High-energy Nuclear Collisions: A Non-perturbative Numerical Description of Transverse Dynamics

or

Trying to make the most of MV model

Raju Venugopalan + A.K.

Reminder of McLerran-Venugopalan model

* Partons in a nucleus are separated into the high-x (valence + hard sea) and low-x parts. The latter part corresponds to central rapidity in HIC where QGP is supposed to form.

* High-x partons are considered recoilless sources of color charge. For a large, Lorentz-contracted nucleus this color charge distribution is Gaussian in the transverse plane.

$$P([\rho]) \propto \exp\left[-\frac{1}{2g^4\mu^2}\int d^2r_t\ \rho^2(r_t)\right]$$

This color charge is static in the transverse plane.

* A nucleus is considered infinitely thin in the longitudinal direction. This assumption can be relaxed if necessary.

* μ is a dimensional parameter which determines the color charge density.
With the inclusion of semi-hard partons

$$\mu^2 = \frac{A^{\frac{1}{3}}}{\pi r_0^2} \int_{x_0}^{1} dx \left(\frac{1}{2N_c} q(x, Q^2) + \frac{N_c}{N_c^2 - 1} g(x, Q^2) \right)$$

$\left(x_0 = \frac{Q}{\sqrt{s}} \right)$

[Gyulassy, McLerran]

* Consider a nuclear collision.

[Kovner, Mc Lerran, Weigert]

solve equation of motion for low-x fields in the random color charge background:

$$D_\mu F_{\mu\nu} = J_\nu$$

where, for an infinitely thin nucleus,

$$J_\nu = \sum_{1,2} \delta_{\nu,\pm} \, \delta(x_\mp) \, \rho_{1,2}(r_+)$$

For a single nucleus the solution is a pure gauge:

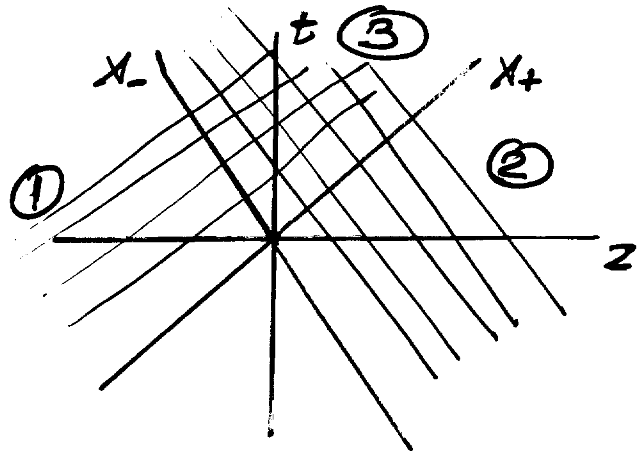
$$A_\pm = 0 \, , \quad A_\perp = i e^{i\Lambda(r_+)} \partial_\perp e^{-i\Lambda(r_+)} \theta(x_\mp)$$

where

$$\nabla^2 \Lambda = \rho$$

(note $\theta(x_\mp)$ ensuring causality)

* It is necessary to match solutions in 3 regions, differing by their causal relation to the nuclei.



In the region # 3 (after the collision) the field strength does not vanish. Using $A_\tau = 0$ and $\nabla \cdot A_\perp = 0$ at $\tau = 0$, [KMW] arrive, perturbatively, at the multiplicity per unit rapidity:

$$N_{k_\perp} \propto \left(\frac{\alpha_s^{3/4}\mu}{k_\perp}\right)^4 \ln\left(\frac{k_\perp}{\alpha_s \mu}\right)$$

This resembles a high-temperature theory (assuming weak coupling)

$$T \longrightarrow \alpha_s^{3/4}\mu$$

$$k \ll T \longrightarrow k \ll \alpha_s^{3/4}\mu \quad \left(\begin{array}{c}\text{classical}\\ \text{physics}\end{array}\right)$$

$$k \sim g^2 T \longrightarrow k \sim \alpha_s \mu \quad \left(\begin{array}{c}\text{non - perturbative}\\ \text{domain!}\end{array}\right)$$

[KNW] not valid

We need a non-perturbative tool for soft low-x modes!

