UNCERTAIN SYSTEM MODELING OF SNS RF CONTROL SYSTEM *

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***Abstract*—**This paper addresses the modeling problem of the linear accelerator RF system for SNS. The cascade of the klystron and the cavity is modeled as a nominal system. In the real world, high voltage power supply ripple, Lorentz Force Detuning, microphonics, cavity RF parameter perturbations, distortions in RF components, and loop time delay imperfection exist inevitably, which must be analyzed. The analysis is based on the accurate modeling of the disturbances and uncertainties. In this paper, a modern control theory is applied for modeling the disturbances, uncertainties, and for analyzing the closed loop system robust performance.

1 INTRODUCTION

The Spallation Neutron Source (SNS) Linac to be built at Oak Ridge National Laboratory (ORNL) consists of a combination of low energy normal conducting (NC) accelerating structures as well as higher energy superconducting RF (SRF) structures. The purpose of RF system modeling is to investigate the various cavity configurations in order to provide the correct requirements for the control system hardware and to specify RF components; verify system design and performance objectives; optimize control parameters; and to provide further insight into the RF control system operation.

In a linear accelerator RF system, there are several sources of the uncertainties and the disturbances. For a klystron, the major disturbance source is the high voltage power supply (HVPS) ripple. This disturbance affects both the output amplitude and the output phase of a klystron. For a SRF cavity, the major disturbances on the cavity characteristics are the Lorentz Force Detuning and the microphonics. Also, the changes of RF parameters should be investigated and be included in the model. In the low level RF control system, many RF components are used and these components are not ideal and have their own uncertainties and latencies. Also, feedback loop time delay, waveguide time delay, and other time delays are modeled. All of these uncertainties, disturbances, and time delays, are modeled as either multiplicative uncertainties, additive uncertainties, or exogenous disturbances [1].

For the perturbed system model, low level RF controllers are synthesized by applying modern control theory such as H_2 control, H_∞ control, loop shaping control, and H_∞ based PI control. Closed loop system stability and performance are analyzed.

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2 KLYSTRON MODEL

A klystron can be expressed as the cascade of the linear subsystem and the nonlinear output subsystem. The linear subsystem represents the 3dB bandwidth of the klystron and the constant gain. The nonlinear output subsystem represents the amplitude saturation curve and the phase saturation curve of the klystron. The nonlinear model of a klystron depicts the nonlinear amplitude saturation curve and the nonlinear phase saturation curve of a klystron. However, the nonlinearity hinders the application of the modern linear control theory both for analysis and synthesis. In order to achieve efficient analysis and synthesis for a klystron, and for the cascade of the klystron and cavity in the linear accelerator, a linear klystron model around each operating point is required where the operating point is determined by the required power of the cavity. A linear parameter varying klystron model can be obtained when the amplitude and phase saturation curves are represented by analytic functions, $\sum_{i=1}^{N} c_i A^i$, $\sum_{i=1}^{N} d_i A^i$, where A is the normalized input voltage and $i = 1,2,\cdots,N$ c_i , d_i , $i = 1,2,\cdots,N$ are coefficients and an analytic function is introduced to express the operating point trajectory. The linear parameter varying klystron model can catch the transient behaviors in the period of cavity filling and in the period of beam loading. In order for that to be possible, it is necessary to continuously measure or estimate the trajectory of the output point (V_{out}, θ_{out}) of a klystron, which is a difficult task. Instead, an operating point A_d is considered and the transfer function matrix model is obtained. The linearized klystron model is given by the following 2nd order system,

$$G_{ko}(s) = C_k(A_d)(sI - A_k)^{-1} B_k , \qquad (1)$$

where A_d is the desired operating input voltage for the normalized amplitude saturation curve obtained from the desired operating output of a klystron $(V_{out}^d, \theta_{out}^d)$. The model (1), is a hybrid model since the input-state equation is adopted from a linear parameter varying model and the state-output equation is adopted from a Lyapunov linearization. The model (1) depicts a klystron with a wider dynamics area than a Lyapunov linearization but a narrower area than a linear parameter varying model.

