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QUANTUM ASPECTS OF BEAM PHYSICS*

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Abstract

The continued demand for higher beam energies, luminosities, and brightness, induces increasing number of beam phenomena that invlove quantum effects. In this paper we review the various quantum aspects of beam physics, with emphasis on their recent advances. These include quantum effects in beam dynamics, electron-photon interaction, beam phenomena under strong fields, fundamental physics under violent acceleration, and quantum methodology in beam physics. We conclude with a future outlook of this very exciting new field by the name *quantum beam physics*.

1 WHERE IS $\hbar$ IN BEAM PHYSICS?

It is common knowledge that quantum effects are pronounced in physical systems where the particles involved exihibit the wave nature, or the (radiation) waves involved exhibit the particle nature. In accelerators the de Broglie wavelength of a high energy beam particle is

$$\lambda\!\!\!^{-}_{db} = \frac{\hbar}{p_\perp} \lesssim \frac{\hbar}{\gamma mc}\frac{p}{p_\perp} = \lambda\!\!\!^{-}_c\sqrt{\frac{\beta}{\gamma\epsilon_n}}, \tag{1}$$

where β and ϵ_n are the β-function and the normalized emittance, respectively. This value is generally much smaller than the typical apertures of the cavities and magnets in the accelerator. In addition, the synchrotron radiation induced by the magnets is typically low-energy and long wavelength, and the number of photons per volume of the wavelength is much larger than unity. Therefore the conventional beam dynamics is essentially classical physics to the leading order.

The ever-increasing demand for higher beam energy, luminosity and brightness in accelerators pushes for ever higher acceleration gradients, smaller apertures, and tighter beam phase space, and quantum effects in beam physics become increasingly important.

2 QUANTUM EFFECTS IN BEAM DYNAMICS

2.1 Ultimate Limit of Phase Space

The basic assumptions in the standard treatment of synchrotron radiation reaction were that the photon emission occurs instantly and the recoil of the particle is equal and opposite to the momentum of the emitted photon. As the photon emission is random, its reaction causes random excitations in the beam phase space. It was found[1], however, that these assumptions are violated in a continuous focusing channel. The radiation formation length can in principle be comparable to the betatron oscillation length, and the focusing channel serves as a third party participating in the overall energy-momentum conservation. As a result, the radiation reaction does not cause any excitation of the transverse momentum, but an *absolute damping* of the emittance. This points to a theoretical minimum action, limited only by the zero-point fluctuations due to the uncertainty principle, i.e., $J_{min} = \hbar/2$, or

$$\epsilon_{n,min} = J_{min}/mc = \lambda\!\!\!^{-}_c/2 \sim 10^{-11}\mathrm{cm}. \tag{2}$$

The above result can be genralized to combined focusing-bending systems where the radiation formation length (ρ/γ) is comparable to the average betatron wavelength (due to a very strong focusing)[2]. Pure bending and pure focusing are the two limiting regimes of the general formalism.

2.2 Classical vs. Quantum Tracking

In conventional treatments in particle tracking each point in phase space is assumed to have a perfect resolution. But due to the uncertainty principle, the phase space cannot have infinite resolution. Heifets and Yan[3] show that in the stochastic regions in phase space where classical trajectories tend to diverge exponentially, trackings of particles could be sensitive to such qauntum granularities.

2.3 Coherence and Bose-Einstein Condensate of Particle Beams

We are interested in better understanding, and hopefully eventually attaining a coldest possible particle beam that Nature would allow. To this end a comparison with photon beams should be helpful. Ordinary lights emit different frquency photons at random. It is well-known that laser photons, on the other hand, are monochromatic and coherent. The evolution of the laser spot size is governed by

$$a^2(s) = \frac{\lambda}{\pi}Z_R\Big(1+\frac{s^2}{Z_R^2}\Big), \tag{3}$$

where Z_R is the Raleigh length.

