The universal freeze-out criterion at SPS and RHIC

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We formulate a freeze-out criterion for ultra-relativistic heavy ion collisions in terms of the pion escape probability from the collision region. We find that the increase in pion phase-space density from SPS to RHIC reported at this conference has a small influence on particle freeze-out because of the small $\pi\pi$ cross-section. Our treatment takes into account dynamical expansion, chemical composition and momentum of the escaping particle for particle freeze-out. It supports a freeze-out at rather low temperature—below 100 MeV—and earlier decoupling of high $p_\perp$ particles.

A precise formulation of the condition under which particles decouple ("freeze-out") from a heavy ion collision is important for understanding the dynamical origin of the final state. The observation of STAR [1] that the pion phase-space density increases at RHIC indicates that freeze-out is not characterised solely by a universal value of *pion phase-space density*, in contrast to a recent suggestion [2]. Also, freeze-out is not characterised solely by a universal value of the *total particle density* since the volume from which particles decouple shows a non-monotonous dependence on collision energy [3].

It has been argued [4] and demonstrated [3] that the chemical composition of the system must be taken into account. For pion freeze-out, the nucleon and anti-nucleon density is more important than that of pions since the pion-nucleon cross-section is larger than the pion-pion one. With increasing collision energy the pion abundance and their contribution to scattering grows, but it is argued that the *momentum-averaged* mean free path of a pion at freeze-out stays constant [3].

This argumentation still ignores the influence of global dynamics on the freeze-out criterion. A strong expansion makes the particle density decrease fast and if the characteristic time scale for density decrease $\tau_{\mathrm{exp}} = (\partial_\mu u^\mu)^{-1}$ is smaller than the mean free time between collisions τ_{scatt} a particle is likely to seize interacting, i.e., to freeze-out [5].

In our work we study—in addition to these two aspects—how the probability for a particle to decouple depends on its momentum [6]. We formulate freeze-out as condition for an individual particle: a *particle freezes-out* at the time of its last scattering. This is in contrast to the "fireball freeze-out" which is understood as a boundary in space-time between interacting matter and free-streaming particles. In the extreme case of a hydrodynamic model the "fireball freeze-out" is usually treated by the Cooper-Frye prescription [7] in which *all* particles decouple along the same three-dimensional hyper-surface when *the medium* fulfils certain condition. Our treatment goes beyond Cooper-Frye: particle freeze-out depends on its *type, position, and momentum* in addition to the characteristics of the medium. This corresponds to freeze-out as described by a cascade event generator.

The freeze-out criterion which we explore is based on the *escape probability* [8]

$$\mathcal{P}(x,p,\tau) = \exp\left(-\int_\tau^\infty d\bar\tau\, \mathcal{R}(x+v\bar\tau,p)\right)\,, \tag{1}$$

which determines the probability that a pion emitted with momentum p from the position $(x,\,\tau)$ escapes from the medium without further interaction. Here, the scattering rate $\mathcal{R}(x,p)$ is integrated along the trajectory of the pion. Freeze-out is assumed to occur when $\mathcal{P}(x,p,\tau)$ reaches a *universal* value of order one.

The role of collective expansion: To illustrate how collective expansion affects particle freeze-out, we consider the case of particles with vanishing momentum in the centre of a fireball with longitudinally boost-invariant and transversely linear expansion velocity profile of the form

$$u^\mu \;=\; (\cosh\eta\,\cosh\eta_t,\,\cos\phi\,\sinh\eta_t,\,\sin\phi\,\sinh\eta_t,\,\sinh\eta\,\cosh\eta_t)\,. \tag{2}$$

Here, $\eta = \mathrm{Arctanh}(z/t)$ and $\eta_t = \xi r$ where $\xi = 0.08\,\mathrm{fm}^{-1}$ is a typical value for the transverse expansion gradient [9]. At the time τ_{em} at which the pion is emitted, the density of scattering centres decreases by the rate

$$-\frac{1}{\rho}\frac{\partial\rho}{\partial\tau}\bigg|_{\tau=\tau_{\mathrm{em}}} = \partial_\mu u^\mu = \frac{1}{\tau_{\mathrm{em}}} + 2\xi\,. \tag{3}$$

A power-law decrease of density with time is consistent with (3) if

$$\rho(\tau) = \rho_{\mathrm{em}}\left(\frac{\tau_{\mathrm{em}}}{\tau}\right)^\alpha\,,\qquad \alpha = 1+2\xi\tau_{\mathrm{em}}\,. \tag{4}$$

