Meson-Baryon Scattering in QCD$_2$ for any Coupling

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Abstract

Extending earlier work on strong-coupling meson-baryon scattering in QCD$_2$, we derive the effective meson-baryon action for any value of the coupling constant, in the large-N_c limit. Colour degrees of freedom play an important role, and we show that meson-baryon scattering can be formulated as a relativistic potential problem. We distinguish two cases that are non-trivial for unequal quark masses, and present the resulting equations for meson-baryon scattering amplitudes.

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1   Introduction

The problem of deriving resonances from QCD is clearly of importance, both as a fundamental test of QCD and of specific non-perturbative methods. Experimentally, this field continues to bring surprises, including enhancements near $\bar{p}p$ thresholds [1, 2, 3], an exotic K^+N resonance [4, 5, 6], and two new mesons containing c and $\bar{s}$ quarks [7, 8]. These discoveries demonstrate that non-perturbative QCD is not yet fully 'solved' [9], and underline the interest in developing new non-perturbative methods. The dynamics governing the formation of resonances in QCD is still imperfectly understood, especially the relation between the spectrum and chiral symmetry breaking (χSB), possible 'exotic' non-$\bar{q}q$ mesons and non-qqq baryons, and combinations of light and heavy quarks.

QCD in one space and one time dimension, QCD$_2$, is an interesting laboratory for studying many of these issues. There is no spontaneous χSB in two dimensions, but a trace of the phenomenon does exist, in the sense that $m_{PS} \propto \sqrt{e_c\, m_q}$, where m_{PS} is the mass of the lightest pseudoscalar meson, e_c is the gauge coupling and m_q is the quark mass. This formula for m_{PS} is analogous to the case in four dimensions, with e_c replacing Λ_{QCD}.

Motivated by this similarity and the tractability of QCD$_2$, we investigated in [10] the problem of resonances in the meson-baryon channel in this model. This question is still relevant both as a crude model for the real world in four dimensions and also as a testing ground for non-perturbative methods that might have relevance there and elsewhere.

In [10] we worked in the strong-coupling limit, $e_c/m_q \gg 1$, namely in the limit where the QCD$_2$ coupling is much larger than the quark mass. This would correspond in QCD$_4$ to quark masses being small compared to the QCD scale, $m_q/\Lambda_{QCD} \ll 1$, the limit where explicit χSB is small. Moreover, we worked in the approximation that the baryons are heavy compared to the mesons, which is justified by the large-N_c limit in QCD. In this double limit, we found no resonances in meson-baryon scattering, only elastic scattering. This result suggest that quark masses are essential complications in QCD, at least in two dimensions.

In the present work we relax one of our two previous assumptions, that of strong coupling, while retaining the assumption that baryons are heavy. This discussion of general coupling may equivalently be regarded as introduc-

ing finite quark masses. We show that this generalization necessitates the re-introduction of colour degrees of freedom, which were decoupled in the previous strong-coupling limit. The scattering problem may be formulated as a relativistic potential problem that differs essentially from the previous strong-coupling, low-mass case. We derive the corresponding new equations for meson-baryon scattering amplitudes, valid for any value of the QCD$_2$ coupling.

In Section 2 we discuss the general formulation for QCD$_2$, in bosonized form, which is necessary for obtaining the meson-baryon scattering amplitude for an arbitrary value of the coupling. We use this formulation in Section 3 to derive the effective meson-baryon action, for arbitrary coupling in the heavy-baryon limit. We show that in Section 4 that, for light mesons and heavy baryons, the scattering problem can be transformed to problem with relativistic potentials, and we evaluate that potential in two cases that are non-trivial for unequal quark masses. Section 5 contains our derivation of the equations for scattering amplitudes, and Section 6 discusses the prospects for future work.