An ultimate, coherent particle beam analogous to a laser would have a minimum emittance associated with its de Broglie wavelength. Then we would have a coherent beam that propagates as

$$\sigma^2(s) = \frac{\lambda\!\!\!^{-}_c}{2\pi\gamma}\beta^*\Big(1+\frac{s^2}{\beta^{*2}}\Big). \tag{4}$$

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Would such coherent particle beam necessarily imply certain kind of condensation? Recent progresses on Bose-Einstein condensate (BEC) and the *atom laser*[4, 5] inspire us to wonder if particle beams in accelerator environments can in principle also form condensates. Although particle beams are typically made of fermions, this possibility may not necessarily be ruled out. Afterall, fermions such as He^3 do exhibit superfluidity at low temperatures.

3 ELECTRON-PHOTON INTERACTION IN BEAM PHYSICS

There emerges a new class of beam phenomena which involve quantum mechanics. This is mainly through the applications of lasers in various beam production, cooling, and monitoring schemes.

3.1 Compton Backscattering

Compton scattering between a high energy electron and a (much lower energy) photon, e.g., that in a laser, will induce a dramatic exchange of energies between the photon and the electron. As a result the final state photon will emerge with much higher energy in the lab frame. This mechanism is by now widely applied in beam physics. In the early 1980s a photon-photon collider concept was introduced[6], in which the high energy photon beams are to be produced through the Compton backscattering process. More recently, studies were made[7] in producing intense X-rays by the same mechanism. Turning the attention to the final-state electron, Telnov[8] suggested that Compton backscattering can also be used to reduce the electron beam energy with little increase in its divergence, and thus an effective reduction in the normalized emittance. Another creative concept, proposed by Shintake[9], is to monitor sub-micron beams by intercepting them with a pre-established laser interference pattern.

3.2 Laser Cooling of Stored Beams

Lasers are also invoked to cool stored beams. As an extension to the celebrated idea of stochastic cooling, Michailichenko and Zolotorev[10] suggested the use of laser that would largely expand the bandwidth for the probe. Extending Telnov's idea, Huang and Ruth[11] proposed repeated Compton-scattering cooling of a stored electron beam. Using the *dispersive* cooling mechanism the coolings of ion beam longitudinal and transverse temperatures are found to be highly efficient[12, 13]. In the extreme limit, one expects that crystalline structure be developed in ion beams[14, 15].

3.3 Quantum Effects in Free Electron Laser

The quantum correction to the classical FEL gain formula becomes important when the photon energy is comparable to, or larger than, the gain bandwidth[16]. Such a correction is small when the FEL gain is small, i.e.,

$$\frac{\hbar\omega_s}{E_e/N_u} \approx \frac{\hbar\omega_s}{\Delta E_e} \ll 1, \tag{5}$$

where $\hbar\omega_s$ is the FEL photon energy, E_e and electron energy, and N_u is number of undulator sections.

However, in the high-gain SASE FEL[17], the recoil effect can in principle become severe. It was found[18, 16], that quantum corrections to SASE noise can be kept small if the emission is non-degenerate:

$$(\gamma\Delta x\Delta\phi)^2(\Delta z\Delta\gamma) \ll N_e(\pi\lambda\!\!\!^{-}_c)^3, \tag{6}$$

where N_e is the number of electrons in the bunch.

4 BEAM PHENOMENA UNDER STRONG FIELDS

For an initial state electron with momentum p_μ traverses an external electromagnetic field, $F_{\mu\nu}$, there is a dimensionless, Lorentz invariant parameter which characterizes the nonlinear QED phenomema:

$$\Upsilon = \frac{|p_\mu F^{\mu\nu} p^\lambda F_{\lambda\nu}|^{1/2}}{mF_c} \quad , \tag{7}$$

where $F_c \equiv m^2c^3/e\hbar \approx 4.4 \times 10^{13}\mathrm{Gauss} = 1.8 \times 10^{18}\mathrm{V/m}$ is the Schwinger critical field strength. The physics involved is essentially classical if $\Upsilon \ll 1$. Quantum effects become dominant when $\Upsilon \gtrsim 1$. It happens that several beam phenomena fall under this condition.

