CERN-TH/2002-239
hep-ph/0209207

Higgs-Mediated $B^0_{s,d} \to \mu\tau, e\tau$ and $\tau \to 3\mu, e\mu\mu$ Decays in Supersymmetric Seesaw Models

Athanasios Dedes[1], **John Ellis**[2] and **Martti Raidal**[2,3]

[1]*Physikalisches Institut der Universität Bonn, Nußallee 12, D-53115 Bonn, Germany*

[2] *TH Division, CERN, CH-1211 Geneva 23, Switzerland*

[3] *National Institute of Chemical Physics and Biophysics, Tallinn 10143, Estonia*

Abstract

We study the rates allowed for the Higgs-mediated decays $B^0_{s,d} \to \mu\tau, e\tau$ and $\tau \to \mu\mu\mu, e\mu\mu$ in supersymmetric seesaw models, assuming that the only source of lepton flavour violation (LFV) is the renormalization of soft supersymmetry-breaking terms due to off-diagonal singlet-neutrino Yukawa interactions. These decays are strongly correlated with, and constrained by, the branching ratios for $B^0_{s,d} \to \mu\mu$ and $\tau \to \mu(e)\gamma$. Parametrizing the singlet-neutrino Yukawa couplings Y_ν and masses M_{N_i} in terms of low-energy neutrino data, and allowing the flavour-universal soft masses for sleptons and for squarks, as well as those for the two Higgs doublets, to be different at the unification scale, we scan systematically over the model parameter space. Neutrino data and the present experimental constraints set upper limits on the Higgs-mediated LFV decay rates $Br(B^0_s \to \mu\tau, e\tau) \lesssim 4 \times 10^{-9}$ and $Br(\tau \to \mu\mu\mu, e\mu\mu) \lesssim 4 \times 10^{-10}$.

CERN-TH/2002-239
September 2002

1. Introduction. In the minimal supersymmetric extension of the Standard Model (MSSM), non-holomorphic Yukawa interactions of the form $D^cQH_2^*$ are generated at one-loop level [1]. At large values of $\tan\beta = v_2/v_1$, the contribution of such loop-suppressed operators to the down-type quark masses may become comparable to those from the usual superpotential terms D^cQH_1. As these two contributions have different flavour structures, because of the up-type quark Yukawa interactions, they cannot be diagonalized simultaneously [2, 3]. This difference leads to potentially large new contributions to Higgs-mediated flavour-changing processes involving down-type quarks, such as $B^0_{s,d} \to \mu\mu$ [4, 5, 6, 7, 8] and B^0-$\bar{B}^0$ mixing [8, 5].

This discussion cannot be generalized directly to the charged-lepton sector, because the MSSM has no right-handed neutrinos at the electroweak scale. However, experimental data convincingly indicate that neutrinos do have masses [9]. Their small values are most naturally explained via the seesaw mechanism [10], which involves super-heavy singlet (right-handed) neutrinos with masses M_{N_i}. In this case, the presence of neutrino Yukawa couplings $N_i^c(Y_\nu)_{ij}L_jH_2$ above the heavy-neutrino decoupling scale induces off-diagonal elements in the left-slepton mass matrix via renormalization [11, 12],

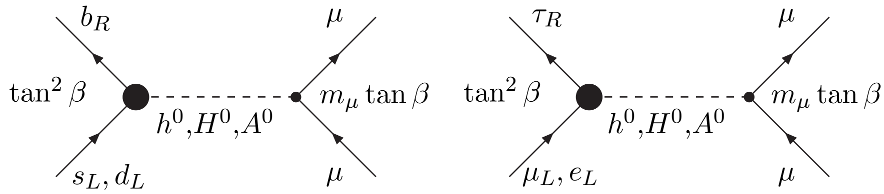
$$(\Delta m^2_{\tilde{L}})_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y^\dagger LY)_{ij}\,, \quad L = \ln\frac{M_{GUT}}{M_{N_i}}\delta_{ij}\,, \qquad (1)$$

even if the initial slepton soft supersymmetry-breaking masses m_0 and trilinear couplings A_0 are flavour-universal at M_{GUT}. This is the only source of lepton-flavour violation (LFV) in supersymmetric seesaw models with flavour-universal soft mass terms. Since it is induced by heavy-neutrino Yukawa interactions, it relates the LFV processes to low-energy neutrino data. At the one-loop level, (1) also gives rise to flavour violation in non-holomorphic interactions of the form $E^cLH_2^*$ and leads to Higgs-mediated LFV processes in the charged-lepton sector.

It is well known that, at large $\tan\beta$, new Higgs-mediated contributions to the decay $B^0_{s,d} \to \mu\mu$ may exceed the Standard Model (SM) contribution by orders of magnitude. The dominant contributions come from the diagrams in Fig. 1 (a) which are determined by Cabibbo-Kobayashi-Maskawa (CKM) matrix elements if the squark mass matrices are flavour-universal at the GUT scale, as suggested by the data on flavour-changing neutral interactions [13]. It was suggested recently [14] that the Higgs-mediated contribution to the LFV process $\tau \to \mu\mu\mu$ depicted in Fig. 1 (b) might be sizeable in supersymmetric seesaw models [1], with a branching ratio as large as $Br(\tau \to \mu\mu\mu) \sim \mathcal{O}(10^{-7})$. This claim was made without a complete study of all related LFV processes.

Since the only source of LFV in this model is (1), and all LFV processes are induced by slepton-neutralino and sneutrino-chargino loops, Higgs-mediated LFV is constrained indirectly by limits on the processes $\tau \to \mu\gamma$, $\tau \to e\gamma$ and $\mu \to e\gamma$, which have been well studied by experiment. Although $\tau \to \mu(e)\gamma$ and Higgs-mediated LFV depend on the model parameters in systematically different ways, the former impose important and unavoidable constraints on the latter processes. Additional constraints are expected to come from lepton-flavour-conserving but quark-flavour-violating processes such as $B^0_s \to \mu\mu$, which

[1]For a discussion of LFV decays in non-supersymmetric models the reader is referred to [15, 16].



