CERN-TH/2002-331
hep-ph/0211282
November 2002

IMPROVING BOUNDS ON γ IN $B^{\pm} \to DK^{\pm}$ AND $B^{\pm,0} \to DX_s^{\pm,0}$

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ABSTRACT

In view of recent experimental progress in rate and CP asymmetry measurements in $B^{\pm} \to DK^{\pm}$, we reconsider information on the weak phase γ which can be obtained from these processes. Model-independent inequalities are proven for $\sin^2\gamma$ in terms of two ratios of partial rates for $B^{\pm,0} \to DX_s^{\pm,0}$, where X_s is any multiparticle charmless state carrying strangeness ± 1. Good prospects are shown to exist for using these inequalities and CP asymmetry measurements in two body and multibody decays in order to improve present bounds on γ.

PACS codes: 12.15.Hh, 12.15.Ji, 13.25.Hw, 14.40.Nd

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The observation of CP violation in decays of B mesons to J/ψ and neutral kaons [1] is in good agreement with the prediction of the the Standard Model, in which CP violation originates in a single phase $\gamma \equiv \mathrm{Arg}V_{ub}^*$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Further measurements of CP asymmetries in other B decay processes are needed in order to establish the CKM hypothesis for CP violation on a firm ground, or to observe deviations from this simple picture. So far CP violation in B decays was observed only in processes involving $B^0 - \bar{B}^0$ mixing, whereas the phase γ has not yet been put to a direct test. It is therefore of great importance to search for *direct* CP violation in processes unaffected by uncertainties due to penguin amplitudes [2], where CP asymmetries have clean theoretical interpretations in terms of the weak phase γ.

One of the very early proposals for a clean measurement of γ is based on decays of the type $B^\pm \to DX_s^\pm$ [3], where $X_s^\pm$ stands for a charged kaon or *any* few particle state with the same flavor quantum numbers as a charged kaon, e.g. $X_s = K, K^*, K\pi, K^*\pi$. The weak phase γ occurs as the relative phase between two B^- decay amplitudes into D^0 and $\bar{D}^0$ flavor states, from $b \to c\bar{u}s$ and $b \to u\bar{c}s$, both contributing in decays to CP eigenstates, $D^0_{\mathrm{CP}\pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$. In the original proposal all three B^- decay amplitudes and a corresponding B^+ decay amplitude for a CP-eigenstate had to be measured in order to determine γ. In the simplest case of two body decays, $X_s = K$, one of the four amplitudes, $A(B^- \to \bar{D}^0 K^-)$, is color-suppressed. Its measurement using hadronic $\bar{D}^0$ decays is prohibited [4] due to interference with a comparable contribution from $B^- \to D^0 K^-$ followed by doubly-Cabibbo-suppressed (DCS) D^0 decays. Nevertheless, it was noted in Ref. [5] that useful constraints on γ can also be obtained without measuring this difficult mode. Several variants of this basic scheme were suggested, some of which rely on hitherto unmeasured and more difficult B and D decay modes [6], and others which require extra assumptions about negligible rescattering effects [7].

The magnitudes of all five amplitudes required for an implementation of this proposal, $A(B^- \to D^0 K^-)$ and the four amplitudes $A(B^\pm \to D^0_{\mathrm{CP}\pm} K^\pm)$, have already been measured. The decay $B^- \to D^0 K^-$ and its charge-conjugate were observed several years ago [8]. Recently branching ratios for the processes $D^0_{\mathrm{CP}} K^\pm$ were measured by the Belle collaboration [9] both for CP-even and odd states, and by the BABAR collaboration [10] for CP-even states. CP asymmetry measurements in decays involving D^0 CP-eigenstates [9, 10] are approaching a level for setting interesting bounds on the asymmetries. In addition, there exists new experimental information [11] indicating that color-suppression of the ratio

$$r \equiv |A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-)| \tag{1}$$

is less effective than anticipated. This improves the feasibility of this method.

