# Rho-Mesons and Dilepton Emission at Finite Temperature and Density

based on:

Eletsky, Ioffe + Kapusta
Eur. J. Phys. A <u>3</u>, 381 (1998)

Eletsky + Kapusta
Phys. Rev. C <u>59</u>, 2757 (1999)

history:

Gale + Kapusta '87
Korpa + Pratt '90
Korpa, Siemens + Ko '90
⋮
Gale + Kapusta '91
⋮
Brown + Rho '91
⋮
Li, Ko + Brown '95
⋮
Rapp, Chanfray + Wambach '96 - '98
Klingl + Weise; Klingl, Weise + Kaiser '96 - '97
Friman + Pirner '97
⋮

CERES/NA 45

PHYS. LETT. B422 405, (1998).

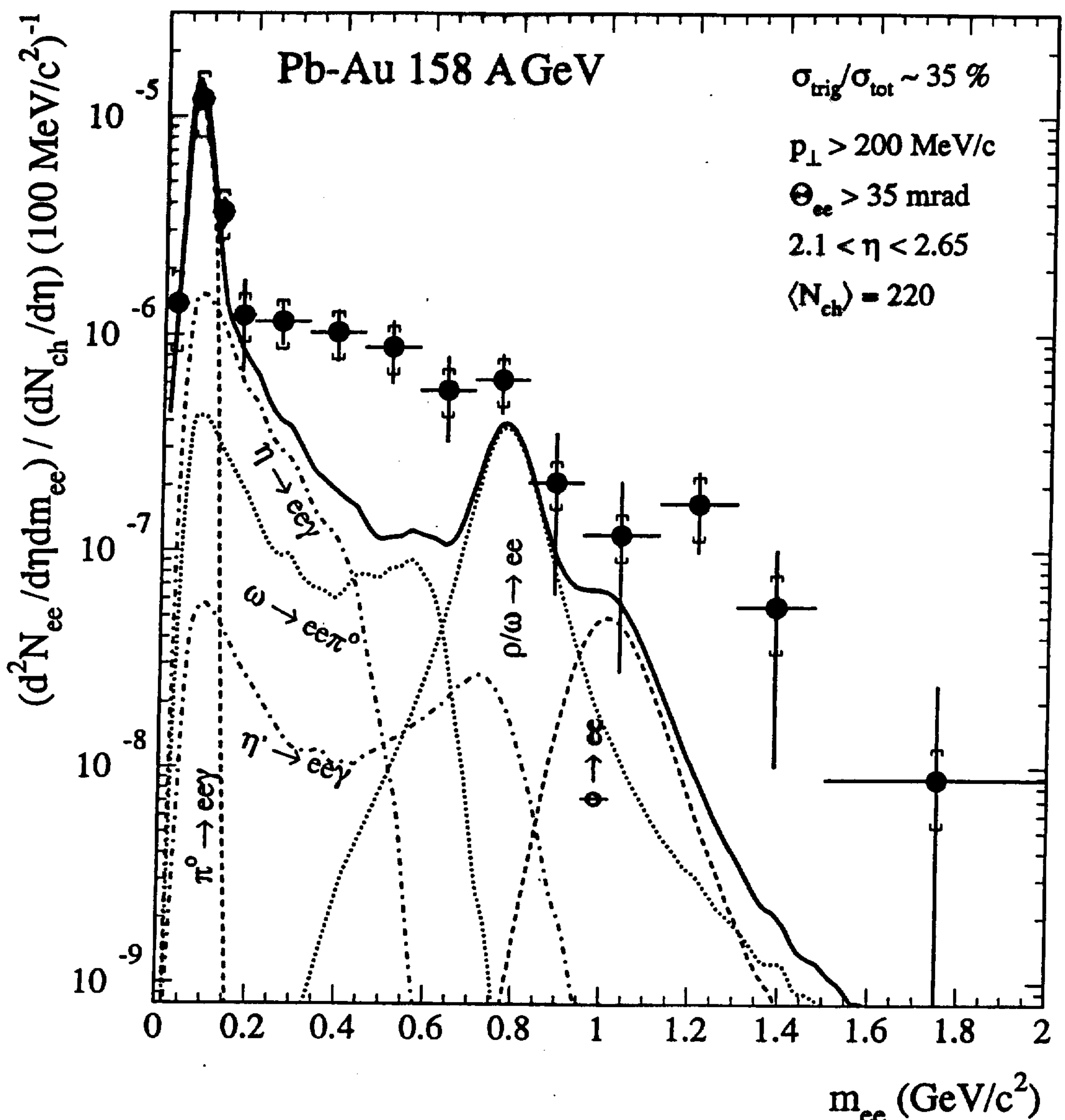



FIG. 2. Inclusive invariant $e^+e^-$ mass spectrum in 158 A GeV Pb-Au collisions normalized to the observed charged-particle density. The statistical errors of the data are shown as bars, the systematic errors are given independently as brackets. The full line represents the $e^+e^-$ yield from hadron decays scaled from p-induced collisions. The contributions of individual decay channels are also shown.

# Thermal Emission Rate of Dileptons

hadronic state $i \rightarrow$ hadronic state $f + \ell^+\ell^-$

rate/volume $\quad R_{fi} = \dfrac{|S_{fi}|^2}{\tau\, V}$

time $\quad$ volume

S-matrix $\quad S_{fi} = \langle f | \int d^4x\, d^4y\, \hat{J}^h_\mu(x)\, D^{\mu\nu}(x-y)\, \hat{J}^\ell_\nu(y) | i \rangle$

hadronic current $\quad$ photon propagator $\quad$ leptonic current

Assume translation invariance. Average over initial states with Boltzmann weight, sum over final states.

$$E_+ E_- \frac{dR}{d^3p_+\, d^3p_-} = -\frac{e^2}{(2\pi)^6}\left[ p_\nu^- p_\beta^+ + p_\beta^- p_\nu^+ - g_{\nu\beta}\left(p_+\cdot p_- + m_\ell^2\right)\right] \cdot D^{\mu\nu}(k)\, D^{\alpha\beta}(-k)\, f^-_{\mu\alpha}(k)$$

