WARPED SUPERSYMMETRY

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Introduction

The two-brane Randall-Sundrum scenario provides an appealing reformulation of the gauge hierarchy problem.

It turns the stabilization of the Higgs mass into a geometrical question: the stabilization of a fifth dimension.

As in four dimensions, supersymmetry might well do the job.

This Talk

In this talk I will describe the minimal supersymmetrization of the original Randall-Sundrum action.

The final result will be a five dimensional bulk-plus-brane action. The action will be invariant under a single four-dimensional supersymmetry.

The low energy effective action will be nothing but ordinary $N = 1$ supergravity.

For now, I will not include the radion multiplet, nor will I include matter on the bulk or the brane.

Minimal approach!

The Procedure

The procedure is as follows:

- Start with pure AdS supergravity in five dimensions.

- Compactify on the orbifold S^1/Z_2.

- Find the supersymmetry left unbroken away from the orbifold points.

- Add a brane action at the orbifold points to achieve a fully supersymmetric theory.

- Derive the supergravity zero modes and the corresponding four-dimensional low-energy effective action.

Supersymmetric Bulk

Pure AdS $D=5$ supergravity:

$$
\begin{aligned}
S_{\text{bulk}} &= \Lambda \int d^5x\, e\Big[-\frac{1}{2\kappa^2}R + 6\,\frac{\Lambda^2}{\kappa^2} \\
&+ \mathrm{i}\epsilon^{MNOPQ}\overline{\Psi}_M \Sigma_{NO} D_P \Psi_Q - \frac{1}{4}F_{MN}F^{MN} \\
&- 3\Lambda\,\bar{\Psi}_M \Sigma^{MN}\Psi_N - \mathrm{i}\kappa\sqrt{\frac{3}{2}}\frac{1}{2}F_{MN}\bar{\Psi}^M\Psi^N \\
&- \kappa\Lambda\sqrt{\frac{3}{2}}\epsilon^{MNOPQ}\bar{\Psi}_M\Sigma_{NO}\Psi_P B_Q + \ldots \Big]
\end{aligned}
$$

Supersymmetry transformations:

$$
\begin{aligned}
\delta e_M{}^A &= \mathrm{i}\kappa\,(\bar{\eta}\Gamma^A\Psi_M - \bar{\psi}_M\Gamma^A\eta) \\
\delta B_M &= -\mathrm{i}\sqrt{\frac{3}{2}}\,(\bar{\eta}\psi_M - \bar{\psi}_M\eta) \\
\delta\psi_M &= \frac{2}{\kappa}D_M\eta + \mathrm{i}\,\frac{\Lambda}{\kappa}\Gamma_M\eta - \mathrm{i}\sqrt{6}\,\Lambda B_M\eta + \ldots
\end{aligned}
$$

Bulk Spinors

The bulk spinors are four-component Dirac:

$$\Psi_M = \begin{pmatrix} \psi^1_{M\alpha} \\ \bar{\psi}^{\dot{\alpha}}_{2M} \end{pmatrix}$$

where

$$\Gamma^a = \begin{pmatrix} 0 & \sigma^a_{\alpha\dot{\alpha}} \\ \bar{\sigma}^{a\dot{\alpha}\alpha} & 0 \end{pmatrix} \qquad \Gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

Also

$$\psi^{\pm}_M = \frac{1}{\sqrt{2}}(\psi^1_M \pm \psi^2_M)$$

and likewise for $\eta^{\pm}$.

Parity

Coordinates:

$$\{\ x_0,\ x_1,\ x_2,\ x_3,\ x_5\ \} \qquad x_5 = r\phi$$

Z_2 parity:

$$x_5 \to -x_5$$

- Even parity fields

$$e_m{}^a, \quad e_{5\hat{5}}, \quad B_5, \quad \psi_m^+, \quad \psi_5^-, \quad \eta^+$$

- Odd parity fields

$$e_5{}^a, \quad e_m{}^{\hat{5}}, \quad B_m, \quad \psi_m^-, \quad \psi_5^+, \quad \eta^-$$

Consistent with supersymmetry!

Unbroken Supersymmetry

Let us first find the unbroken supersymmetry.

We start with the bosonic background, given by Randall and Sundrum:

$$ds^2 = e^{-2\sigma(\phi)}\eta_{mn}\,dx^m dx^n + r^2\,d\phi^2$$

with $\sigma(\phi) = r\Lambda|\phi|$ and all other fields zero.

