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Magnetic fields in Cosmology

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observed galactic magnetic fields $\sim \mu G$

Amplitude of the primordial magnetic field B_0

- If $\underline{B}$ contracted with the cosmic plasma:

$$\frac{B_{Gal}}{B_0} \sim \left(\frac{\rho_{Gal}}{\rho}\right)^{2/3} \sim 10^3 \;, \quad B_0 \sim 10^{-9} G$$

- If $\underline{B}$ has been amplified by non-linear dynamo

$$B_0 \overset{?}{\lesssim} 10^{-9} G \;, \quad B_0 \overset{?}{\sim} 10^{-13} G$$

1) How are large scale coherent magnetic fieds produced in the early universe?

2) How do they evolve in a Friedmann universe?

3) observational consequences, constraints

1 generation of primordial magnetic fields

a) • <u>string cosmology</u>

Dilaton coupling to the elm. field ⇒ "particle creation"

(Gasperini, Giovannini & Veneziano)

b) • <u>electroweak anomaly</u>

can "convert" an initial lepton assymetry into a hypercharge field ⇒ elm. field after electroweak sym. breaking

(Joyce & Shaposhnikov, 1997)

c) • breaking of conformal invariance

d) • nucleosyntesis (Opher & Opher 1997)

<u>causality</u> (b, d): $\langle B_i(k) B_j(k) \rangle$ is analytic

$$\langle B_i(k) B_j(k) \rangle = (k_i k_j - k^2 \delta_{ij}) A(k)$$

$$|\underline{B}(k)|^2 \propto k^n \,, \quad n \geq 2$$

2 Evolution of magnetic fields

The conductivity of the universe is extremely high

$$\sigma = \alpha T \quad , \quad 100 \geq \alpha \gtrsim 1$$

- For $1\,\mathrm{MeV} < T < M_W$, $1 \leq \alpha \leq 7$
- For non-relativistic electrons , $T < \frac{1}{2}\,\mathrm{MeV}$

$$\sigma = \frac{e^2 n \tau_c}{m} \sim 4\,(4\pi)^2 \alpha_{elm} T \sim 4T$$

$$\sigma_0 \sim 4T_0 \sim \frac{40}{\mathrm{cm}}$$

- For wave numbers $k \ll k_{damp}$ $\quad B \propto \frac{1}{a^2}$
- For small enough scales

$$Ba^2 \propto e^{-\frac{k^2 t}{4\pi a \sigma}}$$

$$k_{damp} = \left(\frac{4\pi a \sigma}{t}\right)^{1/2}$$

$$\lambda_{damp} \sim \left(\frac{t}{\sigma}\right)^{1/2} \sim 10^{-5}\,\mathrm{cm}\left(\frac{\mathrm{GeV}}{T}\right)^{3/2}$$

$$(T > T_{eq})$$

- Turbulence may transfer power to larger scales.

3 Observational consequences, constraints

Order of magnitude estimate:

$$\frac{\Omega_B}{\Omega_\gamma} = \frac{\rho_B}{\rho_\gamma} \sim 10^{-7}\left(\frac{B}{10^{-9}G}\right)^2$$

$$\Omega_B \lesssim \Omega_\gamma \cdot 10^{-5} \implies \underline{B \lesssim 10^{-8}\,\text{Gauss}}$$

Current limits: $|B| \lesssim (10^{-9} - 10^{-8})$ Gauss

- Nucleosynthesis (Olinto et al.)
- Large scale structure (Wasserman, Olinto et al.)
- CMB polarization (Faraday rotation) (Kosowsky & Loeb)
  $\langle a^{T*}_{\ell,m}\, a^{P}_{\ell\pm 1,m} \rangle \neq 0$ (Scannapieco & Ferreira)

- CMB anisotropies
  - A constant magnetic field (Barrow, Ferreira & Silk)
  - Magnetosonic plasma waves (Adams et al.)
  - Alfén waves (D., Kahniashvili, Yates)
  - Gravitational effects of $\underline{B}$

<u>Exemple:</u> Alfén waves in a constant magnetic "background" field

$$\underline{B} = \underline{B}_0 + \underline{B}_1(x) \quad , \quad |B_1| < |B_0|$$

$$\langle |\underline{B}|^2 \rangle \quad \propto \quad k^n \qquad (*)$$

$\Rightarrow$ Vortical waves are induced in the Plasma which propagate at speed v_A

$$v_A^2 = \frac{B_0^2}{4\pi(\rho_\gamma + \rho_\beta)} \sim \frac{\Omega_B}{\Omega_\gamma} \ll 1$$

$$|\underline{\Omega}|^2 \sim |\underline{B}_1|^2 \sin^2 \mu k t v_A \quad \sim \quad |\underline{B}_1|^2 k t v_A$$

