Next-to-Next-to-Leading Order Higgs Production at Hadron Colliders

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The Higgs-boson production cross section at pp and $p\bar{p}$ colliders is calculated in QCD at next-to-next-to-leading order (NNLO). We find that the perturbative expansion of the production cross section is well behaved and that scale dependence is reduced relative to the NLO result. These findings give us confidence in the reliability of the prediction. We also report an error in the NNLO correction to Drell-Yan production.

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Introduction.—Gluon fusion will be the most important production channel for Higgs discovery at the CERN Large Hadron Collider (LHC). The Higgs boson should manifest itself in the reaction $pp \to H(\to \gamma\gamma) + X$, where a signal should emerge on top of a very smooth, measurable $\gamma\gamma$ background. At the Tevatron, the focus for Higgs discovery is in associated production modes such as $W/Z + H$ and $t\bar{t} + H$. In a mass window around the WW threshold, however, gluon fusion is important.

Next-to-next-to-leading order (NNLO) corrections to the process $gg \to H$ have been evaluated recently in the heavy top limit and the approximation of soft gluon radiation [1–3], where the partonic center-of-mass energy is close to the Higgs mass, $M_H^2/\hat{s} \equiv x \to 1$. If we write the partonic cross section as an expansion in $(1 - x)$, it has the following form:

$$\begin{aligned}\hat{\sigma}_{ij} &= \sum_{n\geq 0}\left(\frac{\alpha_s}{\pi}\right)^n \hat{\sigma}_{ij}^{(n)},\\ \hat{\sigma}_{ij}^{(n)} &= a^{(n)}\delta(1-x) + \sum_{k=0}^{2n-1} b_k^{(n)}\left[\frac{\ln^k(1-x)}{1-x}\right]_+ \\ &\quad + \sum_{l=0}^{\infty}\sum_{k=0}^{2n-1} c_{lk}^{(n)}(1-x)^l \ln^k(1-x),\end{aligned} \tag{1}$$

where the $[\,]_+$ terms are " +" distributions defined in the usual way (see, e.g., Ref. [2]). References [1–3] contain the coefficients $a^{(n)}$ and $b_k^{(n)}$ up to $n = 2$ of this expansion. However, as anticipated in Ref. [4], these contributions are not sufficient to arrive at a reliable prediction for the total cross section. Using resummation techniques, the authors of Ref. [4] evaluated the coefficient $c_{03}^{(2)}$ at NNLO. It was included in the final results of Refs. [2,3]. However, the unknown subleading terms $c_{0i}^{(2)}$, with $i \leq 2$, were treated in different ways by Refs. [2,3], leading to significant deviations in the numerical results. It is the purpose of the current Letter to report on the analytical evaluation of the coefficients $c_{lk}^{(2)}$ with $k = 0, \ldots, 3$ and $l \geq 0$. In other words, we compute the partonic cross section for Higgs production in terms of an expansion around the soft limit. We find that the series converges very well and conclude that our final results are equivalent to a calculation of the cross section in closed analytic form. We therefore resolve the ambiguities of Refs. [2,3] and provide a realistic prediction for the Higgs production cross section in pp and $p\bar{p}$ collisions.

In checking our methods, we found an error in the NNLO Drell-Yan calculation of Ref. [5]. The correct result is given at the end of the next section.

The calculation.—In the following, we will assume all quark masses to vanish, except for the top quark mass, and neglect all electroweak couplings. In this limit, the Higgs boson can couple to gluons only via a top quark loop. This coupling can be approximated by an effective Lagrangian corresponding to the limit $m_t \to \infty$, which is valid for a large range of M_H, including the currently favored region between 100 and 200 GeV. The effective Lagrangian is

$$\mathcal{L}_{\rm eff} = -\frac{H}{4v} C_1(\alpha_s) G^a_{\mu\nu} G^{a\mu\nu}, \tag{2}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor, H is the Higgs field, $v \approx 246$ GeV is the vacuum expectation value of the Higgs field, and $C_1(\alpha_s)$ is the Wilson coefficient. Renormalization of this Lagrangian has been discussed in Ref. [2], for example, and shall not be repeated here. In the $\overline{MS}$ scheme, the coefficient function $C_1(\alpha_s)$ reads as follows, up to the order required here [6,4]:

$$C_1(\alpha_s) = -\frac{1}{3}\frac{\alpha_s}{\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{2777}{288} + \frac{19}{16}l_{\mu t} + n_f\left(-\frac{67}{96} + \frac{1}{3}l_{\mu t}\right)\right] + \cdots\right\}, \tag{3}$$

where $l_{\mu t} = \ln(\mu_R^2/M_t^2)$. μ_R is the renormalization scale and M_t is the on-shell top quark mass. $\alpha_s \equiv \alpha_s^{(5)}(\mu_R^2)$ is the $\overline{MS}$ renormalized QCD coupling constant for five active flavors, and n_f is the number of massless flavors. In our numerical results, we always set $n_f = 5$.

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The Feynman diagrams to be evaluated for hadronic collisions at NNLO are (i) two-loop virtual diagrams for $gg \to H$; (ii) one-loop single real emission diagrams for $gg \to Hg$, $gq \to Hq$, and $q\bar{q} \to Hg$; (iii) tree-level double real emission diagrams for $gg \to Hgg$, $gg \to Hq\bar{q}$, $gq \to Hgq$, $qq \to Hqq$, $q\bar{q} \to Hgg$, and $q\bar{q} \to Hq\bar{q}$. The coefficients $a^{(2)}$ and $b_k^{(2)}$ in Eq. (1) are determined by the gg subprocess only, while the $c_{lk}^{(2)}$ receive contributions from all subprocesses.

For the single real emission diagrams (ii), the full analytical result for general values of x has been evaluated and will be published elsewhere. It can be expanded trivially in terms of $(1 - x)$. In order to obtain this expansion for the double real emission contribution (iii), we evaluated the squared amplitude and expressed the invariants of incoming and outgoing momenta, as well as the phase space measure in terms of two scattering angles and the dimensionless variables x, y, z, defined by [7]

$$M_H^2 = \hat{s}x, \qquad p_1 \cdot p_H = \frac{\hat{s}}{2}[1 - (1 - x)y],$$
$$p_2 \cdot p_H = \frac{\hat{s}}{2}\left(\frac{x + (1 - x)^2 y(1 - y)(1 - z)}{1 - (1 - x)y}\right). \tag{4}$$

p_1, p_2, and p_H are the momenta of the incoming partons and the (outgoing) Higgs boson, respectively. Modulo powers that vanish as $d \to 4$ (d is the space-time dimension), the resulting expression is expanded as a Laurent series in $(1 - x)$. The leading terms in this expansion are of order $(1 - x)^{-1}$ and give rise to the purely soft contribution obtained in Refs. [2,3]. Here we also keep higher orders in this expansion, $(1 - x)^l$, $l \geq 0$. Aside from a few extra algebraic manipulations of hypergeometric functions, this expansion procedure allows us to perform the phase space integration along the lines of Ref. [7]. Details of the calculations will be presented elsewhere.

There are a number of checks that can be performed on our result. One is to see that all poles in $d - 4$ cancel. We have explicitly verified this cancellation through order $(1 - x)^{16}$. Since we have computed single real emission and the mass factorization counterterms in closed form [as opposed to an expansion in $(1 - x)$], we can also obtain the pole terms for double real emission in closed form by *demanding* that the poles cancel. This allows us to obtain in closed form all finite terms in the cross section that are linked to the poles. These include all terms proportional to $\ln^n(1 - x)$ ($n = 1, 2, 3$) and all explicitly scale-dependent terms.

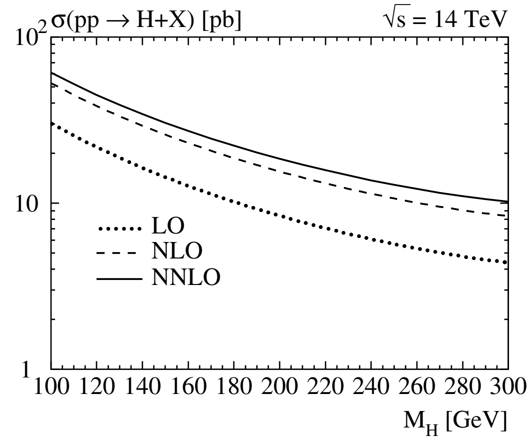

FIG. 1. LO (dotted line), NLO (dashed line), and NNLO (solid line) cross sections for Higgs production at the LHC ($\mu_F = \mu_R = M_H$). In each case, we weight the cross section by the ratio of the LO cross section in the full theory ($M_t = 175$ GeV) to the LO cross section in the effective theory [Eq. (2)].