We then evaluate the supersymmetry transformations in this background:

$$\begin{aligned} \delta\psi_m^{\pm} &= \frac{2}{\kappa}\partial_m\eta^{\pm} \mp \mathrm{i}\,\mathrm{sgn}(\phi)\,\frac{\Lambda}{\kappa}\,\sigma_m\,\bar{\eta}^{\mp} \pm \mathrm{i}\,\frac{\Lambda}{\kappa}\,\sigma_m\bar{\eta}^{\pm} \\ \delta\psi_5^{\pm} &= \frac{2}{\kappa}\partial_5\eta^{\pm} + \frac{\Lambda}{\kappa}\,\eta^{\mp} \end{aligned}$$

Here, $\mathrm{sgn}(\phi) = \pm 1$ is a step function, and $\omega_{mAM}\Sigma^{AM} = \mathrm{sgn}(\phi)\,\Lambda\,\Gamma_m\Gamma^{\hat{5}}$.

Finally, we demand that the supersymmetry transformations vanish.

This gives the Killing spinors:

$$\begin{aligned} \eta^{+} &= \frac{1}{\sqrt{2}}\, e^{-\sigma(\phi)/2}\,\eta(x) \\ \eta^{-} &= \frac{1}{\sqrt{2}}\, e^{-\sigma(\phi)/2}\,\mathrm{sgn}(\phi)\eta(x) \end{aligned}$$

These spinors parameterize the one unbroken supersymmetry of the Randall-Sundrum scenario.

New Transformations

The Killing spinors are almost – but not quite – *bona fide* Killing spinors. They miss by delta functions at the orbifold points.

Therefore we change the gravitino supersymmetry transformations!

We take

$$\delta\psi_5^- = \delta\psi_5^-\Big|_{\text{old}} - \frac{4}{r\kappa}\left[\,\delta(\phi) - \delta(\phi-\pi)\,\right]\eta^+$$

With this change, the Killing spinors satisfy the Killing equations everywhere, even at the orbifold points.

Moreover, the modified transformations still close into the supersymmetry algebra!

Supersymmetric Brane

The bulk action is not invariant under the new supersymmetry transformations.

The variation is determined by the values of the bulk fields on the branes.

These depend on the jump conditions, which are themselves fixed by the brane action.

We assert that the brane action is simply

$$
\begin{aligned}
S_{\text{brane}} \;=\; & \frac{\Lambda}{r\kappa^2}\int d^5x\,\hat{e}\,(-3\Lambda + 2\kappa^2\psi_m^+\sigma^{mn}\psi_n^+) \\
& [\,\delta(\phi) - \delta(\phi - \pi)\,] \;+\; h.c.
\end{aligned}
$$

This implies the jump conditions

$$[\omega_{ma5}] \;=\; \pm 2\Lambda\, e_{ma} \;, \qquad [\psi_m^-] \;=\; \pm 2\psi_m^+$$

with solutions

$$\omega_{ma5} \;=\; \mathrm{sgn}(\phi)\,\Lambda\, e_{ma} \;, \qquad \psi_m^- \;=\; \mathrm{sgn}(\phi)\, \psi_m^+$$

near the branes. The bulk variation is then

$$\begin{aligned} \delta S_{\text{bulk}} \;=\; & \frac{\Lambda}{r\kappa} \int d^5x\, e\, e^{\hat{5}5} \Big[\Big(8\, \eta^+ \sigma^{mn} D_m \psi_n^+ \\ & - \mathrm{i}\, \kappa \sqrt{6}\, F^{\hat{5}m}\, \eta^+ \psi_m^+ \\ & + 6\mathrm{i}\, \Lambda\, (1 - \mathrm{sgn}^2(\phi))\, \eta^+ \sigma^m \bar{\psi}_m^+ \Big) \\ & \qquad [\, \delta(\phi) - \delta(\phi - \pi)\,] \Big] \;+\; h.c. \end{aligned}$$

This must be cancelled by the variation of the brane action!

