A new model-independent way of extracting $|V_{ub}/V_{cb}|$

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Abstract

The ratio between the photon spectrum in $B \to X_s\gamma$ and the differential semileptonic rate wrt the hadronic variable M_X/E_X is a short-distance quantity calculable in perturbation theory and independent of the Fermi motion of the b quark in the B meson. We present a NLO analysis of this ratio and show how it can be used to determine $|V_{ub}/V_{cb}|$ independently of any model for the shape function. We also discuss how this relation can be used to test the validity of the shape-function theory on the data.

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1 Introduction

The accurate determination of the parameters entering the Cabibbo–Kobayashi–Maskawa (CKM) matrix V is one of the main physics goals at beauty factories. The large number of produced B mesons allows for the study of many decay modes and the measurement of branching ratios (BRs) as small as 10^{-6}. Unfortunately, a precise extraction of the CKM parameters from the data is often hindered by uncertainties on the hadronic parameters accounting for low-energy strong interaction effects. There exist, however, observable quantities that are free, at least to some extent, from hadronic uncertainties. They are obviously highly valued and have been intensively looked for in recent years.

In particular, it has been suggested many years ago that one can build a suitable ratio of the lepton-energy spectrum in the semileptonic decay $B \to X_u l \nu$ and the photon spectrum in $B \to X_s \gamma$ from which the dominant non-perturbative QCD effects due to the heavy-quark Fermi motion drop out [1, 2]. This ratio can be used to extract the combination of CKM matrix elements $|V_{ub}/(V_{tb}V_{ts}^*)|$. Theoretically, the cancellation of long-distance effects takes place for hadronic invariant masses as small as $M_X^2 \sim Q\Lambda_{QCD}$, where M_X is the invariant mass of the hadronic system and Q is the hard scale of the process (in the B rest frame $Q = 2E_X$, with E_X denoting the hadronic energy). In this regime, the dominant non-perturbative effects are described by a single universal function, the *shape function*, entering the different spectra. At smaller $M_X^2 \sim \Lambda_{QCD}^2$, however, few resonances dominate the spectra and universality is lost. It remains to be seen whether the region where the shape function provides a good description of hadronic physics is large enough to allow a clean extraction of the CKM parameters.

Several short-distance quantities based on various partially integrated spectra or moments have been proposed in the literature, with a particular attention to the problem of defining the shape function consistently beyond the leading order [3]. Alternatively, one can use a model for the shape function, and estimate the theoretical uncertainty by changing the parameters of the model. This is routinely done in order to implement cuts on M_X useful to identify the $B \to X_u l \nu$ sample [4], an approach which has been actually used to extract V_{ub} from the semileptonic decay rate at LEP [5]. Very recently CLEO has also published a V_{ub} measurement based on a combined analysis of the electron and photon spectra in semileptonic and radiative decays [6]. Other proposals to identify $B \to X_u l \nu$ events with low theoretical uncertainty include a cut on the invariant leptonic mass q^2 [7] and a combination of cuts on q^2 and M_X [8].

In this paper, we propose a phenomenological strategy to investigate the "shape-function region" and possibly to extract the ratio $|V_{ub}/(V_{tb}V_{ts}^*)| \approx |V_{ub}/V_{cb}|$. We use the fact that the differential rates $d\,\Gamma_{rd}/dx$ for the radiative decay $B \to X_s \gamma$ and $d\,\Gamma_{sl}/d\xi$ for the semileptonic decay $B \to X_u l \nu$ probe exactly the same long-distance structure function. The kinematical variables involved are $x = 2E_\gamma/m_B$ and $\xi = 2\,t/(1 + t)$ with $t = \sqrt{1 - M_X^2/E_X^2}$. The ratio of the above two distributions is a short-distance quantity, which can be computed in

perturbation theory. As we will explain in section 4, to a good approximation it provides a determination of $|V_{ub}/V_{cb}|$ in the SM. Including perturbative $O(\alpha_s)$ corrections, one finds

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq C(\alpha_s)\frac{\frac{d}{d\xi}\Gamma_{sl}}{\frac{d}{dx}\Gamma_{rd}\big|_{x=\xi} - \frac{\alpha_s}{\pi}B(\xi)} \tag{1}$$

where the coefficient $C(\alpha_s)$ and the function $B(\xi)$ will be calculated in the following. Clearly, eq. (1) is valid up to higher twist $O(1-x) \sim O(\Lambda_{QCD}/E_X)$ contributions.

Our treatment of the semileptonic and radiative decays follows closely refs. [9–11]. In this approach, the hard scale Q is identified as twice the hadronic energy $2E_X$ in the B rest frame rather than as the heavy-quark mass m_b. Consequently, the simplest kinematical variable for factorizing long-distance effects in this decay turns out to be a function of M_X/E_X and the relevant spectrum is $d\Gamma_{sl}/d\xi$ rather than $d\Gamma_{sl}/dM_X$.

The advantages of our method are the following:

- the cancellation of Landau-pole effects and of non-perturbative Fermi-motion effects in the ratio of spectra in eq. (1) is exact and requires no approximation;

- the shape function is defined in a consistent subtraction scheme for infrared (IR) singularities, avoiding double counting of IR logs;

- at least in principle, our ratio is valid point by point on the spectra, namely for any value of $x = \xi$.

In a realistic case, some smearing, such as partial integration over the spectra, may be necessary as all these calculations, including ours, rely on local quark–hadron duality. One could integrate eq. (1) over different kinematical ranges and check the stability of the results with respect to the smearing procedure. This, in turn, would amount to a test of the underlying assumptions, and most notably of quark–hadron duality.

In this paper, we present a NLO calculation of the perturbative corrections to eq. (1). To make contact with experiments, we also give expressions of the relevant functions in the presence of kinematical cuts on the hadronic invariant mass M_X. After inclusion of the NLO corrections, the ratio in eq. (1) allows for a determination of $|V_{ub}/V_{cb}|$ with a $O(5\%)$ theoretical error.

The paper is organized as follows. In section 2 we introduce the formalism used throughout the paper, while results for the relevant spectra are given in section 3 and in the Appendix. The extraction of $|V_{ub}/V_{cb}|$ and the test of local duality are discussed in section 4. Section 5 contains our summary.

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2 Formalism

The heavy-quark Fermi motion in semi-inclusive decays [12, 13] can be treated in field theory by introducing the shape function $\tilde{f}(k_+)$, also known as the structure or light-cone distribution function of heavy flavours [14, 1, 15, 2].

