CALCULATING TRAPPED MODES IN TESLA CAVITIES WITH TIME AND FREQUENCY DOMAIN METHODS

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Abstract

We report the development of different algorithms for the calculation of quality factors of trapped eigenmodes in accelerating cavities, which have resonance frequencies above the cutoff frequency of the beam tubes. The analysis is based on a discretization of such cavity structures by the Finite Integration Technique (FIT), and the radiation at the open boundaries is systematically taken into account by different approaches in time and frequency domain. Comparison with the conventional method of analyzing closed cavities and identifying modes with little change in frequency as function of boundary condition show qualitative differences. Some modes from the closed cavity model do not exist in the open structure and thus would be misinterpreted as trapped modes when only a closed cavity analysis is employed.

Results indicate that even single cell cavities of the TESLA type show *Q*-values above 10^3. And for the TESLA 9-cell cavity trapped modes with *Q*-values in excess of 10^6 are found, which correspond to recent measurement results from the TESLA Test Facility at DESY.

1 INTRODUCTION

Trapped modes in accelerating cavities have been subject to serious consideration with respect to beam instabilities since many years. While the phenomenon of trapped modes is well known in antenna theory, the analysis of these fields remains a complicated task. In the field of accelerator design the typical analysis tools are eigenmode solvers for closed structures such as the E-module in MAFIA [1]. Such tools *a priori* do not allow the rigorous analysis of trapped modes as the basic feature of continuous loss of energy through travelling modes is not included. The eigenmode analysis can only give some hints on the existence of trapped modes. The same facts limit the usefulness of measurements in which typically the beam tubes are electrically short ended and thus also do not take the key issue of travelling modes into account. In many practical cases this analysis is sufficient as it results in upper limits for the impedance of such modes. However, in the case of superconducting structures these upper limits become intolerable and a more detailed analysis becomes indispensable.

In this paper we present several different methods for the analysis of trapped modes in open structures, which are all based on the discretization with the Finite Integration Technique (FIT) [1,2]. The effect of travelling modes is taken into account by different techniques in time or frequency domain.

Especially in superconducting cavities *Q*-values determined solely by eigenmode computations yield results that are not tolerable from the beam dynamics point of view. Although it still holds, that a closed cavity analysis gives upper limits, these limits are unacceptably high. In this case a careful analysis of trapped modes becomes a crucial issue, as does the design of couplers as the only means to reduce the *Q*-values.

To avoid confusion with the experimental evidence of trapped modes in the TTF [3], it should be pointed out that there exist also other phenomena that lead to trapped modes with high *Q*-values. In the TTF case some of the trapped modes that were found experimentally are most likely to be caused by the effect of the neighbour cavities that did not allow the fields above cut-off frequency to travel away from the main cavity [4]. These trapped modes are externally trapped and thus a different subject than the one treated here, where the beam tubes are assumed to be infinitely long with a constant cross section. In superconducting cavities the *Q*-values of trapped modes are dominated by the radiation losses. Thus we neglect the wall losses in this paper. However, they can easily be taken into account in the numerical analysis by performing a perturbation calculation and adding up these two types of losses.

The conventional eigenmode analysis of trapped modes considers only the influence of changed boundary conditions on the mode frequency. If the eigenfrequency stays nearly constant with perfect electric (PEC) and perfect magnetic (PMC) boundary conditions the mode is considered as trapped.

2 TIME DOMAIN ANALYSIS

A seemingly simple approach to attack the open cavity problem is to use a time domain analysis including waveguide ports. These waveguide boundary ports simulate infinitely long beam tubes and have been successfully used in S-parameter computations for waveguide components [5]. As the orthogonality of the waveguide modes in the boundary plane is exactly taken into account (in the sense of the numerical model), this

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boundary condition is well suited to deal also with high-*Q* cases with small time signals at the ports.

The *Q*-values of trapped modes are determined solely by the ratio of the stored energy in the cavity and the power radiated through the boundary ports. Thus, assuming an excitation of the cavity and freely oscillating fields, the knowledge of the time signals at an arbitrary point in the cavity, monitored by pickup antennas in the simulation, which include the exponential decay of the stored energy, is sufficient for the calculation of Q_{RAD}.

From a discrete Fourier transform (DFT) of the time signals we obtain the cavity impedance (the transfer functions between the beam tube ports and the pickup antennas) and thus the width of the trapped resonances. For the cavity analyzed here, the spectrum clearly shows some sharp resonances.

One serious problem with the time domain analysis as described so far is, that all modes with high Q_{RAD}-factors are related to long settling times, when they are excited by fields at the port modes. The other way round, if they are oscillating in the cavity, the time signals at the ports are very low, and again long simulation times are needed for a numerically robust extraction of the Q_{RAD}-factors.

One possibility to improve the situation in time domain is to apply advanced signal processing techniques, where the spectrum of a signal calculated by the field simulation in time domain is approximated by the system response of a digital filter. Different techniques are available and have been implemented [6].

However, there is still a need for more robust techniques, and this is why we return to the supposedly less efficient frequency domain. Based on the same computational model, they again lead to eigenvalue problems, but now including the open boundary condition at the waveguide ports.

3 FREQUENCY DOMAIN ANALYSIS

A method to calculate the cavity impedance in the frequency domain, including the radiation at the beam tube ports, has already been presented in [7], and in principle the *Q*-factors of trapped modes can be derived from these results in a similar way as mentioned above for the time domain scheme.

However, in this approach a inhomogeneous equation has to be solved for each frequency point, and thus it seems to be more appropriate to pass over to an eigenvalue equation again.