3 PERTURBED KLYSTRON MODEL

The major perturbation of a klystron's output is due to the high voltage power supply (HVPS) ripple. HVPS

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ripple changes both the amplitude and the phase of the klystron output. This results in perturbation of both the In-phase and Quadrature outputs of the klystron. Hence, a perturbed klystron due to HVPS ripple can be represented by a nominal system with exogenous disturbance. In this case, a proper transfer matrix from the HVPS ripple to the klystron output should be obtained.

The perturbed output voltage due to the HVPS ripple is expressed in terms of $A_p = A(1+\Delta_A R)^{1.25}$ where Δ_A is the amplitude perturbation in percentage and $R \in \Re$, $|R| \le 1$, is the normalized ripple signal. The effect of HVPS ripple on the output phase of the klystron is described by two terms: the first is the perturbed output of the phase saturation curve due to A_P, $\sum_{i=1}^{N} d_i A_P^i$, and the second is the direct additive phase perturbation, $\Delta_P R$. Then the perturbed klystron model is defined as an additive uncertainty model:

$$G_{kP}(S) = G_{ko}(s) + G_{ko}(s) W_{Ripple}(s) \overline{\Delta}_R (A, \Delta_A, \Delta_P) \qquad (2)$$

where $W_{Ripple}(s)$ is a frequency-shaping weighting function matrix and $\overline{\Delta}_R$ is a uncertainty matrix satisfying $\left\| \overline{\Delta}_R \right\|_2 \le 1$.

4 SRF CAVITY MODEL

The modeling of a SRF cavity is based on the assumption that the RF generator and the cavity are connected by a transformer. The equivalent circuit of the cavity is transformed to the equivalent circuit of a RF generator with a transmission line (waveguide) and the model is obtained [2]. The minimal realization of a SRF cavity is given by the second order system

$$Y(s) = G_c(s)U(s) + G_B(s)I(s) \qquad (3)$$

where

$$G_c(s) = C_z (sI - A_z(\Delta\omega_L))^{-1} B_z, \qquad (4)$$

$$G_B(s) = C_z (sI - A_z(\Delta\omega_L))^{-1} B_{zI}. \qquad (5)$$

Meanwhile, the Lorentz Force Detuning is written as

$$\dot{\Delta\omega}_L = -\frac{1}{\tau_m}\Delta\omega_L - \frac{2\pi}{\tau_m}\overline{K} y_1^2 - \frac{2\pi}{\tau_m}\overline{K} y_2^2 \qquad (6)$$

Equation (3) shows that from the perspective of a cavity, beam current is an exogenous disturbance. Also, the coefficients of the transfer matrix are dependent upon the Lorentz Force Detuning $\Delta\omega_L$

5 PERTURBED SRF CAVITY MODEL

The SRF cavity model given by (5) is a perturbed model where the perturbation is due to the Lorentz Force Detuning. The nominal SRF cavity model is given when $\Delta\omega_L$ is zero. Microphonics, $\Delta\omega_{MCP}$ contribute a similar perturbation

Another dominant perturbation in the SRF cavity model is due to the external Q, Q_{ext}. Since $Q_o >> Q_{ext}$ in the SRF cavity, $Q_L \approx Q_{ext}$ and the coupling factor β (β>>1) is given by $Q_L = \frac{Q_o}{1+\beta} \approx \frac{Q_o}{\beta}$. Hence, the perturbation of Q_{ext} is equivalently described by the inverse of the perturbation of the coupling factor β. Let β_o represent the nominal value of the coupling factor. Then a multiplicative perturbation of β is expressed as

$$\beta = \beta_o (1+\delta_\beta) \qquad (7)$$

where $|\delta_\beta| \le 1.0$ represents the degree of the perturbation.