4.1 Beam-Beam Interaction in Linear Colliders

During beam-beam interaction in linear colliders (LCs), particles in one beam interaction with the extremely intense collective EM fields of the oncoming beam. This triggers intense radiation of hard photons by the name *beamstrahlung* [19], characterized by the *beamstrahlung parameter* based on the *mean field* of the beam[20]:

$$\Upsilon = \gamma\frac{\langle E+B\rangle}{F_c} \approx \frac{5}{6}\frac{r_e\lambda\!\!\!^{-}_c\gamma N_e}{\sigma_z(\sigma_x+\sigma_y)}. \tag{8}$$

At $\Upsilon \sim 1$, severe beamstrahlung ($\sim 10-30\%$ averaged energy loss) is expected.

It was then discovered[21] that when $\Upsilon \gtrsim 1$, copious e^+e^- pairs will also be produced through the coherent interaction between the beamstrahlung photons and the collective fields of the oncoming beam. One of the pair particles always carries the same sign of charge as that of the oncoming beam, and will thus be deflected unbound, causing potentially severe particle detector backgrounds. Another background[22, 23, 24] is the quantum chromodynamic (QCD) *minijets*. As photon can manifest itself from time to time as a quark-antiquark pair, two beamstrahlung photons can interact hadronically through their quark-gluon contents. Since the $\gamma\gamma$ cross section rises as the center-of-mass energy increases, this background is expected to become more severe in future generations of LCs.

All these backgrounds are directly influenced by the number of beamstrahlung photons emitted per electron throughout the collision[25],

$$n_\gamma \approx 2.54\Big(\frac{\alpha\sigma_z}{\lambda_c\gamma}\Big)\frac{\Upsilon}{(1+\Upsilon^{2/3})^{1/2}}. \tag{9}$$

It is interesting to note that n_γ as a function of Υ peaks at $\Upsilon \sim 10$, and diminishes not only in small but also in large Υ limits, thanks to the quantum nature of hard photon emissions[25]. While the current LC design efforts focus on constraining $\Upsilon \lesssim 1$, it is hardly avoidable that far future LCs would necessarily be operated in the deep quantum regime. Nevertheless, it is conforting that the beamstrahlung backgrounds would not be worsen[3].

4.2 Relativistic Heavy Ion Collisions

For low energy heavy-ion collisions near the Coulomb barrier, quasi-bound molecular states are formed with binding energies dive into the negative energy continuum, resulting in a resonance which subsequently decays into an e^+e^- pair[26]. In the collision between two relativistic heavy ions, a different nonlinear QED effect, analogous to beamstrahlung coherent pair creation, should in principle occur[27], and has indeed been observed experimentally[28]. In the near future, ion energies above 100 GeV/nucleon will be available in the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and the Large Hadron Collider (LHC) at CERN. These should provide opportunities to study nonlinear QED with effective coupling constant $Z\alpha \sim 1$.

4.3 Crystal Channeling of Relativistic Beams

Another physical environment where high energy beams encounter strong fields is crystal channeling[29, 30], where the confining (or focusing) field is as large as $10^{12}\mathrm{V/m}$. Such strong fields are useful in beam handling and production. For example, a bent crystal is able to redirect proton beams in a short distance[31]. For channeling electrons or positron at energies 100 GeV and beyond, there will be copious coherent e^+e^- pairs produced. Because of the same channeling effect, the outcoming positron emittance should be much reduced. This can be invoked for a novel positron source.

