

## Alternative approach to $b \rightarrow s\gamma$ in the uMSSM

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### Abstract

The gluino contributions to the  $C'_{7,8}$  Wilson coefficients for  $b \rightarrow s\gamma$  are calculated within the unconstrained MSSM. New stringent bounds on the  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  mass insertion parameters are obtained in the limit in which the SM and SUSY contributions to  $C_{7,8}$  approximately cancel. Such a cancellation can plausibly appear within several classes of SUSY breaking models in which the trilinear couplings exhibit a factorized structure proportional to the Yukawa matrices. Assuming this cancellation takes place, we perform an analysis of the  $b \rightarrow s\gamma$  decay. We show that in a supersymmetric world such an alternative is reasonable and it is possible to saturate the  $b \rightarrow s\gamma$  branching ratio and produce a CP asymmetry of up to 20%, from only the gluino contribution to  $C'_{7,8}$  coefficients. Using photon polarization a LR asymmetry can be defined that in principle allows for the  $C_{7,8}$  and  $C'_{7,8}$  contributions to the  $b \rightarrow s\gamma$  decay to be disentangled. In this scenario no constraints on the “sign of  $\mu$ ” can be derived.

## 1 Introduction

The precision measurements of the inclusive radiative decay  $B \rightarrow X_s\gamma$  provides an important benchmark for the Standard Model (SM) and New Physics (NP) models at the weak-scale, such as low-energy supersymmetric (SUSY) models. In the SM, flavor changing neutral currents (FCNC) are forbidden at tree level. The first SM contribution to the  $b \rightarrow s\gamma$  transition appears at one loop level due to the CKM flavor changing structure, showing the characteristic Cabibbo suppression. NP contributions to  $b \rightarrow s\gamma$  typically also arise at one loop, and in general can be much larger than the SM contributions if no mechanisms for suppressing the new sources of flavor violation exist.

Experimentally, the inclusive  $B \rightarrow X_s \gamma$  branching ratio has been measured by ALEPH [1], BELLE [2] and CLEO [3] resulting in the current experimental weighted average

$$BR(B \rightarrow X_s \gamma)_{exp} = (3.23 \pm 0.41) \times 10^{-4}, \quad (1)$$

with new results expected shortly from BABAR and BELLE which could further reduce the experimental errors. Squeezing the theoretical uncertainties down to the 10% level has been (and still is) a crucial task. The SM theoretical prediction has been the subject of intensive theoretical investigation in the past several years. From the original calculation at LO [4], impressive progress in the theoretical precision has been achieved with the completion of NLO QCD calculations [6, 7, 8] and the addition of several further refinements [9, 10]. The original complete SM NLO calculation [7] gives the following prediction for  $\sqrt{z} = m_c/m_b = 0.29$ :

$$BR(B \rightarrow X_s \gamma)_{SM} = (3.28 \pm 0.33) \times 10^{-4}. \quad (2)$$

The main source of uncertainty of the previous result is due to NNLO QCD ambiguities. In [11] it is shown that using  $\sqrt{z} = 0.22$  (i.e. the running charm mass instead of the pole mass) is more justifiable and causes an enhancement of about 10% of the  $b \rightarrow s \gamma$  branching ratio, leading to the current preferred value:

$$BR(B \rightarrow X_s \gamma)_{SM} = (3.73 \pm 0.30) \times 10^{-4}. \quad (3)$$

Although these theoretical uncertainties can be addressed only with a complete NNLO calculation, the SM value for the branching ratio is in agreement with the experimental measurement within the  $1 - 2\sigma$  level.

The general agreement between the SM theoretical prediction and the experimental results have provided useful guidelines for constraining the parameter space of models with NP present at the electroweak scale, such as the 2HDM and the minimal supersymmetric standard model (MSSM). In SUSY models superpartners and charged Higgs loops contribute to  $b \rightarrow s \gamma$ , with contributions that typically rival the SM one in size. To get a sense of the typical magnitudes of the SUSY contribution to  $b \rightarrow s \gamma$ , it is illustrative to consider the (unphysical) limit of unbroken SUSY but broken electroweak gauge symmetry, which corresponds to the supersymmetric Higgsino mass parameter  $\mu$  set to zero, and the ratio of Higgs vacuum expectation values  $\tan \beta \equiv v_u/v_d$  set to 1. In this limit SM and SUSY contributions are identical in size and cancel each other [12], due to the usual sign difference between boson and fermion loops. Of course, this limit is unphysical: not only must SUSY be (softly) broken, but  $\mu = 0$  and  $\tan \beta = 1$  have been ruled out by direct and indirect searches at LEP.

