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We predict the  $Z^0$  transverse momentum distribution from proton-proton and nuclear collisions at the LHC. After demonstrating that higher-twist nuclear effects are very small, we propose  $Z^0$  production as a precision test for leading-twist pQCD in the TeV energy region. We also point out that shadowing may result in unexpected phenomenology at the LHC.

The physics plans for the Large Hadron Collider (LHC) at CERN, which is going to be the highest-energy accelerator on Earth, include a heavy-ion program. Quantum Chromodynamics (QCD) for both hadronic and nuclear collision will enter a new era at the LHC, where we hope to discover new physics. However, “standard physics” needs to be tested in the new, high-energy regime.

At LHC energies, perturbative QCD (pQCD) provides a powerful calculational tool [1]. Clearly, an understanding of pQCD at the hadronic collision level is a prerequisite for the discussion of particle production in nuclear collisions. Enhanced power corrections from multiple scattering in both the initial and final states, not to mention potential new physics from the quark-gluon plasma (QGP), make pQCD predictions for nuclear collisions more difficult than for hadronic collisions.

The testing of pQCD in nuclear collisions at the LHC requires “clean processes”, where pQCD works well on the hadron level. One of the important recent advances in pQCD theory is a reorganization of perturbative corrections (folding selected logarithmic contributions at all orders into exact low-order calculations), which is beginning to provide practical applications [2]. The soft-gluon resummation for the inclusive production of colorless massive states [3–5] may be the best understood and best tested among these resummation techniques. For the transverse momentum distribution of heavy bosons of mass  $M$ , when  $p_T \ll M$ , the  $p_T$  distribution calculated order-by-order in  $\alpha_s$  in conventional fixed-order perturbation theory receives a large logarithm,  $\ln(M^2/p_T^2)$ , at every power of  $\alpha_s$ , even in the leading order in  $\alpha_s$ . Therefore, at sufficiently small  $p_T$ , the convergence of the conventional perturbative expansion in powers of  $\alpha_s$  is impaired, and the logarithms must be resummed.

The heavy-ion program at the LHC will make it possible to observe the full  $p_T$  spectra of heavy vector bosons in nuclear collisions and will provide a testing ground for resummation theory in nuclear collisions. In the present paper, we focus on  $Z^0$  production [6,7]. Based on LHC design luminosities [8], we estimate that a month of running will provide  $\sim 4 * 10^5$   $Z^0$  events in a proton-proton ( $pp$ ) collision, and  $\sim 8 * 10^2$   $Z^0$  events in a Pb+Pb collision in a  $p_T$  interval of 0.5 GeV in the peak regions of the corresponding spectra. Due to the large mass of the  $Z^0$ , and no final state rescattering in its production,

nuclear effects from final state interactions are expected to be small. We will show that the power corrections enhanced by initial state rescattering also remain small. Thus, leading twist pQCD should work well here. The only important nuclear effect left is the nuclear modification of parton distribution functions (shadowing). Therefore,  $Z^0$  production could provide a bench mark test for pQCD at the LHC in both  $pp$  and nuclear collisions.

Resummation of the large logarithms in QCD can be carried out either in  $p_T$ -space directly [9], or in the so-called “impact parameter”,  $\tilde{b}$ -space, which is a Fourier conjugate of the  $p_T$ -space. Using the renormalization group equation technique, Collins and Soper improved the  $b$ -space resummation to resum all logarithms as singular as  $\ln^m(M^2/p_T^2)/p_T^2$  with  $m \geq 0$ , when  $p_T \rightarrow 0$  [3]. Collins, Soper, and Sterman (CSS) derived a formalism for the transverse momentum distribution of vector boson production in hadronic collisions [4]. In the CSS formalism, non-perturbative input is needed for the large  $\tilde{b}$  region. The dependence of the pQCD results on the non-perturbative input is not weak if the original extrapolation proposed by CSS is used. Recently, a new extrapolation scheme was proposed, based on solving the renormalization group equation including power corrections [5]. Using the new extrapolation formula, the dependence of the pQCD results on the non-perturbative input was significantly reduced. The results agree with Tevatron CDF [10] and D0 [11] data very well in the entire  $p_T$  interval from  $p_T \lesssim 1$  GeV to  $p_T$  as large as the the vector mass.