(a) **(b)**

Figure 1: *Dominant Higgs penguin diagrams contributing to (a)* $B^0_{s,d} \to \mu\mu$ *and (b)* $\tau \to \mu\mu\mu$ *decays at large* $\tan\beta$*.*

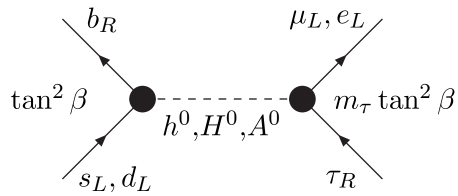


Figure 2: *Dominant Higgs penguin diagrams contributing to* $B^0_{s,d} \to \tau\mu$ *decays at large* $\tan\beta$*.*

depends on Higgs boson masses in the same way as Higgs-mediated $\tau \to \mu\mu\mu$ (see Fig. 1). In addition, there are other Higgs-mediated LFV processes such as $B^0_s \to \mu\tau$ and $B^0_s \to e\tau$, which are induced by double-penguin diagrams, as depicted in Fig. 2, and may also be of experimental interest. The current experimental upper limits on such decays are shown in Table 1. Although the double-penguin diagram is suppressed by an additional loop factor, its amplitude is enhanced by $m_\tau/m_\mu \tan\beta$, as seen by comparing Fig. 2 with Fig. 1. Therefore, at large $\tan\beta$, the rate for $B^0_s \to \mu\tau$ may even exceed that of $\tau \to \mu\mu\mu$.

To quantify these qualitative statements, we adopt for the moment the approximation considered in [14], *i.e.*, we take all the supersymmetry-breaking mass parameters of the model to be equal at low scales, use heavy-neutrino masses that are degenerate with $M_N = 10^{14}$ GeV, and assume that $(Y_\nu^\dagger Y_\nu)_{32,33} = 1$. This approximation is not realistic, but it is useful for comparing the sensitivities of different processes to new physics: a more complete treatment of these processes is presented in the following Sections of this Letter. In the simplified case, working in the mass-insertion approximation and following exactly [14], we obtain

$$Br(\tau \to 3\mu) \simeq 1.6 \times 10^{-8} \left[\frac{\tan\beta}{60}\right]^6 \left[\frac{100\,\mathrm{GeV}}{M_A}\right]^4 , \tag{2}$$

which is about six times smaller than the estimate quoted in that paper. This should be compared with the corresponding estimate

$$Br(\tau \to \mu\gamma) \simeq 1.3 \times 10^{-3} \left[\frac{\tan\beta}{60}\right]^2 \left[\frac{100\,\mathrm{GeV}}{M_S}\right]^4 . \tag{3}$$

Both equations (2) and (3) are valid in the large-$\tan\beta$ limit. Whereas (2) is two orders of

Channel	Expt.	Bound (90% CL)
$B_s \to \mu^\pm \mu^\mp$	CDF [17]	$< 2.0 \times 10^{-6}$
$B_s \to e^\pm \mu^\mp$	CDF [17]	$< 6.1 \times 10^{-6}$
$B_s \to e^\pm \tau^\mp$	—	—
$B_s \to \mu^\pm \tau^\mp$	—	—
$B_d \to \mu^\pm \mu^\mp$	BaBar [18]	$< 2.0 \times 10^{-7}$
$B_d \to e^\pm \mu^\mp$	BaBar [18]	$< 2.1 \times 10^{-7}$
$B_d \to e^\pm \tau^\mp$	CLEO [19]	$< 5.3 \times 10^{-4}$
$B_d \to \mu^\pm \tau^\mp$	CLEO [19]	$< 8.3 \times 10^{-4}$

Table 1: *Current experimental bounds for the branching ratios of the leptonic B decays* $B_{s,d} \to l_i^+ l_j^-$.

magnitude *below* the present experimental bound on $\tau \to 3\mu$, (3) is three orders of magnitude *above* the present bound on $\tau \to \mu\gamma$. There is also the photonic penguin contribution to the decay $\tau \to 3\mu$, which is related to $Br(\tau \to \mu\gamma)$ by

$$Br(\tau \to 3\mu)_\gamma = \frac{\alpha}{3\pi} \left(\ln \frac{m_\tau^2}{m_\mu^2} - \frac{11}{4} \right) Br(\tau \to \mu\gamma). \tag{4}$$

Numerically, (3) leads to

$$Br(\tau \to 3\mu)_\gamma \simeq 3.0 \times 10^{-6} \left[\frac{\tan\beta}{60} \right]^2 \left[\frac{100\,\text{GeV}}{M_S} \right]^4 , \tag{5}$$

which is a factor of 100 larger than (2). Notice also that suppressing (3) by postulating large slepton masses would suppress (2) at the same time, since sleptons enter into both loops. However, suppressing (2) by a large Higgs mass M_A would not affect $\tau \to \mu\gamma$.

With the same assumptions, we obtain

$$Br(B_s^0 \to \mu\mu) \simeq 1.9 \times 10^{-5} \left[\frac{\tan\beta}{60} \right]^6 \left[\frac{100\,\text{GeV}}{M_A} \right]^4 , \tag{6}$$

where only the leading $\tan\beta$ dependence is presented, and

$$Br(B_s^0 \to \tau\mu) \simeq 3.6 \times 10^{-7} \left[\frac{\tan\beta}{60} \right]^8 \left[\frac{100\,\text{GeV}}{M_A} \right]^4 , \tag{7}$$

to be compared with the upper limits in Table 1. In the case of B_d mesons, one should just multiply (7) with $|V_{td}/V_{ts}|^2 \simeq 0.05$. As expected, the Higgs-mediated branching ratio for $B_s^0 \to \tau\mu$ can be larger than the one for $\tau \to 3\mu$.

We comment in passing that the decays $B_{d,s} \to \mu e$ are suppressed by the ratio $m_\mu^2/m_\tau^2 \simeq 0.0036$ compared to $B_{d,s} \to \mu\tau$ and, moreover, they are strongly constrained by the process $\mu \to e\gamma$. We find that their branching ratios are below the range of prospective experimental interest.