In the realistic case of softly broken SUSY, the contributions to  $b \rightarrow s \gamma$  depend strongly on the parameters of the SSB Lagrangian, as well as the values of  $\mu$  and  $\tan \beta$ . In particular, as the origin and dynamical mechanism of SUSY breaking are unknown, there

is no reason *a priori* to expect that the soft parameters will be flavor-blind (or violate flavor in the same way as the SM). Of course, the kaon system has provided strong FCNC constraints for the mixing of the first and second generations which severely limit the possibility of flavor violation in that sector [13, 14]. Note however that the constraints for third generation mixings are significantly weaker, with  $b \rightarrow s\gamma$  providing usually the most stringent constraints.

Nevertheless, for calculational ease one of the following simplified MSSM scenarios have often been assumed:

- The SUSY partners are very heavy and their contribution decouples, so that only the Higgs sector contributes to  $b \rightarrow s\gamma$ . In this scenario, as well in general 2HDMs, NLO calculations have been performed [8, 15, 16]. Due to coherent contributions between SM and Higgs sector, a lower bound on the charged Higgs mass can usually be derived [17] in this class of models. In the large  $\tan\beta$  region the two-loop SUSY correction to the Higgs vertex can produce quite sizeable modifications and should be carefully taken into account [18].
- The SUSY partners as well the extra Higgs bosons have masses of order the electroweak scale, but the only source of flavor violation is in the CKM matrix. This scenario, known as minimal flavor violation (MFV), is motivated for example within minimal supergravity (mSUGRA) models. MFV scenarios have been studied at LO [19, 20, 21, 22], in certain limits at NLO [23], and including large  $\tan\beta$  enhanced two-loop SUSY contributions [18, 24]. In this scenario, the  $b \rightarrow s\gamma$  decay receives a contribution from the chargino sector as well as from the charged Higgs sector. To avoid overproducing  $b \rightarrow s\gamma$ , the charged Higgs and chargino loops must cancel to a good degree. This cancellation can be achieved for a particular “sign of  $\mu$ ” in the mSUGRA parameter space<sup>1</sup> which flips the sign of the chargino contribution relative to the SM and charged Higgs loops, always interfering constructively. Although this cancellation can occur and puts important constraints on the mSUGRA parameter space, it is important to note that it is not due to any known symmetry but rather should be interpreted, in a certain sense, as a fine-tuning.
- There are new sources of flavor violation in the soft breaking terms. In this case, additional SUSY loops involving down-type squarks and gluinos or neutralinos (hereafter neglected compared with the gluino loops due to the weaker coupling) contribute to  $b \rightarrow s\gamma$ . It is well known that the gluino contribution can dominate the amplitude for such nonminimal SUSY models, both due to the  $\alpha_s/\alpha$  enhancement with respect to the other SM and SUSY contributions, and due to the  $m_{\tilde{g}}/m_b$  enhancement from

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<sup>1</sup>Specifically the relative sign between the parameters  $\mu$  and  $A_t$ , and so generally different from the “sign of  $\mu$ ” relevant in the case of the muon  $g - 2$  MSSM contribution [25].

the chirality flip along the gluino line. Thus in this scenario, which is generally noted as the unconstrained MSSM (uMSSM), usually only the gluino contribution is discussed. It has been shown [13, 26] that the 23-LR off-diagonal entry of the down-squark mass matrix is severely constrained by  $b \rightarrow s\gamma$  measurements to be of  $\mathcal{O}(10^{-2})$ . Less stringent bounds can be obtained for the other 23 off-diagonal entries. No known symmetry assures that these constraints can be automatically satisfied; again this fact could be interpreted at the electroweak scale as a fine-tuning.