As another check on our approach, we applied it to the cross section for the Drell-Yan process at NNLO, where the full x dependence is known in analytical form [5]. Detailed comparison with unpublished intermediate results [8] shows that our expansion of the tree-level double real emission terms is in complete agreement with the corresponding expansion of the exact calculation. However, we find differences in the one-loop single real emission terms which we also have computed exactly. We conclude that the NNLO result for the Drell-Yan process in Ref. [5] is incorrect and that the correct result is

$$\begin{aligned}
\Delta_{q\bar{q}}^{(2),C_A} &= \Delta_{q\bar{q}}^{(2),C_A}|_{\text{Ref. [5]}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A C_F\{-8x[2\text{Li}_2(1 - x) + 2\ln(x)\ln(1 - x) - \ln^2(x)]\}, \\
\Delta_{q\bar{q}}^{(2),C_F} &= \Delta_{q\bar{q}}^{(2),C_F}|_{\text{Ref. [5]}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F^2\{-16\ln(x) - 8(3 + x)[2\text{Li}_2(1 - x) + 2\ln(x)\ln(1 - x) - \ln^2(x)]\}, \\
\Delta_{qg}^{(2),C_A} &= \Delta_{qg}^{(2),C_A}|_{\text{Ref. [5]}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A T_f\{-8x\ln(x) + 4x[2\text{Li}_2(1 - x) + 2\ln(x)\ln(1 - x) - \ln^2(x)]\}, \\
\Delta_{qg}^{(2),C_F} &= \Delta_{qg}^{(2),C_F}|_{\text{Ref. [5]}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_f\{-4(3 - x)[2\text{Li}_2(1 - x) + 2\ln(x)\ln(1 - x) - \ln^2(x)] \\
&\qquad + 12(1 - x)[1 - 2\ln(1 - x)] + (28 - 44x)\ln(x)\}.
\end{aligned} \tag{5}$$

The numerical effect of these corrections is rather small and shall be investigated in more detail elsewhere.

Partonic results.—We now present the result for the partonic Higgs production cross sections at NNLO. We define

$$\sigma_0 = \frac{\pi}{576 v^2}\left(\frac{\alpha_s}{\pi}\right)^2, \qquad L(x) = \ln(1-x),$$
$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \frac{\alpha_s}{\pi}\hat{\sigma}_{ij}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \hat{\sigma}_{ij}^{(2)} + \cdots. \tag{6}$$

The lower order terms, $\hat{\sigma}_{ij}^{(0)}$ and $\hat{\sigma}_{ij}^{(1)}$, are given in Refs. [9,10]. If we split the second order terms into "soft" and "hard" pieces,

$$\hat{\sigma}_{ij}^{(2)} = \delta_{ig}\delta_{jg}\hat{\sigma}_{gg}^{(2),\mathrm{soft}} + \hat{\sigma}_{ij}^{(2),\mathrm{h}}, \tag{7}$$

the soft pieces are given in Eq. (25) of Ref.[2], while the hard pieces, $\hat{\sigma}_{ij}^{(n),\mathrm{h}}$ [to order $(1-x)^1$] are

$$\hat{\sigma}_{gg}^{(2),\mathrm{h}} = \sigma_0\Big\{\frac{1453}{12} - 147\zeta_2 - 351\zeta_3 + n_f\Big(-\frac{77}{18} + 4\zeta_2\Big) + L(x)\Big[-\frac{1193}{4} + 180\zeta_2 + \frac{101}{12}n_f\Big] + L^2(x)\Big(\frac{411}{2} - 4n_f\Big) - 144L^3(x) + (1-x)\Big[-\frac{3437}{4} + L(x)\Big(\frac{2379}{2} - 270\zeta_2\Big) - \frac{2385}{4}L^2(x) + 216L^3(x) + \frac{1017}{2}\zeta_2 + \frac{1053}{2}\zeta_3 + n_f\Big(\frac{395}{24} - \frac{45}{2}L(x) + \frac{22}{3}L^2(x) - \frac{22}{3}\zeta_2\Big)\Big] + \cdots\Big\}, \tag{8}$$