In view of these important developments, we wish to reconsider in this Letter the implications which further improvements in these measurements will have on constraining γ. In particular, we make use of two inequalities [5]

$$\sin^2\gamma \leq R_{\mathrm{CP}\pm} \quad , \tag{2}$$

where we define for each of the two CP-eigenstates a ratio of charge-averaged rates

$$R_{\mathrm{CP}\pm} \equiv \frac{2[\Gamma(B^- \to D_{\mathrm{CP}\pm} K^-) + \Gamma(B^+ \to D_{\mathrm{CP}\pm} K^+)]}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \bar{D}^0 K^+)} \quad . \tag{3}$$

We will find that, although these two constraints do not depend explicitly on r, and do not require a knowledge of r, in general they become stronger with increasing values of this parameter. For a reasonable estimate, $r \sim 0.2$, assuming that the relevant final state interaction phase is not very large ($\delta \leq 30^\circ$), which can be verified by CP asymmetry measurements, these constraints improve present bounds on γ.

In the second part of the Letter we proceed to a general discussion of decays of the form $B^\pm \to DX_s^\pm$ and $B^0\,(\bar{B}^0) \to DX_s^0\,(\bar{X}_s^0)$, where $X_s^\pm$ and $X_s^0\,(\bar{X}_s^0)$ are arbitrary charmless mutiparticle states with strangeness ± 1. We will prove a generalization of Eq. (2) to multibody decays of this type. In the absence of color-suppression in most multibody decays, which implies larger values of corresponding r parameters in these processes, these bounds are likely to provide stronger constraints on γ than in the case of two body decays. Our considerations apply to any multibody decay of this kind, for instance $B^\pm \to DK^\pm\pi^0$ and the self-tagged $B^0\,(\bar{B}^0) \to DK^\pm\pi^\mp$, and are model-independent [12].

Using notations for amplitudes as in [3, 5] and disregarding a common strong phase,

$$A(B^- \to D^0 K^-) = |A| \quad , \qquad A(B^- \to \bar{D}^0 K^-) = |\bar{A}| e^{i\delta} e^{-i\gamma} \quad , \tag{4}$$

we define in addition to the two ratios of charge-averaged rates (3) two pseudo asymmetries

$$\mathcal{A}_{\rm CP\pm} \equiv \frac{\Gamma(B^- \to D_{\rm CP\pm} K^-) - \Gamma(B^+ \to D_{\rm CP\pm} K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \bar{D}^0 K^+)} \quad . \tag{5}$$

Expressions of these measurables in terms of $r = |\bar{A}/A|$, δ and γ are readily obtained, neglecting tiny $D^0 - \bar{D}^0$ mixing [13] and using $D^0_{\rm CP\pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$,

$$R_{\rm CP\pm} = 1 + r^2 \pm 2r \cos\delta \cos\gamma \quad , \tag{6}$$

$$\mathcal{A}_{\rm CP\pm} = \pm r \sin\delta \sin\gamma \quad , \tag{7}$$

where (6) implies

$$\frac{1}{2}(R_{\rm CP+} + R_{\rm CP-}) = 1 + r^2 \quad . \tag{8}$$

In principle, the quantities $R_{\rm CP\pm}$ and $\mathcal{A}_{\rm CP\pm}$ hold information from which r, δ and γ can be determined. The parameter r is given by (8), and γ is obtained up to a discrete ambiguity from $R_{\rm CP\pm}$ and $\mathcal{A}_{\rm CP\pm}$:

$$R_{\rm CP\pm} = 1 + r^2 \pm 2\sqrt{r^2\cos^2\gamma - \mathcal{A}^2_{\rm CP\pm}\cot^2\gamma} \quad . \tag{9}$$

Plots of $R_{\rm CP\pm}$ as function of γ, for a few values of r around 0.2 and asymmetries in the range of $0 - 20\%$, may be borrowed from [14] plotting analogous quantities for the processes $B^0 \to K^+\pi^-$ and $B^+ \to K^0\pi^+$, which involve a similar algebra relating γ to $B \to K\pi$ decay rates. These plots can be used to study the precision in r, $R_{\rm CP\pm}$ and $\mathcal{A}_{\rm CP\pm}$ required to measure γ to a given accuracy. In our case the accuracy improves with increasing values of r due to a larger interference between $A(B^- \to D^0K^-)$ and $A(B^- \to \bar{D}^0K^-)$.