1. Complex Eigenvalue Formulation

To this end, the computational model of the FI-discretization including the energy loss through the waveguide boundaries is formulated in frequency domain [8], leading to a linear system of equations for the vector $\widehat{e}$ of electric grid voltages (the state variables of the FI-formulation). Its solutions are the complex eigenvectors $\widehat{e}$ and the corresponding eigenvalues (the squared complex frequencies ω). The *Q*-factor of these modes finally is given by

$$Q_{RAD} = \frac{\mathrm{Re}(\omega)}{2\,\mathrm{Im}(\omega)} \qquad \textbf{(1)}$$

Finally the task is to solve a (complex and nonsymmetrical) linear eigenvalue problem, where a good start solution for iterative solvers is given by the solution of the corresponding real problem.

A typical trapped-modes analysis requires to calculate both several (10-30) solutions of the real problem (with closed boundaries) and some solutions of the complex formulation, and the most severe disadvantage of this approach is the high numerical cost of the complex solvers. Thus, in the next chapter we present an alternative frequency domain approach, which does not require solution of a complex eigenvalue problem.

2. Modal Approach

The following ‘modal’ approach has been used previously for a similar analysis [9]. The waveguide-coupled cavity is interpreted as a multiport system, and the impedance matrix referring to the generalized voltage and current quantities at the ports of this system is calculated. To this end we use a set of modal coefficients, which are related to the 3D cavity modes and their coupling to the 2D waveguide modes at the ports. The desired eigenmodes and their external *Q*-factors can then be derived by means of a simple network consisting of the impedance matrix of the structure and the matched resistive loads of infinitely long waveguides.

The model problem introduced above can be viewed as a one-port system, making use of the third symmetry of the structure. The impedance matrix then reduces to one single impedance quantity related to the input port of the cavity, which is defined as a reference plane in the beam tube. We introduce a PMC boundary condition at this reference plane and perform a conventional (real-valued) eigenmode analysis of the closed structure. From the eigenfrequencies ω of the complete system we find the *Q*-factors of the corresponding modes according to (1).

4 RESULTS

As a validation two test models were calculated with these three methods. We used a single cell model with the shape of a TESLA end cell and a triple cell resonator with TESLA geometry. The results for the *Q*-values of some trapped modes are shown in table 1. For the single cell we received nearly the same numbers with all three methods. For the triple cell the *Q*-values of some modes are already in the 10^4 region. Thus the resolution limit of the time domain calculation was reached and we get only a lower limit. Nevertheless the results are consistent.

The calculation of a TESLA 9-cell cavity in three dimensions would exceed reasonable calculation times. Thus we implemented the modal analysis in two dimensions again with perfect boundary approximation

(PBA) [10]. Here we found some trapped modes. The three modes in the 5th dipole passband around 3.08 GHz have extremely high *Q*-values (see table 2). Especially the third one shows in addition to that a considerable loss factor or transversal shunt impedance respectively. These modes have to be considered as harmful.

In figure 1 the electric field of the first of these three modes is shown. The field energy concentrates on the inner three cells and in the end cells the field amplitude is diminished by factor 100.

Table 1: Comparison of the three methods

Q	*f* /GHz	Time Domain	Complex Eigenvalue Problem	Modal Analysis
Single Cell	3.057	71	62	61
	3.335	1,510	1,558	1,578
Triple Cell	3.070	>20,000	25,579	26,100
	3.342	>12,500	13,489	13,934

Table 2: Trapped modes in the TESLA 9-cell cavity

f /GHz	*Q*	loss factor V/pCm^2	$R_{S\perp}/Q$ Ω
3.078	2,340,000	0.0391	0.0019
3.080	282,070	4.0128	0.1991
3.085	565,640	28.478	1.4055

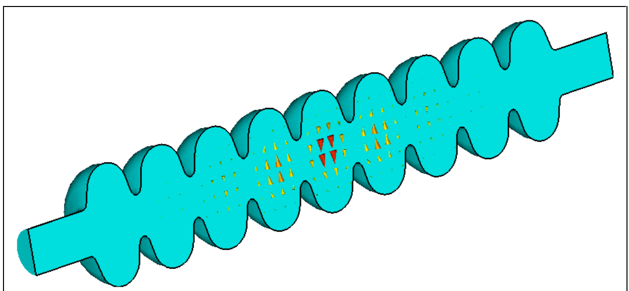

Figure 1: Electric field of a trapped TE-dipole mode

5 CONCLUSION

We calculate the quality factors of trapped dipole modes in accelerating cavities, which have resonance frequencies well above the cut-off frequency of the beam tube. Three different approaches have been presented, where the radiation at the ports is rigorously taken into account. As these approaches follow quite different paths, the calculated *Q*-factors with deviations in the range of some percent can be considered to be reliable results.

Quantitative results show *Q*-values of 1,000 for single cell cavities and more than 10,000 for three cell cavities. Thus the impedance above cut-off frequency may yield a serious contribution to beam instabilities.

The conventional procedure of computing closed cavity modes and comparing the eigenfrequencies when changing the beam tube end boundary condition may give hints for the existence of trapped modes. This analysis yields upper limits for the contribution of the fields above cut-off frequency, which may be sufficient in some practical applications. However, a comparison between closed cavity analysis and open cavity analysis shows, that not all modes that appear to be candidates for trapped modes from the closed cavity analysis are actually real trapped modes. Thus for some modes, the closed cavity analysis does give upper limits. However, for some modes this type of analysis may give wrong results. Especially for superconducting accelerators the closed cavity approximation yields unacceptable values for the impedance because the losses are dominated by the radiation and a rigorous analysis of the open cavity structure is indispensable.

In recent measurements at the TESLA test facility (TTF) trapped modes in the 5th dipole passband with extremely high *Q*-values showed up. The results from this paper show that they exist already in the perfect cavity without production errors.

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