2 General Formulation

We start from the discussion of [11], which presented an effective action for bosonized QCD$_2$ with N_f flavours and N_c colours:

$$S_{eff}[u] = S_0[u] + \frac{e_c^2 N_f}{8\pi^2} \int d^2x \, \text{Tr} \left[\partial_-^{-1} \left(u \, \partial_- u^\dagger \right)_c \right]^2 + m'^2 N_{\tilde{m}} \int d^2x \, \text{Tr} \left(u + u^\dagger \right),$$

$$(1)$$

where the traces are over both colour and flavour. The second term is the result of integrating out the gauge fields, which was made possible by the quadratic dependence on the gauge fields in two dimensions in the light-cone gauge, which we chose. Finally, $S_0[u]$ is the Wess-Zumino-Witten (WZW) action, representing free fermions.

The above action is exact for any value of the gauge coupling e_c. The matrices u represent the bosonic version of the quark bilinears, symbolically $\Psi \otimes \bar{\Psi}$, with both colour and flavour indices. The symmetry groups are $SU(N_c)$ and $U(N_f)$, for colour and flavour, respectively, including the $U(1)$

of baryon number, and

$$m'^2 = m_q C \tilde{m}, \qquad (2)$$

where m_q is the quark mass, $\tilde{m}$ the normal-ordering scale, and $C \equiv \frac{1}{2} e^\gamma \approx 0.891$. For a detailed discussion, see [12]. For the time being, we take all quarks to have the same mass.

When using the product scheme for bosonization

$$u = h \times g \equiv [\; \underbrace{SU(N_c)}_{h} \;]_{N_F} \times [\; \underbrace{U(N_F)}_{g} \;]_{N_c} \qquad (3)$$

and taking the strong-coupling limit $e_c/m_q \rightarrow \infty$, one can eliminate the second term in (1), after first choosing an appropriate normal-ordering scale $\tilde{m}$, as described in [12]. The colour degrees of freedom h are completely eliminated in this limit, and one gets an effective action in terms of flavour degrees of freedom only:

$$\tilde{S}_{eff}[g] \;=\; N_c S_0[g] + m^2 N_m \int d^2 x \left(\mathrm{Tr}_f\, g + \mathrm{Tr}_f\, g^\dagger \right), \qquad (4)$$

$$\text{where} \qquad m \;=\; \left[N_c C m_q \left(\frac{e_c \sqrt{N_f}}{\sqrt{2\pi}} \right)^{\Delta_c} \right]^{\frac{1}{1+\Delta_c}}, \qquad (5)$$

$$\Delta_c \;=\; \frac{N_c^2 - 1}{N_c(N_c + N_F)} \; . \qquad (6)$$

Note that here the traces are over flavour only.

As shown in [10], this action results in elastic scattering only, and is therefore unsuitable for modelling QCD$_4$ realistically. Scattering is a soluble problem in this approximation, with the transmission and reflection coefficients T and R given in [10] for all meson energies.

When dealing with finite coupling, we should not use the product scheme, but rather the 'u-scheme', where the original matrices $u \in U(N_F \times N_c)$. The two schemes are equivalent only when the quark masses are zero, or equivalently in the strong-coupling limit. One important aspect of the difference, at finite mass, is that the 'u-scheme' admits multi-soliton solutions [11] that do not exist in the product scheme. Each of the solitons in these solutions carries both colour and flavour, yet the total multi-soliton solutions are

colour-neutral. These individual coloured solitons were interpreted in [11] as constituent quarks. When the quark masses are equal, the solutions can be obtained analytically, and are given by the sine-Gordon profile. For unequal quark masses, the solutions were obtained numerically.

3 Extension to Arbitrary Coupling

We first consider the second term of (1), for arbitrary coupling:

$$\frac{e_c^2 N_f}{8\pi^2} \int d^2x \, \mathrm{Tr} \, \left[\partial_-^{-1} \left(u \, \partial_- u^\dagger\right)_c\right]^2 \tag{7}$$

where $\left(u \, \partial_- u^\dagger\right)_c$ is the colour part of $M \equiv u\partial_- u^\dagger$, to be computed as

$$M_c = \mathrm{Tr}_f M - 1/N_c \mathrm{Tr}_{f\&c} M, \tag{8}$$

with details to be found in [12]. As already mentioned, this term represents the interactions, as it arises from integrating out the gauge potentials. However, we will see that, for the physical situation we discuss, this term does not contribute to meson-baryon scattering for any coupling. As a result, the latter is described by the effective action $\tilde{S}_{eff}[u]$, whereas in the strong coupling limit it is described by $\tilde{S}_{eff}[g]$.