$k = p^+ + p^-$

essential information content

$$f^{-}_{\mu\nu}(k) = -\frac{1}{Z}\sum_{i,f} e^{-\beta H_i} (2\pi)^4 \delta(p_i - p_f - k)$$

$$\cdot \langle f | \hat{J}^h_\mu(0) | i \rangle \langle i | \hat{J}^h_\nu(0) | f \rangle$$

Relationships:

temperature / Matsubara $\quad f^T_{\mu\nu}(k) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{f^n_{\mu\nu}(\omega', \vec{k})}{\omega' - i\omega_n}$

retarded $\quad f^R_{\mu\nu}(k) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{f^n_{\mu\nu}(\omega', \vec{k})}{\omega' - \omega - i\epsilon}$

spectral density $\quad f^n_{\mu\nu}(\omega, \vec{k}) = -(e^{\beta\omega} - 1) f^{-}_{\mu\nu}(\omega, \vec{k})$

retarded improper photon self-energy $\quad f^R_{\mu\nu} = P^R_{\mu\nu}$

$$D = D_0 + D_0 P D_0 = D_0 + D_0 \Pi D$$

Lowest order in $e^2$

$$E_+ E_- \frac{dR}{d^3p_+ d^3p_-} = \frac{2e^2}{(2\pi)^6} \frac{1}{(k^2)^2} \left[ p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - g^{\mu\nu}(p_+ \cdot p_- + m_l^2) \right]$$

$$\cdot \frac{1}{e^{\beta\omega} - 1} \, \mathrm{Im}\, \Pi^R_{\mu\nu}(k)$$

# Vector Dominance Model

$$\hat{J}_\mu^h = -\frac{e}{g_\rho} m_\rho^2 \hat{\rho}_\mu - \frac{e}{g_\phi} m_\phi^2 \hat{\phi}_\mu - \frac{e}{g_\omega} m_\omega^2 \hat{\omega}_\mu$$

Neglecting $\omega$ + $\phi$ (very narrow):

$$\mathrm{Im}\, \Pi_{\mu\nu}^R = \frac{e^2}{g_\rho^2} m_\rho^4 \,\mathrm{Im}\, D_{(\rho)\mu\nu}^R$$

$$E_+ E_- \frac{dR}{d^3p_+ d^3p_-} \propto \mathrm{Im}\, D_{(\rho)\mu\nu}^R$$

Assumption: Rho-meson scatters from hadrons $b$ in the medium; interference between sequential scatterings is negligible.

$$\Pi_{\rho b}(E,p) = -4\pi \int \frac{d^3k}{(2\pi)^3}\, n_b(\omega)\, \frac{\sqrt{s}}{\omega}\, f_{\rho b}^{(cm)}(s)$$

$$= -\frac{1}{2\pi p} \int_{m_b}^{\infty} d\omega\, n_b(\omega) \int_{s_-}^{s_+} ds \sqrt{s}\, f_{\rho b}^{(cm)}(s)$$

forward scattering amplitude

$$s_\pm = E^2 - p^2 + m_b^2 + 2(E\omega \pm pk)$$

$$\omega = \sqrt{m_b^2 + k^2}$$

$$\sigma = \frac{4\pi}{q_{cm}} \, \mathrm{Im}\, f^{(cm)}(s)$$

$$m_\rho f_{\rho b}^{(\rho \text{ rest frame})} = m_b f_{\rho b}^{(b \text{ rest frame})} = \sqrt{s}\, f_{\rho b}^{(cm)}$$

This embodies the minimal many-body physics in a dilute hadronic system.

Interesting limits:

Target particle $b$ moves nonrelativistically:

$$\omega \simeq m_b \quad \text{and} \quad \Pi_{\rho b} \approx -4\pi f_{\rho b}^{(b\ \text{rest frame})} \rho_b$$

density

Chiral limit, $b = \pi$:

Adler's Thm. says that $f_{\rho\pi}^{(\rho\ \text{rest frame})}$ has two powers of pion momentum.

$$\Pi_{\rho\pi} \sim T^4 \quad (\rho \text{ at rest})$$

Evaluate self-energy in rest frame of $\rho$:

$$\Pi_{\rho b}(E,p) = -\frac{m_\rho^2 T}{\pi p} \int_{m_b}^{\infty} d\omega \, \ln\left[\frac{1-e^{-\omega_+/T}}{1-e^{-\omega_-/T}}\right] f_{\rho b}^{(\rho\ \text{rest frame})}(\omega)$$

$$\omega_\pm = (E\omega \pm p k)/m_\rho$$

for $b$ a boson. Analogous formula for fermions.

