

SCALING LAWS FOR LASER WAKEFIELD ACCELERATORS*

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Abstract

Scaling laws are derived as a function of laser wavelength for various quantities of interest for a single stage of a channel-guided laser wakefield accelerator. Comparisons are made for drive laser pulses of equal energies. Scalings that favor short or long wavelengths depend on the quantity that is to be optimized. For example, holding the number of electrons per bunch constant favors shorter wavelengths, whereas holding the energy gain constant can favor longer wavelengths.

1 INTRODUCTION

Plasma-based accelerators [1] are capable of sustaining ultrahigh electric fields E_z on the order of $E_0 = cm\omega_p/e$, i.e.,

$$E_0 [\text{V/m}] \simeq 96n^{1/2} [\text{cm}^{-3}], \quad (1)$$

where n is the ambient plasma density and $\omega_p = (4\pi ne^2/m)^{1/2}$ is the plasma frequency. Although the accelerating field is very high, the wavelength of the accelerating structure (i.e., the plasma wave) is very short and on the order of the plasma wavelength $\lambda_p = 2\pi c/\omega_p$, i.e.,

$$\lambda_p [\mu\text{m}] \simeq 3.3 \times 10^{10} n^{-1/2} [\text{cm}^{-3}], \quad (2)$$

e.g., $E_0 \sim 30$ GV/m and $\lambda_p \simeq 100 \mu\text{m}$ for $n \simeq 10^{17} \text{cm}^{-3}$. The shortness of λ_p has important consequences. For example, if an electron bunch is to be injected into the plasma wave such that its initial low energy spread is maintained, then it is desirable for the initial bunch length L_b to be short compared to λ_p . Furthermore, the bunch must be injected at the optimal plasma wave phase with femtosecond timing accuracy. These requirements are beyond the state-of-the-art for conventional injector technology (e.g., photo-injectors). Conversely, the shortness of λ_p and the wide-spread availability of ultrashort high power lasers may allow plasma-based accelerators to be developed as a compact source of ultrashort, high energy electron bunches [2-7].

In the standard laser wakefield accelerator (LWFA) a single, intense, short (a pulse length $L \sim \lambda_p$) is used to drive the wakefield. As the pulse propagates through the plasma, its ponderomotive force $F_p \propto \nabla a^2$ expels plasma electrons, thus exciting a large amplitude plasma wave with phase velocity $v_p \simeq c$ in its wake. Here, a^2 is proportional to the laser intensity I ,

$$a^2 \simeq 7.2 \times 10^{-19} \lambda^2 [\mu\text{m}] I [\text{W/cm}^2], \quad (3)$$

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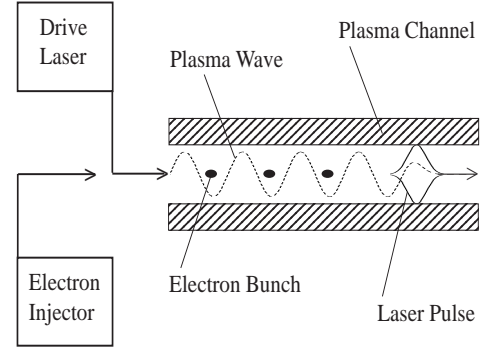


Figure 1: Schematic of a channel-guided LWFA. The plasma channel guides the laser pulse and supports the wakefield, which is generated behind the laser pulse with a phase velocity $v_p \simeq c$. Short electron bunches with length $L_b \ll \lambda_p$ are injected into the wakefield for acceleration to high energy.

and is related to the power P of a Gaussian radial intensity profile $I \propto \exp(-2r^2/r_s^2)$ by $P[\text{GW}] \simeq 21.5a^2 r_s^2/\lambda^2$, where λ is the laser wavelength, r_s is the spot size, and linear polarization has been assumed.