2. Effective Lagrangians and branching ratios for LFV processes. We now present the calculational details we use in arriving at the approximate results of the previous Section and the numerical results to be presented in the next Section. We consider the R-parity conserving superpotential:

$$\begin{aligned} W &= U_i^c (Y_u)_{ij} Q_j H_2 - D_i^c (Y_d)_{ij} Q_j H_1 + \\ & N_i^c (Y_\nu)_{ij} L_j H_2 - E_i^c (Y_e)_{ij} L_j H_1 + \frac{1}{2} N^c{}_i (M_N)_{ij} N_j^c + \mu H_2 H_1 \,, \end{aligned} \tag{8}$$

where the indices i, j run over three generations and M_N is the heavy singlet-neutrino mass matrix. We work in a basis where $(Y_d)_{ij}$, $(Y_e)_{ij}$ and $(M_N)_{ij}$ are real and diagonal. At the one-loop level, this leads to the effective Lagrangian [3, 5, 8, 14].

$$\begin{aligned} -\mathcal{L}^{eff} &= \bar{d}_R^i Y_{di} \left[\delta_{ij} H_1^0 + \left(\epsilon_0 \delta_{ij} + \epsilon_Y \left(Y_u^\dagger Y_u \right)_{ij} \right) H_2^{0*} \right] d_L^j + h.c. + \\ & \bar{l}_R^i Y_{ei} \left[\delta_{ij} H_1^0 + \left(\epsilon_1 \delta_{ij} + \epsilon_2 E_{ij} \right) H_2^{0*} \right] l_L^j + h.c. \,, \end{aligned} \tag{9}$$

where ϵ_0, ϵ_Y, ϵ_1, and ϵ_2 are loop-induced form factors, and $E_{ij} = (\Delta m_{\tilde{L}}^2)_{ij}/m_0^2$ is the unique source of LFV. In the following, we present analytical expressions in the mass-insertion approximation, which is simple and known to reproduce well the full results in the case of large $\tan\beta$. However, in our numerical analyses of the next Section we use the full diagrammatic calculation of the $(\bar{b}s)$-Higgs transition of [4], which we modify according to [5] to include the resummed large-$\tan\beta$ contributions to the down-type Yukawa couplings. Also, the LFV matrix E_{ij} is calculated exactly using the numerical solutions to the full set of renormalization-group equations of the MSSM, including singlet neutrinos [12].

In the mass-insertion approximation, the down-quark form factors are [5, 8]

$$\epsilon_0 = \frac{2\alpha_s}{3\pi} \frac{\mu M_{\tilde{g}}}{m_{\tilde{d}_L}^2} F_2 \left(x_{\tilde{g}\tilde{d}_L}, x_{\tilde{d}_R \tilde{d}_L} \right) \,, \qquad \epsilon_Y = \frac{1}{16\pi^2} \frac{\mu A_u}{m_{\tilde{u}_L}^2} F_2 \left(x_{\mu \tilde{u}_L}, x_{\tilde{u}_R \tilde{u}_L} \right) \,, \tag{10}$$

where $x_{ab} = m_a^2/m_b^2$ and

$$F_2 \left(x, y \right) = - \frac{x \ln x}{(1-x)(x-y)} - \frac{y \ln y}{(1-y)(y-x)} \,. \tag{11}$$

In the lepton sector, the corresponding form factors are [14]

$$\begin{aligned} \epsilon_1 &= \frac{\alpha'}{8\pi} \mu M_1 \left[2F_3 \left(M_1^2, m_{\tilde{l}_L}^2, m_{\tilde{l}_R}^2 \right) - F_3 \left(M_1^2, \mu^2, m_{\tilde{l}_L}^2 \right) + 2F_3 \left(M_1^2, \mu^2, m_{\tilde{l}_R}^2 \right) \right] + \\ & \frac{\alpha_2}{8\pi} \mu M_2 \left[F_3 \left(\mu^2, m_{\tilde{l}_L}^2, M_2^2 \right) + 2F_3 \left(\mu^2, m_{\tilde{\nu}}^2, M_2^2 \right) \right] \,, \end{aligned} \tag{12}$$

$$\begin{aligned} \epsilon_2 &= \frac{\alpha'}{8\pi} m_0^2 \mu M_1 \left[2F_4 \left(M_1^2, m_{\tilde{l}_L}^2, m_{\tilde{\tau}_L}^2, m_{\tilde{\tau}_R}^2 \right) - F_4 \left(\mu^2, m_{\tilde{l}_L}^2, m_{\tilde{\tau}_L}^2, M_1^2 \right) \right] + \\ & \frac{\alpha_2}{8\pi} m_0^2 \mu M_2 \left[F_4 \left(\mu^2, m_{\tilde{l}_L}^2, m_{\tilde{\tau}_L}^2, M_2^2 \right) + 2F_4 \left(\mu^2, m_{\tilde{\nu}_l}^2, m_{\tilde{\nu}_\tau}^2, M_2^2 \right) \right] \,, \end{aligned} \tag{13}$$

where

$$F_3 \left(x, y, z \right) = - \frac{xy \ln(x/y) + yz \ln(y/z) + zx \ln(z/x)}{(x-y)(y-z)(z-x)} \,,$$

$$F_4\,(x,y,z,w) \;=\; -\frac{x\ln x}{(x-y)(x-z)(x-w)} - \frac{y\ln y}{(y-x)(y-z)(y-w)} + (x \leftrightarrow z, y \leftrightarrow w)\,. \tag{14}$$

The resulting effective Lagrangians describing flavour-violating neutral Higgs interactions with down quarks and charged leptons are [5, 14]

$$-\mathcal{L}^{eff}_{d_i\neq d_j} = (2G_F^2)^{1/4}\frac{m_{d_i}\kappa^d_{ij}}{\cos^2\beta}\left(\bar{d}_{iR}\,d_{jL}\right)\left[\cos(\alpha-\beta)h^0 + \sin(\alpha-\beta)H^0 - iA^0\right] + h.c.\,, \tag{15}$$

$$-\mathcal{L}^{eff}_{l_i\neq l_j} = (2G_F^2)^{1/4}\frac{m_{l_i}\kappa^l_{ij}}{\cos^2\beta}\left(\bar{l}_{iR}\,l_{jL}\right)\left[\cos(\alpha-\beta)h^0 + \sin(\alpha-\beta)H^0 - iA^0\right] + h.c.\,, \tag{16}$$

where

$$\kappa^d_{ij} \;=\; \frac{\epsilon_Y Y_t^2 \bar{\lambda}^t_{ij}}{\left[1+(\epsilon_0+\epsilon_Y Y_t^2\delta_{it})\tan\beta\right]\left[1+\epsilon_0\tan\beta\right]}\,, \tag{17}$$

$$\kappa^l_{ij} \;=\; \frac{\epsilon_2 E_{ij}}{\left[1+(\epsilon_1+\epsilon_2 E_{ii})\tan\beta\right]^2}\,. \tag{18}$$

As we are interested in B-meson decays, we have $\bar{\lambda}^t_{bq} = V^*_{tb}V_{tq}$, where the V_{ij} are CKM matrix elements.