# Determination of Scattering Amplitudes for $\pi - N$ Dominated Gas Near Freezeout in Heavy Ion Collisions

$\rho\pi$ at low energy is saturated with resonances

$a_1(1260)$, $\pi(1300)$, $a_2(1320)$, $\omega(1420)$

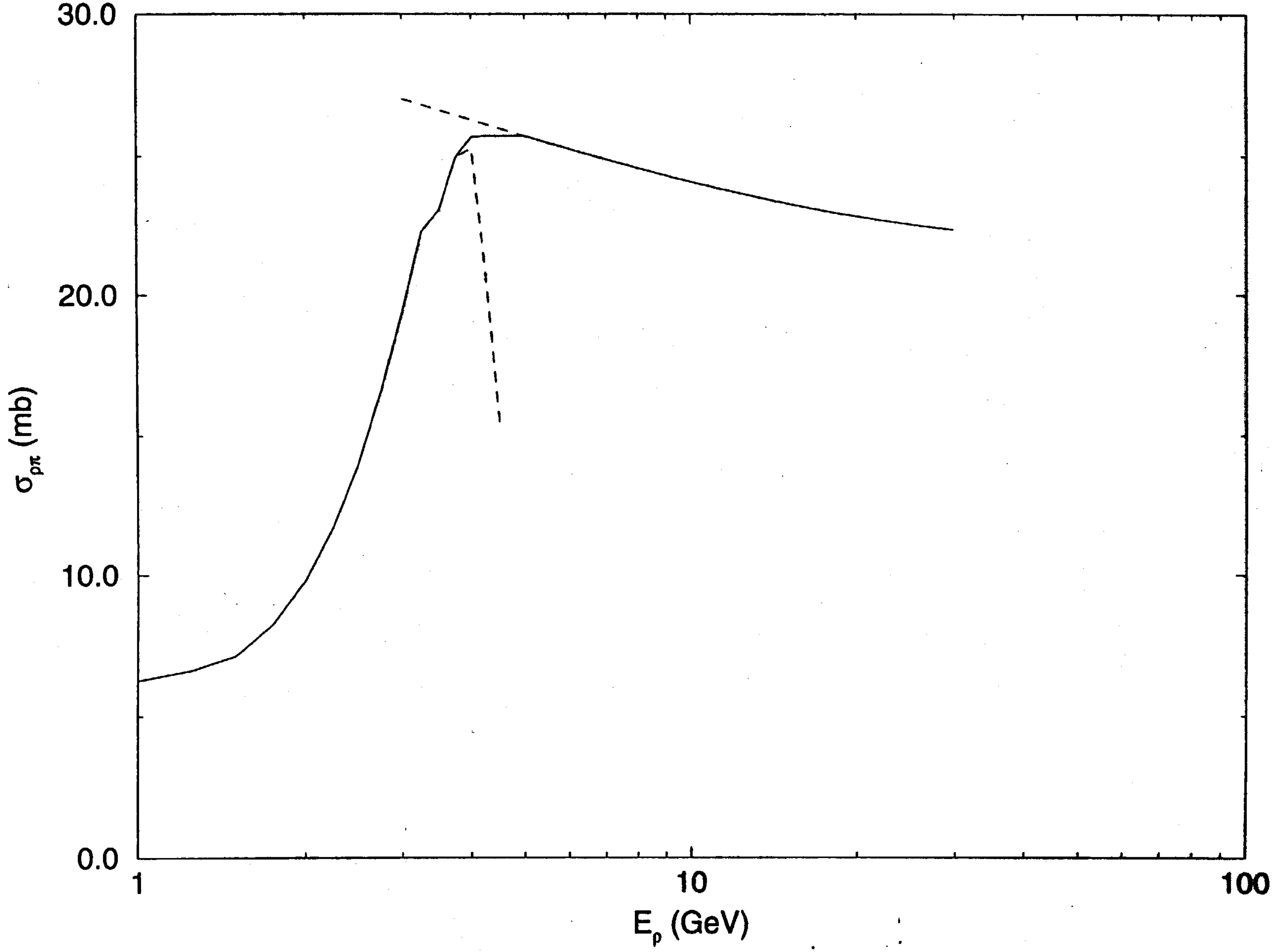
including threshhold factors to satisfy Adler.

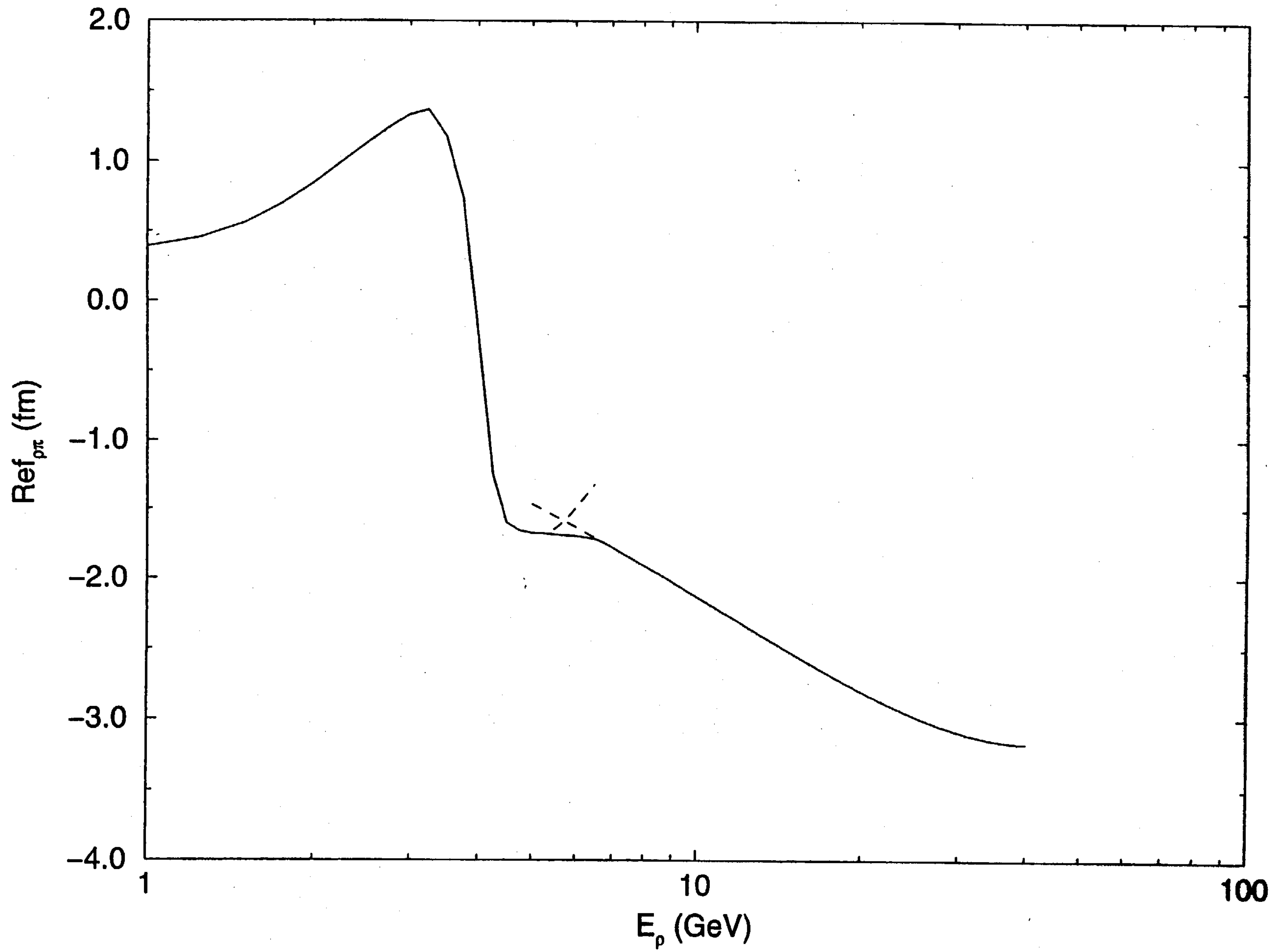
$\rho\pi$ at high energy is described by Regge poles + VDM

$\rho$ + $\rho'$ poles: Intercepts + residues taken from Boreskov, Kaidalov + Ponomarev and Donnachie + Landshoff.

