$K^+ \to \pi^+\pi^0$ decays at next-to-leading order in the chiral expansion on finite volumes *

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We present the ingredients for determining $K^+ \to \pi^+\pi^0$ matrix elements via the combination of lattice QCD and chiral perturbation theory (χPT). By simulating these matrix elements at unphysical kinematics, it is possible to determine all the low-energy constants (LECs) for constructing the physical $K^+ \to \pi^+\pi^0$ amplitudes at next-to-leading order (NLO) in the chiral expansion. In this work, the one-loop chiral corrections are calculated for arbitrary meson four-momenta, in both χPT and quenched χPT (qχPT), and the finite-volume effects are studied.

1. INTRODUCTION

The need for a high-precision prediction for $K \to \pi\pi$ amplitudes is underlined by the recent experimental measurement of $\mathrm{Re}(\epsilon'/\epsilon)$ and the long-standing puzzle, the $\Delta I = 1/2$ rule. Although the finite-volume (FV) techniques developed in Refs. [3–5] can ultimately enable an accurate calculation of $K \to \pi\pi$ decay rates, the most practical approach to the numerical calculation of these decay rates remains the combination of lattice QCD and (quenched and partially quenched) χPT. Apart from a calculation for the CP-conserving, $\Delta I = 3/2$, $K \to \pi\pi$ decay in Ref. [1], all the numerical studies hitherto follow a strategy proposed in Ref. [2], which only allows the determination of these amplitudes at leading-order (LO) in the chiral expansion[3]. Because of the large kaon mass and the presence of final state interactions, non-LO corrections in this expansion are significant. In a recent work [6,7], we have proposed to perform lattice simulations at unphysical kinematics over a range of meson masses and momenta, from which we can determine all the necessary LECs for constructing the physical matrix element $\langle \pi^+\pi^0|\mathcal{O}^{\Delta S=1}|K^+\rangle$ at NLO in the chiral expansion. We have suggested a specific unphysical kinematics[4], the SPQR kinematics as explained in detail in Refs. [6,7], which enables such a procedure, and have studied the following $\Delta S = 1$ operators (α, β are colour indices)

$$\begin{aligned} Q_4 &= (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta - \bar{d}_\beta d_\beta)_L \\ &\quad +(\bar{s}_\alpha u_\alpha)_L(\bar{u}_\beta d_\beta)_L, \\ Q_7 &= \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_R, \\ Q_8 &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_R, \end{aligned} \tag{1}$$

where e_q is the electric charge of q and $(\bar{\psi}_1\psi_2)_{L,R}$ means $\bar{\psi}_1\gamma_\mu(1 \mp \gamma_5)\psi_2$.

2. FINITE-VOLUME EFFECTS

In Ref. [6], we investigate the FV corrections, power-like in $1/L$, which arise from replacing sums by integrals in the one-loop calculation that involves the diagrams in Fig. 1 [5]. It can be shown

*SHEP 02-18.

†Presenter at the conference.

[3] In Ref. [1], the decay amplitude is also obtained at the precision of leading-order in the chiral expansion.

[4] Another choice is considered in Ref. [8].

[5] Such a calculation for $\langle \pi^+\pi^0|Q_4|K^+\rangle$ at two particular kinematics, $M_K = M_\pi$ and $M_K = 2M_\pi$ with all mesons

that a diagram which does not have an imaginary part in Minkowski space will only have FV corrections exponential in L, therefore only diagram (c) contributes to the $1/L^n$ corrections. Because the two-pion final state has $I = 2$, this diagram only contains four-quark intermediate states and there are no disconnected quark-loops in the quark-flow picture. For the same reason, it does not receive contributions from the η' propagator. Hence the $1/L^n$ effects are identical in χPT and quenched χPT (qχPT) for $K^+ \to \pi^+\pi^0$ at this order, and only at this order. This is not true for $\Delta I = 1/2$ decay amplitudes [9,10].

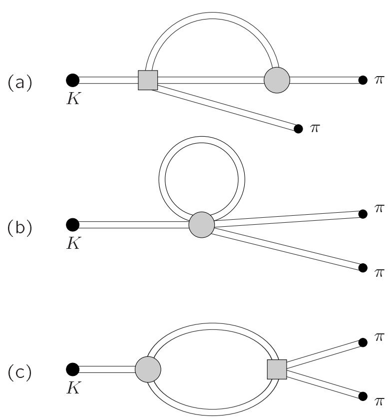

Figure 1. *One-loop diagrams for $K^+ \to \pi^+\pi^0$ amplitudes. The grey circles (squares) are weak (strong) vertices. The diagrams for wavefunction renormalisation are not shown here.*

We have found that in the center-of-mass frame, for all the $K^+ \to \pi^+\pi^0$ amplitudes, these one-loop $1/L^n$ corrections are independent of the weak operators and can be removed by a universal factor derived first by Lellouch and Lüscher in Ref. [3]. This modifies the conclusion of Refs. [11,12], in which a FV effect resulting in the shift of the two-pion total energy in the argument of the tree-level amplitudes is interpreted as a genuine $1/L^n$ correction to the matrix elements, and therefore the FV effects in $\langle \pi^+\pi^0 | Q_4 | K^+ \rangle$ are found to depend on M_K.

at rest, have been performed in Refs. [11,12].

