

SOME RESULTS FROM THE NA27 DATA

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The self-affine analysis and erraticity analysis of pseudorapidity gaps are performed for the data of 400GeV/c pp collisions. The self-affine analysis has been shown to exhibit a better scaling behavior. The self-affine multifractal dimensions and multifractal spectrum have been obtained. The simulated results using FRITIOF program can not reproduce the scaling behavior. The analysis of event-to-event fluctuations has been performed. The increase of event-space moments $C_{p,q}(M)$ with decreasing phase-space scale is dominated by the statistical fluctuations. The erraticity analysis based on measuring the pseudorapidity gaps is also performed. The entropy-like quantities S_q and Σ_q deviate from 1 significantly, implying that both of them are useful to serve as effective measures of erraticity in multiparticle production. The $\ln S_q$ versus q has a quite linear behavior, but the $\ln \Sigma_q$ versus q has only an approximate linear behavior. The FRITIOF simulated results follow the same scaling behavior, but the deviations from the experimental data are rather large.

1 The data

In the present investigation, the angular distribution of charged particles produced in pp collisions at 400GeV/c was measured by using the LEBC films offered by the CERN NA27 collaboration. A total of 3950 non-single-diffractive events ($N \geq 4$) were measured. The accuracy in pseudorapidity in the region of interest ($-2 \leq \eta \leq 2$) is of the order of 0.1 pseudorapidity units. In order to compare with the experimental data, we used a Monte-Carlo (MC) generator FRITIOF version 7.02 and JETSET 7.3 to simulate the multiparticle production in 400GeV/c pp collisions. A total of 4500 non-single-diffractive events ($N \geq 4$) have been created.

2 Self-affine analysis

Since Białas and Peschanski proposed to study nonstatistical fluctuation in multiparticle production by the method of factorial moments $F_q(\delta)$ [1], a large variety of experiments were performed to search for the anomalous scaling behavior $F_q(\delta) \propto (\delta)^{-\phi_q}$ ($\delta \rightarrow 0$). The results have shown that the power-law behavior does not hold exactly for high energy hadron-hadron collisions [2]. The reason for this is that the usual procedure for calculating higher-dimensional factorial moments is to divide phase space into bins with the

same M in each direction. This is called self-similar analysis. However, phase space in high energy multiparticle production is anisotropic. Wu and Liu have proposed a new method to calculate the higher-dimensional factorial moments which is called self-affine analysis [3]. In self-affine analysis, the two-dimensional phase space region $\Delta\eta\Delta\varphi$ is divided by $\lambda_\eta\lambda_\varphi$. The shrinking ratios λ_η and λ_φ are characterized by a parameter $H = \ln \lambda_\eta / \ln \lambda_\varphi$, called Hurst exponent, which can be deduced from the data by fitting two corresponding one-dimensional second-order factorial-moment saturation curves

$$F_2^{(i)}(M_i) = A_i - B_i M_i^{-C_i} \quad (i = \eta, \varphi), \quad H_{\eta\varphi} = (1 + C_\varphi)/(1 + C_\eta) \quad (1)$$

In order to obtain the Hurst exponent from NA27 data, the second order factorial moment for one-dimensional phase space were calculated and are fitted to Eq.(3), cf. Table 1.

Table 1 The fitting parameters obtained according to (3).

Variables	A	B	C	χ^2/NDF
η	1.371 ± 0.025	0.222 ± 0.018	0.425 ± 0.109	8.449/36
φ	1.509 ± 0.018	0.420 ± 0.039	0.057 ± 0.021	11.16/34

From these parameter values, we obtain the Hurst exponent $H_{\eta\varphi} = 0.74 \pm 0.07$. The second order two-dimensional factorial moments are then calculated with the method of continuously varying scale [4] for $H = 0.5, 0.74, M_\eta = M_\varphi^H$; $H = 1.0, 2.0, M_\varphi = M_\eta^{1/H}$. The results are shown in Fig.1 and are fitted both linearly (dashed lines) and quadratically (full lines). The parameters of quadratic fit ($y(x) = a + bx + cx^2$) are shown in Table.2. We can see that the coefficient b which characterizes the strength of anomalous scaling is positive only for $H = 0.74$. The coefficient c of the quadratic term, characterizing the degree of upward-bending is the smallest for $H = 0.74$. These results mean that the self-affine analysis exhibits a better scaling behavior.