• <u>Doppler effect</u>

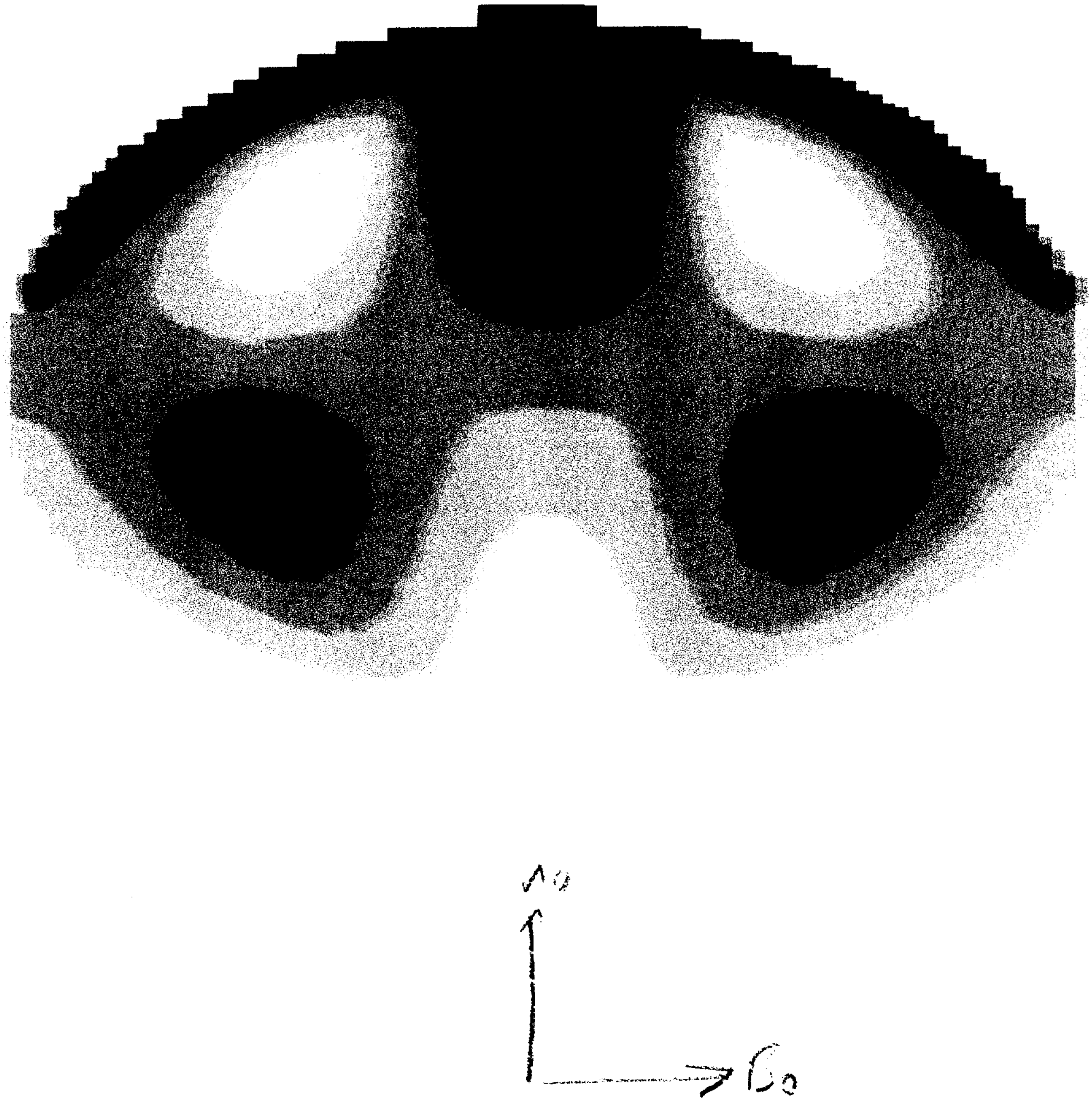
$$\left(\frac{\Delta T}{T}\right)(\underline{x}_0, t_0, \underline{n}) = \underline{\Omega}\,(\underline{x}_{dec}, t_{dec})\,\underline{n}$$

$$\underline{x}_{dec} = \underline{x}_0 + (t_0 - t_{dec})\,\underline{n}$$

For a pure power law $(*)$, the correlations can be calculated analytically

$$C_\ell(m) := \langle a_{\ell m} a^*_{\ell m} \rangle \neq 0 \qquad \frac{D_\ell}{C_\ell} \sim 1$$