Under the assumption $\mathcal{R}\propto\rho$ we obtain for the opacity integral of a particle of zero momentum

$$\int_{\tau_{\mathrm{em}}}^\infty d\tau\,\mathcal{R}(\tau) = \frac{\mathcal{R}_{\mathrm{em}}}{\alpha-1}\tau_{\mathrm{em}} = \frac{\mathcal{R}_{\mathrm{em}}}{2\xi}\,, \tag{5}$$

where $\mathcal{R}_{\mathrm{em}}$ is the scattering rate at τ_{em}. Technically, the calculation of this opacity integral for particles of non-zero momentum p is more complicated, since one has to follow the propagation of the particle through layers of different density. For a quantitative statement, this requires a model of the space-time evolution of the fireball. Qualitatively, however, the general result will be consistent with the main feature of the case $p=0$ considered here: as seen from (5), freeze-out from a denser fireball (larger $\mathcal{R}_{\mathrm{em}}$) is possible if it is compensated by a stronger expansion gradient such that the value of opacity integral stays constant.

The role of chemical composition: To determine the value of $\mathcal{R}_{\mathrm{em}}$ for collisions at $\sqrt{s} = 17\,A\mathrm{GeV}$ (SPS) and $\sqrt{s} = 130\,A\mathrm{GeV}$ (RHIC), we calculate the scattering rate

$$\mathcal{R}(p) = \sum_i \int d^3k\,\rho_i(k)\,\sigma_i'(s)\,|v_\pi - v_i|\,, \tag{6}$$

where σ_i' is the cross-section for pion scattering on species i. For the collinear cross-section we use the parametrisation

$$\sigma_i(\sqrt{s}) = \sum_r \langle j_i,m_i,j_\pi,m_\pi||J_r,M_r\rangle \frac{2S_r+1}{(2S_i+1)(2S_\pi+1)}\frac{\pi}{p_{\mathrm{CMS}}^2}\frac{\Gamma_{r\to\pi i}\Gamma_{i,\mathrm{tot}}}{(M_r-\sqrt{s})^2+\Gamma_{i,\mathrm{tot}}^2/4}\,, \tag{7}$$

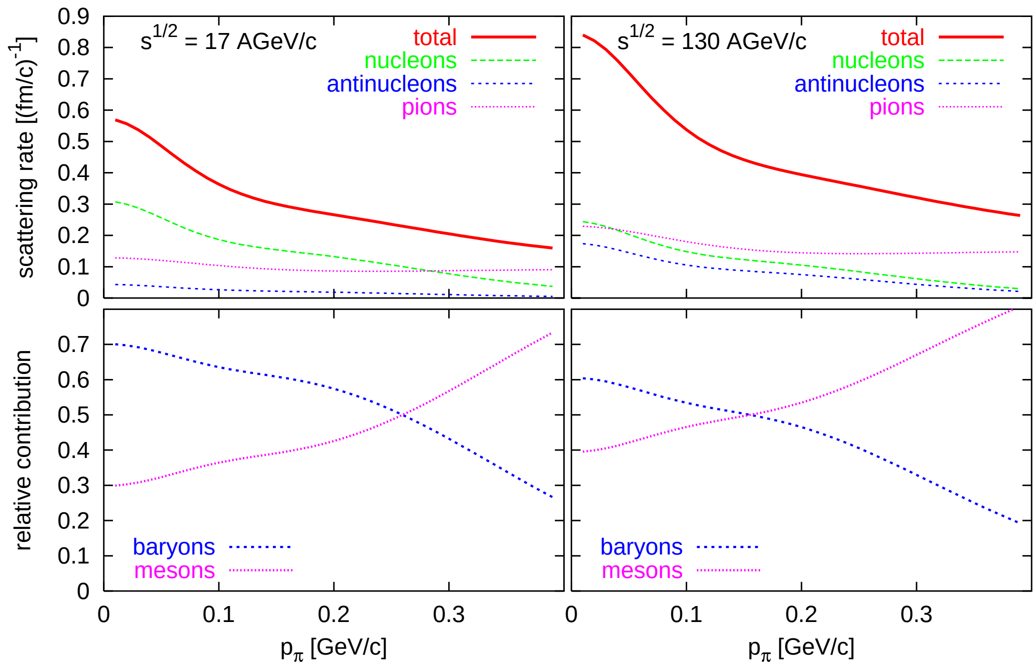

Figure 1. The pion scattering rate as a function of pion momentum with respect to medium calculated at $T = 100\,\mathrm{MeV}$ and the highest estimates of chemical potentials allowed by data from SPS (left column) and RHIC (right column). Contributions to the total scattering rate from scattering on nucleons, anti-nucleons and pions are indicated. The lower row shows the baryonic and mesonic relative contributions.