Lattice Formulation

* The Hamiltonian formalism is better suited for numerical work. In the continuum [Makhlin]

$$H = \frac{\tau}{2}\int d\eta\, d^2r_t \left[p^\eta p^\eta + \frac{1}{\tau^2} p^r p^r + \frac{1}{\tau^2} F_{\eta r} F_{\eta r} + F_{xy} F_{xy} \right]$$

For "perfect pancake" nuclei we only consider boost-invariant configurations. Hence

$$A_r(\tau, \eta, \vec{r}_t) = A_r(\tau, \vec{r}_t) \qquad A_\eta(\tau, \eta, r_t) = \Phi(\tau, r_t)$$

(this resembles a finite-T dimensional reduction – an adjoint scalar emerges)

Per unit rapidity

$$H = \frac{\tau}{2}\int d^2r_t \left[p^\eta p^\eta + \frac{1}{\tau^2} E_r E_r + \frac{1}{\tau^2} (D_r \Phi)(D_r \Phi) + F_{xy} F_{xy} \right]$$

* Discretize (on a 2d lattice)

$$H_L = \frac{1}{2\tau} \sum_{l} E_l E_l + \tau \sum_{\square} \left(1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, U_\square\right) + \frac{\tau}{2} \sum_{j} p_j p_j + \frac{1}{4\tau} \sum_{j,n} \mathrm{Tr}\left(\Phi_j - U_{j,n} \Phi_{j+n} U^\dagger_{j,n}\right)^2$$

and solve (numerically) the resulting equations of motion for $X_\pm > 0$

Interested in soft modes

$\longrightarrow$ use classical approximation

* Just as in the continuum:
  - ☒ Average over the static color charge
  - ☒ Determine initial conditions by matching.

Relation to continuum physics

* 3 dimensional quantities occur in the classical lattice theory:
  - $g^2\mu$ (only in this combination)
  - L, the linear size of a nucleus
  - a, the lattice cutoff

In the units of a, in the continuum limit

$g^2\mu \longrightarrow 0$, $L \longrightarrow \infty$, but

$g^2\mu L$ is constant

- RHIC — $\mu \leq 0.5\,\mathrm{GeV}$
- LHC — $\mu \leq 1\,\mathrm{GeV}$
- Nuclear size: for $A=200$ and p.b.c.

$$L = 11.6\ \mathrm{fm}$$

What to measure?

(a) gluon numbers
(b) energy (done) and energy flow (in progress).
(c) energy by mode

(a) and (c) are gauge-dependent, not well defined

* All the information is there, just come up with the right observables!

At the moment we have:

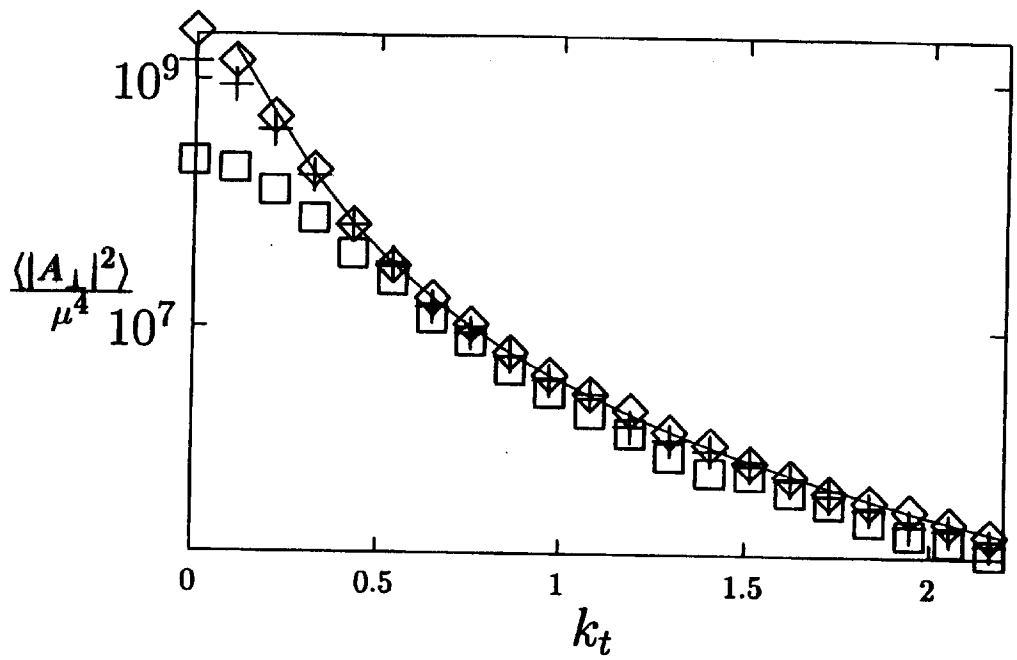


Figure 3: Field intensity over μ^4 as a function of k_t for $\mu = 200$MeV (squares), $\mu = 100$MeV (pluses), and $\mu = 50$MeV (diamonds). Solid line is the LPTh prediction. The field intensity is in arbitrary units and k_t is in GeV.

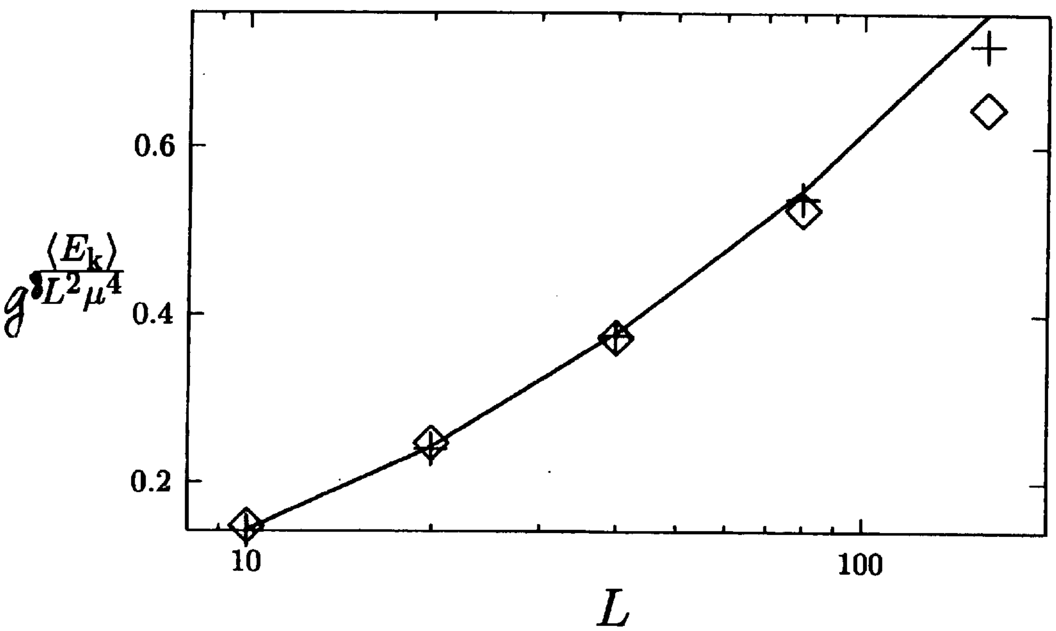
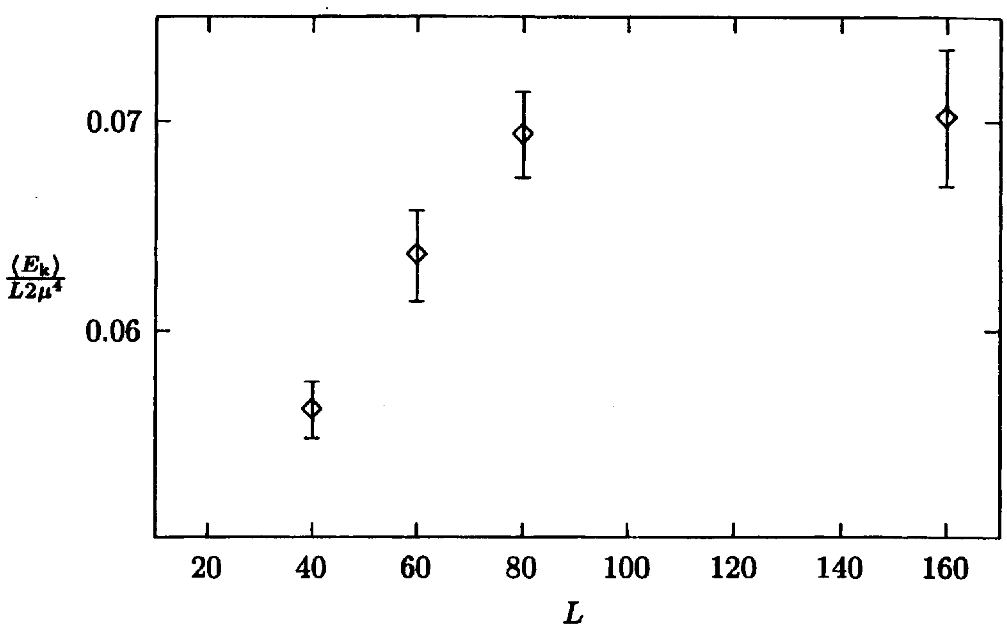