A practical high-energy LWFA requires the use of a plasma channel to guide the laser pulse (see Fig. 1). In vacuum, laser propagation is typically limited by diffraction, the characteristic distance of which is the Rayleigh length $Z_R = \pi r_0^2/\lambda$, where r_0 is the laser spot radius at focus (e.g., $Z_R \simeq 3$ mm for $r_0 = 30 \mu\text{m}$ and $\lambda = 1 \mu\text{m}$). Extending the single-stage interaction (acceleration) length beyond Z_R requires optical guiding [8]. In the standard LWFA, this can be achieved with a preformed plasma density channel [8-15]. The index of refraction in a plasma is approximately $\eta_R \simeq 1 - \lambda^2/2\lambda_p^2$. As in an optical fiber, a plasma channel can provide optical guiding if the index of refraction peaks on axis, $\partial\eta_R/\partial r < 0$, which can be achieved with a plasma density profile that has a local minimum on axis, $\partial n/\partial r > 0$. Specifically, a channel with a radially parabolic density profile of the form $n(r) = n_0 + \Delta n r^2/r_0^2$ can guide a Gaussian laser pulse with a constant spot size r_0 provided the channel depth Δn satisfies $\Delta n = \Delta n_c$, where $\Delta n_c = 1/\pi r_e r_0^2$ is the crit-

ical channel depth [8] and $r_e = e^2/m_e c^2$ is the classical electron radius, i.e.,

$$\Delta n_e [\text{cm}^{-3}] \simeq 1.13 \times 10^{20} / r_0^2 [\mu\text{m}]. \quad (4)$$

The following discussion will be restricted to standard LW-FAs that utilize channel guiding.

The most common sources of high power, short laser pulses are solid-state systems based on the technique of chirped-pulse amplification (CPA) [16]. Such CPA systems typically have $\lambda \sim 1 \mu\text{m}$ and have been used in numerous LWFA and channel-guiding experiments [9-15]. Alternatively, high-power, short-pulse CO₂ laser systems ($\lambda \sim 10 \mu\text{m}$) are being developed and will be available for future LWFA experiments [17]. In this paper, scaling laws with respect to λ for relevant LWFA quantities are derived and discussed.

2 SCALING LAWS

In this section, simple scaling laws for LWFA quantities are presented under idealized assumptions. These idealized scaling laws assume the following:

1. A standard LWFA that is channel guided.
2. The mildly relativistic regime, $a^2 \ll 1$.
3. The acceleration length is limited by electron dephasing.
4. The plasma channel is sufficiently broad such that the formula describing wakefield generation in a uniform plasma apply.
5. The transverse size of the laser pulse is $2c/\omega_p$ and the transverse size of the electron bunch is c/ω_p .
6. The total electrons per bunch is the beam loading limit.

In the following, when equations are presented in practical form (with numerical coefficients), E_z is in V/m, n is in cm^{-3} , λ is in μm , I is in W/cm^2 , W_L is in J, L_d is in m, ΔW is in GeV, Lum_s is in cm^{-2} , and a^2 is dimensionless.

In the mildly relativistic limit within a broad channel, the axial electric field of the wake [1] can be written as $E_z = 0.38a^2 E_0$, where $E_0 = m_e c \omega_p / e = 96n^{1/2}$, i.e.,

$$\begin{aligned} E_z &= 2.7 \times 10^{-17} I \lambda^2 n^{1/2} \\ &= 3.4 \times 10^{-25} W_L \lambda^2 n^2 \end{aligned} \quad (5)$$

This assumes a linearly polarized laser pulse with Gaussian profiles in the radial and axial directions. This also assumes that the laser pulse length is optimized to maximize the wakefield amplitude, i.e., $L = \lambda_p / \sqrt{2\pi} = 0.4\lambda_p$, where the pulse length L is defined such that $W_L = (1/8\pi) A_L E_L^2 L$ is the pulse energy, $\lambda_p = 2\pi c / \omega_p$ is the plasma wavelength, E_L is the peak laser electric field,

$A_L = \pi r_0^2 / 2$ is the cross-sectional area of the Gaussian pulse, and r_0 is the laser spot size.

The laser spot size is assumed to be $r_0 = 2c/\omega_p$ in order to ensure high efficiency of energy transfer between the wake and the accelerated electrons [18], since electrons loaded near the axis will absorb wake energy out to a radius of approximately c/ω_p . Furthermore,

$$a^2 = 9.4 \times 10^{-27} W_L \lambda^2 n^{3/2} \quad (6)$$

The acceleration length is assumed to be equal to the electron dephasing length $L_D = \lambda_p^2 / \lambda^2$,

$$L_d = 3.7 \times 10^{25} \lambda^{-2} n^{-3/2} \quad (7)$$

The ideal maximum energy gain is given by $\Delta W = e E_z L_d$,

$$\Delta W = I/n = 1.3 \times 10^{-8} W_L n^{1/2} \quad (8)$$

The number of electrons accelerated per bunch is assumed to be equal to the beam loading limit [18] $N_b = E_z A_b / 4\pi e$, where A_b is the effective cross-sectional area of the beam which is assumed to be $A_b = \pi c^2 / \omega_p^2$,