Use VDM to relate $\gamma\pi$ to $\rho\pi$.

Strictly speaking our results apply to transversely polarized $\rho$-mesons.


30.0
20.0
10.0
0.0
$\sigma_{p\pi}$ (mb)
1
10
100
$E_p$ (GeV)



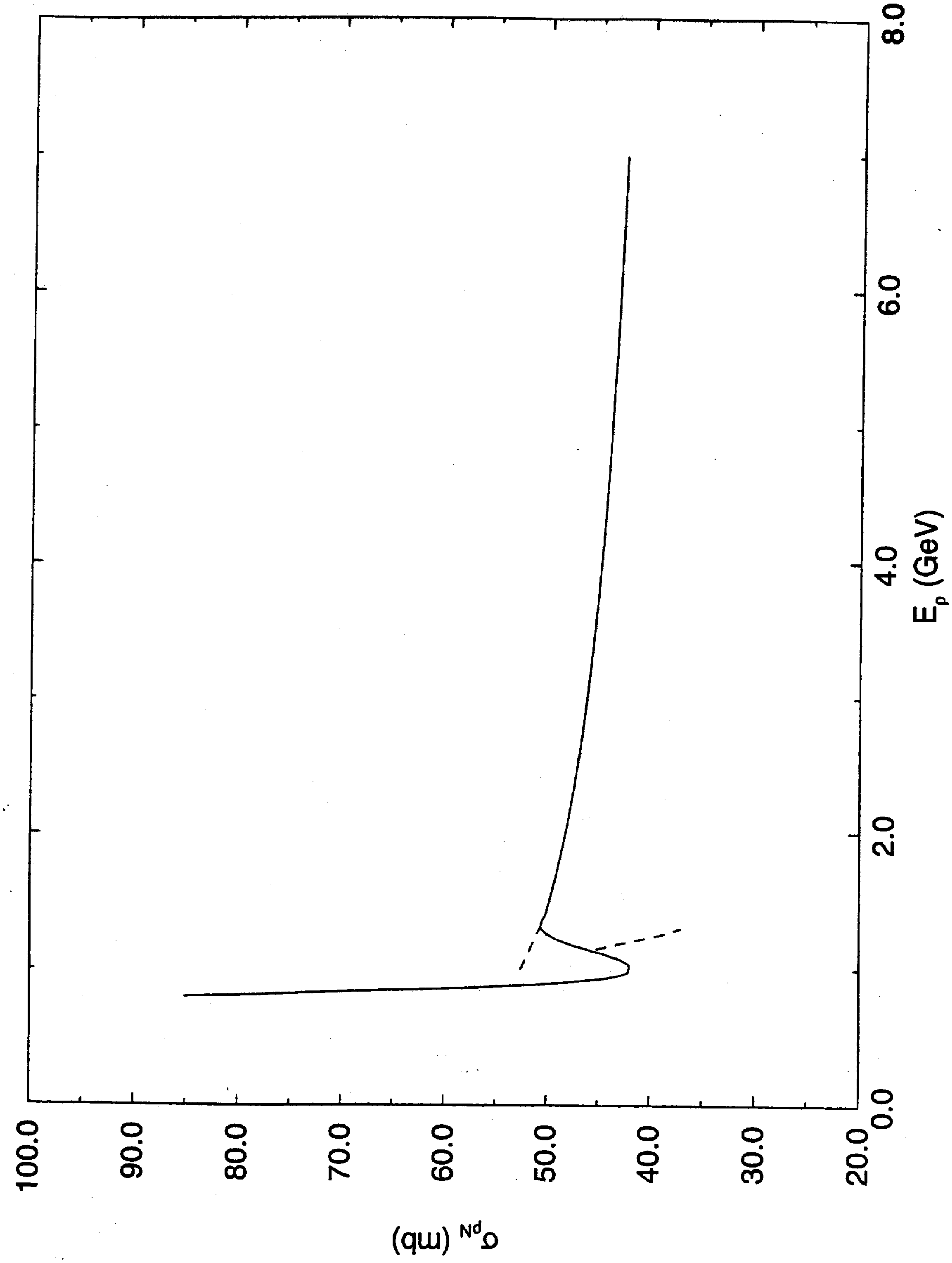
2.0
1.0
0.0
−1.0
−2.0
−3.0
−4.0
$\mathrm{Re}f_{\rho\pi}$ (fm)
1
10
100
$E_\rho$ (GeV)