In order to describe meson-baryon scattering, we follow the four-dimensional example [13], and take u to be of the form

$$u = \exp(-i\Phi_c) \exp(-i\delta\Phi), \tag{9}$$

corresponding to a classical soliton Φ_c representing one of the baryons described in [11], and a small fluctuation $\delta\Phi$ around it, representing the meson. The resulting action is then expanded to second order in $\delta\Phi$, yielding a linear equation of motion for $\delta\Phi$ in the soliton background. The latter serves as an external potential in which the meson is propagating.

We start by evaluating

$$
\begin{aligned}
M &\equiv u\partial_- u^\dagger = \\
&= \exp(-i\Phi_c) \, \partial_- (\exp i\Phi_c) + \exp(-i\Phi_c) \exp(-i\delta\Phi) \left[\partial_- \exp(i\delta\Phi)\right] \exp(i\Phi_c),
\end{aligned}
\tag{10}
$$

and obtain the equations of motion for the meson field by varying with respect to $\delta\Phi$. The variation of (7) with respect to $\delta\Phi$ is proportional to

$$\frac{\delta M_c}{\delta(\delta\Phi)}\,\partial^{-2}M_c. \tag{11}$$

To compute its variation with respect to $\delta\Phi$, we need only the second term M_2 of M, as the first term M_1 is independent of $\delta\Phi$.

We take for the soliton a diagonal ansatz, following [11]:

$$
\begin{aligned}
{[\exp(-i\Phi_c)]}_{\alpha\alpha'jj'} &= \delta_{\alpha\alpha'}\,\delta_{jj'}\exp\left(-i\sqrt{4\pi}\chi_{\alpha j}\right):\\
\alpha &= 1,\ldots,N_c,\\
j &= 1,\ldots,N_f,
\end{aligned}
\tag{12}
$$

so that

$$\{\exp(-i\Phi_c)[\exp(-i\delta\Phi)\,\partial_-\exp(i\delta\Phi)]\exp(i\Phi_c)\}_{\alpha j,\alpha'j'} =$$
$$\exp(-i\sqrt{4\pi}\chi_{\alpha j})\,[\exp(-i\delta\Phi)\,\partial_-\exp(i\delta\Phi)]_{\alpha j,\alpha'j'}\,\exp(i\sqrt{4\pi}\chi_{\alpha'j'}). \tag{13}$$

The part of M that contributes to the effective action is its colour projection (8). We note that $\mathrm{Tr}_{f\&c}M_2 = 0$, and thus

$$[(M_2)_c]_{\alpha,\alpha'} = \sum_j \exp(-i\sqrt{4\pi}\chi_{\alpha j})[\exp(-i\delta\Phi)\,\partial_-\exp(i\delta\Phi)]_{\alpha j,\alpha'j}\exp(i\sqrt{4\pi}\chi_{\alpha'j}).$$
$$\tag{14}$$

The mesons $\delta\Phi$ have to be diagonal in colour, so

$$[(M_2)_c]_{\alpha,\alpha'} = \sum_j [\,\exp(-i\delta\Phi)\,\partial_-\exp(i\delta\Phi)\,]_{\alpha j,\alpha j}\delta_{\alpha,\alpha'}. \tag{15}$$

We recall that the flavour structure of the mesons is independent of their colour indices, and restrict our attention to mesons that have no $U(1)$ flavour part. In this way, we may be sure that classical solutions lead to stable particles, since their non-vanishing flavour quantum numbers put them in a different sector from the vacuum. We then have

$$\sum_j [\,\exp(-i\delta\Phi)\,\partial_-\exp(i\delta\Phi)\,]_{\alpha j,\alpha j} = 0, \tag{16}$$

as advertized earlier, and the effective meson-baryon action is

$$\tilde{S}_{m\text{-}b}[\delta\Phi] = S_0[u] + m^2 N_m \int d^2x\left(\mathrm{Tr}\,u + \mathrm{Tr}\,u^\dagger\right), \tag{17}$$

with u depending on $\delta\Phi$ for fixed Φ_c as in (9).