The brane supersymmetry transformations are inherited from the bulk

$$
\begin{aligned}
\delta e_m{}^a &= \mathsf{i}\kappa\,(1+\mathrm{sgn}^2(\phi))\,\eta^+\sigma^a\bar{\psi}_m^+ \;+\; h.c. \\
\delta\psi_m^+ &= \frac{2}{\kappa}D_m\eta^+ + \;\mathsf{i}\,\frac{\Lambda}{\kappa}(1-\mathrm{sgn}^2(\phi))\,e_m{}^a\,\sigma_a\bar{\eta}^+ \\
&\quad + \mathsf{i}\sqrt{\frac{2}{3}}F_{\hat{5}m}\,\eta^+ \;+\; \mathsf{i}\sqrt{\frac{2}{3}}F^{\hat{5}n}\,\sigma_{mn}\eta^+
\end{aligned}
$$

The variation of S_{brane} cancels the variation of S_{bulk} when

$$
\int_{-\epsilon}^{\epsilon} d\phi\,\frac{d}{d\phi}\mathrm{sgn}^3(\phi) \;=\; 2
$$

In this case

$$
S \;=\; S_{\text{bulk}} + S_{\text{brane}}
$$

is invariant under a single four dimensional supersymmetry!

Zero Modes

At low energies, only zero modes are excited:

$$
\begin{aligned}
e_M{}^A &= \begin{pmatrix} 1 & 0 \\ 0 & e^{-\sigma(\phi)}\bar{e}_m{}^a(x) \end{pmatrix} \\
\psi_m^+ &= \frac{1}{\sqrt{2}} \binom{\bar{\kappa}}{\kappa} e^{-\sigma(\phi)/2}\, \psi_m(x) \\
\psi_m^- &= \frac{1}{\sqrt{2}} \binom{\bar{\kappa}}{\kappa} e^{-\sigma(\phi)/2}\, \mathrm{sgn}(\phi)\, \psi_m(x)
\end{aligned}
$$

The vierbein $\bar{e}_m{}^a$ and gravitino ψ_m are functions of $x^0, ..., x^3$, but not x^5.

The gravitino zero modes satisfy the massless gravitino equations of motion:

$$
\begin{aligned}
\partial_5 \psi_m^+ + \frac{3}{2} \Lambda\, \psi_m^- - \mathrm{sgn}(\phi) \Lambda\, \psi_m^+ &= 0 \\
\partial_5 \psi_m^- + \frac{3}{2} \Lambda\, \psi_m^+ - \mathrm{sgn}(\phi)\, \Lambda \psi_m^- &= \frac{2}{r} [\, \delta(\phi) \\
& \quad - \delta(\phi - \pi)\,]\, \psi_m^+
\end{aligned}
$$

Effective Action

To find the effective action, we substitute the zero-mode expressions into the supersymmetric bulk-plus-brane action, and integrate over the coordinate x^5.

We use the facts that

$$R = e^{2\sigma}\bar{R} + 20\Lambda^2 - 16\frac{\Lambda}{r}\left[\delta(\phi) - \delta(\phi - \pi)\right]$$

and

$$\omega_{mAB}\Sigma^{AB} = \mathrm{sgn}(\phi)\,\Lambda\Gamma_m\Gamma^{\hat{5}} + \bar{\omega}_{mab}\sigma^{ab}$$

to find

$$S_{\mathrm{eff}} = \int d^4x\,\bar{e}\left[-\frac{1}{2\bar{\kappa}^2}\bar{R} + \epsilon^{mnpq}\,\overline{\psi}_m\bar{\sigma}_n D_p\psi_q\right]$$

where $\bar{\kappa}^{-2} = \kappa^{-2}(1 - e^{-2\pi r\Lambda})$.

This is nothing but $N = 1$ supergravity in four dimensions.

SUSY Transformations

The four-dimensional supersymmetry transformations are found by substituting the zero modes into the five-dimensional transformations.

All x^5-dependent terms cancel, leaving

$$\begin{aligned} \delta e_m{}^a &= i\bar{\kappa}\,\eta\sigma^a\bar{\psi}_m + h.c. \\ \delta\psi_m &= \frac{2}{\bar{\kappa}}\,D_m\eta \end{aligned}$$

These are precisely the transformations of $N = 1$ supergravity in four dimensions.

Summary

This work is a first step towards a detailed understanding of supersymmetry in the context of warped compactifications.

To study stability, one would like, of course, to include the radion multiplet, which reduces to $N = 1$ chiral matter in four dimensions.

For phenomenology, one would also like to add supersymmetric matter on the branes and in the bulk – and study the different mechanisms of supersymmetry breaking.

For other points of view, see the talks by:

Gherghetta
Lalak, Pokorski
Kallosh
Louis