$\rho N$ at low energy is saturated with resonances

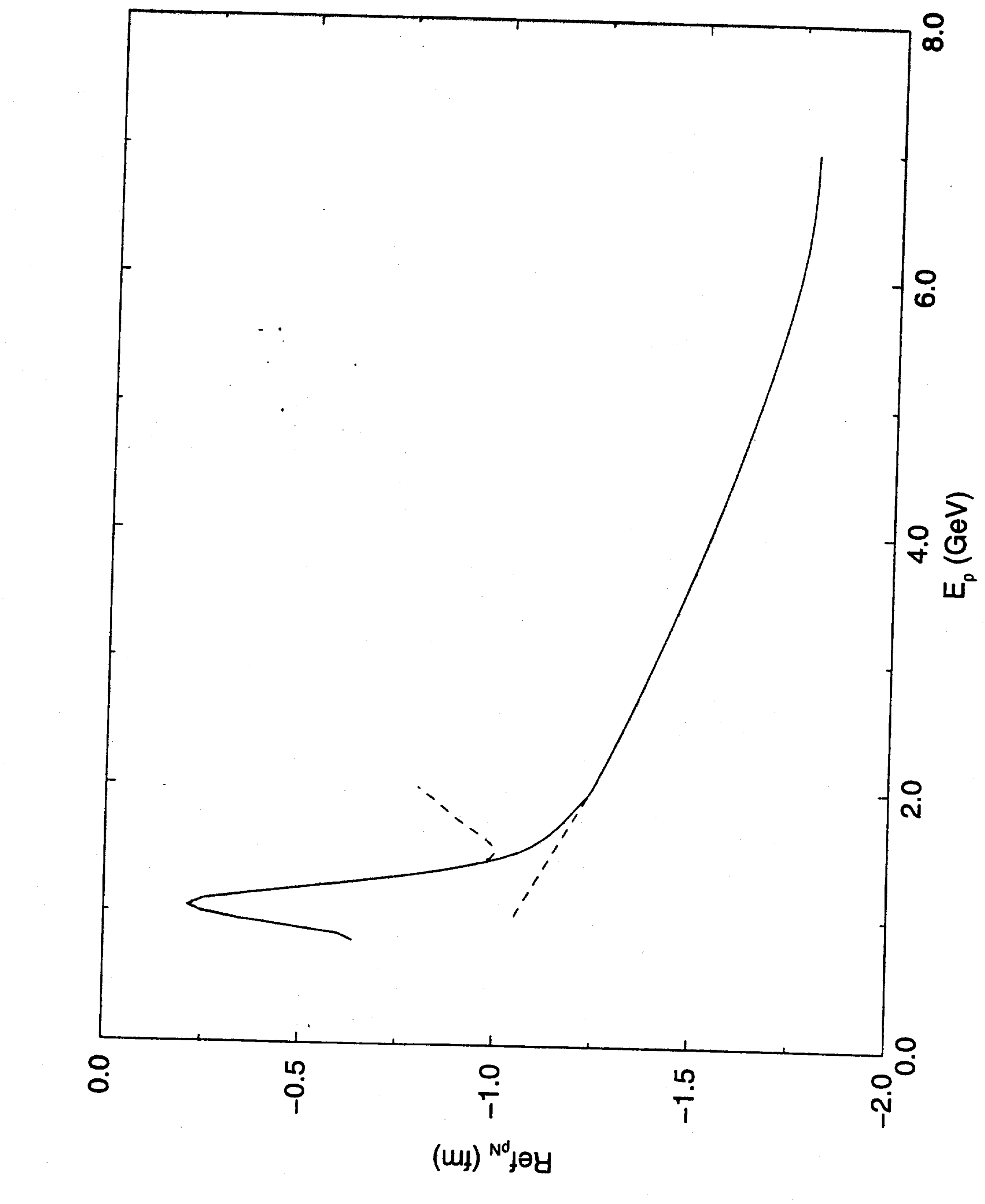
$\Delta(1232)$, $N(1500)$ + 10 $N$ + $\Delta$ resonances above the $\rho N$ threshhold that couple strongly to $\rho N$

$\rho N$ at high energy is obtained from experimental data on $\gamma N$ + VDM and dispersion relation

Elotsky + Ioffe '97



$\sigma_{\rho N}$ (mb)
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
0.0
2.0
4.0
6.0
8.0
$E_p$ (GeV)



0.0
-0.5
-1.0
-1.5
-2.0
$\mathrm{Re}f_{\rho N}$ (fm)
0.0
2.0
4.0
6.0
8.0
$E_p$ (GeV)

# Dispersion Relation

pole of propagator

$$E^2 = m_\rho^2 + p^2 + \Pi_\rho^{vac}(m) + \Pi_{\rho\pi}(p) + \Pi_{\rho N}(p)$$

Gounaris-Sakurai

scattering amplitudes evaluated on the $\rho$ mass-shell

real part

$$E_R^2(p) = m_\rho^2 + p^2 + \mathrm{Re}\,\Pi_{\rho\pi}(p) + \mathrm{Re}\,\Pi_{\rho N}(p)$$

imaginary part

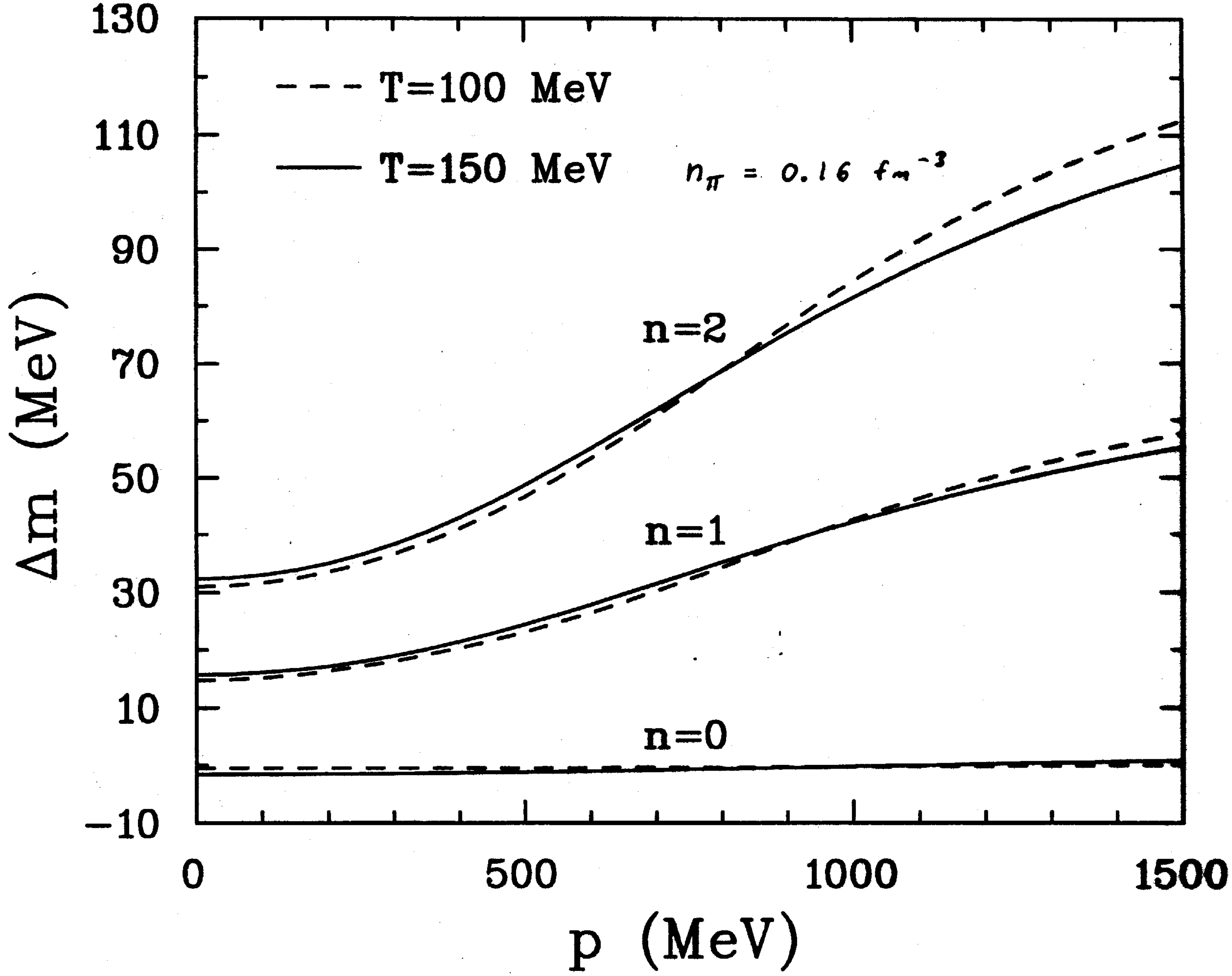
$$\Gamma(p) = -\frac{\mathrm{Im}\,\Pi_\rho^{vac} + \mathrm{Im}\,\Pi_{\rho\pi}(p) + \mathrm{Im}\,\Pi_{\rho N}(p)}{E_R(p)}$$