We are currently investigating the FV effects of these amplitudes in a moving frame. The Lellouch-Lüscher factor has not yet been derived for this, while the modification of Lüscher's quantisation condition [13,14] relating the infinite-volume $\pi\pi$ scattering phase to the FV two-pion energy spectrum, due to the moving frame was obtained in Ref. [15]. As a by-product of our work, we verify that the energy shift obtained in one-loop perturbation theory in a moving frame agrees with the expansion of the quantization condition in Ref. [15] to the same order.

3. ONE-LOOP CHIRAL CORRECTIONS IN INFINITE VOLUME

We evaluate the one-loop correction by using dimensional regularisation and subtracting $\log(4\pi) - \gamma_E + 1 + 2/(4-d)$. The lowest-order amplitudes are all proportional to $1/f^3$, where f is the light pseudoscalar meson decay constant in the chiral limit. At NLO, we choose to express $1/f^3$ in terms of $1/(f_\pi^2 f_K)$. This factor fully absorbs the dependence upon the Gasser-Leutwyler LECs L_4 and L_5 introduced via wavefunction renormalisation.

In Ref. [6], the one-loop diagrams have been calculated for arbitrary external meson four-momenta in both χPT and qχPT. The results are lengthy and are presented on a web site [16]. In Fig. 2, we show an example of these results for $\langle \pi^+\pi^0 | \mathcal{O}_{7,8} | K^+ \rangle$. These plots are the ratios between the one-loop corrections and the lowest-order matrix elements with both final-state pions at rest. Fig. 2a is the result in χPT and Fig. 2b is that in qχPT. This figure suggests that in a quenched numerical calculation of these matrix elements, it is not appropriate in general to perform chiral extrapolation using unquenched χPT results [17]. This is confirmed by numerical data [18].

4. CONCLUSIONS

We have made theoretical progress towards the calculation of $K \to \pi\pi$ decay amplitudes via the combination of lattice QCD and χPT. We find

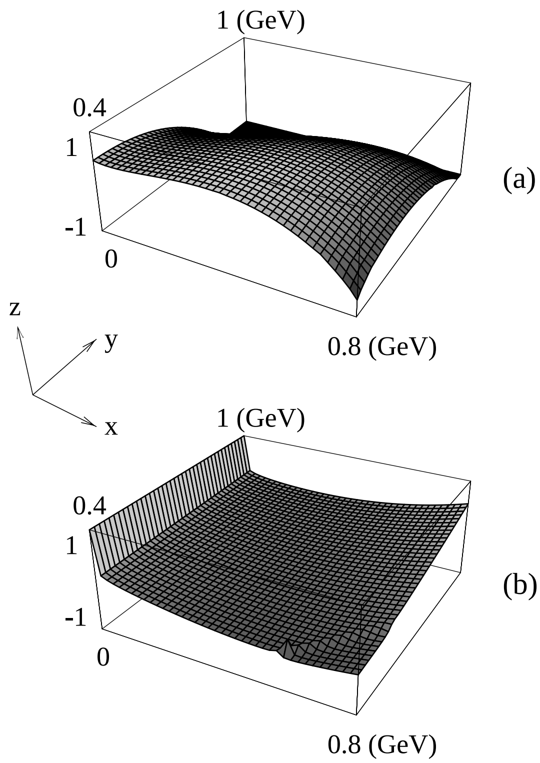

Figure 2. *Ratio between the one-loop correction, at the renormalisation scale 0.7 GeV, and the lowest-order result for* $\langle\pi^+\pi^0|\mathcal{O}_{7,8}|K^+\rangle$ *in (a)* χPT *and (b)* $q\chi PT$*. The x axis is* M_π *and the y axis is* M_K*. In (b), the coupling accompanying the kinetic term of the* η' *propagator is set to zero, and the* η' *mass is taken to be* $M_0 = 0.5$ *GeV. The one-loop results are not very sensitive to these parameters. The singular behaviour along the line* $M_\pi = \sqrt{2}M_K$ *in (b) is due to the fact that when performing the* $q\chi PT$ *calculation, we use a basis in which the pseudo-Goldstone states are* $\bar{q}q'$ *mesons, where* q *and* q' *are* u, d *and* s*, and the* $\bar{s}s$ *meson becomes tachyonic when* $M_\pi > \sqrt{2}M_K$*.*

it feasible to determine all the LECs for constructing $_{I=2}\langle\pi\pi|\mathcal{O}^{\Delta S=1}|K\rangle$ at NLO in the chiral expansion. A quenched numerical study is in progress [18]. As for the $\Delta I = 1/2$ channel, we find the situation to be considerably more complicated [9].

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