The calculation of the spectra in these decays requires the Operator Product Expansion (OPE) of the time-ordered product T of two transition operators (currents or effective Hamiltonians). Schematically, this involves the expansion of the propagator

$$\frac{1}{M_X^2 + 2E_X k_+ + k^2 + i\varepsilon} \simeq \frac{1}{M_X^2 + i\varepsilon} \left[\sum_n \left(\frac{-2E_X k_+}{M_X^2 + i\varepsilon} \right)^n \right] + O\left(\frac{k^2}{M_X^2} \right), \qquad (2)$$

where $k_+ = p_X \cdot k / E_X$ is the virtuality of the heavy quark ($p_b = m_B v + k$) and p_X, M_X and E_X are momentum, mass and energy of the final hadronic state, including the spectator quark[2]. Since it is expected from the confinement dynamics that $k_+ \sim \Lambda_{QCD}$, one can identify three different kinematical regions, depending on the value of M_X^2:

1. When $M_X^2 \sim E_X^2 \gg \Lambda_{QCD}^2$, the OPE converges. At leading twist accuracy, one can safely retain only the lowest order ($n = 0$) term in eq. (2).

2. When $M_X^2 \sim E_X \Lambda_{QCD}$, all the terms in the sum become of order 1. This is precisely the shape-function region. Indeed, the leading-twist terms, i.e. those obtained by neglecting corrections of $O(k^2/M_X^2) \sim O(\Lambda_{QCD}/E_X)$, are resummed by introducing the shape function, formally defined as

$$\tilde{f}(k_+) = \frac{\langle B | \bar{h}_v \delta(k_+ - iD_+) h_v | B \rangle}{\langle B | \bar{h}_v h_v | B \rangle}, \qquad (3)$$

where D_+ is the light-cone plus-component of the covariant derivative and h_v is the heavy quark field in the Heavy Quark Effective Theory; $\tilde{f}(k_+)$ gives the probability of finding a heavy-quark with a light-cone residual momentum k_+ inside the meson. Note that neglecting the terms of order k^2/M_X^2 in eq. (2) is equivalent to assuming the local quark–hadron duality. As the rate is proportional to the imaginary part of eq. (2), it follows that the distribution in k_+ coincides with that in $-M_X^2/2E_X$ and is therefore experimentally accessible.

3. When $M_X^2 \sim \Lambda_{QCD}^2$, even the resummation fails and the contribution of single resonances to the spectra is not negligible. In this region, terms of $O(\Lambda_{QCD}/E_X)$ are not negligible and the shape function cannot be defined.

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[2]At the leading-twist, the use of the meson mass m_B instead of the quark mass m_b in the decomposition of heavy quark momentum p_b only shifts the integration range of k_+ [16].

A different framework to factorize long-distance effects is provided by the resummation theory in perturbative QCD [17]. The connection between the two approaches is discussed in refs. [9,10]. In this framework, a factorized formula for a given distribution, for example the photon spectrum $d\Gamma_{rd}/dx$ in $B \to X_s\gamma$, reads

$$\frac{d\Gamma_{rd}}{dx} = |V_{tb}V_{ts}^*|^2 \, \Gamma_{rd}^0 \, [K_{rd}(\alpha_s)f(x;\alpha_s) + D_{rd}(x;\alpha_s)] \, , \tag{4}$$

where V_{tb} and V_{ts} are the relevant CKM elements, Γ_{rd}^0 is a normalization factor, x is a properly chosen factorization variable, $x = 2E_\gamma/m_B = 1 - M_X^2/m_B^2$ in this example, while the coefficient K_{rd} and the remainder function $D_{rd}(x)$ are short-distance quantities computable in perturbation theory. The form factor $f(x)$ contains all the IR logarithms of the form

$$\alpha_s(2E_X)^k \left(\frac{\ln^n(1-x)}{1-x}\right)_+ \qquad \text{with } 0 \le n \le 2k - 1 \, , \tag{5}$$

where the plus-distribution is defined as

$$\left(\frac{\ln^n(1-x)}{1-x}\right)_+ = \frac{\ln^n(1-x)}{1-x} - \delta(1-x) \int_0^1 dy \frac{\ln^n(1-y)}{1-y} \, . \tag{6}$$

The function $f(x)$ is well defined for $1 - x \gg \Lambda_{QCD}/m_B$. The universality of $f(x)$ stems from the general properties of the soft and the collinear radiation which produce the log-enhanced terms in eq. (5). On the contrary, when $1 - x \lesssim \Lambda_{QCD}/m_B$, $f(x)$ is no longer calculable in perturbation theory. However, $f(x)$ is related to the shape function $\tilde{f}(k_+)$ in the following way [10]

$$f(k_+) = \int dk_+' C(k_+ - k_+'; \alpha_s)\tilde{f}(k_+') \, , \tag{7}$$

where $k_+ = -m_B(1 - x)$ (in the case of $B \to X_s\gamma$, the hard scale is $2E_X = m_B$). The coefficient function $C(k_+ - k_+'; \alpha_s)$ remains perturbative also for $1 - x \sim \Lambda_{QCD}/m_B$ and allows for a definition of a non-perturbative form factor in this region.

3 Results

In this section we present a NLO determination of the short-distance functions entering the formulae of the photon energy spectrum in $B \to X_s\gamma$ and of the z-spectrum in $B \to X_u l\nu$, written in the factorized form of eq. (4). In the latter case, we also give expressions for the same functions in the presence of kinematical cuts.

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3.1 Photon spectrum in $B \to X_s \gamma$

The photon spectrum in the radiative $B \to X_s \gamma$ decay can be expressed as in eq. (4). The normalization factor Γ^0_{rd} for the $b \to s\gamma$ rate is defined as

$$\Gamma^0_{rd} \equiv \frac{\alpha}{\pi} \frac{G_F^2 \, m_b^3 \, m_{b,\overline{MS}}^2 \, (m_b)}{32\pi^3} ,\tag{8}$$

where $\alpha = 1/137.036$ is the fine structure constant. The NLO relation of the $\overline{MS}$ mass, $m_{b,\overline{MS}}(m_b) = 4.26 \pm 0.09$ GeV [18], with the pole mass, $m_b \simeq 4.7$ GeV, both appearing in eq. (8), is

$$\frac{m_{b,\overline{MS}}(m_b)}{m_b} = 1 - \frac{\alpha_s(m_b) \, C_F}{\pi} + O\left(\alpha_s^2\right) ,\tag{9}$$

where $C_F = (N^2 - 1)/(2N)$ and N is the number of colours.