mass shift

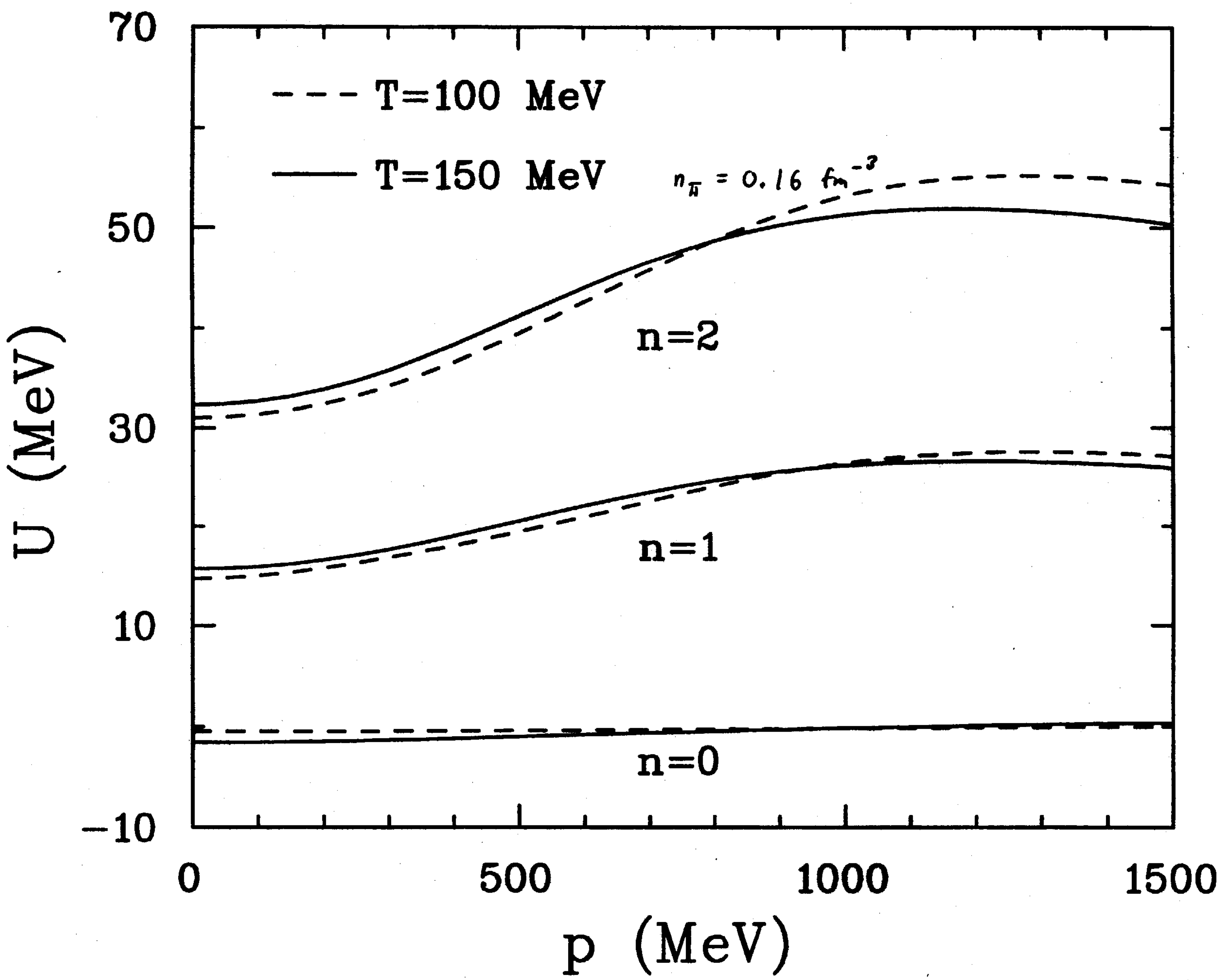
$$\Delta m_\rho(p) = \sqrt{m_\rho^2 + \mathrm{Re}\,\Pi_{\rho\pi}(p) + \mathrm{Re}\,\Pi_{\rho N}(p)} - m_\rho$$