$$N_b = 1.7 \times 10^{-9} W_L \lambda^2 n \quad (9)$$

Another figure of merit is the luminosity $Lum = (k_b f_b / 4\pi) N_b^2 / \sigma_x \sigma_y$, where k_b is the number of bunches per linac, f_b is the linac rep rate, and $\sigma_{x,y}$ are the transverse rms bunch sizes, which are assumed to be equal to c/ω_p . For scaling purposes, it is convenient to define the "single bunch" luminosity as $Lum_s = N_b^2 / \sigma_x \sigma_y$,

$$Lum_s = 9.9 \times 10^{-30} W_L^2 \lambda^4 n^3 \quad (10)$$

3 EXAMPLES

Next, to determine scaling with wavelength, several examples are given. In all these examples, the laser pulse energy W_L is assumed constant.

3.1 Constant E_z

The axial electric field of the wake is held fixed (in addition to the pulse energy). This implies:

$$\begin{aligned} n &\propto \lambda^{-1}, \quad L_d \propto \lambda^{-1/2}, \quad \Delta W \propto \lambda^{-1/2}, \\ N_b &\propto \lambda, \quad Lum_s \propto \lambda \end{aligned} \quad (11)$$

3.2 Constant L_d

The acceleration length is held fixed (in addition to the pulse energy). This implies:

$$\begin{aligned} n &\propto \lambda^{-4/3}, \quad E_z \propto \lambda^{-2/3}, \quad \Delta W \propto \lambda^{-2/3}, \\ N_b &\propto \lambda^{2/3}, \quad Lum_s \propto \text{constant} \end{aligned} \quad (12)$$

3.3 Constant ΔW

The electron energy gain is held fixed (in addition to the pulse energy). This implies:

$$\begin{aligned} n \propto \text{constant}, E_z \propto \lambda^2, L_d \propto \lambda^{-2}, \\ N_b \propto \lambda^2, Lum_s \propto \lambda^4 \end{aligned} \quad (13)$$

3.4 Constant N_b

The number of electrons per bunch is held fixed (in addition to the pulse energy). This implies:

$$\begin{aligned} n \propto \lambda^{-2}, E_z \propto \lambda^{-2}, L_d \propto \lambda, \\ \Delta W \propto \lambda^{-1}, Lum_s \propto \lambda^{-2} \end{aligned} \quad (14)$$

3.5 Constant Lum_s

The single bunch luminosity is held fixed (in addition to the pulse energy). This implies:

$$\begin{aligned} n \propto \lambda^{-4/3}, E_z \propto \lambda^{-2/3}, L_d \propto \text{constant}, \\ \Delta W \propto \lambda^{-2/3}, N_b \propto \lambda^{2/3} \end{aligned} \quad (15)$$

4 DISCUSSION

In making comparisons between $\lambda = 1 \mu\text{m}$ and $\lambda = 10 \mu\text{m}$ drivers, care must be taken so as not to violate the above assumptions, in particular, $a^2 \ll 1$. Note that $a^2 \propto W_L \lambda^2 n^{2/3}$. Hence, when making comparisons at constant density and pulse energy, as in Case 3.3, the assumption $a^2 \ll 1$ may be violated at long wavelengths. On the other hand, for short wavelengths, operation at high density is valid.

A definitive conclusion regarding an optimum driver wavelength is problematic. For example, at sufficiently low density (such that $a^2 \ll 1$), a design for a fixed energy gain favors longer wavelengths, as implied by Case 3.3. On the other hand, a design for a fixed number of electrons per bunch favors short wavelengths, as implied by Case 3.4. Furthermore, a design for a fixed acceleration distance (and fixed luminosity) allows higher energies to be obtained for short wavelengths, however, a higher bunch number is obtained for long wavelengths. The above scaling laws all assume a fixed laser pulse energy. A rigorous study of a LWFA for various wavelength drivers must also include other properties of the driver, such as repetition rate, pulse stability, and average power. Since laser technology is rapidly progressing, a rigorous design study is premature. In terms of physics experiments, invaluable information can be obtained at both 1 and 10 micron.

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