For vector boson ( $V$ ) production in a hadron collision  $h_A + h_B$ , the CSS resummation formalism yields [4]:

$$\frac{d\sigma(h_A + h_B \rightarrow V + X)}{dM^2 dy dp_T^2} = \frac{1}{(2\pi)^2} \int d^2\tilde{b} e^{i\tilde{p}_T \cdot \tilde{b}} \tilde{W}(\tilde{b}, M, x_A, x_B) + Y(p_T, M, x_A, x_B) , \quad (1)$$

where  $x_A = e^y M/\sqrt{s}$  and  $x_B = e^{-y} M/\sqrt{s}$ , with rapidity  $y$  and collision energy  $\sqrt{s}$ . In Eq. (1), the  $\tilde{W}$  term dominates the  $p_T$  distributions when  $p_T \ll M$ , and the  $Y$  term gives corrections that are negligible for small  $p_T$ , but become important when  $p_T \sim M$ .

The function  $\tilde{W}(\tilde{b}, M, x_A, x_B)$  can be calculated perturbatively for small  $\tilde{b}$ , but an extrapolation to the large  $\tilde{b}$  region requiring nonperturbative input is necessary in order to complete the Fourier transform in Eq. (1). In order

to improve the situation, a new form was proposed [5] by solving the renormalization equation including power corrections. In the new formalism,  $\tilde{W}(\tilde{b}, M, x_A, x_B) = \tilde{W}^{pert}(\tilde{b}, M, x_A, x_B)$ , when  $\tilde{b} \leq \tilde{b}_{max}$ , with

$$\tilde{W}^{pert}(\tilde{b}, M, x_A, x_B) = e^{S(\tilde{b}, M)} \tilde{w}(\tilde{b}, c/\tilde{b}, x_A, x_B) , \quad (2)$$

where all large logarithms from  $\ln(1/\tilde{b}^2)$  to  $\ln(M^2)$  have been completely resummed into the exponential factor  $S(\tilde{b}, M)$ , and  $c$  is a constant of order unity [4].

$$\tilde{W}(\tilde{b}, M, x_A, x_B) = \tilde{W}^{pert}(\tilde{b}_{max}) F^{NP}(\tilde{b}; \tilde{b}_{max}) , \quad (3)$$

where the nonperturbative function  $F^{NP}$  is given by

$$F^{NP} = \exp\left\{-\ln(M^2 \tilde{b}_{max}^2 / c^2) \left[ g_1 \left( (\tilde{b}^2)^\alpha - (\tilde{b}_{max}^2)^\alpha \right) + g_2 \left( \tilde{b}^2 - \tilde{b}_{max}^2 \right) \right] - \bar{g}_2 \left( \tilde{b}^2 - \tilde{b}_{max}^2 \right) \right\}. \quad (4)$$

In Eq.(3)  $\tilde{b}_{max}$  is a parameter to separate the perturbatively calculated part from the non-perturbative input. Unlike in the original CSS formalism,  $\tilde{W}(\tilde{b}, M, x_A, x_B)$  is not altered when  $\tilde{b} < \tilde{b}_{max}$ , and is independent of the nonperturbative parameters. In addition, the  $\tilde{b}$ -dependence in Eq. (4) is separated according to different physics origins. The  $(\tilde{b}^2)^\alpha$ -dependence mimics the summation of the perturbatively calculable leading power contributions to the renormalization group equations to all orders in the running coupling constant  $\alpha_s(\mu)$ . The  $\tilde{b}^2$ -dependence of the  $g_2$  term is a direct consequence of dynamical power corrections to the renormalization group equations and has an explicit dependence on  $M$ . The  $\bar{g}_2$  term represents the effect of the non-vanishing intrinsic parton transverse momentum.