The coefficient function K_{rd} reads

$$K_{rd}(\alpha_s) = C_7^{(0)}(\mu_b)^2 \left[1 + \frac{\alpha_s(\mu_b)}{2\pi} \sum_{i=1}^{8} \frac{C_i^{(0)}(\mu_b)}{C_7^{(0)}(\mu_b)} \left(\mathrm{Re}\, r_i + \gamma_{i7}^{(0)} \ln \frac{m_b}{\mu_b} \right) \right. \tag{10}$$
$$\left. + \frac{\alpha_s(\mu_b)}{2\pi} \frac{C_7^{(1)}(\mu_b)}{C_7^{(0)}(\mu_b)} + O\left(\alpha_s^2\right) \right] ;$$

the definitions of r_i are collected in the Appendix. We have denoted by $C_i^{(0)}$ and $C_i^{(1)}$ the LO and NLO contributions to the *effective* Wilson coefficients of the $\Delta B = 1$ effective Hamiltonian [19,20]. The scale μ_b is a renormalization scale of $O(m_b)$ and $\gamma_{i7}^{(0)}$ are elements of the LO anomalous dimension matrix. The terms proportional to $\ln(m_b/\mu_b)$ make the coefficient K_{rd} renormalization scale-independent at the NLO. K_{rd} depends on the top mass: for $M_t = 174.3 \pm 5.1$ GeV we find

$$K_{rd} = 0.1134 \left[1 \pm 0.014 \pm 0.053^{+0.001}_{-0.060} \right] ,\tag{11}$$

where the first two errors come from the top uncertainty and from the treatment of the charm quark mass in the matrix elements containing charm loops. We have followed [21] and used $m_c/m_b = 0.22 \pm 0.04$ in r_i (see Appendix). The central value of eq. (11) has been obtained for $\mu_b = m_b$ and the last error by varying μ_b between $m_b/2$ and $2m_b$. We have also included in K_{rd} a power correction $-\lambda_2 \left(C_2^{(0)} - C_1^{(0)}/6 \right) C_7^{(0)}/9m_c^2$ related to long-distance contributions from $(c\bar{c})$ intermediate states [22] and the electroweak effects following Ref. [23]. We have adopted a light Higgs boson mass, m_h, as suggested by global fits, but the sensitivity to m_h is very small. The latter two contributions modify K_{rd} by about $+2.4\%$ and -3.8%.

As is well known, NLO corrections have an important numerical impact in radiative decays: they increase the inclusive branching ratio by up to more than 30% [20]. In

eq. (11) the effect of NLO corrections is about $(+20 \pm 30)\%$, according to the value of μ_b between $m_b/2$ and $2m_b$. Recently, it has been shown [21] that the dominant reason for the enhancement of the inclusive BR is the running of the b quark mass, which affects differently the light quarks and the top loops. Following this observation, it is possible to stabilize the perturbative expansion for $B \to X_s \gamma$ by a rearrangement of higher-order effects. In this approach, NLO corrections to K_{rd} are not smaller, but have a different sign,

$$K_{rd} = 0.1448\,(1 - 0.99\,\alpha_s - 0.013) = 0.1119\left[1 \pm 0.014 \pm 0.053^{+0.044}_{-0.088}\right].\qquad(12)$$

Here the second correction is due to electroweak and $1/m_c^2$ effects and the errors have the same meaning as in the previous equation. Notice that now the NLO correction is predominantly due to the matrix elements of the effective operators. For this reason, it is numerically close to the NLO corrections to the coefficient in the semileptonic decay, as we will see later on. In particular, the large scale dependence observed in eq. (12) is mostly due to the large $O(\alpha_s)$ corrections to the matrix elements and is dominated by the scale dependence of $\alpha_s(\mu_b)$ and not by the scale dependence of the Wilson coefficients, in contrast to the situation in the preceding equation. In the following numerical studies, we will employ eq. (12).

We stress that the definition of the coefficient function in eq. (10) and of the remainder function given below corresponds to a particular factorization scheme. Specifically, following the convention of [11], we absorb in $f(x)$ *only* the IR logarithms of eq. (5).

The remainder function is given by (the functions $d_{ij}(x)$ are defined in the Appendix)

$$D_{rd}\,(x;\alpha_s) = \frac{\alpha_s\,(\mu_b)}{\pi} \sum_{i \leq j}^{1,8} C_i^{(0)}\,(\mu_b)\,C_j^{(0)}\,(\mu_b)\,d_{ij}\,(x) + O\left(\alpha_s^2\right).\qquad(13)$$

Let us comment on the behaviour of the remainder function at the end points $x = 0$ and 1. The functions $d_{77}\,(x)$ and $d_{88}\,(x)$ have a logarithmic singularity $\ln(1-x)$ for $x \to 1$, coming from power-suppressed long-distance contributions. They are not factorized into the function $f(x)$, which contains only terms of the form $\ln^n(1-x)/(1-x)$. At the lower end-point $x = 0$, the functions $d_{77}\,(x)$ and $d_{78}\,(x)$ are finite, while $d_{88}\,(x)$ has a power singularity $d_{88}\,(x \to 0) \sim 1/x$. This limit corresponds to a soft (real) photon and the singularity would be regulated by including virtual photon contributions so that $1/x \to (1/x)_+$. These contributions are not relevant to our analysis. Finally, the function $d_{88}\,(x)$ has a logarithmic singularity for $m_s \to 0$ because a non-zero photon energy regulates the soft but not the collinear singularity. In this case, the strange mass has to be retained (we adopt $m_s = m_b/50$). The function $D_{rd}(x)$ is shown in fig. 1 for different values of μ_b.

We also present results for the partially integrated rate, for which a factorized formula also holds,

$$\Gamma_{rd}(x) \equiv \int_x^1 \frac{d\Gamma_{rd}}{dx'}dx' = |V_{tb}V_{ts}^*|^2\,\Gamma_{rd}^0\left[K_{rd}\,(\alpha_s)\,F\,(x;\alpha_s) + E_{rd}\,(x;\alpha_s)\right].\qquad(14)$$
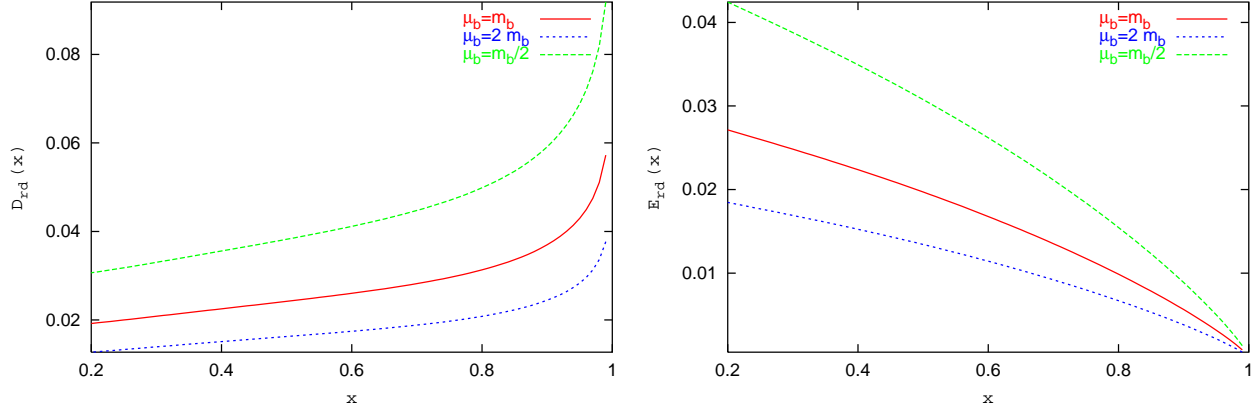
Figure 1: The remainder functions $D_{rd}(x)$ and $E_{rd}(x)$ for $M_t = 174.3$ GeV and different values of the renormalization scale μ_b.