A discussion of the  $b \rightarrow s\gamma$  process in the general unconstrained MSSM is in principle possible, but it is necessary to deal with two unavoidable problems: (i) a large number of free, essentially unconstrained parameters, and (ii) the need to achieve a quite accurate cancellation between the sizeable different contributions (SM, Higgs, chargino/neutralino and gluino) to the Wilson coefficient  $C_7$  associated with the  $Q_7 \propto m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$  operator in such a way that the experimental measurement, which approximately saturated solely by the SM result, is satisfied. Moreover, in general MSSM models with nonminimal flavor violation the gluino loop can also contribute significantly to the Wilson coefficient  $C'_7$  associated with the chirality-flipped operator,  $Q'_7 \propto m_b \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu}$ , as has been recently emphasized in the literature [27, 28]. However, as the SM, Higgs, and chargino contributions to  $C'_7$  are typically suppressed by a factor of  $O(m_s/m_b)$ , it is not possible in general to achieve a cancellation between the different terms in  $C'_7$  and thus a stronger fine-tuning has to be imposed.

However, it has been recently shown [29] that in many classes of SUSY breaking models a particular structure of the soft trilinear couplings  $\tilde{A}$  of the soft-breaking Lagrangian can be derived which can alleviate these constraints. Writing these couplings as  $\tilde{A}_{ij} = A_{ij} Y_{ij}$  (in which  $Y$  denotes the fermion Yukawa matrices), the matrices  $A$  for the up and down sector are given respectively by:

$$A_{ij}^{(u)} = A_{ii}^L + A_{jj}^{R,u} \quad , \quad A_{ij}^{(d)} = A_{ii}^L + A_{jj}^{R,d} \quad . \quad (4)$$

As shown in [29], this factorization holds quite generally in string models, for example in Calabi-Yau models in the large  $T$  limit or in Type I models [30], as well as in gauge-mediated [31] and anomaly-mediated models [32, 33, 34]. If eq.(4) holds, specific relations can be derived for the off-diagonal LR entries in squark mass matrix. In particular, the leading contribution to the entries of interest for the  $b \rightarrow s\gamma$  process are given in the SCKM basis as:

$$\tilde{A}_{23}^{(u)} \propto m_t \left[ (A_{22}^L - A_{11}^L)(V_L^{(u)})_{22}(V_L^{(u)})_{32}^* + (A_{33}^L - A_{11}^L)(V_L^{(u)})_{23}(V_L^{(u)})_{33}^* \right] \quad , \quad (5)$$

$$\tilde{A}_{32}^{(u)} \propto m_t \left[ (A_{22}^{R,u} - A_{11}^{R,u})(V_R^{(u)})_{32}(V_R^{(u)})_{22}^* + (A_{33}^{R,u} - A_{11}^{R,u})(V_R^{(u)})_{33}(V_R^{(u)})_{23}^* \right] \quad , \quad (6)$$

$$\tilde{A}_{23}^{(d)} \propto m_b \left[ (A_{22}^L - A_{11}^L)(V_L^{(d)})_{22}(V_L^{(d)})_{32}^* + (A_{33}^L - A_{11}^L)(V_L^{(d)})_{23}(V_L^{(d)})_{33}^* \right] \quad , \quad (7)$$

$$\tilde{A}_{32}^{(d)} \propto m_b \left[ (A_{22}^{R,d} - A_{11}^{R,d})(V_R^{(d)})_{32}(V_R^{(d)})_{22}^* + (A_{33}^{R,d} - A_{11}^{R,d})(V_R^{(d)})_{33}(V_R^{(d)})_{23}^* \right] \quad , \quad (8)$$

with  $V_{L,R}^{(u,d)}$  the rotation matrices for the up and down quark sector from the interaction to the mass eigenstate<sup>2</sup>. From eqs.(5-8), one can realize first that the down-sector LR off-diagonal entries are naturally suppressed by a factor of  $O(m_b/m_t)$  compared with the up-squark sector ones due to the particular factorization of the soft trilinear couplings given in eq.(4). Second, in these classes of models both the 23 and 32 entries are of the same order and proportional to the largest mass (up or down). Consequently, in these classes of models,  $\mathcal{O}(10^{-2})$  off-diagonal entries in the down-squark sector along with  $\mathcal{O}(1)$  off-diagonal entries in the up-squark can be considered in some sense as a prediction of the underlying fundamental theory<sup>3</sup>. This fact implies comparable chargino and gluino contributions to  $b \rightarrow s\gamma$ , making the possibility of cancellations between the W and the different SUSY contributions to the  $Q_7$  operator less unnatural. The constraints on the gluino contribution to  $Q'_7$  are simultaneously alleviated. This flavor structure holds in essentially all attempts to build string-motivated models of the soft-breaking Lagrangian.

