Determination of weak phases ϕ_2 and ϕ_3 from $B \to \pi\pi, K\pi$ in the pQCD method

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Abstract

We look at methods to determine the weak phases ϕ_2 and ϕ_3 from $B \to \pi\pi$ and $K\pi$ decays within the perturbative QCD approach. We obtain quite interesting bounds on ϕ_2 and ϕ_3 from experimental measurement in B-factory: $55^o \le \phi_2 \le 100^o$ and $51^o \le \phi_3 \le 129^o$. Specially we predict the possibility of large direct CP violation effect in $B^0 \to \pi^+\pi^- \ (23 \pm 7\%)$ decay.

1 INTRODUCTION

One of the most exciting aspect of present high energy physics is the exploration of CP violation in B-meson decays, allowing us to overconstrain both sides and the three weak phases $\phi_1(=\beta)$, $\phi_2(=\alpha)$ and $\phi_3(=\gamma)$ of the unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] and to check the possibility of New Physics.

Beside the "gold-plated" mode $B_d \to J/\psi K_s$[2] which allow us to determine ϕ_1 without any hadron uncertainty, recently measured by BaBar and Belle collaborations[3]. There are many interesting channels with which we may achieve this goal by the dermination of ϕ_2 and ϕ_3[4].

In this letter, we focus on the $B \to \pi^+\pi^-$ and $K\pi$ processes, providing promising strategies to determine the weak phases of ϕ_2 and ϕ_3, by using the perturbative QCD method.

The perturbative QCD method (pQCD) has a sucessful predictive power in exclusive 2 body B-meson decays, specially in charmless B-meson decay processes[5]. By introducing parton transverse momenta $k_\perp$, we can generate naturally the sudakov suppression effect due to resummation of large double logarithms $Exp[-\frac{\alpha_s C_F}{4\pi} \ln^2(\frac{Q^2}{k_\perp^2})]$, which suppress the long-distance contributions in the small $k_\perp$ region and give a sizable average $< k_\perp^2 > \sim \bar{\Lambda} M_B$. This can resolve the end point singularity problem and allow the applicability of pQCD to exclusive decays. We found that almost all of the contribution to the exclusive matrix elements come from the integration region where $\alpha_s/\pi < 0.3$ and the pertubative treatment can be justified.

In the pQCD approach, we can predict the contribution of non-factorizable term and annihilation diagram on the same footing as factorizable one. A folklore for annihilation contributions is that they are negligible compare to W-emission ones due to helicity suppression. However the operators $O_{5,6}$ with helicity structure $(S-P)(S+P)$ are not suppressed and give dominant imaginary values, which is the main source of strong phase in the PQCD approach. So we have a large direct CP violation in $B \to \pi^\pm\pi^\mp, K^\pm\pi^\mp$, since large strong phase comes from the factorized annihilation diagram, which can distinguish pQCD from other models[6, 7].