The partially integrated form factor is defined as

$$F(x; \alpha_s) \equiv \int_x^1 dx' f(x'; \alpha_s) , \qquad (15)$$

while the cumulative remainder function is given by

$$E_{rd}(x; \alpha_s) \equiv \int_x^1 dx' D_{rd}(x') = \frac{\alpha_s(\mu_b)}{\pi} \sum_{i \leq j}^{1,8} C_i^{(0)}(\mu_b) \, C_j^{(0)}(\mu_b) \, f_{ij}(x) + O\left(\alpha_s^2(\mu_b)\right) .$$

Note that, contrary to the d_{ij} case, the functions f_{ij} vanish at the end-point $x = 1$. The explicit expressions of the functions $f_{ij}(x)$ are given in the Appendix, and the cumulative remainder function is shown in fig. 1. Considering that $K_{rd} F(x) \approx 0.1$ for small x, the plot on the left side of fig. 1 shows that the scale dependence of the remainder function can be numerically important. However, the scale dependence of $E_{rd}(x)$ is almost entirely due to the coupling constant $\alpha_s(\mu)$. Given the expression of the partially integrated rate in eq. (14), we observe a significant cancellation between the $O(\alpha_s)$ correction in eq. (12) and $E_{rd}(x)$. It follows, in agreement with [21], that the NLO result for total rate is perturbatively quite stable.

3.2 ξ distribution in semileptonic decay

The starting point for dealing with spectra in $B \to X_u l \nu$ is the triple-differential distribution, which has been computed at order α_s in ref. [24]. As demonstrated in ref. [11], the resummation of threshold logs takes a particularly simple form,

$$\frac{d^3\Gamma_{sl}}{dw dx_l dz} = |V_{ub}|^2 \, \Gamma_{sl}^0 \left[K_{sl}(w, x_l; \alpha_s) \, f(z; \alpha_s) + D_{sl}(w, x_l, z; \alpha_s) \right] , \qquad (16)$$

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when one adopts the kinematical variables

$$w = 2\frac{E_X}{m_B}, \quad x_l = 2\frac{E_l}{m_B}, \quad z = 1 - \frac{M_X^2}{4E_X^2}. \tag{17}$$

The main point is that all IR logs are factorized in a function of the single variable z. Notice that z is only defined between $3/4$ and unity. As already discussed, this formula can be generalized in the non-perturbative region corresponding to $M_X^2 \sim E_X \Lambda_{QCD}$, where $f(z)$ is related to the shape function defined in the effective theory by a short-distance factor. The $O(\alpha_s)$ expressions for $K_{sl}(w, x_l; \alpha_s)$ and $D_{sl}(w, x_l, z; \alpha_s)$ can be found in ref. [11]. The integration over w and x_l involves only short-distance functions and can be easily carried out [25]. The resulting distribution reads

$$\frac{d\Gamma_{sl}}{dz} = |V_{ub}|^2 \Gamma_{sl}^0 [K_{sl}(\alpha_s) f(z; \alpha_s) + D_{sl}^z(z; \alpha_s)] \tag{18}$$

where

$$\Gamma_{sl}^0 = \frac{G_F^2 m_b^3 m_{b,\overline{MS}}^2(m_b)}{192\pi^3}. \tag{19}$$

We have chosen a somewhat unusual normalization factor Γ_{sl}^0, in analogy with the radiative decay case. With our choice, the perturbative expansion of the total inclusive rate [26] has a good convergence pattern. The coefficient is given by

$$K_{sl}(\alpha_s) = 1 + \frac{\alpha_s C_F}{\pi} \left(\frac{403}{144} - \frac{\pi^2}{2} \right) + O(\alpha_s^2) \simeq 1 - 0.91\,\alpha_s. \tag{20}$$

Let us now consider a change of the factorization variable of the kind

$$1 - z \longrightarrow 1 - z' = 1 - z + O(1-z)^2. \tag{21}$$

It simply amounts to a rearrangement of higher-twist contributions and affects only the remainder function. Any ambiguity due to the choice of variable is related to the underlying leading-twist approximation and is common to all leading-twist treatments of the shape function[3]. The new factorized differential rate in terms of z' is

$$\frac{d\Gamma_{sl}}{dz'} = |V_{ub}|^2 \Gamma_{sl}^0 \left[K_{sl}(\alpha_s) f(z', \alpha_s) + D_{sl}^{z'}(z', \alpha_s) \right], \tag{22}$$

where

$$D_{sl}^{z'}(z'; \alpha_s) = \frac{dz}{dz'} D_{sl}(z; \alpha_s) + \frac{\alpha_s C_F}{\pi} \left[\frac{\ln(1-z') + \frac{7}{4}}{1-z'} - \frac{dz}{dz'} \frac{\ln(1-z) + \frac{7}{4}}{1-z} \right]. \tag{23}$$

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[3]A first discussion of subleading twist effects in the shape function region has appeared recently [27]. In $B \to X_s\gamma$ they seem to be small for $E_\gamma < 2.3$ GeV ($x \lesssim 0.87$).
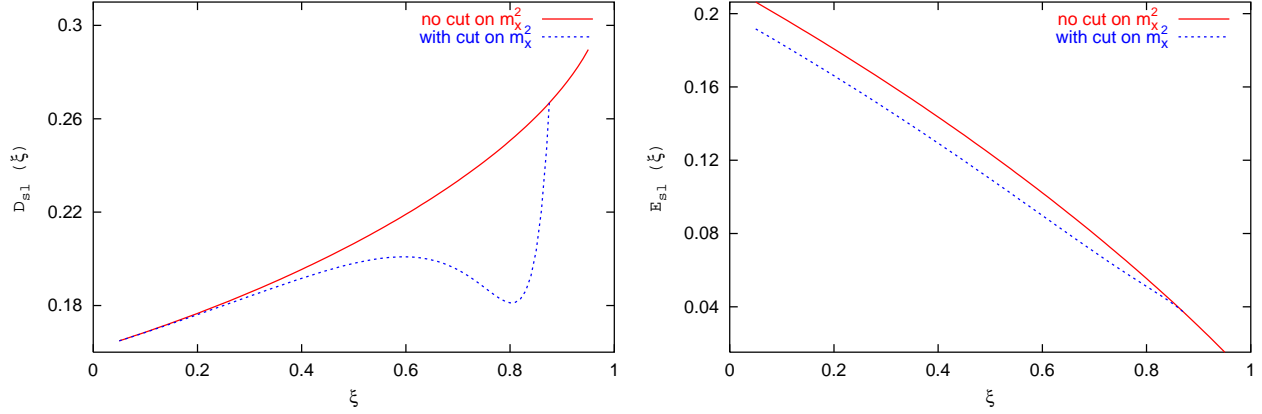
Figure 2: The remainder functions $D_{sl}(\xi)$ and $E_{sl}(\xi)$ with and without a cut on the invariant hadronic mass.