The structure of the paper is as follows. In section 2, we briefly summarize the theoretical framework for the calculation of the  $b \rightarrow s\gamma$  branching ratio at LO and NLO. In section 3, we derive useful mass insertion (MI) formulas for the gluino contributions to the Wilson coefficients  $C_{7,8}$  and  $C'_{7,8}$ . We demonstrate explicitly that in the large  $\tan\beta$  region, a good understanding of these expressions is obtained only by retaining terms in the MI expansion through the second order. For  $\mu$  of the same order as the common squark mass parameter and large  $\tan\beta$ , new (previously overlooked) off-diagonal terms become relevant in the  $b \rightarrow s\gamma$  process. We then devote our attention in section 4 to the analysis of the gluino contribution to  $C'_{7,8}$  in the general uMSSM. In particular, we ask the question of whether the contribution to  $C'_7$  alone can saturate the  $b \rightarrow s\gamma$  branching ratio, assuming that the SM and SUSY contributions to  $C_7$  cancel each other to an extent that the effects of  $C_7$  are subleading. While this scenario may initially appear to be unnatural, we will argue that sufficient cancellations in  $C_7$  do not involve significantly more fine tuning than the usual cancellation required in MFV scenarios. With this analysis, we thus provide an alternative interpretation of  $b \rightarrow s\gamma$  which is at least as viable as any supersymmetric one. This analysis also provides more general mass insertion bounds on  $\delta_{23}^{RL}$  than those obtained recently [28], where the SM (and sometimes Higgs and chargino) contributions to  $C_7$  are always retained. As we are generally interested in moderate to large values of  $\tan\beta$ , we

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<sup>2</sup>In this notation the CKM matrix is  $V_{CKM} = V_L^{(u)}(V_L^{(d)})^\dagger$ .

<sup>3</sup>It is important to note however that the off-diagonal entries of  $\tilde{A}$  in the SCKM basis contain terms proportional to the products of entries of the left-handed and right-handed quark rotation matrices, which are largely unconstrained (except for the CKM constraint for the left-handed up and down quark rotation matrices which enter (for example)  $\tilde{A}_{23}$ ). The quark rotation matrices are highly model-dependent. While the diagonal entries can in general safely be taken  $\mathcal{O}(1)$ , it is typically assumed that the off-diagonal quark rotation matrices are suppressed by powers of the Cabibbo angle in a way that mirrors the CKM matrix (see e.g. [29]). Note though that this assumption is not required, particularly for the right-handed quark rotation matrices which enter  $\tilde{A}_{32}$  which are of particular relevance for this paper.

are able to put rather stringent bounds on the mass insertion parameter  $\delta_{23}^{RR}$ . In section 4.3, we study the branching ratio and CP asymmetry as functions of the SUSY parameter space within this scenario, assuming complex off-diagonal MIs. Throughout the paper, to avoid EDM constraints we set the relevant reparameterization invariant combinations of the flavor-independent phases to zero. Finally in section 4.4 we show that if the photon polarization will be measured, it is possible to distinguish such a scenario from the usual  $C_7$  dominated scenario through the definition of a LR asymmetry.

Since we are interested in analyzing a supersymmetric world where the one-loop SUSY effects are of the same order as the SM loops, we assume relatively light superpartner masses. Specifically we choose the gluino mass  $\tilde{m}_{\tilde{g}} = 350$  GeV and the common diagonal down-squark mass  $\tilde{m}_D = 500$  GeV, with the lightest down-squark mass in the 250 – 500 GeV range. All of the other sfermion masses, as well the chargino and neutralino masses, do not enter directly in our analysis and (some of them) can be taken to be reasonably light as suggested by [35]. Motivated by the lower limit on the Higgs boson mass [36] (which suggests  $|\cos 2\beta| \approx 1$ ) and by the muon  $g - 2$  excess, we focus to some extent on moderate to large values of  $\tan \beta$ , though our formulas and much of the analysis hold in general.