4.4 Electron Interaction with Ultra-Intense Laser

If a laser is very intense, there is a finite probability that multiple photons can involve in one Compton scattering process. Such multiphoton QED processes are characterized by an additional Lorentz invariant parameter:

$$a_0 = \frac{eE_0}{m\omega_0 c} = \frac{eE_0\lambda\!\!\!^{-}_c}{\hbar\omega_0}, \tag{10}$$

where E_0 is the amplitude and ω_0 is the frequency of the laser.

The multiphoton Compton scattering tends to degrade the spectrum and the polarization of the high energy backscattered photons. It thus imposes a contraint on the various applications of the Compton backscattering mentioned above[3]. On the other hand, these multiphoton QED processes are fundamentally interesting for its own right. Indeed, experimentally they were never observed until recent years. The SLAC experiment E144[32] was dedicated particularly for that purpose, and has provided important data on the phenomena.

4.5 Spontaneous and Stimulated Breakdowns of the Vacuum

One issue in nonlinear QED that remains unclear regards the nature of the breakdown of the QED vacuum. In an attempt to clerify the issue, Chen and Pellegrini[3] borrow the terms "spontaneous" and "stimulated" to distinguish two different types of vacuum breakdowns. The stimulated breakdown, examplified by the coherent beamstrahlung pair creation process [21] and the multiphoton pair production process[32], requires an initial state particle that interacts with the external EM field.

On the other hand, the QED vacuum can also breakdown by a pure classical EM field without any initial state particle. The penetration of the vacuum-fluctuated pairs through the potential barrier is spontaneous in this case. The Lorentz invariant parameters involved are[33]

$$\begin{cases} \mathcal{F}^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu} = \vec{B}^2 - \vec{E}^2, \\ \mathcal{G}^2 = \frac{1}{4}F_{\mu\nu}F^{*\mu\nu} = \vec{E}\cdot\vec{B}, \end{cases} \tag{11}$$

where $F^*_{\mu\nu}$ is the dual field-strength tensor. Therefore in the case of a cross field with $|E| = |B|$ (or a plane wave), both $\mathcal{F}$ and $\mathcal{G}$ vanish, and this nonlinear effect would never occur. This is in sharp contrast with the stimulated process under the same EM field. It occurs that the proposed Linear Coherent Light Source (LCLS) (a free electron laser) at SLAC should have the right intensity for a test on the spontaneous process in the near future[3].

5 FUNDAMENTAL PHYSICS UNDER VIOLENT ACCELERATION

When a laser is ultra-intense, i.e., $a_0 = eE/mc\omega_0 \gg 1$, an electron under the direct influence of the laser can be accelerated and decelerated intermittantly during every laser cycle. Since it occurs within a laser cycle, the acceleration gradient can be as high as[34]:

$$G_l \sim 10\mathrm{TeV/m} \sim 10^{25} g_\oplus. \tag{12}$$

While such intermittant acceleration is not useful for bringing electrons to ultra-high energy, it has been recently suggested [34] that this may be used for studying fundamental physics related to General Relativity, based on the Equivalence Principle.

5.1 The Hawking-Unruh Effect

Bell and Leinaas (BL)[35] first suggested that the well-known phenomenon of the equilibrium spin polarization in electron storage rings may be interpreted as a manifestation of the Unruh effect.

A uniformly accelerated object sees the vacuum fluctuations as a thermal bath, with a temperature given by[36]

$$kT = \frac{\hbar a}{2\pi c}, \tag{13}$$

where a is the object's proper acceleration. Historically this temperature was decuded as an extension of the seminal discovery by Hawking[37] on the blackbody radiation of black holes.

The spin of a circularly accelerated electron serves as a detector where its populations at the two spin levels would follow the Boltzmann distributions. Barber and Mane[38] showed that the BL formulation is equivalent to that of Derbenev-Kondratenko, and the known result of synchrotron radiation power can be reporduced using the Unruh picture. BL also observed that the resultant temperature is higher than that predicted for the linear acceleration. Most recently, Unruh[3] reinvestigated into this issue, and confirmed the BL findings. He explains that the seemingly higher effective temperature in the case of circular acceleration is not due to a supposed nonthermal nature of the heat bath, but rather due to the time-dependence of the spin-orbit coupling.