An important question is therefore the actual value of r. New experimental information exists which relates to this value. Previously arguments based on naïve factorization [15] seemed to imply that the amplitude $A(B^- \to \bar{D}^0 K^-)$ involves a suppression factor, $|a_2/a_1| = 0.25$ [16], for the fact that the quark and antiquark making the kaon in $B^- \to \bar{D}^0 K^-$ do not originate in the same weak current of the effective Hamiltonian describing $b \to u\bar{c}s$. This has led to a commonly accepted estimate $r \approx (|V_{ub}V^*_{cs}|/|V_{cb}V^*_{us}|)(|a_2/a_1|) \approx 0.1$. Recent measurements [11] of the color-suppressed process $\bar{B}^0 \to D^0\pi^0$ show, however, that color-suppression is less effective in this process than anticipated [17], implying $a_2/a_1 \simeq 0.44$. Therefore, a more reasonable estimate is

$$r \sim 0.2 \quad . \tag{10}$$

As noted in the past [5], it is difficult to associate a theoretical uncertainty with this value. While the amplitude for $\bar{B}^0 \to D^0\pi^0$ involves a $b \to c$ transition with a heavy quark in the final state, $B^- \to \bar{D}^0 K^-$ follows from a $b \to u$ transition with a light quark in the final state. The different flavor structure of the two operators and the different kinematics with which the heavy and light quarks emerge from the weak interaction imply different hadronic final state interaction effects in the two cases. This is expected to result in different color-suppression factors. Thus, while we will be using the value (10) as a guide for testing the sensitivity of this method, one should not exclude different values of r. An important task of future studies is to determine r experimentally without measuring $B^- \to \bar{D}^0 K^-$ [18]. If r is as small as (10) it will be very difficult to determine its value from the ratio of rates defined in (3), since the right-hand side of (8) is expected to be only a few percent larger than one. Setting upper bounds on r would also be useful.

Assuming then that r is too small to be measured from (8), one may still obtain useful constraints on γ from the asymmetries $\mathcal{A}_{\rm CP\pm}$ and the two ratios $R_{\rm CP\pm}$. The information obtained from these pairs of quantities is complementary to each other. While the asymmetries become larger for large values of $\sin\delta$, the deviation of $R_{\rm CP\pm}$ from one increases with $\cos\delta$. A sizable asymmetry at a level of 20%, which could soon be observed [9, 10], requires a large value of $|\sin\delta|$. Measurements of strong phases in $B \to \bar{D}\pi$ decays [11], and QCD considerations implying that final state interaction phases are either perturbative or power suppressed in $1/m_b$ [19], both indicate that $|\delta|$ may not be larger than about 30° (mod π). We will take this as a conservative assumption. A value $r = 0.2$ and $\gamma \leq 80^\circ$, taken from CKM fits [20], imply that the two CP asymmetries $\mathcal{A}_{\rm CP\pm}$ may not be larger than 10%. Since the asymmetries for even and odd CP states are equal in magnitude and opposite in sign, they may be combined to yield an overall asymmetry, $|\mathcal{A}_{\rm CP+} - \mathcal{A}_{\rm CP-}| \leq 0.20$. Measuring such an asymmetry requires some reduction of present experimental errors [9, 10]. Setting bounds on asymmetries is important by itself [21], since it would give us information about the strong phase δ.

Let us consider now what can be learned about γ from $R_{\rm CP\pm}$, in particular if experimental limits on asymmetries restrict δ to small values, e.g. $\delta \leq 30^\circ$ (mod π). Rewriting

$$R_{\rm CP\pm} = \sin^2\gamma + (r \pm \cos\delta\cos\gamma)^2 + \sin^2\delta\cos^2\gamma \quad , \tag{11}$$

Table 1: Upper bounds on γ (in degrees) obtained using Eqs. (6) and (12). Numbers in parentheses are corresponding maximal values of $R_{\rm CP\pm}$ for one of the two CP-eigenstates.