## 2 $b \rightarrow s\gamma$ branching ratio at NLO

For the purpose of presentation, we summarize the theoretical framework for evaluating the  $b \rightarrow s\gamma$  branching ratio at NLO. A complete and detailed discussion can be found for example in [7, 8, 9]. The starting point in the calculation of the B meson decay rates is the low-energy effective Hamiltonian, at the bottom mass scale  $\mu_b$ :

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu_b) Q_i(\mu_b) . \quad (9)$$

The operators relevant to the  $b \rightarrow s\gamma$  process<sup>4</sup> are:

$$\begin{aligned} Q_2 &= \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L , \\ Q_7 &= \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu} , \\ Q_8 &= \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} G_{\mu\nu}^a T_a b_R . \end{aligned} \quad (10)$$

and their  $L \leftrightarrow R$  chirality counterpart:

$$Q'_2 = \bar{s}_R \gamma_\mu c_R \bar{c}_R \gamma^\mu b_R ,$$

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<sup>4</sup>This of course depends on the basis chosen; we have chosen the one easiest for our discussion.

$$\begin{aligned}
Q'_7 &= \frac{e}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu} , \\
Q'_8 &= \frac{e}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} G_{\mu\nu}^a T_a b_L .
\end{aligned} \tag{11}$$

The Wilson coefficients  $C_{2,7,8}^{(\prime)}$  are initially evaluated at the electroweak or soft SUSY breaking scale, which we generically denote as  $\mu_0$ , and then evolved down to the bottom mass scale  $\mu_b$ . The standard<sup>5</sup> RG equations for the  $C_{2,7,8}$  operators from the electroweak scale ( $\mu_W < m_t$ ) to the low-energy scale  $\mu_b$  is given by:

$$C_2(\mu_b) = \frac{1}{2} \left( \eta^{-\frac{12}{23}} + \eta^{\frac{6}{23}} \right) C_2(\mu_W) , \tag{12}$$

$$C_7(\mu_b) = \eta^{\frac{16}{23}} C_7(\mu_W) + \frac{8}{3} \left( \eta^{\frac{14}{23}} - \eta^{\frac{16}{23}} \right) C_8(\mu_W) + \sum_{i=1}^8 h_i \eta^{a_i} , \tag{13}$$

$$C_8(\mu_b) = \eta^{\frac{14}{23}} C_8(\mu_W) + \sum_{i=1}^8 \bar{h}_i \eta^{a_i} , \tag{14}$$

where  $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$  and  $h_i, \bar{h}_i$  and  $a_i$  are constants (see [7] for details). The  $C'_{2,7,8}$  coefficients obey the same running as their chirality conjugate counterparts. If the NP scale is much higher than  $m_t$ , the running from  $\mu_{SUSY}$  to  $\mu_W$  with six quarks should also be taken into account (see the first paper of [18]). The coefficient  $C_2$  is dominated by a SM tree-level diagram and is normalized such that  $C_2(\mu_W) = 1$ . Its chirality conjugate,  $C'_2$ , has no SM contribution at tree level and can thus be safely set to zero. The NP contributions to  $C_2$  and  $C'_2$  appear at one-loop order and are negligible. The Wilson coefficients  $C_7$  and  $C'_7$  are the only coefficients that contribute directly to the  $b \rightarrow s\gamma$  branching ratio at the lowest QCD order ( $\alpha_s^0$ ). These coefficients receive contributions both from the SM and NP at one-loop order. The coefficients  $C_8$  and  $C'_8$  receive one-loop SM and NP contributions through the same types of diagrams as  $C_7$  and  $C'_7$ , but with the external photon line substituted by a gluon line. When the QCD running from the matching scale  $\mu_0$  to  $\mu_b$  is performed, these different coefficients mix, as shown in eqs.(12-14), so that the “effective” low-energy coefficients  $C_{2,7,8}(\mu_b)$  receives contributions from different operators.