Figure 1: The lattice size dependence of the scalar kinetic energy density, expressed in units of μ^4 for $\mu = 0.0177$ (pluses) and $\mu = 0.035$ (diamonds). The solid line is the LPTh prediction. The error bars are smaller than the plotting symbols.

PT: $\frac{\langle E_k \rangle}{g^8 \mu^4 L^2} \propto \ln^2(L)$

(L in units of a)

- Both UV and IR divergent


0.07
0.06
$\frac{\langle E_k \rangle}{L2\mu^4}$
20
40
60
80
100
120
140
160
L

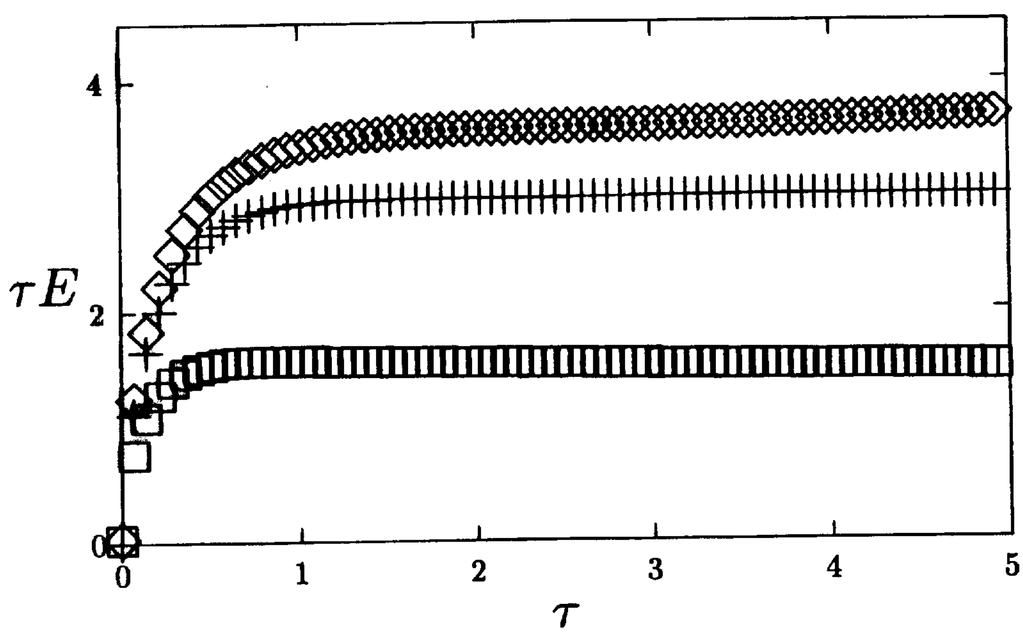


Figure 5: Time history of the energy density in units of μ^4 for $\mu = 200\text{MeV}$ (squares), $\mu = 100\text{MeV}$ (pluses), and $\mu = 50\text{MeV}$ (diamonds). Error bars are smaller than the plotting symbols. Proper time τ is in fm.