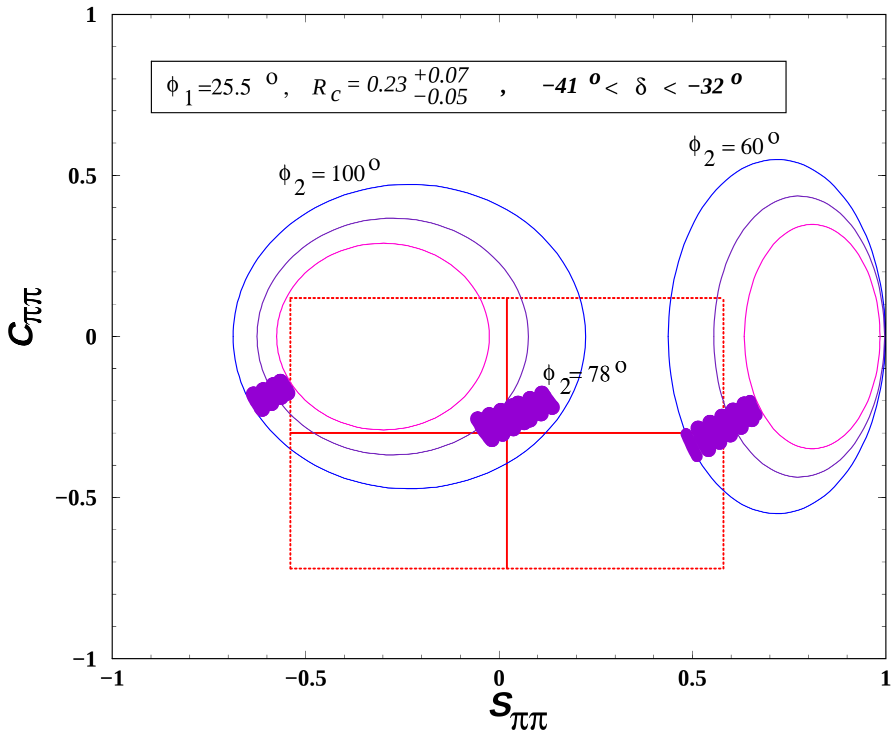

Fig. 1: Plot of $C_{\pi\pi}$ versus $S_{\pi\pi}$ for various values of ϕ_2 with $\phi_1 = 25.5^o$, $0.18 < R_c < 0.30$ and $-41^o < \delta < -32^o$ in the pQCD method. Here we consider allowed experimental ranges of BaBar measurment whinin 90% C.L. Dark areas is allowed regions in the pQCD method for different ϕ_2 values.

2 Extraction of $\phi_2(=\alpha)$ from $B \to \pi^+\pi^-$

Even though isospin analysis of $B \to \pi\pi$ can provide a clean way to determine ϕ_2, it might be difficult in practice because of the small branching ratio of $B^0 \to \pi^0\pi^0$. In reality to determine ϕ_2, we can use the time-dependent rate of $B^0(t) \to \pi^+\pi^-$ including sizable penguin contributions. In our analysis we use the c-convention. The amplitude can be written as:

$$\begin{aligned} A(B^0 \to \pi^+\pi^-) &= V_{ub}^* V_{ud} A_u + V_{cb}^* V_{cd} A_c + V_{tb}^* V_{td} A_t, \\ &= V_{ub}^* V_{ud}\, (A_u - A_t) + V_{cb}^* V_{cd} (A_c - A_t), \\ &= -(|T_c|\, e^{i\delta_T}\, e^{i\phi_3} + |P_c|\, e^{i\delta_P}) \end{aligned} \quad (1)$$

Pengun term carries a different weak phase than the dominant tree amplitude, which leads to generalized form of the time-dependent asymmetry:

$$A(t) \equiv \frac{\Gamma(\bar{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\bar{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S_{\pi\pi}\ sin(\Delta m t) - C_{\pi\pi}\ cos(\Delta m t) \quad (2)$$

where

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \qquad S_{\pi\pi} = \frac{2\, Im(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} \quad (3)$$

satisfies the relation of $C_{\pi\pi}^2 + S_{\pi\pi}^2 \leq 1$. Here

$$\lambda_{\pi\pi} = |\lambda_{\pi\pi}|\, e^{2i(\phi_2 + \Delta\phi_2)} = e^{2i\phi_2} \left[\frac{1 + R_c e^{i\delta}\, e^{i\phi_3}}{1 + R_c e^{i\delta}\, e^{-i\phi_3}} \right] \quad (4)$$

with $R_c = |P_c/T_c|$ and the strong phase difference between penguin and tree amplitudes $\delta = \delta_P - \delta_T$. The time-dependent asymmetry measurement provides two equations for $C_{\pi\pi}$ and $S_{\pi\pi}$ in terms of R_c, δ and ϕ_2.