A remarkable feature of the  $\tilde{b}$ -space resummation formalism is that the resummed exponential factor  $\exp[S(\tilde{b}, M)]$  suppresses the  $\tilde{b}$ -integral when  $\tilde{b}$  is larger than  $1/M$ . Therefore, it can be shown using the saddle point method that, for a large enough  $M$ , QCD perturbation theory is valid even at  $p_T = 0$  [12,4]. As discussed in Ref.s [5,13], the value of the saddle point strongly depends on the collision energy  $\sqrt{s}$ , in addition to its well-known  $M^2$  dependence. Because of the steep evolution of parton distributions at small  $x$ , the  $\sqrt{s}$  dependence of  $\tilde{W}$  in Eq. (1) significantly decreases the value of the saddle point and improves the predictive power of the  $\tilde{b}$ -space resummation formalism at collider energies, in particular at the LHC.

In  $Z^0$  production, since final state interactions are negligible, power correction can arise only from initial state multiple scattering. Power corrections directly to the physical observables are proportional to powers of  $\Lambda_{QCD}/Q$  ( $Q$  being the physical large scale). These corrections are small for  $Z^0$  production as a result of the large mass of the  $Z^0$ . Power corrections to the evolution of the renormalization group equations are proportional to powers of  $\Lambda_{QCD}/\mu$ , with evolution scale  $\mu$ . Therefore, physical observables carry the effect of the latter

type power corrections for all  $\mu[Q_0, Q]$ , with the boundary condition at the scale  $Q_0$ . Even with large mass,  $Z^0$  can still carry a large effect of these power corrections.

Equations (3) and (4) represent the most general form of  $\tilde{W}$ , and thus (apart from isospin and shadowing effects, which will be discussed later), the only way nuclear modifications associated with scale evolution enter the  $\tilde{W}$  term is through the coefficient  $g_2$ . (Since the  $\bar{g}_2$  term of Eq. (4) is related to the partons' intrinsic transverse momentum, it should not have a strong nuclear dependence.)

The parameters  $g_1$  and  $\alpha$  of Eq. (4) are fixed by the requirement of continuity of the function  $\tilde{W}(\tilde{b})$  and its derivative at  $\tilde{b} = \tilde{b}_{max}$ . (The results are insensitive to changes of  $\tilde{b}_{max}$  in the interval  $0.3 \text{ GeV}^{-1} \lesssim \tilde{b}_{max} \lesssim 0.7 \text{ GeV}^{-1}$ . We use  $\tilde{b}_{max} = 0.5 \text{ GeV}^{-1}$ .) The value of  $g_2$  and  $\bar{g}_2$  can be obtained by fitting the low-energy Drell-Yan data. These data can be fitted with about equal precision if the values  $\bar{g}_2 = 0.25 \pm 0.05 \text{ GeV}^2$  and  $g_2 = 0.01 \pm 0.005 \text{ GeV}^2$  are taken. As the  $\tilde{b}$  dependence of the  $g_2$  and  $\bar{g}_2$  terms in Eq. (4) is identical, it is convenient to combine these terms and define

$$G_2 = \ln\left(\frac{M^2 \tilde{b}_{max}^2}{c^2}\right) g_2 + \bar{g}_2 . \quad (5)$$

Using the values of the parameters listed above, we get  $G_2 = 0.33 \pm 0.07 \text{ GeV}^2$  for  $Z^0$  production in  $pp$  collisions. The parameter  $G_2$  can be considered the only free parameter in the non-perturbative input in Eq. (4), arising from the power corrections in the renormalization group equations. An impression about the importance of power corrections can be obtained by comparing results with the above value of  $G_2$  to those with power corrections turned off ( $G_2 = 0$ ). We therefore define the ratio

$$R_{G_2}(p_T) \equiv \frac{d\sigma^{(G_2)}(p_T)}{dp_T} \bigg/ \frac{d\sigma(p_T)}{dp_T} . \quad (6)$$

The cross sections in the above equation and in the results presented in this paper have been integrated over rapidity ( $-2.4 \leq y \leq 2.4$ ) and invariant mass squared. For the parton distribution functions, we use the CTEQ5M set [14] in the present work.