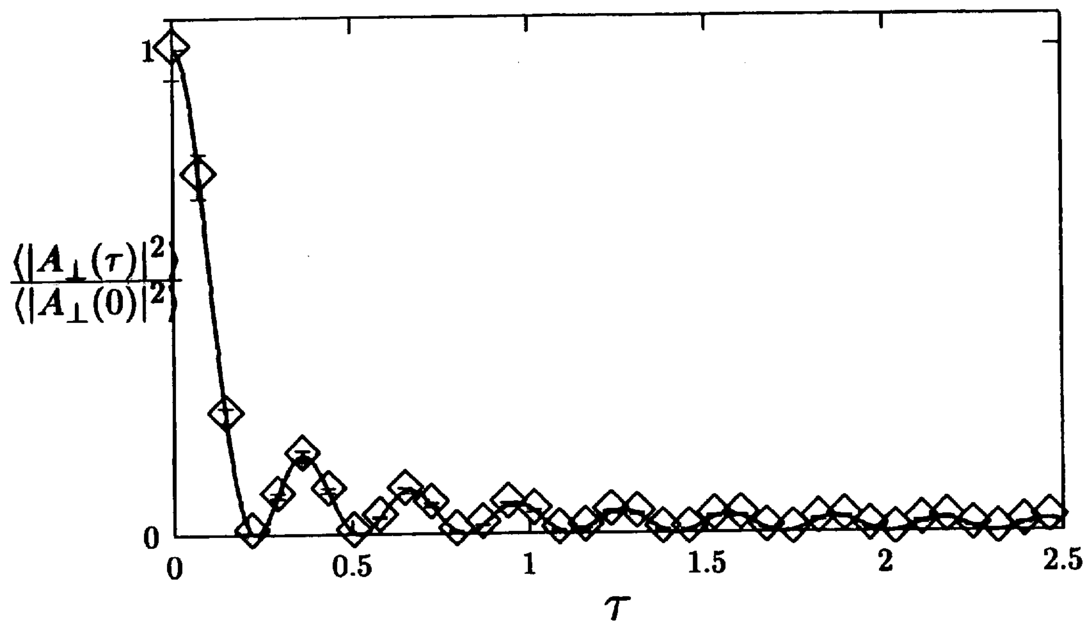


Figure 2: Normalized field intensity of a hard ($k_t = 2.16$GeV) mode vs proper time τ in units of fm (diamonds). Solid line is the LPTh prediction.

