Principles, Progress and Problems in Inflationary Cosmology

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Inflationary cosmology has become one of the cornerstones of modern cosmology. Inflation was the first theory within which it was possible to make predictions about the structure of the Universe on large scales, based on causal physics. The development of the inflationary Universe scenario has opened up a new and extremely promising avenue for connecting fundamental physics with experiment. This article summarizes the principles of inflationary cosmology, discusses progress in the field, focusing in particular on the mechanism by which initial quantum vacuum fluctuations develop into the seeds for the large-scale structure in the Universe, and highlights the important unsolved problems of the scenario. The case is made that new input from fundamental physics is needed in order to solve these problems, and that thus early Universe cosmology can become the testing ground for trans-Planckian physics.

I. INTRODUCTION

With the recent high-accuracy measurements of the spectrum of the cosmic microwave background (CMB) (see Fig. 1 [1]), cosmology has become a quantitative science. There is now a wealth of new data on the structure of the Universe as deduced from precision maps of the cosmic microwave background anisotropies, from cosmological redshift surveys, from redshift-magnitude diagrams of supernovae, and from many other sources. Standard Big Bang (SBB) cosmology provides the framework for describing the present data. The interpretation and explanation of the existing data, however, requires us to go beyond SBB cosmology and to consider scenarios like the Inflationary Universe in which space-time evolution in the very early Universe differs in crucial ways from what is predicted by the SBB theory. Since inflationary cosmology at later times smoothly connects with the SBB picture, we must begin this article with a short review of the framework of SBB cosmology.

Standard big bang cosmology rests on three theoretical pillars: the cosmological principle, Einstein's general theory of relativity, and the assumption that matter is a classical perfect fluid. The cosmological principle concerns the symmetry of space-time and states that on large distance scales space is homogeneous and isotropic. This implies that the metric of space-time can be written in Friedmann-Robertson-Walker (FRW) form. For simplicity, we consider the case of a spatially flat Universe:

$$
ds^{2} = dt^{2} - a(t)^{2} \left[dr^{2} + r^{2} (d\vartheta^{2} + \sin^{2} \vartheta d\varphi^{2}) \right]. \quad (1)
$$

Here, t, r, θ and φ are the space-time coordinates, and ds gives the proper time between events in space-time. The coordinates r, ϑ and φ are "comoving" spherical coordinates, and t is the physical time coordinate. Space-time curves with constant comoving coordinates correspond to the trajectories of particles at rest. If the Universe is expanding, i.e. the scale factor $a(t)$ is increasing, then the physical distance $\Delta x_p(t)$ between two points at rest with fixed comoving distance Δx_c grows as $\Delta x_p = a(t)\Delta x_c$.

FIG. 1. Compilation of the spectrum of the CMB. In the region around the peak (the region probed with greatest precision by the COBE satellite) the error bars are smaller than the size of the data points.

The dynamics of an expanding Universe is determined by the Einstein equations, which relate the expansion rate to the matter content, specifically to the energy density ρ and pressure p. For a Universe obeying the cosmological principle (and neglecting the possible presence of a cosmological constant) the Einstein equations reduce to the Friedmann-Robertson-Walker (FRW) equations

$$
\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3}\rho\tag{2}
$$

$$
\dot{\rho} = -3H(\rho + p),\tag{3}
$$

where $H = \dot{a}/a$ is the Hubble expansion rate, and an overdot denotes the derivative with respect to time t.