Figure 1 displays the differential cross sections and the corresponding  $R_{G_2}$  ratio (with the limiting values of  $G_2 = 0.26 \text{ GeV}^2$  (dashed) and  $G_2 = 0.40 \text{ GeV}^2$  (solid)) for  $Z^0$  production as functions of  $p_T$  at  $\sqrt{s} = 14 \text{ TeV}$ . The deviation of  $R_{G_2}$  from unity decreases rapidly as  $p_T$  increases, and it is smaller than one percent for both  $\sqrt{s} = 5.5 \text{ TeV}$  (not shown) and  $\sqrt{s} = 14 \text{ TeV}$  in  $pp$  collisions, even when  $p_T = 0$ . In other words, the effect of power corrections is very small at the LHC for the whole  $p_T$  region.

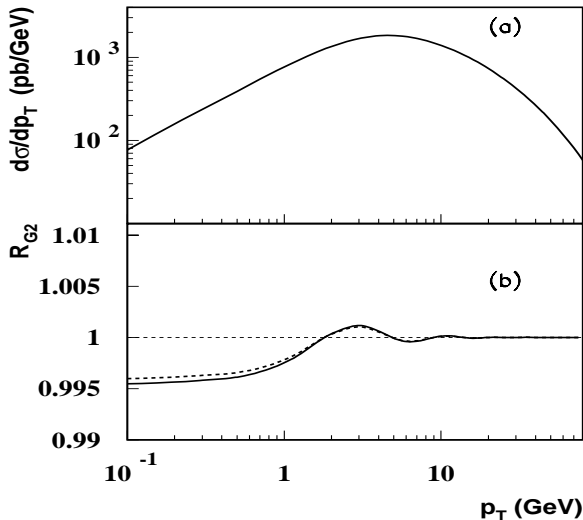


FIG. 1. (a) Cross section  $d\sigma/dp_T$  for  $Z^0$  production in  $pp$  collisions at the LHC with  $\sqrt{s} = 14$  TeV; (b)  $R_{G_2}$  defined in Eq. (6) with  $G_2 = 0.26$  GeV<sup>2</sup> (dashed) and 0.40 GeV<sup>2</sup> (solid).

In lack of nuclear effects on the hard collision, the production of heavy vector bosons in nucleus-nucleus ( $AB$ ) collisions should scale, compared to the production in  $pp$  collisions, as the number of hard collisions,  $AB$ . However, there are several additional nuclear effects on the hard collision in a heavy-ion reaction. First of all, since the parton distribution of neutrons is different from that of the protons, the production cross section of heavy vector bosons in proton-neutron interactions differs from the corresponding production cross section in  $pp$  collisions. This difference is the source of the so-called isospin effects. At LHC,  $x \sim 0.02$ , and the magnitude of the isospin effects is about 2%. This is because when  $x$  is in this range, the  $u - d$  asymmetry is very small [15].

The dynamical power corrections entering the parameter  $g_2$  should be enhanced by the nuclear size, i.e. proportional to  $A^{1/3}$ . Taking into account the  $A$ -dependence, we obtain  $G_2 = 1.15 \pm 0.35$  GeV<sup>2</sup> for Pb+Pb reactions. We find that with this larger value of  $G_2$ , the effects of power corrections appear to be enhanced by a factor of about three from  $pp$  to Pb+Pb collisions at the LHC. Thus, even the enhanced power corrections remain under 1% when  $3 \text{ GeV} \lesssim p_T \lesssim 80 \text{ GeV}$ . This small effect is taken into account in the following nuclear calculations.