Table 2 The fitting parameters obtained according to (4).

H	a	b	c
0.50	0.181 ± 0.007	-0.0213 ± 0.0051	0.0131 ± 0.0015
0.74	0.144 ± 0.014	0.0243 ± 0.0075	0.0049 ± 0.0014
1.00	0.194 ± 0.006	-0.0197 ± 0.0031	0.0130 ± 0.0011
2.00	0.192 ± 0.015	-0.0062 ± 0.0020	0.0100 ± 0.0030

In order to obtain the self-affine multifractal spectrum, the two-dimensional factorial moments of continuous order [5] (q from -1 to 4 with step 0.2) are calculated and shown in Fig.2. The dotted lines in the figure are from FRITIOF Monte-Carlo, the dashed lines are from another MC having the

same multiplicity distribution and same number of events as the experimental data, but no correlations. It can be seen that no intermittency behavior can be observed in both MC's.

Using the intermittency exponents $\phi(q)$ for continuous order q obtained by fitting $\ln F(q, M)$ versus $\ln M$, the multifractal dimension $D(q)$ and multifractal spectrum $f(\alpha)$ of the self-affine fractal can be calculated through the following relations

$$\tau(q) = q - 1 - \phi(q), \quad D(q) = \tau(q)/(q - 1), \quad \alpha = d\tau(q)/dq, \quad f(\alpha) = q\alpha - \tau(q).$$

$D(q)$ versus q is shown in Fig.4. $D(q)$ decreases with increasing q . The self-affine multifractal spectrum $f(\alpha)$ is shown in Fig.5. It is concave downward with a maximum at $q = 0$, $f(\alpha(0)) = D(0) = 1$. These mean that multiparticle production at 400GeV/c pp collisions is a self-affine multifractal process.

The index μ for Levy stable law, defined by the equation: $\phi(q)/\phi(2) = (q^\mu - q)/(2^\mu - 2)$ can be obtained using a method proposed by Hu Yuan et al. [6]. From Eqs.(5) and (6) we can get the following relationship [6]

$$1 - f(\alpha) \propto (B - \alpha)^{\mu/(\mu-1)}, \quad \text{for } \alpha < B. \quad (2)$$

where B is the value of α when $f(\alpha) = 1$. From Fig.4, the value of α for $f(\alpha) = 1$ is found to be $B = 1.0260$. The $\ln(1-f(\alpha))$ versus $\ln(B - \alpha)$ are shown in Fig.5. The slope $C = 2.0997 \pm 0.0098$ and the Levy index $\mu = C/(C - 1) = 1.91 \pm 0.01$ are obtained through a linear fit.

3. The analysis of event-to-event fluctuations

The investigation of non-linear phenomena in high energy collisions has lasted a long time. Cao and Hwa [7]. proposed to characterize the spacial pattern of an event by using the horizontally normalized factorial moments. However, their method is meaningful only for high multiplicity events [8].

When the event multiplicity N is low and the number of bins is high, only a few events have $n_m \geq q$, so the statistical fluctuation may be very large and very little information can be obtained. In Fig.6 the event space moments $\ln C_{p,q}(M)$ versus $\ln M$ from NA27 data are plotted on the left side and $\ln C_{p,q}(M)$ versus $\ln C_{2,2}(M)$ on the right. In order to see whether the variation of the $\ln C_{p,q}(M)$ versus $\ln M$ is caused by the statistical fluctuations, we created a MC event sample which has same multiplicity distributions within $\Delta\eta$ as experimental data, but the particles are randomly distributed with equal probability. The results are shown in Fig.6 as dashed lines. We can see that the increase of the moments $C_{p,q}(M)$ with decreasing phase-space scale is indeed dominated by the statistical fluctuations.