The  $b \rightarrow s\gamma$  branching ratio is usually defined by normalizing it to the semileptonic  $b \rightarrow c e^- \bar{\nu}_e$  branching ratio, giving:

$$BR(B \rightarrow X_s \gamma) |_{E_\gamma > (1-\delta) E_\gamma^{max}} = BR(B \rightarrow X_c e \bar{\nu}) \frac{6\alpha}{\pi f(z)} \left| \frac{V_{tb} V_{ts}^*}{V_{cb}} \right|^2 K(\delta, z) . \tag{15}$$

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<sup>5</sup>In a recent paper [27] it has been pointed out that the gluino contribution (and the same argument holds also for the chargino and neutralino contributions) is the sum of two different pieces, one proportional to the bottom mass and one proportional to the gluino mass, which have a different RG evolution. We have found that at LO, this is equivalent to the usual SM evolution once the running bottom mass  $m_b(\mu_0)$  is used instead of the pole mass in the  $C_i(\mu_0)$  WC.

Here  $f(z)$  is a phase space function and should be calculated for on-shell masses, namely  $\sqrt{z} = m_c/m_b = 0.29$ .  $\delta$  is the experimental photon detection threshold, which for comparison between experimental data and theoretical prediction is usually set to 0.9 [9]. The dependence of  $K_{NLO}$  from the Wilson coefficients  $C_i$  and  $C'_i$  at NLO is given by [9]:

$$K_{NLO}(\delta, z) = \sum_{i \leq j=2,7,8} k_{ij}^{(0)}(\delta, z) \left\{ \text{Re}[C_i^{(0)}(\mu_b)C_j^{(0)*}(\mu_b)] + (C_{i,j} \rightarrow C'_{i,j}) \right\} + k_{77}^{(1)}(\delta, z) \left\{ \text{Re}[C_7^{(1)}(\mu_b)C_7^{(0)*}(\mu_b)] + (C_7 \rightarrow C'_7) \right\}. \quad (16)$$

In the previous expression  $C_i^{(0)}$  and  $C_i^{(1)}$  refer respectively to the LO and NLO contributions to the Wilson coefficients  $C_i$  defined as:

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \mathcal{O}(\alpha, \alpha_s^2). \quad (17)$$

As in the following we are deriving only one-loop formulas for the Wilson coefficients  $C_{7,8}^{(\prime)}$ ,  $C_i \equiv C_i^{(0)}$ . We will briefly discuss the effects of including  $C_7^{(1)}$  in section 4.1. The coefficients  $k_{ij}(\delta, z)$  used in the our analysis are calculated for  $\delta = 0.9$  and  $\sqrt{z} = 0.22$  using the formulas derived in [7, 9]. The LO branching ratio expression can be easily derived from eq.(16) setting  $k_{77}^{(0)} = 1$  and all the other  $k_{ij}^{(0,1)} = 0$ , giving:

$$K_{LO} = |C_7(\mu_b)|^2 + |C'_7(\mu_b)|^2, \quad (18)$$

independently of the choice of  $\delta$  and  $z$ .

### 3 $C_{7,8}$ and $C'_{7,8}$ gluino contributions to $b \rightarrow s\gamma$

In the following we will focus on the gluino contribution to the Wilson coefficients  $C_{7,8}$  and  $C'_{7,8}$ . There is only one gluino diagram that contributes to  $C_7$  and  $C'_7$ , with the external photon line attached to the down-squark line, while two diagrams can contribute to the  $C_8$  and  $C'_8$  coefficients, as the gluon external line can be attached to the squark or the gluino lines. The one-loop gluino contributions to the  $C_{7,8}$  and  $C'_{7,8}$  coefficients are given respectively by:

$$C_7^{\tilde{g}}(\mu_W) = \frac{4g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \sum_A \frac{m_W^2}{\tilde{m}_A^2} \left\{ L_b L_s^* F_2(x_A^g) + \frac{\tilde{m}_{\tilde{g}}}{m_b} R_b L_s^* F_4(x_A^g) \right\} \quad (19)$$

$$C_8^{\tilde{g}}(\mu_W) = -\frac{g_s^2}{6g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \sum_A \frac{m_W^2}{\tilde{m}_A^2} \left\{ L_b L_s^* F_{21}(x_A^g) + \frac{\tilde{m}_{\tilde{g}}}{m_b} R_b L_s^* F_{43}(x_A^g) \right\} \quad (20)$$

$$C_7^{\prime\tilde{g}}(\mu_W) = \frac{4g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \sum_A \frac{m_W^2}{\tilde{m}_A^2} \left\{ R_b R_s^* F_2(x_A^g) + \frac{\tilde{m}_{\tilde{g}}}{m_b} L_b R_s^* F_4(x_A^g) \right\} \quad (21)$$