$$D_\ell(m) := \langle a_{\ell-1,m}\, a^*_{\ell+1,m} \rangle \neq 0$$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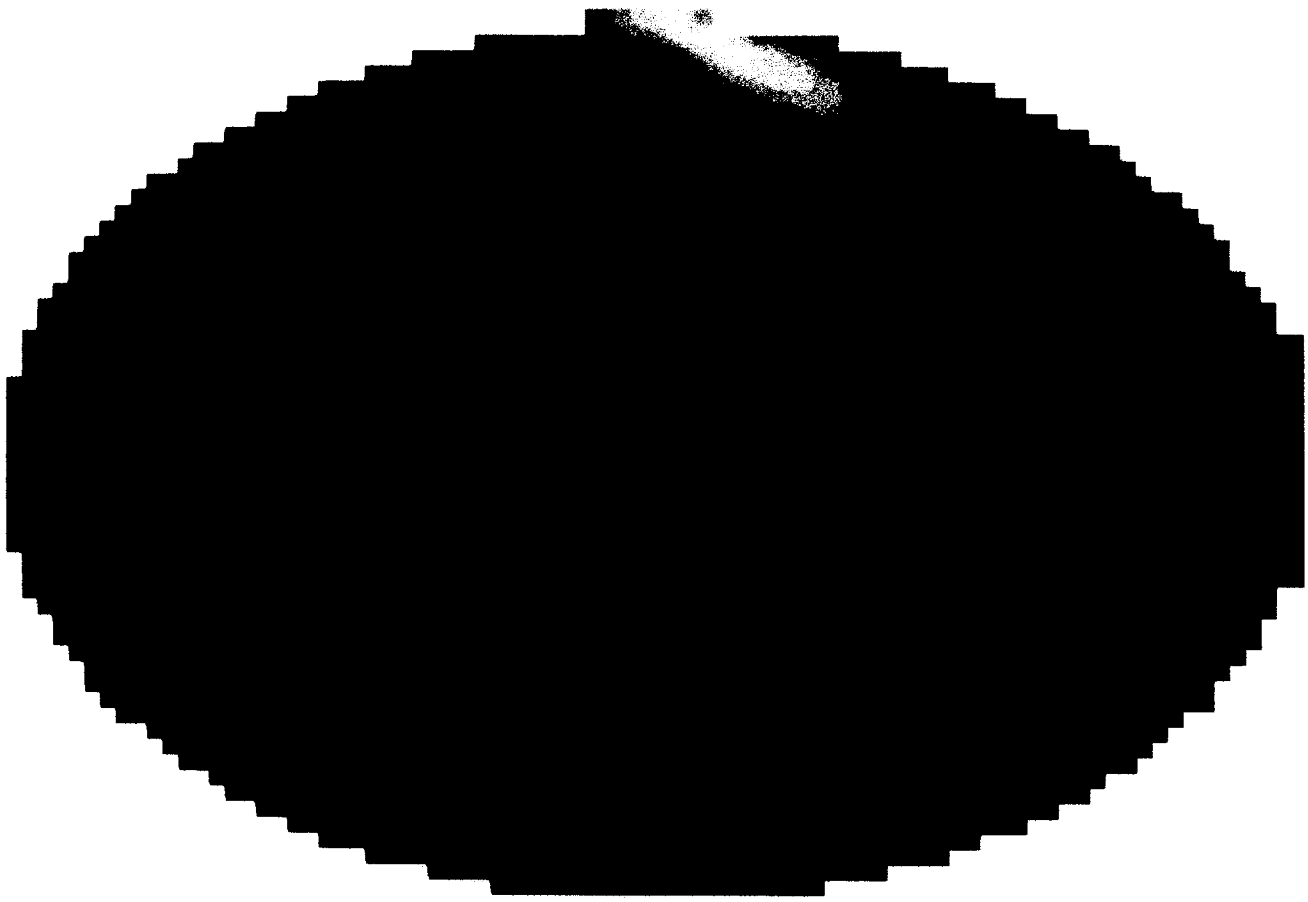
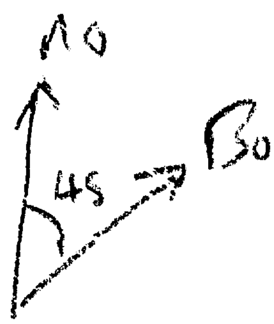
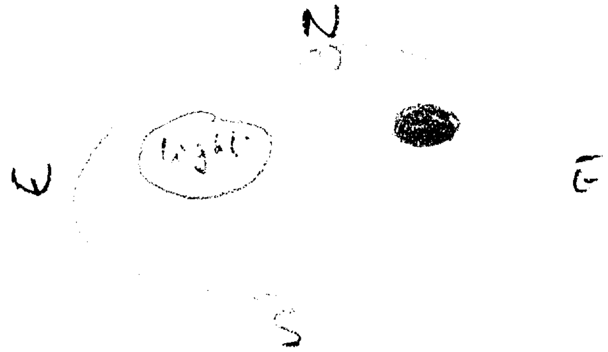
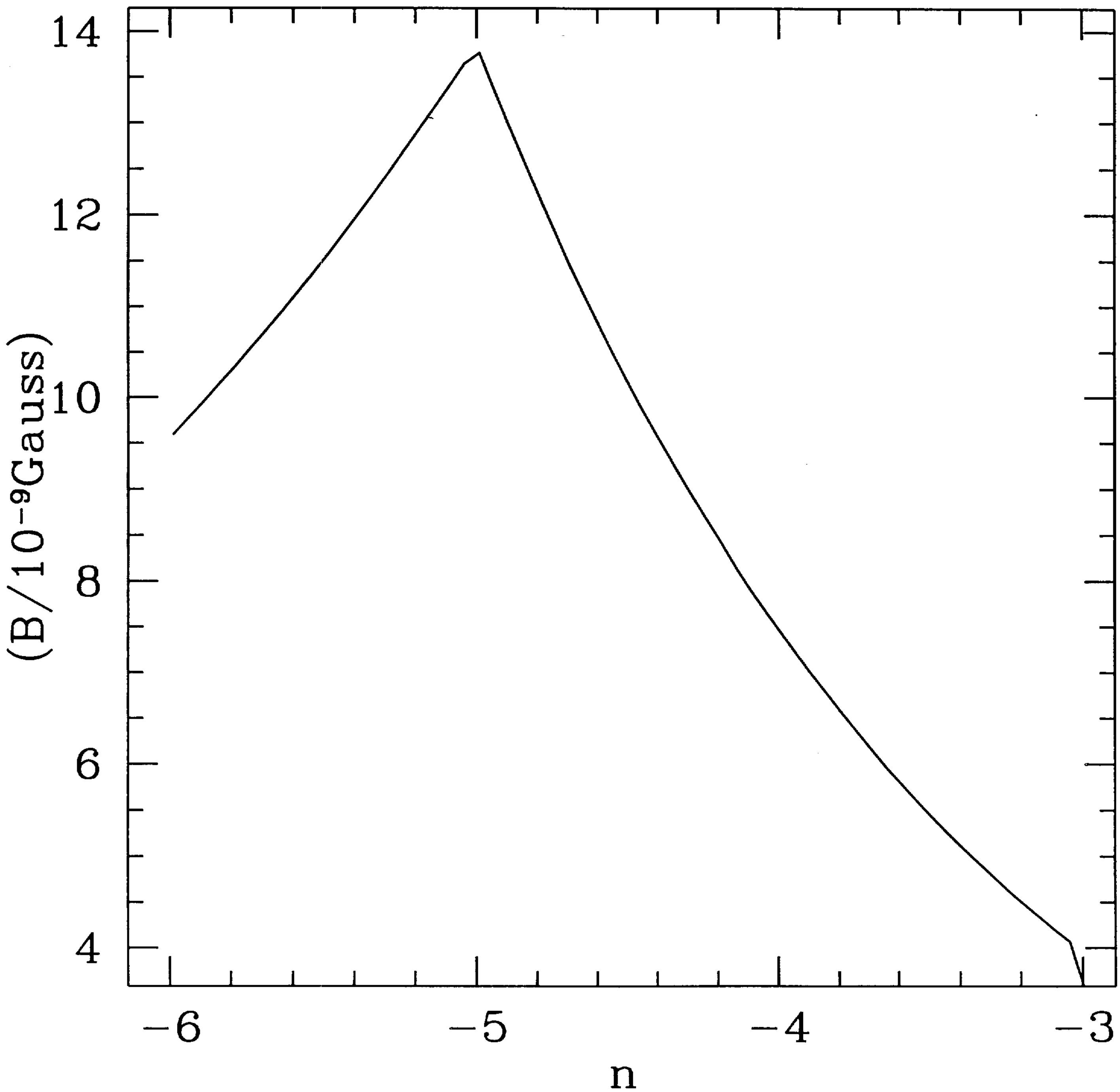






$$\ell^2 \bar{C}_\ell \cong \ell^2 \bar{D}_\ell \cong \begin{cases} v_A^4 \ell^{n+5} \begin{cases} \left(\frac{t_{dec}}{t_0}\right)^{n+5} & -6 < n \leq -3 \\ \left(\frac{t_{dec}}{t_0}\right)^2 (k_{max} t_0)^{-(n+3)}, & -3 \leq n \leq -1 \end{cases} \\ v_A^4 \ell^4 \left(\frac{t_{dec}}{t_0}\right)^2 (k_{max} t_0)^{-2}, & -1 \leq n \end{cases}$$


14
12
10
8
6
4
(B/10⁻⁹Gauss)
−6
−5
−4
−3
n