When we define $R_{\pi\pi} = \overline{Br}(B^0 \to \pi^+\pi^-)/\overline{Br}(B^0 \to \pi^+\pi^-)|_{tree}$, where $\overline{Br}$ stands for a branching ratio averaged over B^0 and $\bar{B}^0$, the explicit expression for $S_{\pi\pi}$ and $C_{\pi\pi}$ are given by:

$$R_{\pi\pi} = 1 - 2\, R_c\, cos\delta\, cos(\phi_1 + \phi_2) + R_c^2, \quad (5)$$

$$R_{\pi\pi} S_{\pi\pi} = sin2\phi_2 + 2\, R_c\, sin(\phi_1 - \phi_2)\, cos\delta - R_c^2 sin2\phi_1, \quad (6)$$

$$R_{\pi\pi} C_{\pi\pi} = 2\, R_c\, sin(\phi_1 + \phi_2)\, sin\delta. \quad (7)$$

If we know R_c and δ, we can determine ϕ_2 from the experimental data on $C_{\pi\pi}$ versus $S_{\pi\pi}$.

Since the pQCD method provides $R_c = 0.23^{+0.07}_{-0.05}$ and $-41^o < \delta < -32^o$, the allowed range of ϕ_2 at present stage is determined as $55^o < \phi_2 < 100^o$ as shown in Figure 1. Since we have a relatively large strong phase than QCD-factorization ($\delta \sim 0^o$), we predict large direct CP violation effect of $A_{cp}(B^0 \to \pi^+\pi^-) = (23 \pm 7)\%$ which will be tested by more precise experimental measurement in future. In our numerical analysis, since the data by Belle collaboration[8] is placed ourside allowed physical regions, we only considered the recent BaBar measurement[9] with 90% C.L. interval taking into account the systematic errors:

- $S_{\pi\pi} = \quad 0.02 \pm 0.34 \pm 0.05$ [-0.54, +0.58]
- $C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04$ [-0.72, +0.12].

The central point of BaBar data corresponds to $\phi_2 = 78^o$ in pQCD method.

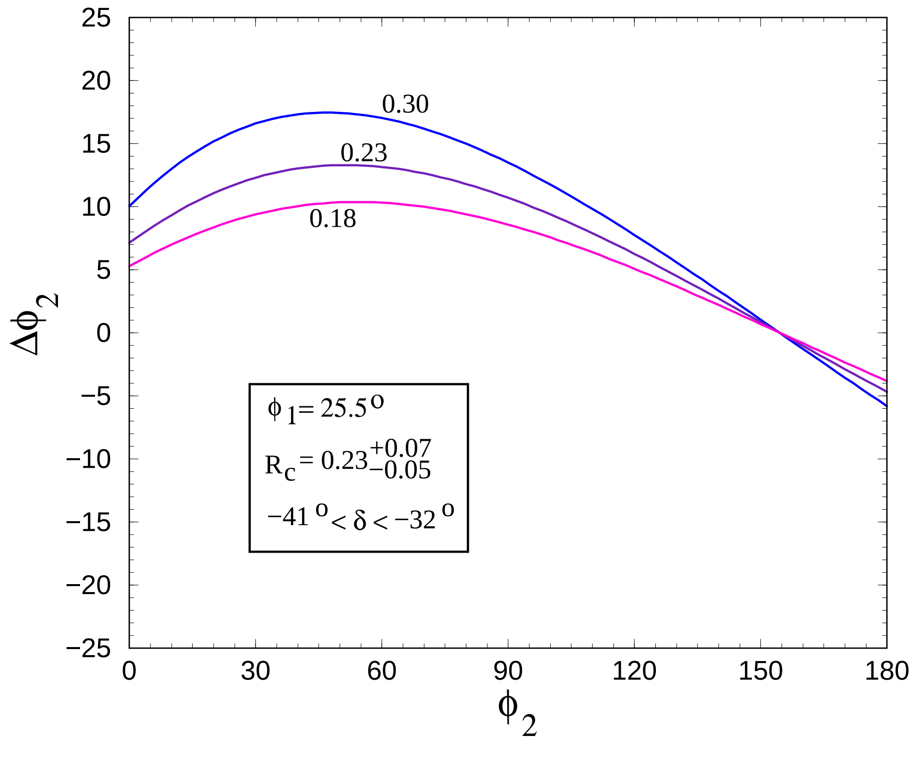

Fig. 2: Plot of $\Delta\phi_2$ versus ϕ_2 with $\phi_1 = 25.5^o$, $0.18 < R_c < 0.30$ and $-41^o < \delta < -32^o$ in the pQCD method.