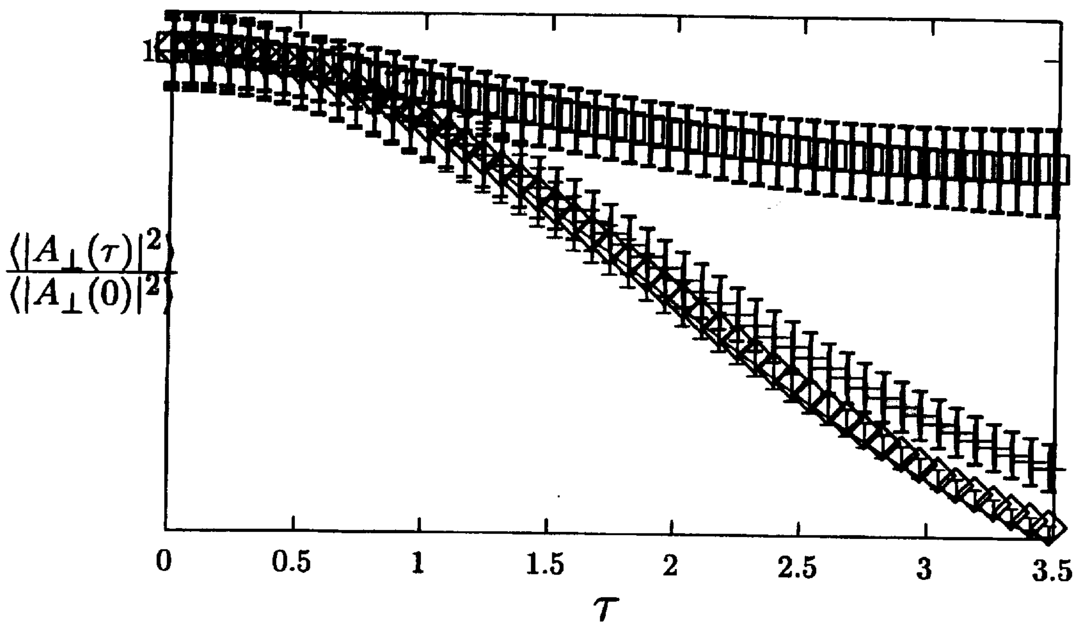
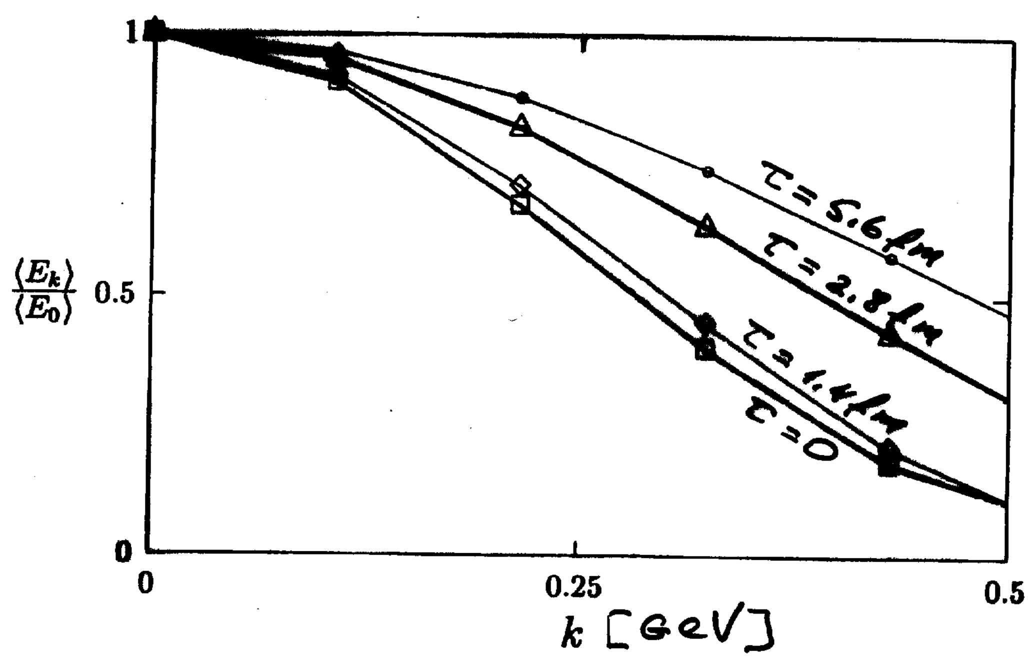


Figure 4: Normalized field intensity of a soft ($k_t = 108$MeV) mode vs proper time τ (in units of fm) for $\mu = 200$MeV (squares), $\mu = 100$MeV (pluses), and $\mu = 50$MeV (diamonds). Solid line, nearly coinciding with the $\mu = 50$MeV curve, is the LPTh prediction.

$\mu = 0.41\,\mathrm{GeV}$

$L = 11.6\,\mathrm{fm}$



Energy of mode k vs k
(relative to k=0)

Summary and outlook

1. We have at our disposal a fully non-perturbative tool for low-x physics in HIC.

2. We have made contact with the perturbative regime (for $k \gg \alpha_s \mu$).

3. Non-perturbative effects are clearly visible (in particular, no infrared divergencies, unlike in PT).

4. Other quantities, such as energy and momentum flux, lend themselves easily to a numerical study.

5. Other interesting observables?

6. The exact boost invariance can be relaxed if necessary.

7. Does thermal equilibration occur, and how do we detect it?