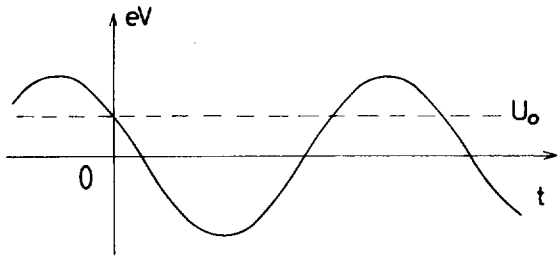


Fig. 1 Energy gain at the RF cavity versus time

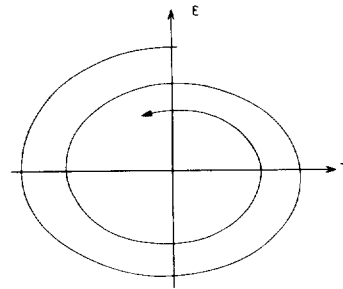


Fig. 2 Position of particle relative to synchronous particle

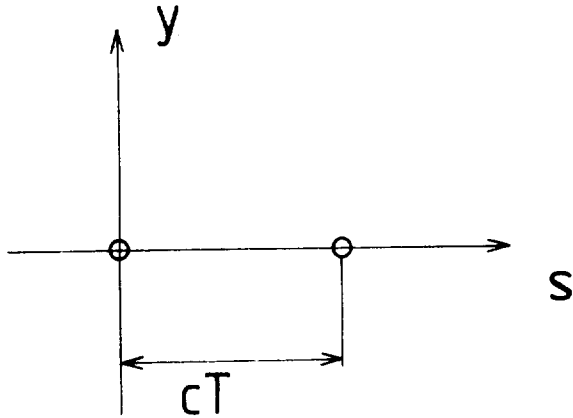


Fig. 3 Damped energy oscillation in longitudinal phase space. The time t is parameter on the curve.

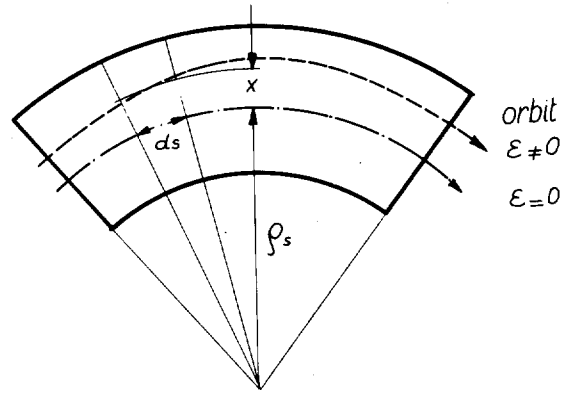


Fig. 4 Particle trajectories in a sector magnet

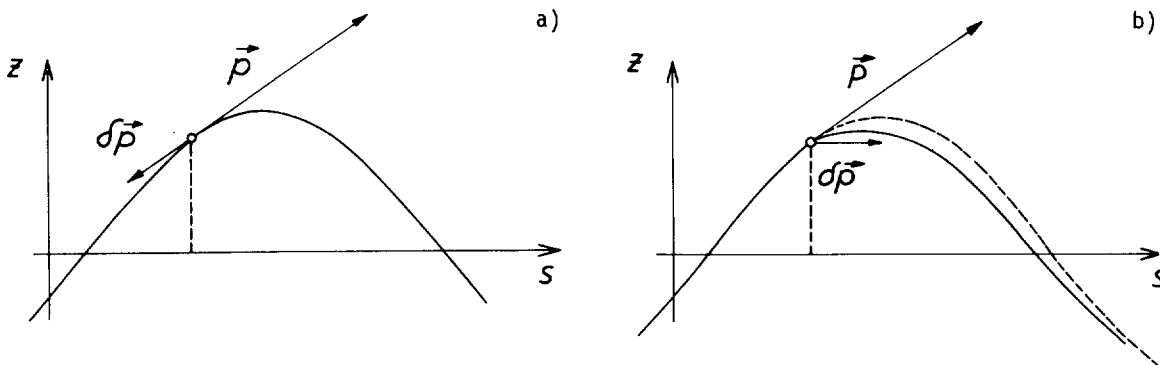


Fig. 5 Vertical motion
 a) quantum emission
 b) compensation of energy loss by the RF cavity

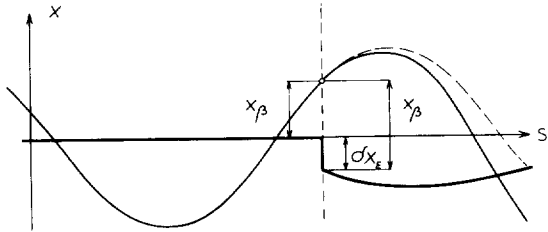


Fig. 6 Excitation of horizontal betatron oscillation at $D_x \neq 0$ by quantum emission