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4 Evaluation of the Potential

The equation of motion for $\delta\Phi$ is obtained from (17), by first varying with respect to u and then varying u with respect to $\delta\Phi$. To first order in $\delta\Phi$, we find

$$\delta u = -i[\exp(-i\Phi_c)]\delta\Phi. \tag{18}$$

The resulting equation of motion is then

$$\frac{1}{4\pi}\partial_+\left[(\partial_- u)\,u^\dagger\right] + \left(um^2 - m^2 u^\dagger\right) = 0, \tag{19}$$

where m is the diagonal mass matrix: $m = \delta_{ij}m_j$ with (possibly different) entries m_j corresponding to flavours j. We note that there is the possibility of an overall scale ambiguity in m, since, when the masses are different, there is a question which normal-ordering scale to use, as discussed in the Appendix of [11]. The resulting equation of motion for $\delta\Phi$ is

$$\delta\Phi - i\left(\partial_+\Phi_c\right)\left(\partial_-\delta\Phi\right) + i\left(\partial_-\delta\Phi\right)\left(\partial_+\Phi_c\right) +$$
$$\frac{1}{2}\left[\delta\Phi\mu^2\exp(-i\Phi_c) + \exp(i\Phi_c)\mu^2\delta\Phi\right] = 0, \tag{20}$$

where $\mu \equiv m\sqrt{8\pi}$.

As discussed before, both Φ_c and $\delta\Phi$ are diagonal in colour. Moreover, Φ_c is diagonal in flavour too. So, taking the $\alpha\alpha jj'$ matrix element of the equation of motion (20), we find

$$\delta\Phi_{\alpha jj'} - i(\partial_+\Phi_c)_{\alpha j}(\partial_-\delta\Phi)_{\alpha jj'} + i\left(\partial_-\delta\Phi\right)_{\alpha jj'}(\partial_+\Phi_c)_{\alpha j'} +$$
$$\frac{1}{2}\{\delta\Phi_{\alpha jj'}\mu^2{}_{j'}[\exp(-i\Phi_c)]_{\alpha j'} + [\exp(i\Phi_c)]_{\alpha j}\mu^2{}_j\delta\Phi_{\alpha jj'}\} = 0. \tag{21}$$

Examining the classical solutions for the quark solitons inside the baryons as in [11], we see that, for a given colour index α, there is only one flavour for which Φ_c is non-zero. We can now distinguish three cases.

• The first is when an index α and indices j and j' are chosen in such a way that both $(\Phi_c)_{\alpha j}$ and $(\Phi_c)_{\alpha j'}$ are zero. In such a case,

$$\delta\Phi_{\alpha jj'} + \frac{1}{2}[\mu^2{}_j + \mu^2{}_{j'}]\delta\Phi_{\alpha jj'} = 0, \tag{22}$$
$$\text{where}(\Phi_c)_{\alpha j} = 0 \text{ and } (\Phi_c)_{\alpha j'} = 0.$$

Thus $\delta\Phi_{\alpha jj'}$ is a free field with squared mass given by the average of m_j^2 and $m_{j'}^2$ in this case, which we do not discuss further.

• The second case is that of $j = j'$, with α such that $(\Phi_c)_{\alpha j}$ is a quark soliton inside the baryon. In this case,

$$\delta\Phi_{\alpha jj} + \mu^2{}_j \cos[(\Phi_c)_{\alpha j}]\delta\Phi_{\alpha jj} = 0. \tag{23}$$

This case is analogous to the case in Section 2 of [10], with the difference that here the potential is determined by Φ_c which, for the case of unequal masses, is not necessarily of the sine-Gordon type.