Using (16), one easily obtains the branching ratio for the decay $\tau \to \mu\mu\mu$, given by

$$\begin{aligned} Br(\tau\to 3\mu) \;&=\; \frac{G_F^2 m_\mu^2 m_\tau^7 \tau_\tau}{1536\pi^3\cos^6\beta}|\kappa^l_{\tau\mu}|^2\left[\left(\frac{\sin(\alpha-\beta)\cos\alpha}{M^2_{H^0}} - \frac{\cos(\alpha-\beta)\sin\alpha}{M^2_{h^0}}\right)^2 + \frac{\sin^2\beta}{M_A^4}\right] \\ &\approx\; \frac{G_F^2 m_\mu^2 m_\tau^7\tau_\tau}{768\pi^3 M_A^4}|\kappa^l_{\tau\mu}|^2\tan^6\beta\,, \end{aligned} \tag{19}$$

where τ_τ is the τ lifetime and the large-$\tan\beta$ limit is taken in the last step. This is the result that was used to derive the estimate (2).

The branching ratio for $\tau \to \mu\gamma$ in the mass-insertion approximation reads [12]:

$$Br(\tau\to\mu\gamma) = \frac{\alpha}{4}m_\tau^5\tau_\tau|A_L|^2\,, \tag{20}$$

where

$$\begin{aligned} A_L \;&\approx\; (\Delta m^2_{\tilde{L}})_{\tau\mu}\frac{\alpha_2}{4\pi}\mu M_2\tan\beta \\ &\times\; D\left[D\left[\frac{1}{m^2}\left\{f_c(x_{Mm}) - \frac{1}{4}f_n(x_{Mm})\right\};M^2\right](M_2^2,\mu^2);m^2\right](m^2_{\tilde{\nu}_\mu},m^2_{\tilde{\nu}_\tau})\,, \end{aligned} \tag{21}$$

where $D[f(x);x](x_1,x_2) = (f(x_1)-f(x_2))/(x_1-x_2)$, and

$$f_c(x) \;=\; -\frac{1}{2(1-x)^3}(3-4x+x^2+2\ln x)\,,$$

$$f_n(x) \;=\; \frac{1}{(1-x)^3}(1-x^2+2x\ln x)\,.$$

This is the result that was used to derive the estimate (3).

The dominant operators at large $\tan\beta$ in the effective Hamiltonian describing the transition $\bar{b} \to \bar{s} l_i^+ l_j^-$ are

$$\mathcal{H} = -\frac{G_F^2 M_W^2}{\pi^2} V_{tb}^* V_{ts} \Big[c_S^{ij} \mathcal{O}_S^{ij} + c_P^{ij} \mathcal{O}_P^{ij} + c_{10}^{ij} \mathcal{O}_{10}^{ij} \Big] + h.c. \;, \tag{22}$$

where V is the CKM matrix and G_F the Fermi coupling constant. Here $c_{S,P,10}$ and $\mathcal{O}_{S,P,10}$ are, respectively, the Wilson coefficients and the local operators [2], which are given by

$$\begin{aligned} \mathcal{O}_{10}^{ij} &= (\bar{b}_R \gamma^\mu s_L)(\bar{l}_i \gamma_\mu \gamma_5 l_j) \;, \\ \mathcal{O}_S^{ij} &= m_b (\bar{b}_R s_L) \bar{l}_i l_j \;, \\ \mathcal{O}_P^{ij} &= m_b (\bar{b}_R s_L) \bar{l}_i \gamma_5 l_j \;. \end{aligned} \tag{23}$$

In the SM with the seesaw mechanism, the only non-zero coefficients are the c_{10}. For $i \neq j$, they scale like the square of the inverse mass of the singlet neutrinos, and are completely negligible. In supersymmetric seesaw models, there are two additional operators in (22), the scalar $\mathcal{O}_S$ and the pseudoscalar $\mathcal{O}_P$, whose coefficients dominate over the SM contributions. In particular, for $i \neq j$ they are suppressed by the scale of supersymmetry-breaking soft masses only. At large $\tan\beta$ the dominant contributions to c_S^{ij} and c_P^{ij} come from the penguin and double-penguin diagrams presented in Fig. 1 and Fig. 2, respectively. We should note here, however, that when the Higgs masses are $\mathcal{O}(\text{TeV})$, and the sneutrinos are very light, the box diagrams may dominate. In this case we obtain in general small branching ratios, $Br(B_s \to \mu\tau) \lesssim 10^{-14}$, when we take the $\tau \to \mu\gamma$ constraint into account.

By setting c_{10} to zero in (22), we obtain the following branching ratio [3],

$$\begin{aligned} \text{Br}(B_s \to l_i l_j) &= \frac{G_F^4 M_W^4}{8\pi^5} |V_{tb}^* V_{ts}|^2 M_{B_s}^5 f_{B_s}^2 \tau_{B_s} \Big(\frac{m_b}{m_b + m_s} \Big)^2 \\ &\times \sqrt{\Big[1 - \frac{(m_{l_i} + m_{l_j})^2}{M_{B_s}^2} \Big] \Big[1 - \frac{(m_{l_i} - m_{l_j})^2}{M_{B_s}^2} \Big]} \\ &\times \Big\{ \Big(1 - \frac{(m_{l_i} + m_{l_j})^2}{M_{B_s}^2} \Big) |c_S^{ij}|^2 + \Big(1 - \frac{(m_{l_i} - m_{l_j})^2}{M_{B_s}^2} \Big) |c_P^{ij}|^2 \Big\} \;, \end{aligned} \tag{24}$$

where M_{B_s} and τ_{B_s} are the mass and lifetime of the B_s meson, and $f_{B_s} = 230 \pm 30$ GeV [20] is its decay constant.

For $B_s \to \mu\mu$ decay, the form factors are [5, 8]:

$$c_S^{\mu\mu} = \frac{\sqrt{2}\pi^2}{G_F M_W^2} \frac{m_\mu \kappa_{bs}^d}{\cos^3\beta \bar{\lambda}_{bs}^t} \left[\frac{\sin(\alpha - \beta)\cos\alpha}{M_{H^0}^2} - \frac{\cos(\alpha - \beta)\sin\alpha}{M_{h^0}^2} \right]$$

[2] In (22) we have omitted operators proportional to the strange-quark mass, since they are subleading for our processes.