In order to get the erraticity behavior of the low multiplicity system, a new method based on measuring the rapidity gaps has been proposed by Hwa and Zhang [9]. They have proposed the moments

$$G_q = \frac{1}{N+1} \sum_{i=0}^n x_i^q, \quad \text{and} \quad H_q = \frac{1}{N+1} \sum_{i=0}^n (1-x_i)^{-q} \quad (3)$$

as measures of spatial patterns in terms of rapidity gaps $x_i = X_{i+1} - X_i$, $i = 0, \dots, N$, with $X_0 = 0$ and $X_{N+1} = 1$. From this we can obtain the entropy-like quantities

$$S_q = \frac{\langle G_q \ln G_q \rangle}{\langle G_q^{st} \ln G_q^{st} \rangle}, \quad \text{and} \quad \Sigma_q = \frac{\langle H_q \ln H_q \rangle}{\langle H_q^{st} \ln H_q^{st} \rangle}. \quad (4)$$

where G_q and H_q for experimental data, G_q^{st} and H_q^{st} for pure statistical fluctuations. How much degree of S_q and Σ_q deviate from 1 is a measure of erraticity in multiparticle production based on rapidity gaps. We calculate the S_q and Σ_q only in central region ($-2 \leq \eta \leq 2$) and drop the events which has less than six particles in this region. A total of 2515 events for NA27 data and 2723 events for FRITIOF have been selected.

The results for S_q are shown in Fig.7a. It can be seen that S_q deviate from 1 significantly and the $\ln S_q$ versus q has a quite linear behavior. This means that S_q satisfies the exponential relationship $S_q \propto e^{\alpha q}$. The straight lines are the linear fit to the experimental data. The fit parameter is listed in Table 3. This result is different from the power law behaviour claimed by Hwa and Zhang [9], $S_q \propto q^{\alpha_1}$, cf. the linear fit to the $\ln S_q$ versus $\ln q$ plotted in Fig.7b and the fitting parameter listed in Table 3. The results from FRITIOF MC are also shown in Fig.8 (open circles) and Table 3. We can see that there is a same scaling behavior for the FRITIOF MC, although the values of S_q deviate from experimental data sufficiently large.

Table 3 The fit parameters obtained according to (13) and (14).

EVENT SAMPLE	α	χ^2/NDF	α_1	χ^2/NDF
experimental data	0.133 ± 0.002	0.42	0.66 ± 0.06	18.8
FRITIOF	0.078 ± 0.002	0.92	0.38 ± 0.04	10.2

The results for Σ_q are shown in Fig.8. It can be seen that Σ_q deviate from 1 significantly, but $\ln \Sigma_q$ versus q only has an approximately linear behavior $\Sigma_q \propto e^{\beta q}$, when $q \geq 2$. For the experimental data, $\beta = 0.30 \pm 0.04$. For the FRITIOF Monte-Carlo event sample, $\beta = 0.15 \pm 0.03$.

The value of β (0.15 ± 0.03) for the FRITIOF MC is much smaller than that ($\beta = 0.30 \pm 0.04$) for the experimental data. The S_q and Σ_q deviate

from 1 significantly implies that both of them are useful to serve as effective measures of erraticity in multiparticle production.

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Figure Captions

- Fig.1. $\ln F_2$ versus $\ln(M_\eta M_\varphi)$ for different values of the Hurst-exponent.
- Fig.2 $\ln F_q$ versus $\ln M$ for continuous order q .
- Fig.3 $D(q)$ versus q .
- Fig.4 $f(\alpha)$ versus α .
- Fig.5 $\ln(1 - f(\alpha))$ versus $\ln(B - \alpha)$.
- Fig.6 The event space moments $\ln C_{p,q}(M)$ versus $\ln M$.
- Fig.7 $\ln S_q$ versus q and $\ln q$.
- Fig.8 $\ln \Sigma_q$ versus q .