Here we have left implicit the functional dependence of $z = z(z')$. The differential distributions $d\Gamma_{sl}/dz$ and $d\Gamma_{sl}/dz'$ are two examples of observables which differ only by higher-twist effects. In principle, a comparison between observables of this type could provide some information on the size of higher-twist contributions.

As we are going to directly compare the semileptonic and radiative decays, it is preferable to choose the factorization variable for the two cases in a similar way. In $B \to X_s \gamma$ the variable $z = 1 - M_X^2/(4E_X^2)$ can be expressed in terms of the photon energy fraction x as $z = 1 - \frac{1-x}{(2-x)^2}$. Since we have factorized IR effects in the radiative decay in terms of a function $f(x)$, it seems natural to adopt here a variable ξ such that $1 - z = (1-\xi)/(2-\xi)^2$. Solving for ξ we then find

$$1 - \xi = \frac{1-t}{1+t}, \tag{24}$$

where we have introduced the variable $(0 \le t \le 1)$

$$t \equiv \sqrt{1 - 4(1-z)} = \sqrt{1 - M_X^2/E_X^2}. \tag{25}$$

Unlike z, the variable ξ is defined between 0 and 1. Using $dz/d\xi = \xi/(2-\xi)^3$ and the formulae given in [11], it is straightforward to obtain $D_{sl}(\xi) \equiv D_{sl}^\xi(\xi)$, which is also shown in fig. 3:

$$D_{sl}(\xi; \alpha_s) = \frac{\alpha_s C_F}{\pi} \left[\frac{21 + 21\,\xi - 5\,\xi^2 - \xi^3}{12} + \frac{70 - 90\,\xi + 5\,\xi^2 + 8\,\xi^3}{70} \ln(1-\xi) \right]. \tag{26}$$

The higher-twist logarithms contained in the remainder functions are process-dependent and in general cannot be factorized in the function $f(\xi)$. It is nonetheless interesting to note that the IR logarithm for $\xi \to 1$ has a very small coefficient in eq. (26), much smaller than the corresponding coefficient of $\ln(1-z)$ in $D_{sl}^z(z)$.

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One can also consider the partially integrated rate

$$\Gamma_{sl}(\xi) \equiv \int_\xi^1 \frac{d\Gamma_{sl}}{d\xi'} d\xi' = |V_{ub}|^2 \, \Gamma_{sl}^0 \left[K_{sl}\left(\alpha_s\right) \, F\left(\xi; \alpha_s\right) + E_{sl}\left(\xi, \alpha_s\right) \right] , \qquad (27)$$

where at $O(\alpha_s)$ we find

$$E_{sl}(\xi; \alpha_s) = \frac{\alpha_s C_F}{\pi} \left[\frac{11725 - 6756\,\xi - 5898\,\xi^2 + 788\,\xi^3 + 141\,\xi^4}{5040} \right.$$
$$\left. + \frac{86 - 210\,\xi + 135\,\xi^2 - 5\,\xi^3 - 6\,\xi^4}{210} \ln(1 - \xi) \right]. \qquad (28)$$

If a cut on the hadronic mass $M_X < m_D$ is imposed, as often required in a realistic environment, the remainder function in eq. (26) is modified in the region

$$\xi < \xi_0 \equiv 1 - c, \qquad c = \frac{m_D^2}{m_B^2}.$$

The new expression is

$$D_{sl}\left(\xi; \alpha_s\right)|_{\xi<\xi_0} = \frac{\alpha_s C_F}{\pi} \left\{ \frac{c^{\frac{3}{2}}\,\xi}{(1-\xi)^{\frac{5}{2}}} \left[\frac{(7+6\,c)\,(\xi-2)\,\xi}{2} \right. \right. \qquad (29)$$
$$\left. - \frac{70\,(2-\xi)^2 + 10\,c^2\,(1-\xi) + 7\,c\,(35 - 35\,\xi + 8\,\xi^2)}{35} \ln(1-\xi) \right]$$
$$+ \frac{21 - 42\,\xi + (21 + 114\,c^2 + 16\,c^3)\,\xi^2 - 2\,c^2\,(57 + 8\,c)\,\xi^3 + 21\,c^2\,\xi^4}{12\,(1-\xi)^3}$$
$$\left. + \left[(1-\xi)^2 + c^3\,\xi\,(2 - 3\xi + \xi^2) + \frac{c^2\,\xi\,(22 - 33\xi + 15\,\xi^2 - 2\,\xi^3)}{2} \right] \frac{\ln(1-\xi)}{(1-\xi)^3} \right\} .$$

In the partially integrated case, the remainder function of eq. (28) becomes

$$E_{sl}|_{\xi<\xi_0}(\xi; \alpha_s) = E_{sl}|_{\xi\geq\xi_0}(\xi_0; \alpha_s) + \Delta(\xi_0; c) - \Delta(\xi; c) \qquad (30)$$

for $\xi < \xi_0$, with

$$\Delta(\xi; c) \equiv \int_0^\xi D_{sl}(\xi')\, d\xi'$$
$$= \frac{\alpha_s\,C_F}{\pi} \left\{ c^2\,(2-\xi)\,\xi \frac{56\,c\,(1-\xi) + 9\,(22 - 22\,\xi + 3\,\xi^2)}{24\,(1-\xi)^2} - \frac{\ln(1-\xi)^2}{2} \qquad (31) \right.$$
$$\left. - \left[\frac{7}{4} - c^2 \frac{2c(14 - 28\xi + 17\xi^2 - 3\,\xi^3) + 3\,(33 - 66\xi + 44\,\xi^2 - 11\xi^3 + \xi^4)}{6\,(1-\xi)^2} \right] \ln(1-\xi) \right.$$

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$$+ \quad c^{\frac{3}{2}}\,\xi\,\frac{360\,c^2\,(\xi-1) - 35\,(96 - 96\,\xi + 29\,\xi^2) - 14\,c\,(246 - 246\,\xi + 61\,\xi^2)}{315\,(1-\xi)^{\frac{3}{2}}}$$

$$+ \quad 2\,c^{\frac{3}{2}}\,(\xi-2)\,\frac{70\,(2-\xi)^2 + 30\,c^2\,(1-\xi) + 7\,c\,(41 - 41\,\xi + 8\,\xi^2)}{105\,(1-\xi)^{\frac{3}{2}}}\,\ln(1-\xi)\Bigg\}\,.$$

More complicated experimental cuts can be applied taking as a starting point the triple differential distribution [11] and employing eq. (23).