[3] We note that (24) can be straightforwardly extended to B_d LFV decays by replacing $s \to d$.

$$\approx \quad -\frac{4\pi^2 m_\mu m_t^2}{M_W^2}\frac{\epsilon_Y \ \tan^3\beta}{[1+(\epsilon_0+\epsilon_Y Y_t^2)\tan\beta]\,[1+\epsilon_0\tan\beta]}\left[\frac{1}{M_{A^0}^2}\right], \tag{25}$$

$$c_P^{\mu\mu} \quad = \quad -\frac{\sqrt{2}\pi^2}{G_F M_W^2}\frac{m_\mu \kappa_{bs}^d}{\cos^3\beta\bar{\lambda}_{bs}^t}\left[\frac{\sin\beta}{M_{A^0}^2}\right] \approx c_S^{\mu\mu}, \tag{26}$$

and, for the double-penguin contribution to $B_s \to \mu\tau$ decay, we obtain from (15) and (16) the form factors

$$c_S^{\mu\tau} = c_P^{\mu\tau} \quad = \quad \frac{\sqrt{2}\pi^2}{G_F M_W^2}\frac{m_\tau \kappa_{bs}^d \kappa_{\tau\mu}^{*l}}{\cos^4\beta\bar{\lambda}_{bs}^t}\left[\frac{\sin^2(\alpha-\beta)}{M_{H^0}^2}+\frac{\cos^2(\alpha-\beta)}{M_{h^0}^2}+\frac{1}{M_{A^0}^2}\right]$$

$$\approx \quad \frac{8\pi^2 m_\tau m_t^2}{M_W^2}\frac{\epsilon_Y \ \kappa_{\tau\mu}^{*l} \ \tan^4\beta}{[1+(\epsilon_0+\epsilon_Y Y_t^2)\tan\beta]\,[1+\epsilon_0\tan\beta]}\left[\frac{1}{M_{A^0}^2}\right]. \tag{27}$$

The last two results are those that were used to derive the estimates (6) and (7). In the calculation for the ratio $B_s \to \mu\tau$ one also has contributions from the operators $(\bar{b}P_{L(R)}s)(\bar{\mu}P_{L(R)}\tau)$. However, their contribution is proportional to $\left[\frac{\sin^2(\alpha-\beta)}{M_{H^0}^2}+\frac{\cos^2(\alpha-\beta)}{M_{h^0}^2}-\frac{1}{M_{A^0}^2}\right]$ which vanishes approximately at large $\tan\beta$. Furthermore, the operator $(\bar{b}P_R s)(\bar{\mu}P_L\tau)$ is proportional to $m_s m_\mu$ and thus subdominant to $(\bar{b}P_L s)(\bar{\mu}P_R\tau)$ we consider here. For the same reason, the leptonic CP-asymmetries in $B_s \to \mu\tau$ decays due to the complex $\kappa_{\tau\mu}^l$ in (27) are of order m_μ/m_τ.

3. Numerical Analyses. The purpose of this work is to study in a complete way the allowed rates for the Higgs-mediated LFV processes in supersymmetric seesaw models in which the only source of LFV is the renormalization of the soft supersymmetry-breaking mass parameters above M_{N_i}, due to the singlet-neutrino Yukawa couplings. We work with two models. *First*, we study the constrained MSSM (CMSSM) which has just two universal mass parameters at GUT scale, $M_{1/2}$ for gauginos and m_0 for all scalars, taking for simplicity $A_0 = 0$ and not expecting our results to be sensitive to its exact value. *Secondly*, we study the more general flavour-universal MSSM (GFU-MSSM) in which the universal masses for squarks, sleptons and the Higgs doublets H_1 and H_2 are different from each other. This permits different mass scales for squarks and sleptons which, in turn, are independent of the Higgs boson masses. However, we always require that squark and slepton mass matrices at the GUT scale are each proportional to unit matrices. If one goes beyond this assumption, arbitrary sources of flavour violation appear in the soft supersymmetry-breaking sector, and the model loses all the predictivity, in particular the connection between the LFV and the neutrino masses and mixings. Moreover, LFV rates generally exceed experimental limits [13].

We parametrize the singlet-neutrino Yukawa couplings Y_ν and masses M_{N_i} in terms of low energy neutrino data according to [21]. We generate all the free parameters of the model randomly and calculate the low-energy sparticle masses and mixings by solving numerically the one-loop renormalization-group equations and imposing the requirement of electroweak symmetry breaking. We calculate the rates for the LFV processes as described in the last Section. For the decays $l_i \to l_j\gamma$, we use the exact diagrammatic formulae in [12].

We fix the light-neutrino parameters by [22] $\Delta m^2_{32} = 3 \times 10^{-3}$ eV^2, $\Delta m^2_{21} = 5 \times 10^{-5}$ eV^2, $\tan^2\theta_{23} = 1$, $\tan^2\theta_{12} = 0.4$ and $\sin\theta_{13} = 0.1$, corresponding to the LMA solution for the solar neutrino anomaly. The neutrino mixing phase is taken to be maximal: $\delta = \pi/2$. The overall light-neutrino mass scale is randomly generated in the range $(10^{-6} \to 10^{-1})$ eV, with a constant logarithmic measure. We assume the normal mass hierarchy, which is favoured by leptogenesis arguments [23]. The rest of the neutrino parameters are calculated from the randomly generated textures H_1 (which maximizes $\tau - \mu$ violation) and H_2 (which maximizes $\tau - e$ violation), that were introduced in [21].

We fix $\tan\beta = 60$, which is the largest value for which electroweak symmetry breaking is obtained generically. In the case of the CMSSM, the mass parameters $M_{1/2}$ and m_0 for sleptons, squarks and both Higgses are generated randomly in the range $(0 \to 700)$ GeV. In the case of the GFU-MSSM, we have four scalar mass parameters $m_0^{\tilde{q}}$, $m_0^{\tilde{l}}$, $m_0^{H_1}$ and $m_0^{H_2}$ for squarks, sleptons and Higgs doublets, respectively. We generate each of them randomly and independently in the range $(0 \to 700)$ GeV. We impose the experimental upper bounds $Br(\tau \to \mu\gamma) < 2 \times 10^{-6}$, $Br(\tau \to e\gamma) < 2 \times 10^{-6}$ and $Br(\mu \to e\gamma) < 1.2 \times 10^{-11}$, and the lower bounds on unobserved particle masses are taken to be $m_{\tilde{l}}, m_{\tilde{q}}, M_A > 100$ GeV, and no other phenomenological bounds are imposed. Thus, our analysis should give an accurate comparison of different LFV decays and $B_{s,d} \to \mu\mu$ with each other, but may be somewhat optimistic concerning the overall rates.