$$C_8^{\prime\tilde{g}}(\mu_W) = -\frac{g_s^2}{6g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \sum_A \frac{m_W^2}{\tilde{m}_A^2} \left\{ R_b R_s^* F_{21}(x_A^g) + \frac{\tilde{m}_{\tilde{g}}}{m_b} L_b R_s^* F_{43}(x_A^g) \right\}, \quad (22)$$



in which  $x_A^g = \tilde{m}_{\tilde{g}}^2/\tilde{m}_A^2$ , with  $\tilde{m}_{\tilde{g}}$  the gluino mass, and  $\tilde{m}_A$  the mass of the A-th down squark eigenstate.  $L_d$  and  $R_d$  are the Left and Right gluino couplings to a generic down quark  $d$  given by:

$$L_d^{\tilde{g}} = -\sqrt{2} U_{A,d} \quad , \quad R_d^{\tilde{g}} = \sqrt{2} U_{A,d+3} \quad , \quad (23)$$

in which  $U$  is the  $6 \times 6$  down-squark rotation matrix. The loop integrals  $F_{12}$  and  $F_{43}$  are defined as:

$$F_{21} = F_2(x) + 9F_1(x) \quad , \quad F_{43} = F_4(x) + 9F_3(x) \quad , \quad (24)$$

using the conventions for the integrals  $F_i(x)$  as in [19] for an easier connection with the standard convention in the literature.

It is illustrative to write the gluino contribution to the  $C_{7,8}$  and  $C'_{7,8}$  Wilson coefficients using the MI approximation. First, note that the set of integrals used in [19] is not the most appropriate for dealing with the MI formulas. However, for the sake of simplicity we will retain these conventions and further define the integrals  $F_i$  and their ‘‘derivatives’’ through the following self-consistent relations:

$$F_i\left(\frac{x}{y}\right) \equiv \frac{1}{y} f_i(x, y) \quad , \quad F_i^{(1)}\left(\frac{x}{y}\right) \equiv \frac{1}{y^2} \frac{\partial}{\partial y} f_i(x, y) \quad , \quad \dots \quad , \quad F_i^{(n)}\left(\frac{x}{y}\right) \equiv \frac{1}{n!} \frac{1}{y^{n+1}} \frac{\partial^n}{\partial y^n} f_i(x, y) \quad .$$

Using this notation, the first and second order terms in the MI expansion for the  $C_{7,8}$  and  $C'_{7,8}$  coefficients are given respectively by:

$$C_7^{\tilde{g}}(1) = \frac{8g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{LL} F_2^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{LR} F_4^{(1)}(x_D^g) \right\} \quad , \quad (25)$$

$$C_8^{\tilde{g}}(1) = -\frac{g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{LL} F_{21}^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{LR} F_{43}^{(1)}(x_D^g) \right\} \quad , \quad (26)$$

$$C_7^{\prime\tilde{g}}(1) = \frac{8g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{RR} F_2^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RL} F_4^{(1)}(x_D^g) \right\} \quad , \quad (27)$$

$$C_8^{\prime\tilde{g}}(1) = -\frac{g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{RR} F_{21}^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RL} F_{43}^{(1)}(x_D^g) \right\} \quad , \quad (28)$$

and

$$C_7^{\tilde{g}}(2) = \frac{4g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{LR} F_2^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{LL} F_4^{(2)}(x_D^g) \right\} \quad , \quad (29)$$

$$C_8^{\tilde{g}}(2) = -\frac{g_s^2}{6g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{LR} F_{21}^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{LL} F_{43}^{(2)}(x_D^g) \right\} \quad , \quad (30)$$

$$C_7^{\prime\tilde{g}}(2) = \frac{4g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{RL} F_2^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RR} F_4^{(2)}(x_D^g) \right\} \quad , \quad (31)$$

$$C_8^{\prime\tilde{g}}(2) = -\frac{g_s^2}{6g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{RL} F_{21}^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RR} F_{43}^{(2)}(x_D^g) \right\} \quad . \quad (32)$$