4 Test of local duality and determination of $|V_{ub}/V_{cb}|$

In this section we construct a quantity that has a perturbative expansion in powers of α_s free from IR logs. In the shape-function region, this also guarantees the cancellation of non-perturbative effects related to the Fermi motion. Using the universality of the function $f(x)$, it follows from eqs. (4) and (18) that

$$\frac{1}{K_{sl}(\alpha_s)}\left[\frac{1}{|V_{ub}|^2\,\Gamma_{sl}^0}\frac{d\Gamma_{sl}}{d\xi} - D_{sl}(\xi;\alpha_s)\right] = \frac{1}{K_{rd}(\alpha_s)}\left[\frac{1}{|V_{tb}V_{ts}^*|^2\,\Gamma_{rd}^0}\frac{d\Gamma_{rd}}{dx} - D_{rd}(x;\alpha_s)\right]_{x=\xi}. \tag{32}$$

Normalizing the experimental differential rates to the total semileptonic rate for $B \to X_c l\nu$, the previous equation can be rewritten as

$$R \equiv \frac{\frac{d}{d\xi}\mathrm{BR}_{sl,u} - \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\mathrm{BR}_{sl,c}}{g_{sl}} D_{sl}(\xi;\alpha_s)}{\frac{d}{dx}\mathrm{BR}_{rd} - \frac{6\alpha}{\pi}\left|\frac{V_{tb}V_{ts}^*}{V_{cb}}\right|^2 \frac{\mathrm{BR}_{sl,c}}{g_{sl}} D_{rd}(x;\alpha_s)}\Bigg|_{x=\xi} = \frac{K_{sl}(\alpha_s)}{K_{rd}(\alpha_s)}\frac{\pi}{6\alpha}\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2, \tag{33}$$

where $\mathrm{BR}_{sl,u}$ and BR_{rd} are the branching ratios of the semileptonic and radiative decay, respectively. $\mathrm{BR}_{sl,c}$ denotes the experimental value for the branching ratio of $B \to X_c l\nu$, $\mathrm{BR}_{sl,c} \approx \mathrm{BR}_{sl,tot} = 0.1045 \pm 0.0021$ [28], while $g_{sl} = g(m_c^2/m_b^2)$ is the phase-space function for the semileptonic $b \to c$ decay, with $g(x) = 1 - 8x + 8x^3 - x^4 - 12x^2\ln x$. Although it is sufficient to employ the tree-level expression for g_{sl} in the above equations, there is an important uncertainty from quark masses. A very accurate determination of this phase-space factor, which could be adopted above, is $g_{sl} = 0.575 \pm 0.017$ [21]. The CKM matrix elements appearing in eq. (33) can be expressed in terms of the Wolfenstein parameters $\lambda = |V_{us}| \simeq 0.22$, $\bar{\rho}$, and $\bar{\eta}$ as

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 = \lambda\sqrt{\bar{\rho}^2 + \bar{\eta}^2} + O(\lambda^3), \quad \left|\frac{V_{ts}^*V_{tb}}{V_{cb}}\right|^2 = 1 + \lambda^2(2\bar{\rho} - 1) + O(\lambda^4) \approx 0.97. \tag{34}$$

The r.h.s. of eq. (33) is a purely short-distance quantity calculable in perturbation theory and is independent of the point $x = \xi$ where the l.h.s. is evaluated. Hence one expects to

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find an experimental plateau in the range of ξ where the shape function provides a good description of the spectra. In practice, this allows a study of the extension of the shape-function region and a test of the validity of the local quark–hadron duality hypothesis.

The numerical value of R depends on the CKM matrix elements. Let us introduce

$$C(\alpha_s) = \frac{6\alpha}{\pi} \frac{K_{rd}(\alpha_s)}{K_{sl}(\alpha_s)} \simeq (1.94 \pm 0.16) \times 10^{-3}, \tag{35}$$

where we have estimated the theoretical error due to higher-order QCD corrections and to the top and charm masses on the basis of the results of the previous section, combining different uncertainties in quadrature. The ratio R is then given by

$$R = \frac{1}{C(\alpha_s)} \left| \frac{V_{ub}}{V_{tb}V_{ts}^*} \right|^2 = 5.15 \times 10^2 \, (1 \pm 0.08) \left| \frac{V_{ub}}{V_{tb}V_{ts}^*} \right|^2, \tag{36}$$

Of course, the theoretical error on R is not the only perturbative uncertainty affecting eq. (33), as higher-order corrections to the remainder functions D_{sl} and D_{rd} might not be negligible, although they largely cancel against each other in the ratio R. In particular, we have seen that the remainder functions are relatively sizeable. The error on the r.h.s. of eq. (33) induced in this way depends on the precise shape of the spectra and cannot easily be estimated.

Provided a plateau can be experimentally identified, eq. (33) provides a stringent constraint on the parameters of the unitarity triangle. In fact, it allows for a model-independent extraction of the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$. This is apparent from the second of eq. (34), which shows that $|V_{tb}V_{ts}^*/V_{cb}|^2$ depends only weakly on $\bar{\rho}$ and $\bar{\eta}$ (whose numerical values are taken from a recent global fit [29]). Therefore, to very good approximation, a measurement of the two spectra represents a measurement of $|V_{ub}/V_{cb}|$. The master formula is

$$\left| \frac{V_{ub}}{V_{cb}} \right|^2 \simeq C(\alpha_s) \frac{\frac{d}{d\xi}\mathrm{BR}_{sl,u}}{\frac{d}{dx}\mathrm{BR}_{rd}\big|_{x=\xi} - h(\xi;\alpha_s)} \simeq C(\alpha_s) \frac{\frac{d}{d\xi}\mathrm{BR}_{sl,u}}{\frac{d}{dx}\mathrm{BR}_{rd}} \bigg|_{x=\xi}, \tag{37}$$

where $C(\alpha_s)$ has been defined in eq. (35) and

$$h(\xi;\alpha_s) = \frac{6\,\alpha}{\pi} \frac{\mathrm{BR}_{sl,c}}{g_{sl}} \left[D_{rd}(\xi;\alpha_s) - \frac{K_{rd}}{K_{sl}} D_{sl}(\xi;\alpha_s) \right]. \tag{38}$$

Whenever kinematical cuts have to be applied, the remainder functions have to be modified accordingly. In the case of a cut on the invariant hadronic mass, the relevant formulae are given in the previous section.