In Fig. 3 we present scatter plots of Higgs-mediated $Br(\tau \to 3\mu)$ against $Br(\tau \to \mu\gamma)$ in (a) the CMSSM and (b) the GFU-MSSM. Whilst $Br(\tau \to 3\mu) < 10^{-11}$ in the CMSSM, in the GFU-MSSM the present $Br(\tau \to \mu\gamma)$ bound allows $Br(\tau \to 3\mu) < 4 \times 10^{-10}$. This increase could have been expected, since in the GFU-MSSM the Higgs masses and slepton masses are independent of each other. The points with the highest possible $Br(\tau \to 3\mu)$ in In Fig. 3 correspond to $M_A \approx 100$ GeV. However, for $\tan\beta = 60$, such low values for M_A are constrained by $B_s \to \mu\mu$. To see this, we plot in Fig. 4 $Br(\tau \to 3\mu)$ against $Br(B_s \to \mu\mu)$ in both models we consider. It follows that, in the GFU-MSSM, the present bound $Br(B_s \to \mu\mu) < 2 \times 10^{-6}$ constrains $Br(\tau \to 3\mu) < 1 \times 10^{-10}$. This should be compared with the rate for the photonic penguin contribution to $Br(\tau \to 3\mu)$, which is given by (5). Our results show that, *after imposing the* $\tau \to \mu\gamma$ *and* $B_s \to \mu\mu$ *bounds*, the photonic penguin dominates over the Higgs-mediated contribution.

We emphasize that we have been seeking to maximize LFV effects in these figures. We also note that the lower bounds on the LFV processes in the plots are artificial, being due to the lower bounds on the generated values of Y_ν.

Proceeding with the B-meson decays, we plot in Fig. 5 the Higgs-mediated $Br(B_s \to \mu\tau)$ against $Br(B_s \to \mu\mu)$ in (a) the CMSSM and (b) the GFU-MSSM. Again, there is a sharp upper bound on $Br(B_s \to \mu\tau)$ in both models. In the GFU-MSSM the maximum allowed value for $Br(B_s \to \mu\tau)$ is 4×10^{-9}, which is larger than that for $\tau \to 3\mu$. However, detecting τ leptons is experimentally challenging, particularly in a high-rate environment such as the LHC. To see the dependence of $B_s \to \mu\tau$ on M_A, we plot in Fig. 6 the value of $Br(B_s \to \mu\tau)$ against M_A. We emphasize that M_A can be very small in the GFU-MSSM [24], whilst in the CMSSM there is an approximate lower bound $M_A > 200$ GeV, because it is related to the slepton masses. The points with the largest $Br(B_s \to \mu\tau)$ in Fig. 6, as well as the points with the largest $Br(\tau \to 3\mu)$ in Fig. 3, 4, correspond to $M_A = \mathcal{O}(100)$ GeV.

Similar results are also valid for the decays $\tau \to e\mu\mu$ and $B_s \to e\tau$. Since phenomenologically the $\tau - e$ transition can be as large as the $\tau - \mu$ one [21], we have the same bounds $Br(\tau \to e\mu\mu) < 4 \times 10^{-10}$ and $Br(B_s \to e\tau) < 4 \times 10^{-9}$. The scatter plots for those processes are indistinguishable from Figs. 3 to 6, so we do not present them here. We have also studied the decays $B^0_{s,d} \to \mu e$ and $\mu \to eee$. The Higgs-mediated contributions to $Br(B^0_{s,d} \to \mu e)$ and $Br(\mu \to eee)$ are suppressed by the $\mu \to e\gamma$ constraint and small Yukawa couplings to be below 10^{-15} and 10^{-21}, respectively.

4. Conclusions.

In this Letter we have studied the allowed rates for Higgs-mediated LFV decays in supersymmetric seesaw models, assuming that LFV is generated entirely by the renormalization effects of neutrino Yukawa couplings. Even if we allow the mass scales for squarks, sleptons and two Higgs doublets to differ from each other, the bounds due to the decays $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\mu \to e\gamma$ and $B_s \to \mu\mu$ are very constraining. We obtain the following bounds on the Higgs-mediates processes: $Br(B^0_s \to \mu\tau, e\tau) \lesssim 4 \times 10^{-9}$ and $Br(\tau \to \mu\mu\mu, e\mu\mu) \lesssim 4 \times 10^{-10}$. We have discussed these numerical results in terms of approximate analytical expressions, but our calculations are more exact. We conclude that the Higgs-mediated contributions to $\tau \to \mu\mu\mu, e\mu\mu$ are subleading compared to the photonic penguin ones.

Acknowledgements

A.D would like to thank Maxim Pospelov for illuminating discussions during the SUSY 02 conference in Hamburg. This work was partially supported by EU TMR contract No. HPMF-CT-2000-00460, by ESF grant No. 5135. A.D. would also like to acknowledge financial support from the CERN Theory Division and the Network RTN European Program HPRN-CT-2000-00148 'Physics Across the Present Energy Frontier: Probing the Origin of Mass'.

References

[1] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50**, 7048 (1994) [arXiv:hep-ph/9306309]; T. Blazek, S. Raby and S. Pokorski, Phys. Rev. D **52**, 4151 (1995) [arXiv:hep-ph/9504364]; M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Nucl. Phys. B **577**, 88 (2000) [arXiv:hep-ph/9912516].

[2] C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D **59**, 095005 (1999) [hep-ph/9807350].

[3] K. S. Babu and C. Kolda, Phys. Rev. Lett. **84**, 228 (2000) [hep-ph/9909476].

[4] C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D **64**, 074014 (2001) [arXiv:hep-ph/0104284]; arXiv:hep-ph/0204225.

[5] G. Isidori and A. Retico, JHEP **0111**, 001 (2001) [arXiv:hep-ph/0110121]; arXiv:hep-ph/0208159.

[6] S. R. Choudhury and N. Gaur, Phys. Lett. B **451**, 86 (1999) [arXiv:hep-ph/9810307]. C. S. Huang, W. Liao, Q. S. Yan and S. H. Zhu, Phys. Rev. D **63**, 114021 (2001) [Erratum-ibid. D **64**, 059902 (2001)] [arXiv:hep-ph/0006250]; P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63**, 054012 (2001) [hep-ph/0008046]; C. Bobeth, A. J. Buras, F. Kruger and J. Urban, Nucl. Phys. B **630**, 87 (2002) [arXiv:hep-ph/0112305].