optical potential

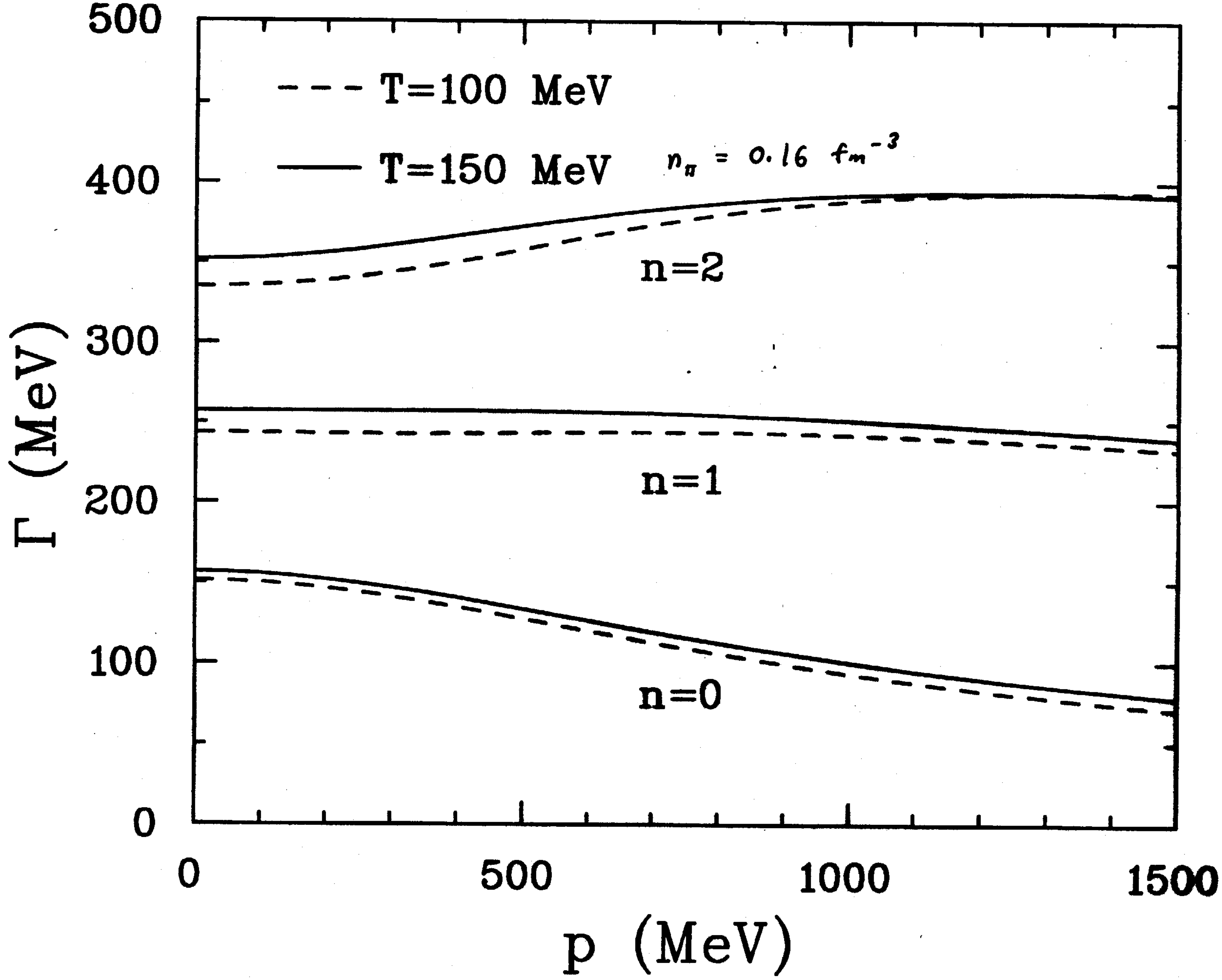
$$U(p) = E_R(p) - \sqrt{m_\rho^2 + p^2}$$