The $\Delta\phi_2$ is the deviation of ϕ_2 due to the penguin contribution, derived from Eq.(4), can be determined with known values of R_c and δ by using the relation $\phi_3 = 180 - \phi_1 - \phi_2$. In figure 2 we show our pQCD prediction on the relation $\Delta\phi_2$ versus ϕ_2. For allowed regions of $\phi_2 = (55 \sim 100)^o$, $\Delta\phi_2 = (8 \sim 16)^o$ and main uncertainties come from the uncertainty of $|V_{ub}|$. The non-zero value of $\Delta\phi_2$ demonstrates sizable penguin contributions in $B^0 \to \pi^+\pi^-$ decay.

3 Extraction of $\phi_3(=\gamma)$ from $B^0 \to K^+\pi^-$ and $B^+ \to K^0\pi^+$ Processes

By using tree-penguin interference in $B^0 \to K^+\pi^-(\sim T^{'} + P^{'})$ versus $B^+ \to K^0\pi^+(\sim P^{'})$, CP-averaged $B \to K\pi$ branching fraction may lead to non-trivial constaints on the ϕ_3 angle[10]. In order to determine ϕ_3, we need one more useful information on CP-violating rate differences[11]. Let's introduce the following observables :

$$\begin{aligned} R_K &= \frac{\overline{Br}(B^0 \to K^+\pi^-)\ \tau_+}{\overline{Br}(B^+ \to K^0\pi^+)\ \tau_0} = 1 - 2\ r_K\, cos\delta\ cos\phi_3 + r_K^2 \\ &\qquad\qquad\qquad\qquad\quad \geq sin^2\phi_3 \end{aligned} \tag{8}$$

$$\begin{aligned} A_0 &= \frac{\Gamma(\bar{B}^0 \to K^-\pi^+ - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(B^- \to \bar{K}^0\pi^-) + \Gamma(B^+ \to \bar{K}^0\pi^+)} \\ &= A_{cp}(B^0 \to K^+\pi^-)\ R_K = -2r_K\, sin\phi_3\, sin\delta. \end{aligned} \tag{9}$$

where $r_K = |T^{'}/P^{'}|$ is the ratio of tree to penguin amplitudes and $\delta = \delta_{T'} - \delta_{P'}$ is the strong phase difference between tree and penguin amplitides. After eliminate $sin\delta$ in Eq.(8)-(9), we have

$$R_K = 1 + r_K^2 \pm \sqrt{(}4r_K^2 cos^2\phi_3 - A_0^2 cot^2\phi_3). \tag{10}$$

Here we obtain $r_K = 0.201 \pm 0.037$ from the pQCD analysis[5] and $A_0 = -0.11 \pm 0.065$ by combining recent BaBar measurement on CP asymmetry of $B^0 \to K^+\pi^-$: $A_{cp}(B^0 \to K^+\pi^-) = -10.2 \pm 5.0 \pm 1.6\%$ [9] with present world averaged value of $R_K = 1.10 \pm 0.15$[12].

As shown in Figure 3, we can constrain ϕ_3 with $1\,\sigma$ range of World Averaged R_K as follows:

- For $cos\delta > 0$, $r_K = 0.164$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $24^o \leq \phi_3 \leq 75^0$.
- For $cos\delta > 0$, $r_K = 0.201$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $27^o \leq \phi_3 \leq 75^0$.
- For $cos\delta > 0$, $r_K = 0.238$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $34^o \leq \phi_3 \leq 75^0$.
- For $cos\delta < 0$, $r_K = 0.164$: we can exclude $0^o \leq \phi_3 \leq 6^0$.
- For $cos\delta < 0$, $r_K = 0.201$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $35^o \leq \phi_3 \leq 51^0$.
- For $cos\delta < 0$, $r_K = 0.238$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $24^o \leq \phi_3 \leq 62^0$.