[7] A. Dedes, H. K. Dreiner and U. Nierste, Phys. Rev. Lett. **87**, 251804 (2001) [arXiv:hep-ph/0108037]; A. Dedes, H. K. Dreiner, U. Nierste and P. Richardson, arXiv:hep-ph/0207026; R. Arnowitt, B. Dutta, T. Kamon and M. Tanaka, arXiv:hep-ph/0203069; S. w. Baek, P. Ko and W. Y. Song, arXiv:hep-ph/0205259; J. K. Mizukoshi, X. Tata and Y. Wang, arXiv:hep-ph/0208078; T. Ibrahim and P. Nath, arXiv:hep-ph/0208142.

[8] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **619**, 434 (2001) [arXiv:hep-ph/0107048]; arXiv:hep-ph/0207241; G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, arXiv:hep-ph/0207036.

[9] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 1562 (1998) [arXiv:hep-ex/9807003]; Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **89**, 011301 (2002) [arXiv:nucl-ex/0204008].

[10] M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (eds. A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[11] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).

[12] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996) [arXiv:hep-ph/9510309]; J. Hisano and D. Nomura, Phys. Rev. D **59**, 116005 (1999) [arXiv:hep-ph/9810479].

[13] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B **110**, 44 (1982); R. Barbieri and R. Gatto, Phys. Lett. B **110**, 211 (1982).

[14] K. S. Babu and C. Kolda, arXiv:hep-ph/0206310; In the same perspective, the branching ratio of $\tau \to \mu\eta$ has been recently studied by M. Sher, arXiv:hep-ph/0207136, with similar results to $\tau \to 3\mu$ branching ratio.

[15] LFV in doublet-Higgs decays in heavy Majorana and Dirac neutrino mass models has been studied in A. Pilaftsis, Phys. Lett. B **285**, 68 (1992); A. Ilakovac and A. Pilaftsis, Nucl. Phys. B **437**, 491 (1995) [arXiv:hep-ph/9403398]; J. L. Diaz-Cruz and

J. J. Toscano, Phys. Rev. D **62**, 116005 (2000) [arXiv:hep-ph/9910233]. LFV in triplet-Higgs neutrino mass models has been studied in E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. **85**, 3769 (2000) [arXiv:hep-ph/0006046]; E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001) [Erratum-ibid. **87**, 159901 (2001)] [arXiv:hep-ph/0102255].

[16] A model-independent analysis of $\tau - \mu$ LFV in B-meson decays has been carried out in D. Black, T. Han, H. J. He and M. Sher, arXiv:hep-ph/0206056.

[17] F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **57**, 3811 (1998).

[18] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0207083.

[19] T. Bergfeld *et al.* [CLEO Collaboration], Phys. Rev. D **62**, 091102 (2000) [arXiv:hep-ex/0007042].

[20] C. W. Bernard, Nucl. Phys. Proc. Suppl. **94**, 159 (2001) [arXiv:hep-lat/0011064].

[21] J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, arXiv:hep-ph/0206110.

[22] C. Giunti, arXiv:hep-ph/0209103.

[23] J. R. Ellis and M. Raidal, arXiv:hep-ph/0206174.

[24] J. R. Ellis, K. A. Olive and Y. Santoso, Phys. Lett. B **539**, 107 (2002) [arXiv:hep-ph/0204192]; J. R. Ellis, T. Falk, K. A. Olive and Y. Santoso, CERN preprint CERN-TH/2002-238 (in preparation).

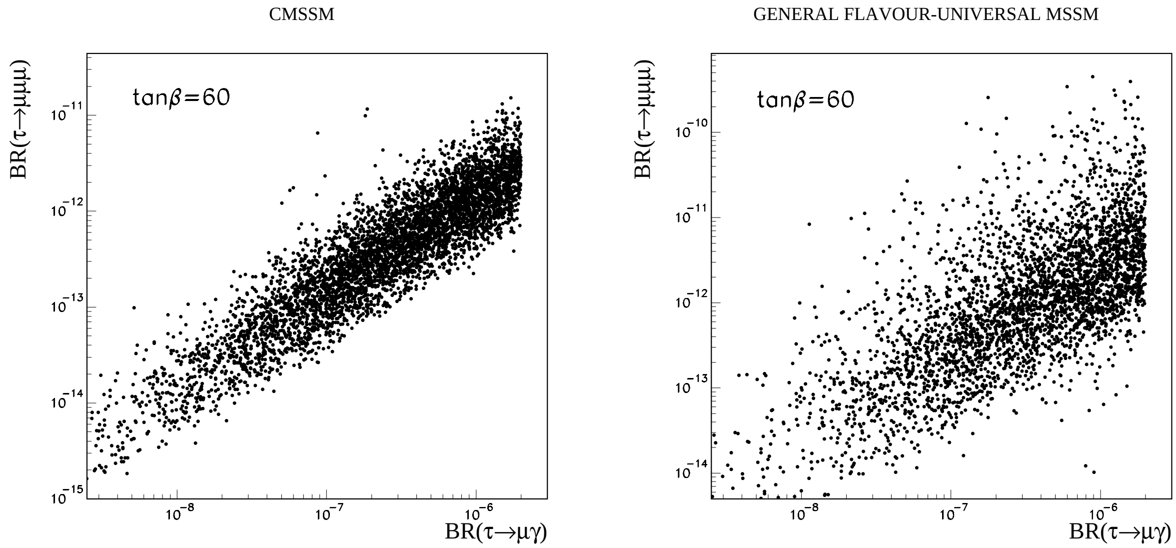


Figure 3: *Scatter plot of Higgs-mediated* $Br(\tau \to 3\mu)$ *against* $Br(\tau \to \mu\gamma)$ *in (a) the CMSSM and (b) the GFU-MSSM.*