$$\hat{\sigma}_{gq}^{(2),\mathrm{h}} = \sigma_0\Big\{\frac{11}{27} + \frac{29}{6}\zeta_2 + \frac{311}{18}\zeta_3 + \frac{13}{81}n_f + L(x)\Big[\frac{341}{18} - \frac{50}{9}\zeta_2 - \frac{2}{3}n_f\Big] + L^2(x)\Big(\frac{85}{36} + \frac{1}{18}n_f\Big) + \frac{367}{54}L^3(x) + (1-x)\Big[-\frac{959}{18} + \frac{433}{9}L(x) - \frac{33}{2}L^2(x) + 8\zeta_2 + \frac{4}{9}n_f L(x)\Big] + \cdots\Big\}, \tag{9}$$

and

$$\hat{\sigma}_{q\bar{q},\mathrm{NS}}^{(2),\mathrm{h}} = \hat{\sigma}_{q\bar{q},\mathrm{S}}^{(2),\mathrm{h}} = \hat{\sigma}_{qq,\mathrm{NS}}^{(2),\mathrm{h}} = \hat{\sigma}_{qq,\mathrm{S}}^{(2),\mathrm{h}} = \sigma_0\Big\{(1-x)\Big[\frac{20}{9} - \frac{16}{9}L(x) + \frac{16}{9}L^2(x) - \frac{16}{9}\zeta_2\Big] + \cdots\Big\}. \tag{10}$$

For the sake of brevity, we have suppressed explicitly scale dependent terms by setting $\mu_F = \mu_R = M_H$ (they can be readily reconstructed using scale invariance) and displayed terms only to order $(1-x)^1$. Terms to order $(1-x)^1$ dominate the corrections (see Fig. 2), but we include terms to order $(1-x)^{16}$ for all subprocesses in our numerical analysis. The labels "NS" and "S" in Eq. (10) denote the flavor nonsinglet and singlet quark contributions, respectively. The four contributions are equal only to order $(1-x)^1$; their expansions differ at higher orders of $(1-x)$ (except that $\hat{\sigma}_{q\bar{q},\mathrm{S}}^{(2),\mathrm{h}} = \hat{\sigma}_{qq,\mathrm{S}}^{(2),\mathrm{h}}$ exactly). We note in passing that our explicit calculation confirms the value for the coefficient $c_{03}^{(2)}$ for the gluon-gluon subprocess derived in Ref. [4].

Hadronic results.—The hadronic cross section σ is related to the partonic cross section through a convolution with the parton distribution functions. It has been argued [11] that convergence is improved by pulling out a factor of x from $\hat{\sigma}_{ij}$ before expanding in $(1-x)$. We indeed observe a more stable behavior at low orders of $(1-x)$ and will adopt this prescription in what follows. Beyond fifth order, however, it is irrelevant which is used.

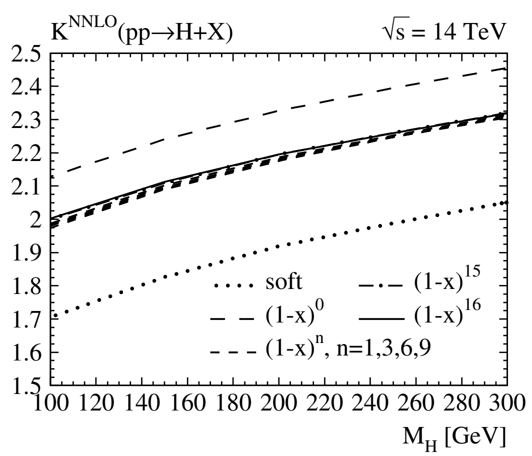

FIG. 2. K factor for Higgs production at the LHC. Each line corresponds to a different order in the expansion in $(1-x)$. The renormalization and factorization scales are set to M_H.