The input-output SRF cavity model with perturbation can be represented by a linear fractional transformation (LFT) [3]. First, let the input-output relation of the perturbed system be expressed as a transfer function matrix $\overline{G}_{SRF}$.

$$\begin{bmatrix} v \\ y \end{bmatrix} = \overline{G}_{SRF} \begin{bmatrix} w \\ u \end{bmatrix}, \qquad (8)$$

$$w = \Delta_{SRF} v, \qquad (9)$$

$$\Delta_{SRF} = \left\{ diag\left[\delta_\beta I_2, \Delta\omega_L I_2, \Delta\omega_{MCP} I_2\right] : \delta_i \in \Re \right\}$$

$$\overline{G}_{SRF} = \begin{bmatrix} G_{SRF22} & G_{SRF21} \\ G_{SRF12} & G_{SRF11} \end{bmatrix}. \qquad (10)$$

The upper linear fractional transformation (LFT) representation of a perturbed SRF cavity is

$$y = F_U(\overline{G}_{SRF}, \Delta_{SRF}) u \qquad (11)$$

where

$$F_U(\overline{G}_{SRF}, \Delta_{SRF}) = G_{SRF12}\Delta_{SRF}\left(I - G_{SRF22}\Delta_{SRF}\right)^{-1} G_{SRF21} + G_{SRF11}. \qquad (12)$$

This is indicated in Figure 1.

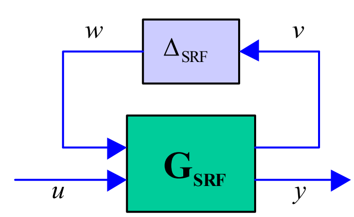

Figure 1: Perturbed SRF cavity LFT representation

6 OTHER UNCERTAINTIES

The analog signals in the cavity are fed back to the control system digital signal processor for several purposes such as low level RF control signal generation, data display, and data storage. RF components such as the RF switch, directional coupler, mixer, I/Q demodulator, preamplifier, bandpass filter, and transformer comprise that feedback loop. Since these components are not perfect, there are amplitude distortions and phase distortions. These distortions are characterized in the frequency domain. Meanwhile, there exist uncertainties in the forward path from the digital signal processor output to the klystron. In this forward loop, RF components such as the I/Q modulator, low power amplifier, bandpass filter, medium power amplifier, directional coupler, and switch are placed and these components inevitably generate amplitude distortions and phase distortions. Also, there is significant time delay due to the feedback cable delay and certain RF components (e.g.: FIR filter).The uncertainty in the RF components in the feedback loop, forward loop, and the time delay are modeled as the multiplicative uncertainty

$$\begin{bmatrix} I_{out} \\ Q_{out} \end{bmatrix} = \left(I + W(s)\Delta(\Delta_A, \Delta_\theta)\right)\begin{bmatrix} I_{in} \\ Q_{in} \end{bmatrix}. \qquad (13)$$

where Δ_A is the multiplicative amplitude pertubation, Δ_θ is the additive phase perturbation, $W(s)$ is the weighting function matrix, and $\Delta(\Delta_A, \Delta_\theta)$ is the uncertainty block satisfying $\left\|\Delta(\Delta_A, \Delta_\theta)\right\|_\infty \le 1,\ \forall\omega$.

7 APPLICATIONS

The perturbed klystron, cavity, and other perturbations are integrated together and result in a perturbed open loop system. The effect of the perturbation on the closed loop system performance is analyzed with a PI feedback controller given by

$$K_P = \begin{bmatrix} 33.2401 & 0.0165 \\ -0.0165 & 33.2401 \end{bmatrix}, \quad K_I = 500e3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

First, HVPS ripple effect is analyzed [1]. The magnitude response from HVPS ripple to the tracking error when amplitude ripple is 1.2% and the phase ripple is 11.75 degrees is shown in figure 2. Figure 2 shows that there is an upper limit from below and lower limit from above in the frequency of the HVPS ripple which guarantees the robust performance. Figure 2 shows that the ripple of the frequency range [5903.9 Hz 35966 Hz] cannot be rejected with the given PI controller.