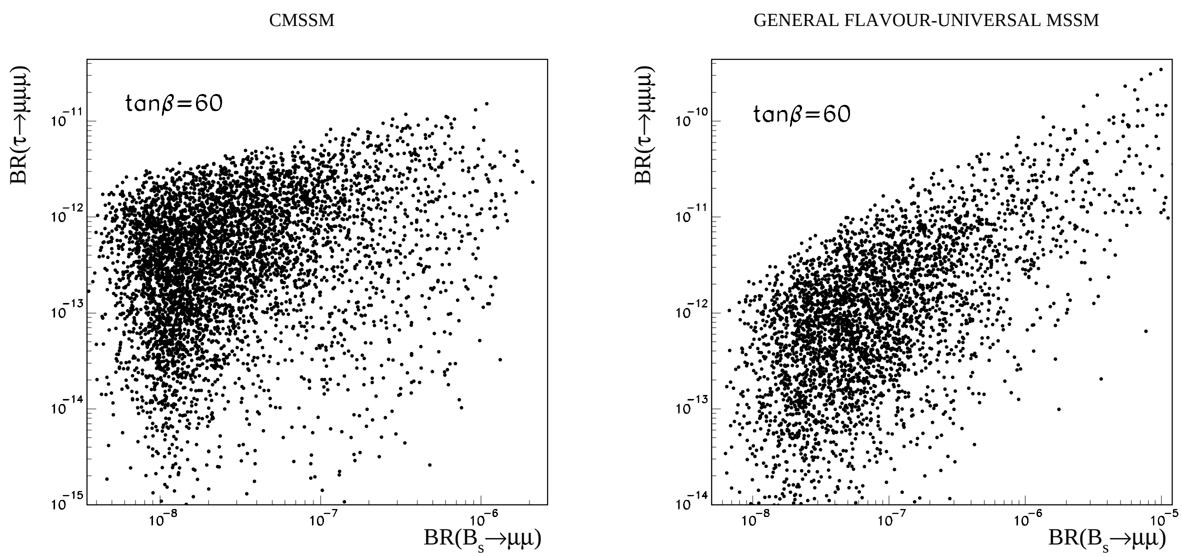


Figure 4: *Scatter plot of Higgs-mediated* $Br(\tau \to 3\mu)$ *against* $Br(B_s \to \mu\mu)$ *in (a) the CMSSM and (b) the GFU-MSSM.*

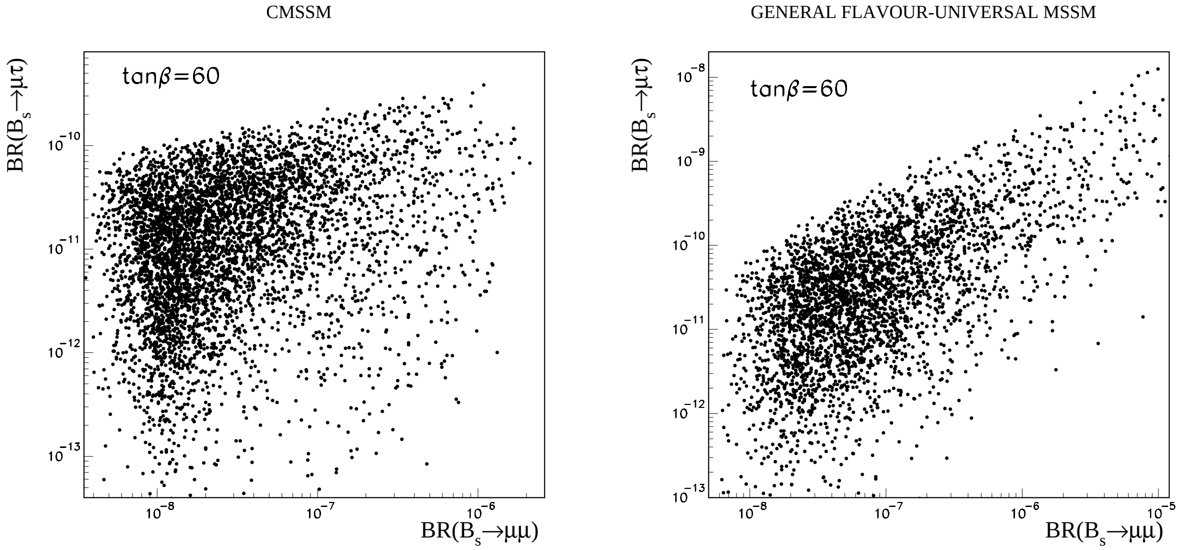


Figure 5: *Scatter plot of Higgs-mediated* $Br(B_s \to \mu\tau)$ *against* $Br(B_s \to \mu\mu)$ *in (a) the CMSSM and (b) the GFU-MSSM.*

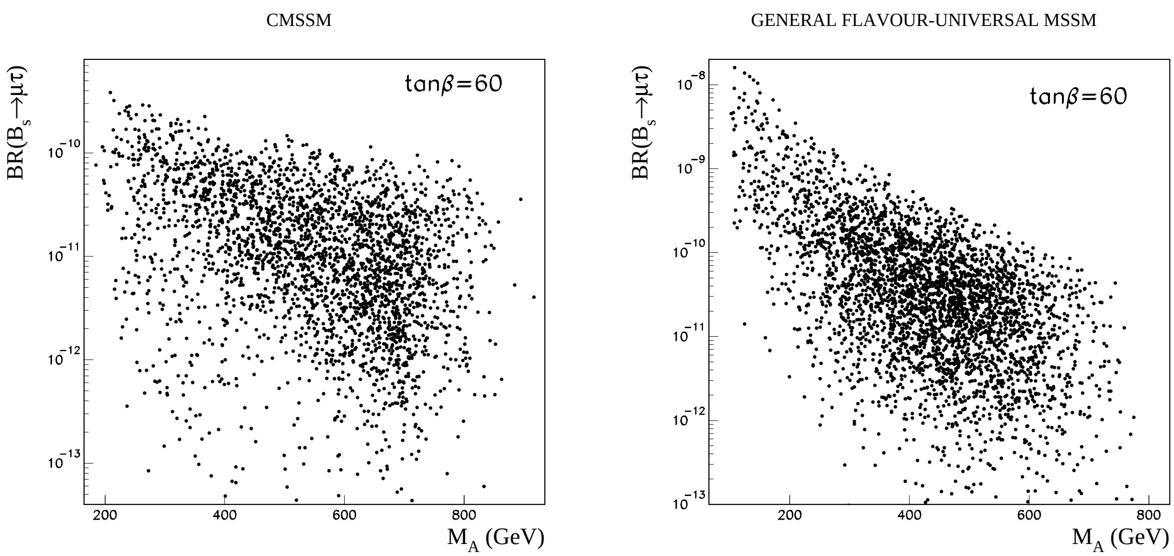


Figure 6: *Scatter plot of Higgs-mediated* $Br(B_s \to \mu\tau)$ *against the pseudoscalar Higgs mass* M_A *in (a) the CMSSM and (b) the GFU-MSSM.*