We plot the function $h(\xi)$ in fig. 3. Quite remarkably, the two remainder functions cancel each other almost completely in the combination $h(\xi)$. In the range, $\xi \gtrsim 0.77$, corresponding to the photon spectrum measured by CLEO [30], the function $h(\xi)$ is small with respect to the measured spectrum. Higher order corrections to the remainder functions could
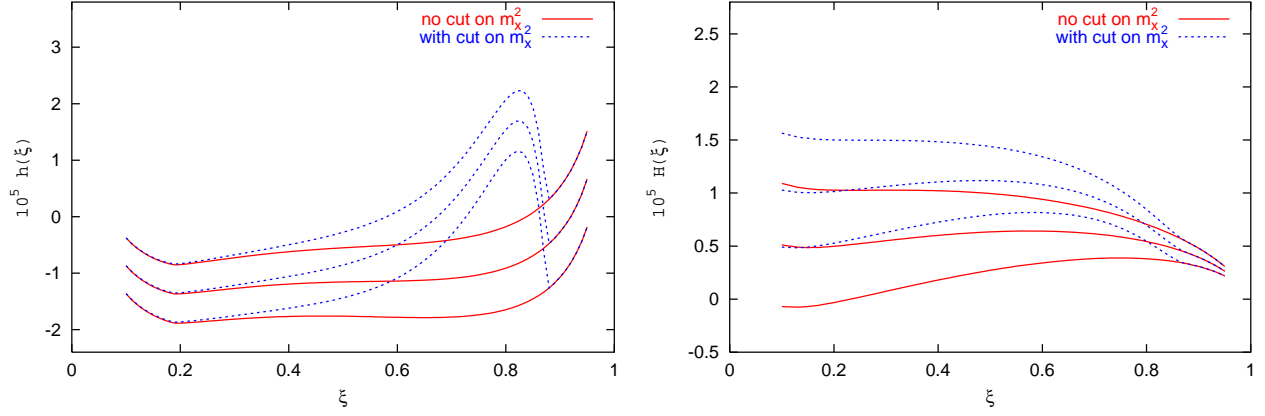
Figure 3: The functions $h(\xi)$ and $H(\xi)$ with and without a cut on the invariant hadronic mass $(\mu_b = m_b)$. For each case, the three lines correspond to values of K_{rd}/K_{sl} equal to our central value $\pm 8\%$, see eq. (35).

partially disrupt the cancellation only if they were very different in the matrix elements of the semileptonic and radiative decays. An additional source of uncertainty is the factor K_{rd}/K_{sl} in eq. (38). The effect of the error on this factor given in eq. (35) is also shown in fig. 3. Although it is difficult to assess the impact of higher-order corrections on $h(\xi)$, we can certainly neglect it in the final expression as a first approximation. These considerations are fairly independent of the presence of a cut on the hadronic invariant mass. Of course, it is straightforward to include in the analysis the small correction due to the fact that $|V_{tb}V_{ts}^*/V_{cb}|$ is not exactly one.

In a realistic analysis, it may be necessary to introduce some smearing over the spectra before applying eq. (37). Some smearing can be provided by partial integration over the spectra. In this case our ratio takes the form

$$\frac{\mathrm{BR}_{sl,u}(\xi) - \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\mathrm{BR}_{sl,c}}{g_{sl}} E_{sl}(\xi)}{\mathrm{BR}_{rd}(x) - \left|\frac{V_{tb}V_{ts}^*}{V_{cb}}\right|^2 \frac{\mathrm{BR}_{sl,c}}{g_{sl}} E_{rd}(x)}\Bigg|_{x=\xi} = R, \qquad (39)$$

where $\mathrm{BR}_{sl,u}(\xi)$ and $\mathrm{BR}_{rd}(x)$ are the partially integrated rates. The value of R is given in eq. (36). In analogy to the above discussion, one can vary the value of $x = \xi$ where eq. (39) is evaluated and look for the presence of a plateau. The master formula of eq. (37) then becomes

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq C(\alpha_s)\frac{\mathrm{BR}_{sl,u}(\xi)}{\mathrm{BR}_{rd}(x) - H(\xi;\alpha_s)}\Bigg|_{x=\xi} \simeq C(\alpha_s)\frac{\mathrm{BR}_{sl,u}(\xi)}{\mathrm{BR}_{rd}(x)}\Bigg|_{x=\xi} \qquad (40)$$

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where again we have neglected

$$H(\xi; \alpha_s) = \frac{6\,\alpha}{\pi} \frac{\mathrm{BR}_{sl,c}}{g_{sl}} \left[E_{rd}(\xi; \alpha_s) - \frac{K_{rd}}{K_{sl}} E_{sl}(\xi; \alpha_s) \right]. \tag{41}$$

Indeed, as can be seen in fig. 3, $H(\xi, \alpha_s)$ is much smaller than the partially integrated branching fraction for $B \to X_s \gamma$ (we recall that $\mathrm{BR}_{rd}(0.77) \approx 3 \times 10^{-4}$ [30]). Although the uncertainty in the determination of $H(\xi)$ is large in relative terms, it seems unlikely that it adds much to the total uncertainty in the extraction of $|V_{ub}/V_{cb}|$, which is therefore dominated by the error on $C(\alpha_s)$.

We stress that, since relation (32) is valid point by point, it is also possible to take into account a non-uniform experimental efficiency in a model independent way, provided that they are similar for the two rates. For example, in the ideal case of a common efficiency $\epsilon(x = \xi)$, eq. (40) will involve $\int dx'\, \epsilon(x')\, d\mathrm{BR}_{rd,sl}/dx'$. In the presence of different efficiencies for the semileptonic and the radiative decays, eq. (40) cannot be used in its present form and alternative startegies should be envisaged, using for instance the same equation translated in the N-moment space or deconvoluting the efficiencies.

5 Conclusions

We have constructed a ratio of spectra of semileptonic and radiative B decays, from which non-perturbative Fermi-motion effects completely cancel out. This is a short-distance quantity with an α_s expansion not involving any IR logs. In spite of the semi-inclusive character of the spectra involved, the expansion is similar to those of inclusive quantities, such as the total semileptonic width. Combined with the experimental determination of the relevant spectra, our calculation allows for a test of the theory of the shape function and of the local quark–hadron duality hypothesis. If a kinematical "shape-function region", where the shape function describes well the observed spectra, exists, the ratio R introduced in eq. (33) should be independent of the variable $x = \xi$. The quantity R involves the photon spectrum of radiative decays and the differential semileptonic charmless rate with respect to a certain function ξ of the hadronic mass over energy, M_X/E_X.