• The third case is when j is different from j', now with one of the Φ_c being a soliton and the other vanishing. Taking $(\Phi_c)_{\alpha j}$ to be the soliton, we obtain

$$\delta\Phi_{\alpha jj'} - i(\partial_+\Phi_c)_{\alpha j}(\partial_-\delta\Phi)_{\alpha jj'} + \frac{1}{2}\{\mu^2{}_{j'} + \mu^2{}_j[\exp(i\Phi_c)]_{\alpha j}\}\delta\Phi_{\alpha jj'} = 0, \tag{24}$$

where $\qquad\qquad\qquad\qquad j' \neq j$ and $(\Phi_c)_{\alpha j'} = 0$ $\qquad\qquad\qquad\qquad$.
This case is analogous to Section 4 of [10], but with the same difference as in (23), i.e., that for unequal masses the soliton is not of the sine-Gordon type.

5 Evaluation of Meson-Baryon Scattering

The equations which determine the static solution $(\Phi_c)_{\alpha j}$ were derived in [11]. For completeness, we include them here too. First one defines

$$(\Phi_c)_{\alpha j} = \sqrt{4\pi}(\chi_c)_{\alpha j}, \tag{25}$$

where the $(\chi_c)_{\alpha j}$ are canonical fields, whose equations of motion are

$$\chi''_{\alpha j} - 4\alpha_c\left(\sum_l \chi_{\alpha l} - \frac{1}{N_c}\sum_{\beta l}\chi_{\beta l}\right) - 2\sqrt{4\pi}m_j^2 \sin\sqrt{4\pi}\chi_{\alpha j} = 0.$$

Note the extra factor 2 in front of the mass term, as compared with Eq. (22) of [11], due to an error in this reference.

Choosing the boundary conditions $\chi_{\alpha j}(-\infty) = 0$, we get as constraints for $\chi_{\alpha j}(+\infty)$, denoted hereafter simply by $\chi_{\alpha j}$,

$$\frac{1}{\sqrt{\pi}}\chi_{\alpha j} = n_{\alpha j} \qquad \text{integers,} \tag{26}$$

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and
$$\sum_l n_{\alpha l} = n \qquad \text{independent of } \alpha. \tag{27}$$

The baryon number [§] associated with any given flavour l is given by
$$B_l = \sum_\alpha n_{\alpha l}.$$

Combining the last two equations, we find
$$B = \sum_l B_l = nN_c$$

for the total baryon number.

We now continue in a similar manner to [10], starting with the first non-trivial case (23) identified above. As the soliton solutions are such that there is a unique correspondence between the colour index α and the flavour index j, we suppress α in what follows. Putting
$$\delta\Phi_{jj} = e^{-i\omega_j t} u_j(x) \tag{28}$$

with
$$u_j(x) \xrightarrow[x\to\infty]{} e^{ikx}, \tag{29}$$

we find
$$\omega_j^2 = k^2 + \mu_j^2, \tag{30}$$

and the equation for $u_j(x)$ is
$$u_j''(x) + \omega_j{}^2 u_j - \mu^2{}_j[\cos{(\Phi_c)}_j]u_j = 0. \tag{31}$$

We define the potential V_j for this scattering process via
$$u_j''(x) + \omega_j{}^2 u_j - V_j u_j = 0, \tag{32}$$

and find
$$V_j = \mu^2{}_j[\cos{(\Phi_c)}_j]. \tag{33}$$

[§]In our normalization, a single quark carries one unit of baryon number.

In our normalization the outgoing wave has coefficient 1, which is more convenient for numerical calculations, and the wave for $x \to -\infty$ is now

$$u_j(x) = \frac{1}{T_j} e^{ikx} + \frac{R_j}{T_j} e^{-ikx}, \qquad x \to -\infty \qquad (34)$$

in this case.