When a system is perturbed, additional energy should be provided by an energy source. In the frequency domain, the additional power can be estimated by the magnitude response for the worst case perturbation. The LFT representation of a perturbed system or the standard form of a multiplicative uncertainty are suitable tools for energy interpretation. For a stable transfer matrix $G(s)$, the H_∞ norm is defined as the input/output RMS energy gain, i.e., for $\|u\|_2$ defined by $\|u\|_2^2 = \int_0^\infty \|u\|^2 dt$,

$$\|G(s)\|_\infty = \sup_\omega \sigma_{\max}\left(G(j\omega)\right) = \max_{\substack{u\in L_2 \\ u\neq 0}} \frac{\|y\|_2}{\|u\|_2} \qquad (18)$$

where L_2 is the space of signals with finite energy . For a fixed frequency ω_1, the maximum singular value of $G(j\omega_1)$ is the largest energy gain at the frequency ω_1. In order to calculate the power margin required for the Lorentz Force Detuning whose value at the end of 1.3 $m$sec RF pulse is –165 Hz (68.75% of -1.2×14.1^2), $\sigma_{\max}\left(F_U(\overline{G}_{SRF}(j\omega, \Delta_{SRF})\right)$ is calculated with $\Delta_{SRF} = \{diag[0_2, \Delta\omega_L I_2, 0_2] : \delta_i \in \Re\}$, which is shown in figure 3. The nominal system's singular value is 1.9362, which changes to 1.7562 due to the system perturbation resulting from the Lorentz Force Detuning (-165 Hz). In order to recover to its nominal value, 9.3 % additional energy is necessary at low frequency. In the case of the closed loop system, the additional energy is scaled by 0.5225 (as given in figure 4), and the additional energy is 4.9% [1], [4].

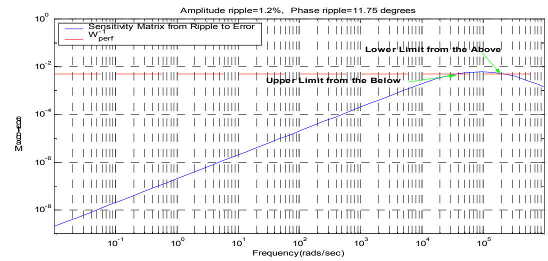

Figure 2: Transfer Matrix from the scaled High Voltage Power Supply (HVPS) ripple to the tracking.

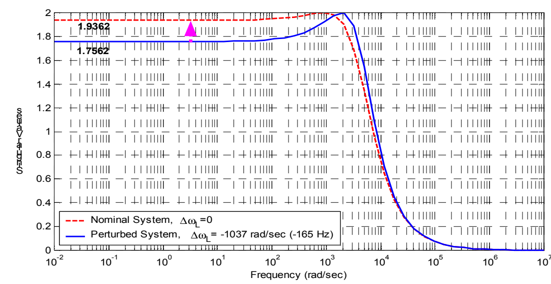

Figure 3: Maximum singular value plot of the open loop nominal system and the open loop perturbed system for a SRF cavity.

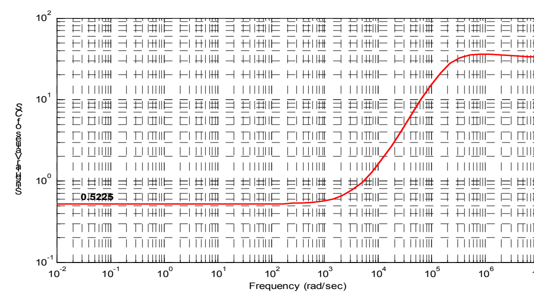

Figure 4: Maximum singular value plot of the scaling transfer matrix $C(s)S(s)$: $C(s)$ is the transfer matrix of the PI feedback controller and $S(s)$ is the sensitivity matrix.

8 REFERENCES

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