Input value of γ	Upper bound on γ assuming		$\lvert\delta\rvert \le 30^\circ (\mathrm{mod}\ \pi)$	$\delta = 0\ (\mathrm{mod}\ \pi)$
	$r = 0.1$	$r = 0.2$	$r = 0.4$	$r = 0.4$
50	71 (0.90)	65 (0.82)	58 (0.71)	53.5 (0.65)
60	74 (0.92)	69 (0.87)	64 (0.81)	60.7 (0.76)
70	77 (0.95)	74 (0.92)	74 (0.92)	70.3 (0.89)
80	82 (0.98)	82 (0.98)	– (1.04)	– (1.02)

one obtains the two simultaneous inequalities

$$\sin^2\gamma \le R_{\rm CP\pm} \quad . \tag{12}$$

These inequalities become useful when $R_{\rm CP\pm} < 1$ holds for either even or odd CP states. This condition is fufilled in a major part of the $r,\ \delta,\gamma$ parameter space because of the two opposite signs of the last term in (6). The condition is equivalent to a rather weak requirement, $|\cos\delta\cos\gamma| > r/2$. Namely, Eqs. (12) imply nontrivial constraints on γ when neither γ nor δ lies too close to $\pi/2$. Since we are assuming this to be true for the strong phase δ, Eq. (12) provides useful bounds on γ for values different from $\pi/2$. The bounds depend on the value of r.

For illustration, we calculate in Table 1 upper bounds on γ obtained from Eqs. (6) and (12) for three values of r, $r = 0.1,\ 0.2,\ 0.4$. The value $r = 0.4$, which may be an overestimate for the case of two body decays, is a realistic value for multibody decays which we discuss below. We include it in the present discussion for a later reference. The bounds are computed for input values of γ in the range $50^\circ \le \gamma \le 80^\circ$ permitted by CKM fits [20]. In computing these bounds we assume $|\delta| \le 30^\circ$ (mod π) which can be verified by CP asymmetry measurements. Also given in parentheses are corresponding maximal values of $R_{\rm CP\pm}$ for one of the two CP-eigenstates, which are obtained for the largest value assumed for δ, $\delta_{\rm max} = 30^\circ$ (mod π). Smaller values of $R_{\rm CP\pm}$, and corresponding stronger upper bounds on γ, are obtained for a smaller strong phase. This is shown by the last column in Table 1 displaying for $r = 0.4$ and $\delta = 0$ (mod π) upper bounds on γ and corresponding values of $R_{\rm CP\pm}$.

We note that as r increases the bounds on γ become stronger. For $r = 0.2$, as estimated in (10), the upper limits on γ already provide useful information on the weak phase beyond CKM fits. In this case the largest deviation of $R_{\rm CP\pm}$ from one is 18%. For $r = 0.4$ the limits are quite close to the input values of γ, in particular for $\delta = 0$ (mod π). If the actual value of γ is near 50° then the bound fixes the weak phase to a very narrow range of several degrees. In this case the deviation of $R_{\rm CP\pm}$ from one is large, 29% for $\delta = 30^\circ$ (mod π) and 35% for $\delta = 0$ (mod π). For a precise measurement of $R_{\rm CP\pm}$ one needs accurate decay branching ratio measurements of D^0 into CP-eigenstates. This is the case, for instance, in $D^0 \to K^+K^-$, where the decay branching ratio is already known to 3.4% [22]. We conclude that, although r may not be determined accurately experimentally, there exist good prospects, in terms of reasonable values of r and δ, for improving present bounds on γ using measured values of $R_{\rm CP\pm}$.

The small value of $r \equiv |A(B^- \to \bar D^0 K^-)/\ A(B^- \to D^0 K^-)|$ in (10) follows from color-suppression in the two body decay $B^- \to \bar D^0 K^-$ on top of a modest CKM-suppression. No color-suppression occurs in most multibody decays of the type $B^-\ (\bar B^0) \to \bar D^0 X_s^{-,0}$ [23]. For instance, none of the processes $B^- \to \bar D^0 K^- \pi^0, B^- \to \bar D^0 K^- \pi^+\pi^-$ and the self-tagged $\bar B^0 \to \bar D^0 K^- \pi^+$ is color-suppressed. Consequently, one expects the corresponding ratio of $\bar D^0$ and D^0 amplitudes in most multibody processes to be larger than in two body decays. As we saw now, the two inequalities (12) become stronger as r increases. An interesting question is therefore whether inequalities similar to (12) hold also in multibody decays. If this were the case, then one would be able to apply such inequalities to these decays in order to obtain stronger constraints on γ. In the remaining part of this Letter we will prove that such equalities hold in general and we will study their consequences.