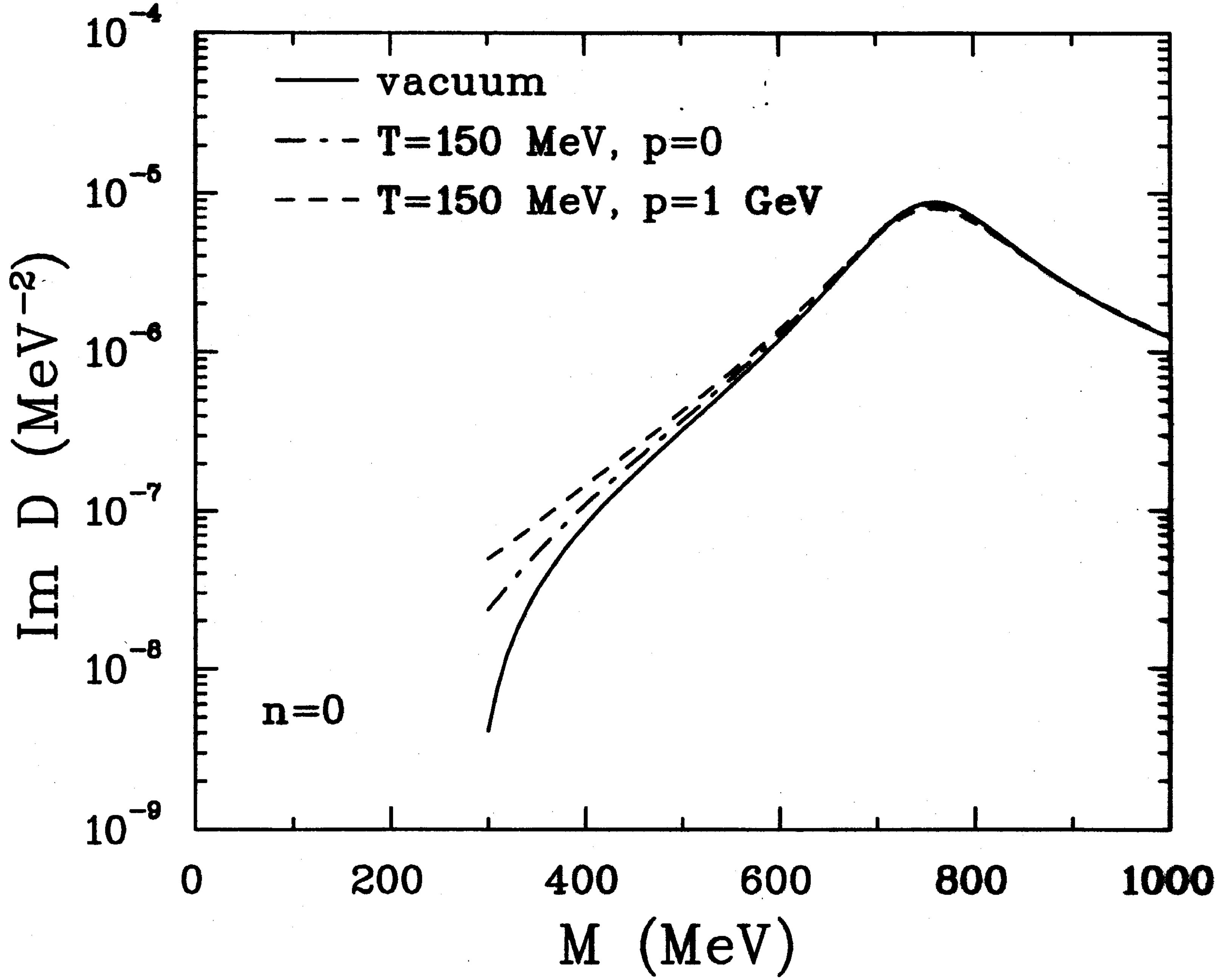


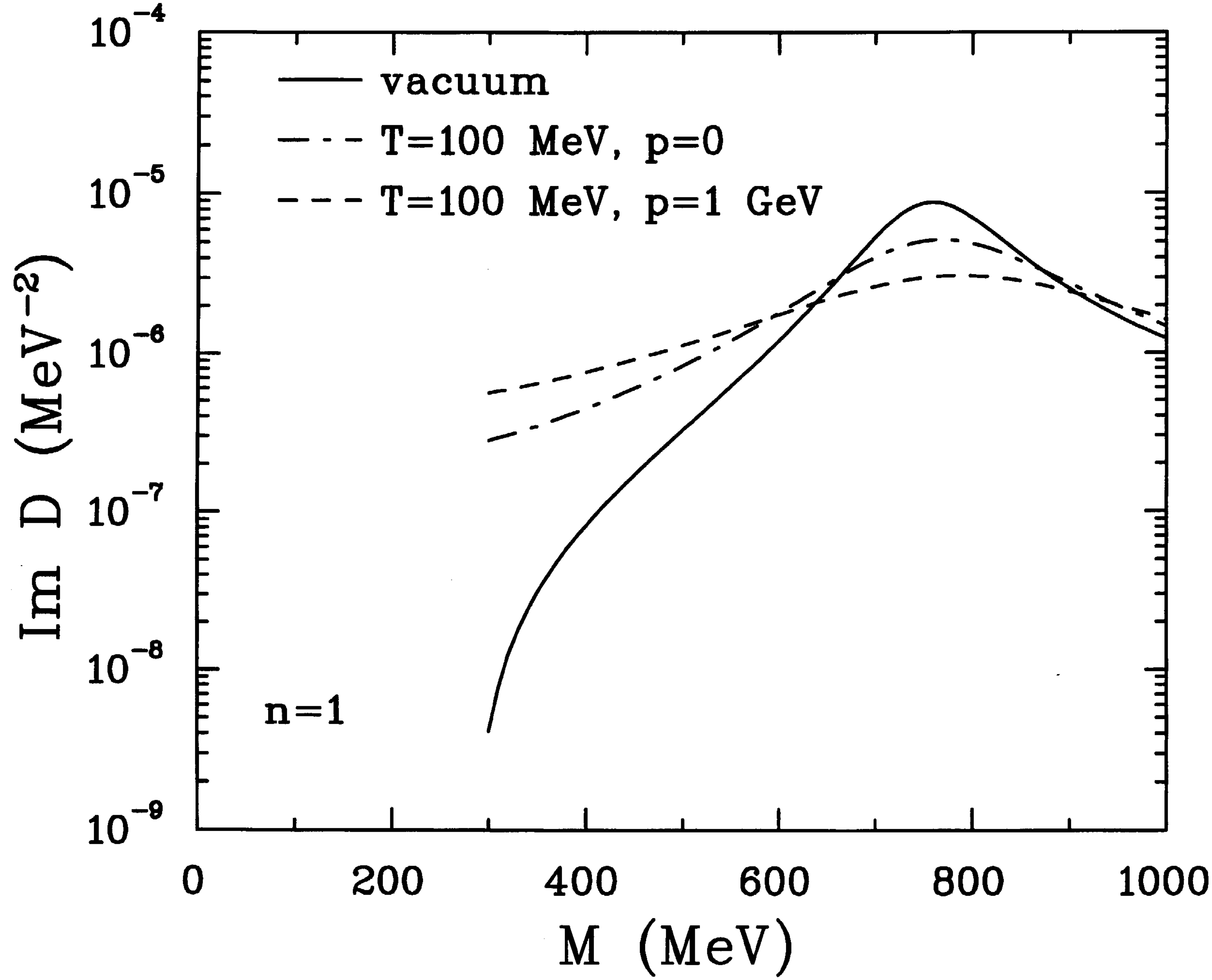
T=100 MeV
T=150 MeV
$n_\pi = 0.16\ fm^{-3}$
n=2
n=1
n=0
130
110
90
70
50
30
10
−10
0
500
1000
1500
Δm (MeV)
p (MeV)


T=100 MeV
T=150 MeV
$n_\pi = 0.16\ fm^{-3}$
n=2
n=1
n=0
U (MeV)
p (MeV)
70
50
30
10
−10
0
500
1000
1500

Time dilation: $\Gamma(p) = \frac{m_\rho}{E}\,\Gamma(0)$




vacuum
T=150 MeV, p=0
T=150 MeV, p=1 GeV
n=0
Im D (MeV$^{-2}$)
M (MeV)
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
$10^{-9}$
0
200
400
600
800
1000


vacuum
T=100 MeV, p=0
T=100 MeV, p=1 GeV
n=1
Im D (MeV^-2)
M (MeV)
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
0
200
400
600
800
1000

## Conclusion

For low to moderate densities the $\rho$-meson dispersion relation can be determined unambiguously in terms of the scattering amplitudes on the constituents of the medium.

For $T \sim 100$ to $150$ MeV and $n \sim n_0$ nucleons are more important than pions in modifying the $\rho$-meson dispersion relation. The mass goes up slightly but more importantly, the width increases dramatically.

The above is in qualitative agreement with CERES and HELIOS-3 data and with theoretical analyses of Rapp, Chanfray + Wambach; Klingl + Weise; Friman + Pirner.

Detailed comparison remains to be done.