From the table 2 of ref.[13], we obtain $\delta_{P'} = 157^o$ and $\delta_{T'} = 1.4^o$, therfore the value of $cos\delta$ becomes negative, $cos\delta = -0.91$. The maximum value of the constraint bound for the ϕ_3 is strongly depend on the value of $|V_{ub}|$. When we take the central value of $r_K = 0.201$, ϕ_3 is allowed within the ranges of $51^o \leq \phi_3 \leq 129^o$, which is consistent with the results by the model-independent CKM-fit in the (ρ, η) plane.

4 CONCLUSION

We discuss two methods to determine weak phases ϕ_2 and ϕ_3 within the pQCD approach through 1) Time-dependent asymmetries in $B^0 \to \pi^+\pi^-$, 2) $B \to K\pi$ processes via penguin-tree interference. Already we can get interesting bounds on ϕ_2 and ϕ_3 from present experimental

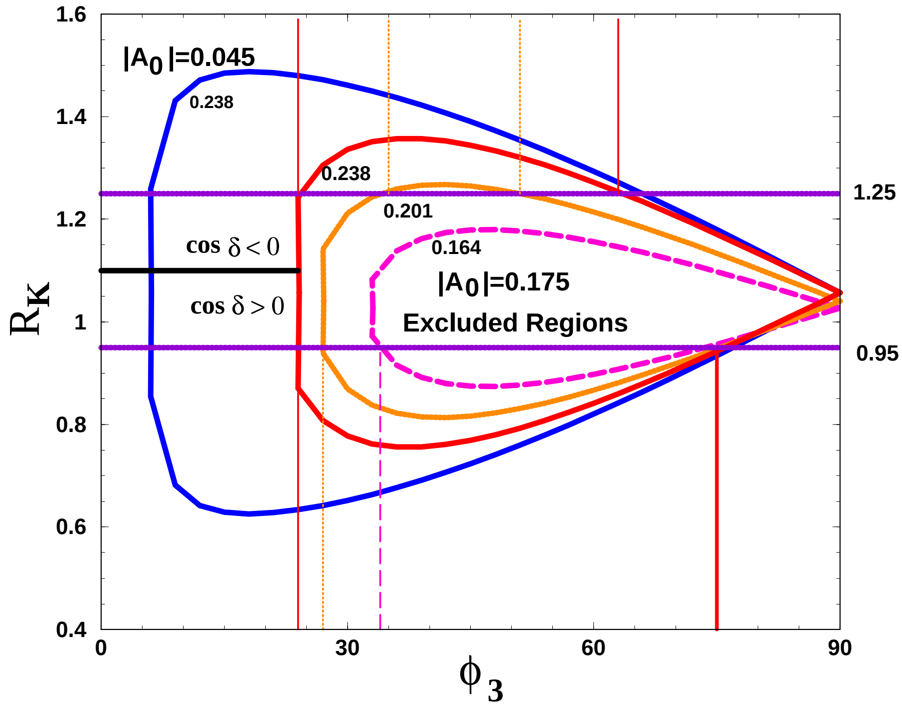

Fig. 3: Plot of R_K versus ϕ_3 with $r_K = 0.164, 0.201$ and 0.238.

measurements. Our predictions within pQCD method is well agreed with present experimental measurements in charmless B-decays. Specially our pQCD method predicted a large direct CP asymmetry in $B^0 \to \pi^+\pi^-$ decay, which will be a crucial touch stone to distinguish our approach from others in future precise measurement. More detail works on other methods in $B \to K\pi$ and $D^{(*)}\pi$ processes will be appeared elsewhere. [13].

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