In a multibody decay of the type $B^- \to D X_s^-$, and in a similar neutral B decay $\bar B^0 \to D X_s^0$, one may generalize Eqs. (4) to hold at any point p in the multibody phase space,

$$A(B^- \to (D^0 X_s^-)_p) = A_p \quad , \qquad A(B^- \to (\bar D^0 X_s^-)_p) = \bar A_p e^{-i\gamma} \quad , \tag{13}$$

and consequently

$$A(B^- \to (D_{\rm CP\pm} X_s^-)_p) = \frac{1}{\sqrt{2}}(A_p \pm \bar A_p e^{-i\gamma}) \quad , \tag{14}$$

where A_p and $\bar A_p$ are complex amplitudes involving final state interaction phases which depend on the point p in phase space. For instance, in $B^- \to D K^- \pi^0$ p is a point in a Dalitz plot and the magnitudes and complex phases of A_p and $\bar A_p$, which depend on resonance structures in the two channels, vary from one point to another. This seems to pose a serious problem in applying the method [3] or any of its variants to multibody decays in order to determine γ. Any such attempt would be strongly model-dependent, since it requires modeling the amplitudes A_p and $\bar A_p$ as functions of p in terms of assumed resonance structures in the two channels. For a very recent attempt, see Ref. [12]. Our following considerations are, however, model-independent.

Let us consider partial rates for the four processes in Eqs. (13) and (14),

$$\Gamma(B^- \to D^0 X_s^-) = \int dp\, |A_p|^2 \quad , \qquad \Gamma(B^- \to \bar D^0 X_s^-) = \int dp\, |\bar A_p|^2 \quad , \tag{15}$$

$$\Gamma(B^- \to D^0_{\rm CP\pm} X_s^-) = \frac{1}{2}\left(\int dp\, |A_p|^2 + \int dp\, |\bar A_p|^2\right) \pm \int dp\, {\rm Re}(A_p \bar A_p^* e^{i\gamma}) \quad , \tag{16}$$

where integration over phase space may be either complete or partial. The corresponding B^+ decay rates for decays to CP-eigenstates are obtained by changing the sign of γ in Eq. (16). Defining ratios of partial rates,

$$r_s^2 \equiv \frac{\Gamma(B^- \to \bar D^0 X_s^-)}{\Gamma(B^- \to D^0 X_s^-)} \quad , \tag{17}$$

$$R_{\mathrm{CP}\pm}(X_s) \equiv \frac{2[\Gamma(B^- \to D_{\mathrm{CP}\pm}X_s^-) + \Gamma(B^+ \to D_{\mathrm{CP}\pm}X_s^+)]}{\Gamma(B^- \to D^0 X_s^-) + \Gamma(B^+ \to \bar{D}^0 X_s^+)} \quad , \tag{18}$$

we find

$$R_{\mathrm{CP}\pm}(X_s) = 1 + r_s^2 \pm 2\cos\gamma \, \frac{\mathrm{Re}(\int dp \, A_p \bar{A}_p^*)}{\int dp \, |A_p|^2} \quad . \tag{19}$$

The Schwarz inequality for the two infinite-dimensional Hilbert-space vectors in phase space, A_p and $\bar{A}_p$,

$$|\int dp \, A_p \bar{A}_p^* | \le \sqrt{\int dp \, |A_p|^2 \int dp \, |\bar{A}_p|^2} \quad , \tag{20}$$

implies

$$\frac{|\mathrm{Re}(\int dp \, A_p \bar{A}_p^*)|}{\int dp \, |A_p|^2} \le \frac{|\int dp \, A_p \bar{A}_p^*|}{\int dp \, |A_p|^2} \le r_s \quad , \tag{21}$$