If indeed R is constant over a sufficiently extended kinematical region, our formalism permits a transparent extraction of $|V_{ub}/V_{cb}|$. This is illustrated by a very simple formula, eq. (37), that links this CKM combination to the above two spectra. Thanks in part to a remarkable cancellation of NLO effects, eq. (37) is a very clean probe of the CKM structure. As this relation is valid point by point, the effects of experimental cuts and of a non-uniform experimental efficiency can in principle be incorporated. Assuming that the data confirm the hypothesis of small higher twist effects, a determination of $|V_{ub}/V_{cb}|$ with a $\sim 5\%$ theoretical error can be eventually obtained without using any model for the shape function.

The program we have just outlined can already be carried out using CLEO and LEP data, in parallel to the other $|V_{ub}/V_{cb}|$ analyses, based on the electron energy and the hadronic invariant mass spectra. Unfortunately, the available photon spectrum [30] is provided in the lab frame; this effective smearing may hamper a precise comparison with the ξ distribution. Moreover, LEP has a small sample of events and a low signal to background ratio, and therefore the ξ distribution cannot be measured very precisely. We hope that some of these problems will be overcome at the B factories. Even with the present limitations, however, our method could provide a welcome check of the alternative techniques and we urge our experimental colleagues to implement it.

Acknowledgements

Appendix

In this appendix we give all the relevant constants and functions necessary for the calculation of the differential spectrum $d\Gamma_{rd}/dx$. The non-vanishing functions $d_{ij}(x)$ are given by [31, 32]

$$
\begin{aligned}
d_{11}(x) &= \frac{1}{36} d_{22}(x) , & d_{12}(x) &= -\frac{1}{3} d_{22}(x) , \\
d_{17}(x) &= -\frac{1}{6} d_{27}(x) , & d_{18}(x) &= -\frac{1}{6} d_{28}(x) , \\
d_{22}(x) &= \frac{16c}{27} \int_0^{x/c} dt \, (1 - c\,t) \left| \frac{G(t)}{t} + \frac{1}{2} \right|^2 , \\
d_{27}(x) &= -\frac{8c^2}{9} \int_0^{x/c} dt \, \mathrm{Re} \left[G(t) + \frac{t}{2} \right] , \\
d_{28}(x) &= -\frac{1}{3} d_{27}(x) , \\
d_{77}(x) &= \frac{1}{3} \left[7 + x - 2x^2 - 2(1 + x) \ln(1 - x) \right] , \\
d_{78}(x) &= \frac{2}{9} \left[4 + x^2 + 4(1 - x) \frac{\ln(1 - x)}{x} \right] , \\
d_{88}(x) &= \frac{1}{27x} \left[-8 + 8x - x^2 - 2x^3 + 2\left(2 - 2x + x^2 \right) \ln \frac{m_b^2(1 - x)}{m_s^2} \right] .
\end{aligned}
\tag{42}
$$

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The constants r_i are given by

$$r_1 = -\frac{1}{6}r_2, \qquad \mathrm{Re}\ r_2 = -4.092 - 12.78\,(0.29 - m_c/m_b),$$

$$r_7 = -\frac{10}{3} - \frac{8}{9}\pi^2, \qquad \mathrm{Re}\ r_8 = \frac{4}{27}\left(33 - 2\pi^2\right). \tag{43}$$

The remaining r_i with $i = 3\text{-}6$ have been calculated recently [33], but their effect is very small because the corresponding Wilson coefficients are small. They can therefore be safely neglected. The constants r_1 and r_2 depend very sensitively on the precise value of the charm mass. Following the arguments given in [21], we use $m_c/m_b = 0.22 \pm 0.04$ in $r_{1,2}$ and obtain $\mathrm{Re}\ r_2 = -4.99 \pm 0.50$. For completeness, we also give the value of $\gamma_{i7}^{(0)}$ entering eq. (10),

$$\gamma_{i7}^{(0)} = \left(-\frac{208}{243}, \frac{416}{81}, -\frac{176}{81}, -\frac{152}{243}, -\frac{6272}{81}, \frac{4624}{243}, \frac{32}{3}, -\frac{32}{9}\right). \tag{44}$$

The functions $f_{ij}(x)$ entering the partially integrated spectrum read

$$f_{11}(x) = \frac{1}{36}f_{22}(x), \qquad f_{12}(x) = -\frac{1}{3}f_{22}(x),$$

$$f_{17}(x) = -\frac{1}{6}f_{27}(x), \qquad f_{18}(x) = -\frac{1}{6}f_{28}(x),$$

$$f_{22}(x) = \frac{16c}{27}\left\{(1-x)\int_0^{x/c} dt\,(1-ct)\left|\frac{G(t)}{t} + \frac{1}{2}\right|^2 + \int_{x/c}^{1/c} dt\,(1-ct)^2\left|\frac{G(t)}{t} + \frac{1}{2}\right|^2\right\},$$

$$f_{27}(x) = -\frac{8c^2}{9}\left\{(1-x)\int_0^{x/c} dt\,\mathrm{Re}\left[G(t) + \frac{t}{2}\right] + \int_{x/c}^{1/c} dt\,(1-ct)\,\mathrm{Re}\left[G(t) + \frac{t}{2}\right]\right\},$$

$$f_{28}(x) = -\frac{1}{3}f_{27}(x),$$

$$f_{77}(x) = \frac{1}{9}(1-x)\left[31 + x - 2x^2 - 3(3+x)\ln(1-x)\right], \tag{45}$$

$$f_{78}(x) = \frac{2}{27}\left[12\,\mathrm{Li}_2(x) - 12(1-x)\ln(1-x) + 25 - 2\pi^2 - 24x - x^3\right],$$

$$f_{88}(x) = \frac{1}{81}\left\{-6\ln\frac{m_b}{m_s}\left(3 - 4x + x^2 + 4\ln x\right) + 12\,\mathrm{Li}_2(x) + 24\ln x + \right.$$
$$\left. -3(1-x)(3-x)\ln(1-x) + (1-x)\left(28 - 5x - 2x^2\right) - 2\pi^2\right\}.$$

The function $G(t)$ in the integrand of $f_{22}(x)$ and $f_{27}(x)$ is given by

$$G(t) = \begin{cases} -2\arctan^2\sqrt{\frac{t}{4-t}} & \text{for}\quad t < 4 \\ 2\ln^2\frac{\sqrt{t}+\sqrt{t-4}}{2} - 2\pi i\ln\frac{\sqrt{t}+\sqrt{t-4}}{2} - \frac{\pi^2}{2} & \text{for}\quad t \geq 4. \end{cases} \tag{46}$$

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The above integrals can be performed analytically with the substitutions $t = 4\sin^2 y$ for $t \leq 4$ and $t = 4\cosh^2 y$ for $t \geq 4$. The resulting expressions are quite long and are not reported here. For practical purposes, the compact integral representation given above is adequate.

The relation between the d_{ij} and f_{ij} functions is simply given by

$$d_{ij}(x) = -\frac{df_{ij}}{dx}(x) . \tag{47}$$

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