In the second non-trivial case (24), we put

$$\delta\Phi_{jj'} = e^{-i\omega_{jj'}t} u_{jj'}(x), \qquad (35)$$

so that

$$u''_{jj'}(x) - i(\Phi_c)'_j(x) u'_{jj'}(x)+$$

$$+\{\omega^2_{jj'} + \omega_{jj'}(\Phi_c)'_j(x) - \frac{1}{2}\{\mu^2{}_{j'} + \mu^2{}_j[\exp(i\Phi_c)]_j\}\} u_{jj'} = 0. \qquad (36)$$

To eliminate the first derivative term in u, we substitute

$$u_{jj'} = [\exp(\frac{i}{2}\Phi_c)]_j \, v_{jj'}. \qquad (37)$$

This results in

$$v''_{jj'}(x) + \{\omega^2_{jj'} + \omega_{jj'}(\Phi_c)'_j(x) - \mu^2{}_j[\cos(\Phi_c)]_j\} v_{jj'}+$$

$$+\frac{1}{2}(\mu^2{}_j - \mu^2{}_{j'}) v_{jj'}+$$

$$+\{\frac{1}{4}[(\Phi_c)'_j(x)]^2 - \frac{1}{2}\mu^2{}_j(1 - [\cos(\Phi_c)]_j)\} v_{jj'}+$$

$$+\frac{i}{2}\{(\Phi_c)_j{}''(x) - \mu^2{}_j[\sin(\Phi_c)_j]\} v_{jj'} = 0. \qquad (38)$$

We note that the last three lines vanish when all the quark masses are equal, as then the soliton is a sine-Gordon one. Thus, the scattering would then be only elastic, as found in [10].

The potential of the scattering is defined here via

$$v''_{jj'}(x) + \omega^2_{jj'} v_{jj'} - V_{jj'} v_{jj'} = 0, \qquad (39)$$

so that

$$V_{jj'} = -\omega_{jj'}(\Phi_c)'_j(x) + \mu^2{}_j[\cos(\Phi_c)]_j$$

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$$-\frac{1}{2}(\mu^2{}_j - \mu^2{}_{j'})$$

$$-\{\frac{1}{4}[(\Phi_c)'_j(x)]^2 - \frac{1}{2}\mu^2{}_j(1 - [\cos(\Phi_c)]_j)\}$$

$$-\frac{i}{2}\{(\Phi_c)_j{}''(x) - \mu^2{}_j[\sin(\Phi_c)_j]\} \tag{40}$$

Taking again

$$v_{jj'}(x) \xrightarrow[x\to\infty]{} e^{ikx}, \tag{41}$$

we get

$$\omega_{jj'} = \frac{1}{2}(\mu^2{}_j + \mu^2{}_{j'}), \tag{42}$$

and the wave for $x \to -\infty$ is

$$v_{jj'}(x) = \frac{1}{T_{jj'}}e^{ikx} + \frac{R_{jj'}}{T_{jj'}}e^{-ikx}, \qquad x \to -\infty \tag{43}$$

in this case.

6 Discussion

We have shown that meson-baryon scattering in QCD_2 in the large-N_c limit is non-trivial for non-zero quark masses, and is described by two distinct effective potentials when the quark masses are unequal. These effective potentials are not of the sine-Gordon type found in previous cases, and we expect the scattering amplitudes also to be non-trivial. Their calculation will require numerical analysis, that we postpone for a future occasion.

Clearly QCD_2 is not a complete laboratory for studying non-perturbative physics in QCD_4. However, it is already quite a rich system, and full QCD can only be richer still. We have already pointed out the existence of constituent quark solitons in QCD_2 [11], and this numerical analysis may cast light on their importance in scattering, where the additive quark model has long been an intriguing approximation.

It is intriguing that the introduction of unequal quark masses is an essential complication. We recall that the light-cone wave functions for mesons containing unequal quark masses are expected not to be symmetric, resulting in a net colour field that underlies this effect. One task of numerical

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analysis will be to see what new physics this produces in meson-baryon scattering and possibly in resonant states. We note that two of the recent new puzzles in non-perturbative QCD concern systems with unequal quark masses [4, 5, 7, 8]. It would be hubristic to suggest that the continuation of our QCD_2 studies will cast light on these puzzles, but they will provide some extra motivation for our future work in this direction.

Acknowledgments

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