where Eq. (20) and the second inequality in Eq. (21) become equalities if and only if the vectors A_p and $\bar{A}_p$ are proportional to each other. Writing

$$\frac{\mathrm{Re}(\int dp \, A_p \bar{A}_p^*)}{\int dp \, |A_p|^2} = r_s \cos\delta_s \quad , \tag{22}$$

one obtains from (19)

$$R_{\mathrm{CP}\pm}(X_s) = 1 + r_s^2 \pm 2 r_s \cos\delta_s \cos\gamma \quad , \tag{23}$$

in analogy with Eq. (6), from which follows the mentioned inequality:

$$\sin^2\gamma \le R_{\mathrm{CP}\pm}(X_s) \quad . \tag{24}$$

Defining pseudo asymmetries in analogy with (5),

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) \equiv \frac{\Gamma(B^- \to D_{\mathrm{CP}\pm}X_s^-) - \Gamma(B^+ \to D_{\mathrm{CP}\pm}X_s^+)}{\Gamma(B^- \to D^0 X_s^-) + \Gamma(B^+ \to \bar{D}^0 X_s^+)} \quad , \tag{25}$$

we also find

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) = \pm \sin\gamma \frac{\mathrm{Im}(\int dp \, A_p \bar{A}_p^*)}{\int dp \, |A_p|^2} \quad . \tag{26}$$

We note that in the special case that A_p and $\bar{A}_p$ are proportional to each other δ_s is the phase of $\int dp \, A_p \bar{A}_p^*$, and the asymmetry obtains an expression similar to that of two body decays,

$$\mathcal{A}_{\mathrm{CP}\pm}(X_s) = \pm r_s \sin\delta_s \sin\gamma \quad . \tag{27}$$

Eqs. (23) and (24) are quite powerful. They apply to any multibody decay of the type under discussion and to an arbitrary choice of phase space over which one integrates. Their advantage over the corresponding relations in two body decays, aside from involving larger branching ratios, is threefold:

1. Since most multibody decays of the type $B^- \to \bar{D}^0 X_s^-$ and $\bar{B}^0 \to \bar{D}^0 X_s^0$ are not color-suppressed, their measurements using hadronic $\bar{D}^0$ decays are less affected by interference with doubly Cabibbo-suppressed D^0 decays in $B^- \to D^0 X_s^-$ and $\bar{B}^0 \to D^0 X_s^0$, respectively. This allows a reasonably accurate direct measurement of r_s in these processes.

2. In the above cases the parameter r_s is expected to be larger than r which is color-suppressed. A typical estimate, based only on CKM factors, is $r_s \approx |V_{ub}V^*_{cs}|/|V_{cb}V^*_{us}| \approx 0.4$. Upper bounds on γ were calculated for this value in the last two columns of Table 1 and were shown to be stronger than for $r = 0.2$.

3. In given multibody decays one may be able to choose judiciously regions of phase space in order to maximize the value of r_s while keeping the phase δ_s as small as possible. This has the effect of improving considerably bounds on γ, as demonstrated in Table 1. For regions of phase space in which A_p and $\bar{A}_p$ are proportional to each other, where both Eqs. (23) and (27) apply, an algebraic solution for γ can be obtained as shown in Eq. (9). In this case the largest CP asymmetry is obtained when maximizing both r_s and δ_s.

In conclusion, we have shown that several two body decays $B^\pm \to DK^\pm$, for which data exist, have the potential of improving present bounds on the weak phase γ. We argued that multibody decays of this class, for both charged and neutral B mesons, are expected to even do better. Measuring $R_{\rm CP\pm}(X_s) = 0.60 \pm 0.05$ for one of the two CP-eigenstates in any of these decays would determine γ to within several degrees, $\gamma = 51^\circ \pm 3^\circ$, approaching the present level of precision in β [20]. On the other hand, a measurement $R_{\rm CP\pm}(X_s) < 0.5$, corresponding to $\gamma < 45^\circ$, would be a signature for physics beyond the Standard Model.

I am grateful to the CERN Theory Division for its kind hospitality, and I wish to thank J. L. Rosner for useful comments.