Next we turn to the phenomenon of shadowing, expected to be a function of  $x$ , the scale  $\mu$ , and of the position in the nucleus. The latter dependence means that in heavy-ion collisions, shadowing should be impact parameter ( $b$ ) dependent. The parameterizations of shadowing in the literature take into account some of these effects, but no complete parameterization exists to date to our knowledge. For example, the HIJING parameterization includes impact parameter dependence, but does not deal with the scale dependence [16,17]. On the other hand, the EKS98 [18] and HKM [19] parameterizations have a scale dependence, but do not consider impact parameter dependence. (The latter parameterizations have been

compared recently [20].) In this paper we concentrate on impact-parameter integrated results, where the effect of the  $b$ -dependence of shadowing is relatively unimportant [21], and we focus more attention on scale dependence. We therefore use EKS98 shadowing [18] in this work.

To quantify the effect of shadowing, we define

$$R_{sh}(p_T) \equiv \frac{d\sigma^{(sh)}(p_T, Z_A/A, Z_B/B)}{dp_T} \bigg/ \frac{d\sigma(pp)}{dp_T}, \quad (7)$$

where  $Z_A$  and  $Z_B$  are the atomic numbers and  $A$  and  $B$  are the mass numbers of the colliding nuclei, and the cross section  $d\sigma(p_T, Z_A/A, Z_B/B)/dp_T$  has been averaged over  $AB$ , while  $d\sigma(pp)/dp_T$  is the  $pp$  cross section. We have seen above that shadowing remains to be the only significant effect responsible for nuclear modifications.

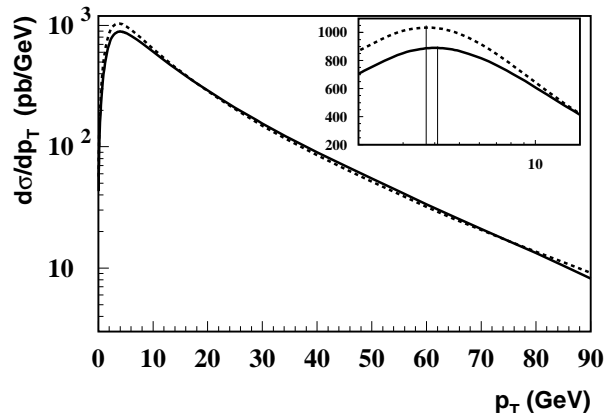


FIG. 2. Cross section  $d\sigma^{(sh)}(p_T, Z_A/A, Z_B/B)/dp_T$  for  $Z^0$  production averaged over  $AB$  (solid line), compared to the proton-proton cross section  $d\sigma(pp)/dp_T$  (dashed line).

Figure 2 presents  $d\sigma^{(sh)}(p_T, Z_A/A, Z_B/B)/dp_T$  (solid line) compared to  $d\sigma(pp)/dp_T$  in  $pp$  collisions (dashed) for  $Z^0$  production at  $\sqrt{s} = 5.5$  TeV. The insert, which is a magnified view of the peak region of the cross section on a logarithmic  $p_T$  scale, emphasizes that the shape of the distribution changes from  $pp$  collisions. Most importantly, the peak moves from 3.7 GeV to 4.1 GeV. This small shift may be difficult to detect experimentally. However, the peak position plays an important role in shadowing, due to the steepness of the cross section. This can be seen in Fig. 3(a), which shows the shadowing ratio (7) (full line) for  $Z^0$  production at  $\sqrt{s} = 5.5$  TeV. In Fig. 3(b) we show the  $R_{G_2}$  ratio defined in Eq. (6) for Pb+Pb collisions for the limiting values of  $G_2 = 0.8$  GeV<sup>2</sup> (dashed) and  $G_2 = 1.5$  GeV<sup>2</sup> (solid), respectively. Since Fig. 3(b) provides a good measure of the overall uncertainty on the shadowing ratio, and this uncertainty is less than 2%, the characteristic shape of  $R_{sh}$  may be easier to confirm experimentally by comparing the full  $p_T$  spectra in Pb+Pb versus  $pp$  collisions at the same energy.