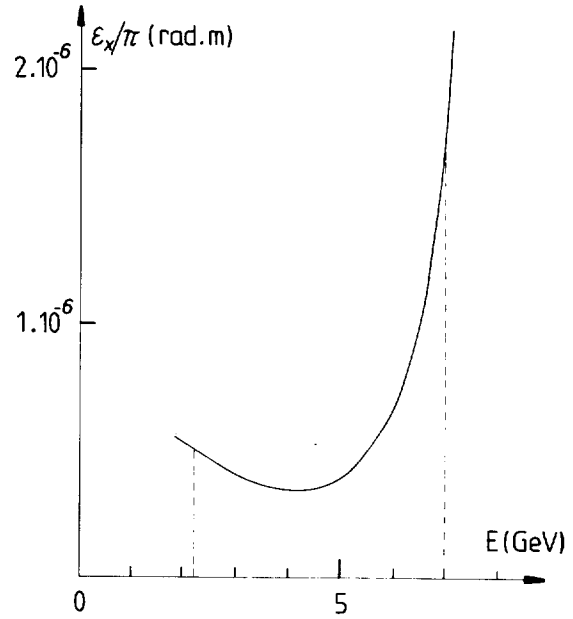


Fig. 7 Evolution of horizontal emittance during acceleration in the electron synchrotron DESY

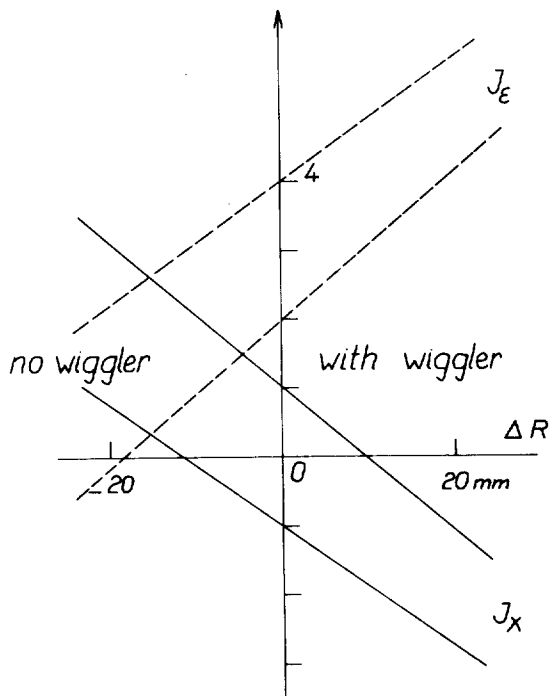


Fig. 8 Damping partition numbers in the PS versus change in average radius without and with wiggler producing $\Delta J_x = 2$. Central orbit $\Delta R = 0$.

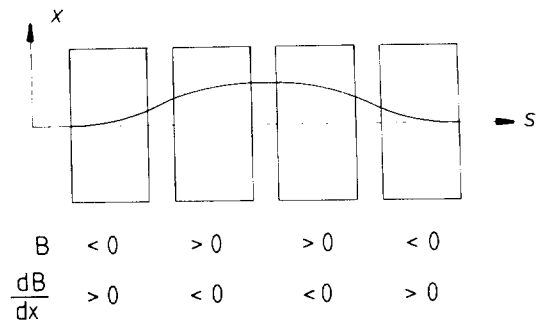


Fig. 9 Orbit in a Robinson wiggler consisting of four magnet blocks

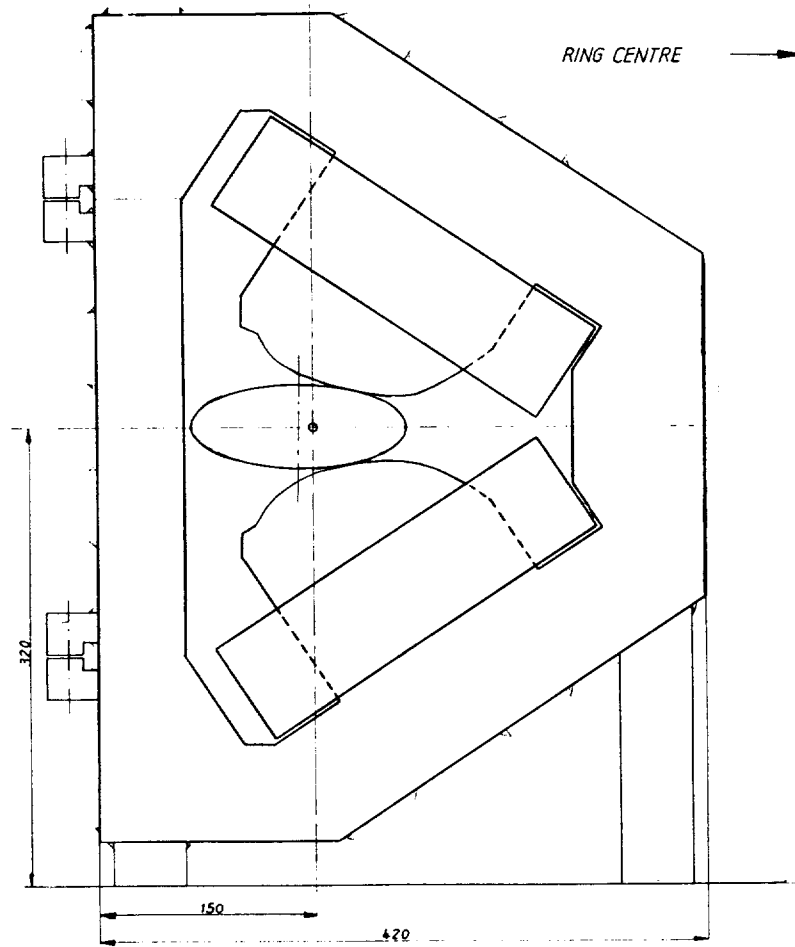


Fig. 10 Cross-section of Robinson wiggler for the PS