To avoid the complications caused by the spin-orbit coupling, Chen and Tajima (CT)[34] investigated the Hawking-Unruh effect under linear, albeit time-varying, acceleration. They proposed that by using an ultra-intense laser with, the sought-after signal should be above the Larmor radiation background. This Hawking-Unruh radiation has also been studied by McDonald earlier[39].

6 QUANTUM METHODOLOGY IN BEAM PHYSICS

There are abundant applications of the theoretical formulations initially developed for quantum mechanics and quantum field theory in beam physics. Some of these efforts do deal with the quantum effects in beam physics. Some others, however, aim at applying the quantum formulations to solve beam physics problems that are essentially classical.

6.1 Quantum Approach to Beam Optics

Jagannathan and co-workers[40] have developed a fully quantum mechanical formalism (Dirac-Pauli equation) for charged particle beam optics. While the leading order recovers the conventional beam optics, the higher orders describe effects such as spin-orbit interaction. Such activity also has practical implications. For example, Pusterla et al.[3] show that the Stern-Gerlach force has been studied under this formalism for a potential application to produce polarized beams.

6.2 Schroedinger Equation for Phase Space Dynamics

When mutual interactions among beam particles are included, two different approaches have been developed. The standard treatment relies on the Fokker-Planck equation for describing the time evolution of the beam density, while the *thermal wave model* more recently introduced[41], is based on a mathematical coarse-graining of the Vlasov equation, which leads to a quantum-like Schroedinger equation, with the normalized emittance playing the role of the Planck constant. Pestroni et al.[3] show that the stochastic dynamics of the Nelson type[42] provides a physical foundation to the quantum-like models that invoke Schroedinger equation.

Rosenzweig demonstrates[3] that the formal similarity of the linear Fokker-Planck equation to the Schroedinger equation for the simple-harmonic oscillator also helps to elucidate certain beam phenomena such as the stochastic cooling of the beam longitudinal momentum spectrum.

6.3 Wigner Function and Beam Distribution

When the analogy between the Wigner function and the Liouville function is invoked, the unitary trasnformation in quantum mechanics is recognized as the counter-part of the symplectic map in classical beam dynamics. Dragt and Hibib show[3] that while Wigner and Liouville functions do transform in an identical way under linear symplectic maps, in general they do not transform identically under nonlinear symplectic maps. Instead there are "quantum corrections" whose $\hbar \to 0$ limit may be very complicated.

Another challenge has been that the Wigner function can in principle be negative, while the classical distribution function has to be positive definite. By invoking a novel *tomography* technique, Fedele and Man'ko[3] are able to develop a *marginal distribution* for the classical particle beam transport, that contains all the information of the Wigner function.

6.4 Supersymmetry in Beam Dynamics

Using the formalism of stochastic quantization in quantum field theory, Bjorken and Chen[43] recently demonstrated that the longitudinal phase space (classical) dynamics in proton storage rings, under the influence of the nonlinear RF potential and its random noise, exhibits the property of *supersymmetry*! Its physical implications are currently under further investigations.

7 FUTURE OUTLOOK

The major issues, as well as the future challenge, in *quantum beam physics* can be summarized as follows:

- What is the fundamental limitation on phase space?
- Can high energy charged particle beams ever be condensed?

• Do we fully understand all aspects of physics under strong fields?

• What highest acceleration gradient can we ever attain, and what can we do with it in the laboratory studies of fundamental physics?

• What are the uses of quantum formulations in beam physics?

There are undoubtedly other important QM effects than we can poorly envision here. But even with this rather limited scope, it is hopefully evident that this new subject, *quantum beam physics*, will only become more prominent in the next century.

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