References

[1] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 091801 (2001); Phys. Rev. D **66**, 032003 (2002); hep-ex/0207042; Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **87**, 091802 (2001); Phys. Rev. D **66**, 032007 (2002); Phys. Rev. D **66**, 071102 (2002).

[2] M. Gronau, Phys. Rev. Lett. **63**, 145 (1989); D. London and R. D. Peccei, Phys. Lett. B **223**, 257 (1989); B. Grinstein, Phys. Lett. B **229**, 280 (1989).

[3] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991).

[4] I. Dunietz, Phys. Lett. B **270**, 75 (1991).

[5] M. Gronau, Phys. Rev. D **58**, 037301 (1998).

[6] D. Atwood, G. Eilam, M. Gronau and A. Soni, Phys. Lett. B **341**, 372 (1995); D. Atwood, I . Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997); Phys. Rev. D **63**, 036005 (2001); B. Kayser and D. London, Phys. Rev. D **61**, 116013 (2000); D. London, N. Sinha and R. Sinha, Phys. Rev. Lett. **85**, 1805 (2000); Y. Grossman, Z. Ligeti and A. Soffer, hep-ph/0210433.

[7] Z. Z. Xing, Phys. Rev. D **58**, 093005 (1998); M. Gronau and J. L. Rosner, Phys. Lett. B **439**, 171 (1998); J. H. Jang and P. Ko, Phys. Rev. D **58**, 111302 (1998).

[8] CLEO Collaboration, M. Athanas *et al.*, Phys. Rev. Lett. **80**, 5493 (1998); Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **87**, 111801 (2001); BABAR Collaboration, K. Aubert *et al.*, hep-ex/0207087.

[9] Belle Collaboration, K. Abe *et al.*, hep-ex/0207012.

[10] BABAR Collaboration, B. Aubert *et al.*, Ref. [8].

[11] Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **88**, 052002 (2002); CLEO Collaboration, T.E. Coan *et al.*, Phys. Rev. Lett. **88**, 062001 (2002); BABAR Collaboration, B. Aubert *et al.*, hep-ex/0207092.

[12] For a recent model-dependent study of $B^\pm \to DK^\pm\pi^0$, see R. Aleksan, T. C. Peterson and A. Soffer, hep-ph/0209194.

[13] C. C. Meca and J. P. Silva, Phys. Rev. Lett. **81**, 1377 (1998); J. P. Silva and A. Soffer, Phys. Rev. D **61**, 112001 (2000).

[14] M. Gronau and J. L. Rosner, Phys. Rev. D **57**, 6843 (1998).

[15] M. Bauer, B. Stech and M. Wirbel, Zeit. Phys. C **34**, 103 (1987).

[16] T. E. Browder, K. Honscheid and D. Pedrini, Ann. Rev. Nucl. Part. Sci. **46**, 395 (1997); see also M. Neubert and B. Stech, in *Heavy Flavours II*, ed. A. J. Buras and M. Lindner (World Scientific 1998), p. 294.

[17] M. Neubert and A. Petrov, Phys. Lett. B **519**, 50 (2001); H. Y. Cheng, Phys. Rev. D **65**, 094012 (2002); C. K. Chua, W. S. hu Hou and K. C. Yang Phys. Rev. D **65**, 096007 (2002).

[18] The ratio r may also be determined in some variants of this method. See, D. Atwood, I. Dunietz and A. Soni, Ref. [6]; Y. Grossman, Z. Ligeti and A. Soffer, Ref. [6].

[19] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591**, 313 (2000).

[20] See, for instance, A. J. Buras, F. Parodi and A. Stocchi, hep-ph/0207101.

[21] M. Gronau and J. L. Rosner, Phys. Rev. D **65**, 013004 (2002).

[22] K. Hagiwara *et al.*, Particle Data Group, Phys. Rev. D **66**, 010001 (2002).

[23] An exception is $B^- \to \bar{D}^0 K_S \pi^-$ which obtains only a color-suppressed contribution. We are assuming that color-suppression in multibody decays can be inferred from quark diagrams in which extra light $q\bar{q}$ pairs are created. Our conclusions are, however, independent of the level of suppression since we are discussing only color-allowed decays.