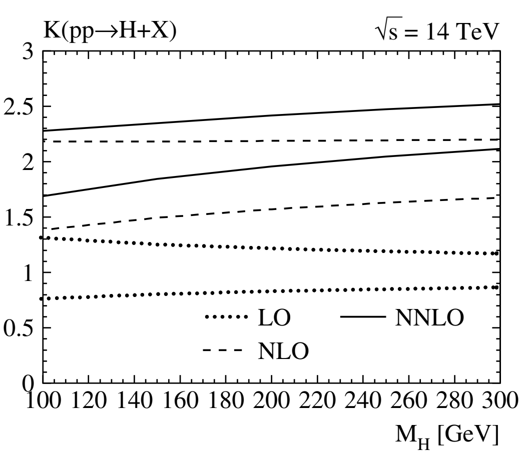

FIG. 3. Scale dependence at the LHC. The lower curve of each pair corresponds to $\mu_R = 2M_H$, $\mu_F = M_H/2$, the upper to $\mu_R = M_H/2$, $\mu_F = 2M_H$. The K factor is computed with respect to the LO cross section at $\mu_R = \mu_F = M_H$.

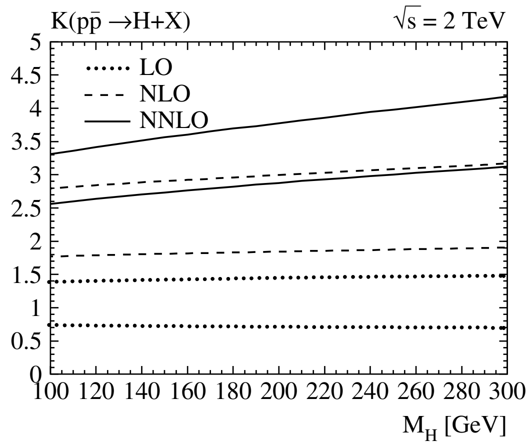

FIG. 4. Scale dependence for Tevatron Run II. The lower curve of each pair corresponds to $\mu_R = \mu_F = 2M_H$, the upper to $\mu_R = \mu_F = M_H/2$.

In Fig. 1, we show the cross section at LO, NLO, and NNLO. At each order, we use the corresponding MRST parton distribution set [12–14]. The NNLO distributions are based upon approximations of the three-loop splitting functions [15]. Studies using other parton distributions, including the NNLO distributions of Alekhin [16] will be presented elsewhere.

We next look at the quality of the expansion that we use for the evaluation of the NNLO corrections. Figure 2 shows the NNLO K factor ($K^{\rm NNLO} \equiv \sigma^{\rm NNLO}/\sigma^{\rm LO}$) for the LHC starting from the purely soft limit $\propto (1-x)^{-1}$ and adding successively higher orders in the expansion in $(1-x)$ up to order $(1-x)^{16}$. Clearly, the convergence is very good: Beyond order $(1-x)^1$, the curves differ by less than 1%. Observe that the purely soft contribution underestimates the true result by about 10%–15%, while the next term in the expansion, $\propto (1-x)^0$, overestimates it by about 5%. Note that the approximation up to $(1-x)^0$ is not the same as the "soft + sl" result of Ref. [2] or the "SVC" result of Ref. [3], since these include only the $\ln^3(1-x)$ terms at that order.

We next consider the renormalization scale (μ_R) and factorization scale (μ_F) dependence of the K factors. At the LHC, we observe that the μ_F and μ_R dependence has the opposite sign. In order to arrive at a conservative estimate of the scale dependence, we display two curves corresponding to the values $(\mu_R, \mu_F) = (2M_H, M_H/2)$ and $(M_H/2, 2M_H)$ (see Fig. 3). The scale dependence is reduced when going from NLO to NNLO and, in contrast, to the results in Ref. [2], the perturbative series up to NNLO appears to be well behaved. The reason is that both the newly calculated contributions from hard radiation and the effect of the previously unavailable set of NNLO parton distribution functions reduce the NNLO cross section. Detailed studies of the individual effects will be presented in a forthcoming paper.

Figure 4 shows the results for the Tevatron at a center-of-mass energy of $\sqrt{s} = 2$ TeV. Here, the dependence on μ_R and μ_F has the same sign, so we set $\mu_R = \mu_F \equiv \mu$ and vary μ between $M_H/2$ and $2M_H$. The K factor is larger than for the LHC, but the perturbative convergence and the scale dependence are satisfactory.

Conclusions.—We have computed the NNLO corrections to inclusive Higgs production at hadron colliders. We find reasonable perturbative convergence and reduced scale dependence.

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