where the sum runs over all resonances in the πi scattering. Particle densities $\rho_i(k)$ are assumed to be thermal. For realistic temperatures, only low-lying resonances are relevant and we include $i = \pi$, N, $\bar{N}$, K, ρ, Δ, $\bar{\Delta}$ when summing in (6) over scattering partners. Since the freeze-out temperature is not known a priori, we scan three values: $T = 90$, 100, 120 MeV. Chemical potentials for pions are estimated from data on phase-space density, those for other species from measured ratios of dN/dy at mid-rapidity (see [6] for more details on the estimates).

Figure 1 shows the scattering rates as a function of the pion momentum *relative to the heat bath* calculated for SPS and RHIC, as well as the most important contributions to $\mathcal{R}$. In spite of the strong increase of the pion phase-space density at RHIC [1], the relative meson contribution to the total scattering rate does not grow proportionally since the $\pi\pi$ cross-section is much smaller than the one for πN. The total baryonic contribution changes very little since the smaller amount of baryons at RHIC is roughly compensated by anti-baryons. In summary, there is about 10% of the relative contribution shifted from baryons to mesons when going from SPS to RHIC.

As seen in Figure 2, the scattering rate depends strongly on the assumed freeze-out temperature since the pion-nucleon scattering is typically dominated by total CMS energy below the Δ resonance peak. At higher temperature, average CMS energies move closer to the peak value thus leading to increased scattering cross-sections. Estimating the pion escape probability (1) from eq. (5) and $\xi = 0.08\,\mathrm{fm}^{-1}$, one finds that a pion only has a

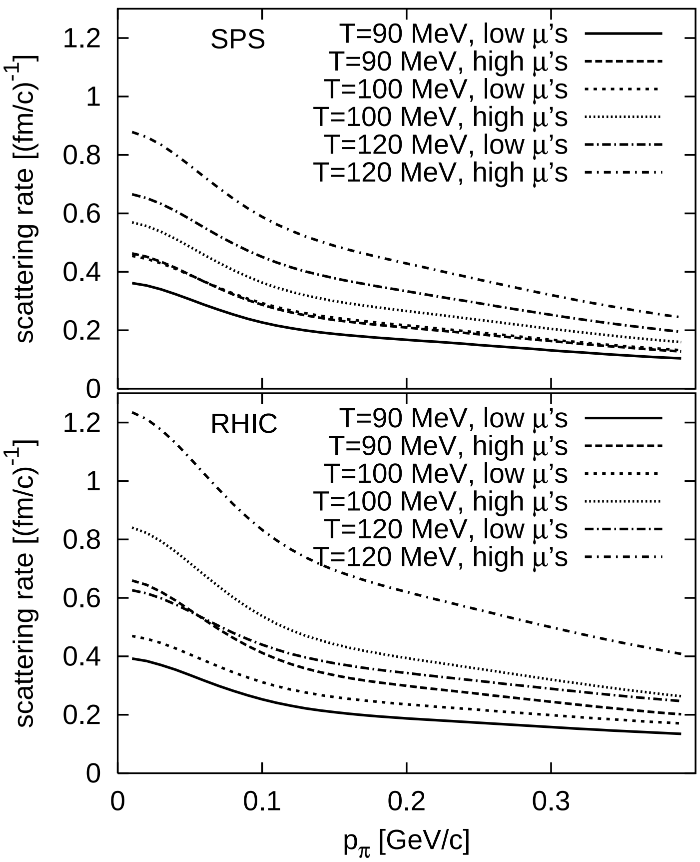

Figure 2. The pion scattering rate as a function of pion momentum with respect to the medium calculated for different temperatures and a range of chemical potentials allowed by data (see [6]). New results of STAR collaboration [1] correspond rather to the "high μ's" estimates.

reasonable chance to escape (of order 50%) when the temperature is 100 MeV at most.

We finally point out that the scattering rates in Figure 2 generically decreases with increasing pion momentum. This leads us to conjecture that high $p_\perp$ pions may decouple earlier, from hotter and smaller system. This may provide a novel as yet unexplored contribution to the observed $M_\perp$ dependence of HBT radii.

The freeze-out criterion studied here differs from the Cooper-Frye formalism typically employed in hydrodynamical model studies. It will be interesting to explore to what extent this affects the predictions of hydrodynamical simulations for the collision evolution.

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