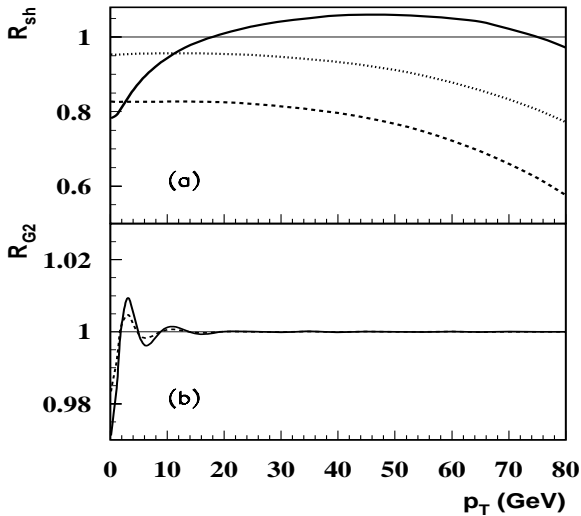


FIG. 3. Cross section ratios for  $Z^0$  production in Pb+Pb collisions at  $\sqrt{s} = 5.5$  TeV: (a)  $R_{sh}$  defined in Eq. (7) (solid line), and  $R_{sh}$  with the scale fixed at 5 GeV (dashed) and 90 GeV (dotted); (b)  $R_{G_2}$  defined in Eq. (6) with  $G_2 = 0.8$  GeV<sup>2</sup> (dashed) and 1.5 GeV<sup>2</sup> (solid).

The appearance of  $R_{sh}$  is surprising, because even at  $p_T = 90$  GeV,  $x \sim 0.05$ , and we are still in the “strict shadowing” region. Therefore, the fact that  $R_{sh} > 1$  for  $20 \text{ GeV} \lesssim p_T \lesssim 70 \text{ GeV}$  is not “anti-shadowing”; rather, it is a consequence of the change of the shape of the cross section from  $pp$  to  $AB$  reactions shown in Fig. 2. To better understand the shape of the ratio as a function of  $p_T$ , we also show  $R_{sh}$  with the scale fixed at the values 5 GeV (dashed line) and 90 GeV (dotted), respectively, in Fig. 3(a). In other words, the nuclear modification to the parton distribution function is only a function of  $x$  and flavor in the calculations represented by the dashed and dotted lines. These two curves are similar in shape, but rather different from the solid line. In  $\tilde{b}$  space,  $\tilde{W}(\tilde{b}, M, x_A, x_B)$  is almost equally suppressed in the whole  $\tilde{b}$  region if the fixed scale shadowing is used. However, with scale-dependent shadowing, the suppression increases as  $\tilde{b}$  increases, as a result of the scale  $\mu \sim 1/\tilde{b}$  in the nuclear parton distribution. We can say that the scale dependence re-distributes the shadowing effect. In the present case, the re-distribution brings  $R_{sh}$  above unity for  $20 \text{ GeV} \lesssim p_T \lesssim 70 \text{ GeV}$ . When  $p_T$  increases further, the contribution from the  $Y$  term starts to be important, and  $R_{sh}$  dips back below one to match the fixed order pQCD result.

We see from Fig. 3 that the shadowing effects in the  $p_T$  distribution of  $Z^0$  bosons at the LHC are intimately related to the scale dependence of the nuclear parton distributions, on which we have only very limited data [18]. Theoretical studies (such as EKS98) are based on the assumption that the nuclear parton distribution functions differ from the parton distributions in the free proton, but obey the same DGLAP evolution [18]. Therefore, the transverse momentum distribution of heavy bosons at

the LHC in Pb+Pb collisions can provide a further test of our understanding of the nuclear parton distributions.

In summary, higher-twist nuclear effects appear to be negligible in  $Z^0$  production at LHC energies. (The results for  $W^\pm$  production are very similar [13].) We have demonstrated that the scale dependence of shadowing effects may lead to unexpected phenomenology of shadowing at these energies. Overall, the  $Z^0$  transverse momentum distributions calculated in this paper can be used as a precision test for leading-twist pQCD in the TeV energy region for both, proton-proton and nuclear collisions. We propose that measurements of  $Z^0$  spectra be very high priority at the LHC.

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