In the previous formulas  $x_D^g = \tilde{m}_g^2/\tilde{m}_D^2$ , with  $\tilde{m}_D$  the average down-squark mass related to the down-squark mass eigenstates via the relation  $\tilde{m}_A^2 = \tilde{m}_D^2 + \delta m_A^2$ . The definitions of the MI parameters are:

$$\begin{aligned} \delta_{ij}^{LL} &= \frac{1}{\tilde{m}_D^2} \sum_{A=1}^6 U_{i,A}^\dagger \delta m_A^2 U_{A,j} & , & & \delta_{ij}^{RR} &= \frac{1}{\tilde{m}_D^2} \sum_{A=1}^6 U_{i+3,A}^\dagger \delta m_A^2 U_{A,j+3} , \\ \delta_{ij}^{LR} &= \frac{1}{\tilde{m}_D^2} \sum_{A=1}^6 U_{i,A}^\dagger \delta m_A^2 U_{A,j+3} & , & & \delta_{ij}^{RL} &= \frac{1}{\tilde{m}_D^2} \sum_{A=1}^6 U_{i+3,A}^\dagger \delta m_A^2 U_{A,j} . \end{aligned} \quad (33)$$

In deriving eqs.(29-32) to the second order in the MI parameters, we have kept only the dominant term proportional to  $\tan\beta$  (the  $A_b$  term is retained in the above expression for defining our convention for the  $\mu$  term; see later), and neglected all of the other off-diagonal mass insertions. Clearly the dominant terms in eqs. (25-32) are those proportional to the gluino chirality flip, such that the gluino contribution to  $C_7$  ( $C'_7$ ) at first order depends only on the MI term  $\delta_{23}^{LR}$  ( $\delta_{23}^{RL}$ ). However, for large  $\tan\beta$  and  $\mu \approx \tilde{m}_A$ , the second order MI terms in eqs. (29-32) can become comparable in size with the first order mass insertions. Thus, two different MI parameters are relevant in the L/R sectors:  $(\delta_{23}^{LR}, \delta_{23}^{LL})$  and  $(\delta_{23}^{RL}, \delta_{23}^{RR})$ , contrary to common wisdom. To which extent the LL and RR MIs are relevant depends of course on the values chosen for  $\mu$  and  $\tan\beta$ , but in a large part of the allowed SUSY parameter space they cannot in general be neglected. Moreover, the fact that the gluino Wilson coefficients depend on two different MI parameters will have important consequences in the study of the  $b \rightarrow s\gamma$  CP asymmetry<sup>6</sup>.

## 4 Alternative solution to $b \rightarrow s\gamma$ branching ratio

In the majority of the previous studies of the  $b \rightarrow s\gamma$  process, the main focus was to calculate the SM and NP contributions to the  $C_{7,8}$  coefficients. The contributions to  $b \rightarrow s\gamma$  coming from  $C'_{7,8}$  have usually been neglected on the assumption that they are suppressed compared to  $C_{7,8}$  by the ratio  $m_s/m_b$ . While this assumption is always valid for the SM and for the Higgs-sector contributions, in the case of the uMSSM this is not generally the case. It is only within specific MSSM scenarios (such as MFV) that the gluino and chargino contributions to the  $C'_{7,8}$  coefficients can be neglected due to the  $m_s/m_b$  suppression factor. In the general uMSSM this suppression can be absent and, in particular, the gluino contributions to  $C_{7,8}$  and  $C'_{7,8}$  are naturally of the same order [27, 28].

Therefore, in the following we present an alternative approach to the  $b \rightarrow s\gamma$  process in supersymmetric models. We assume a particular scenario in which the *total* contribution

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<sup>6</sup>Specifically, if only the first order term in the MI is taken the  $b \rightarrow s\gamma$  CP asymmetry vanishes, as discussed in greater detail in section 4.3.

to  $C_{7,8}$  is negligible and the main contribution to the  $b \rightarrow s\gamma$  branching ratio is given by  $C'_{7,8}$ . This “ $C'_7$  dominated” scenario is realized when the chargino, neutralino, and gluino contributions to  $C_{7,8}$  sum up in such a way as to cancel the W and Higgs contributions almost completely<sup>7</sup>. In our opinion this situation does not require substantially more fine tuning than what is required in the usual MFV scenario, where conversely the NP contributions to  $C_{7,8}$  essentially cancel between themselves (or are almost decoupled) so that all the measured  $b \rightarrow s\gamma$  branching ratio is produced by the W diagram. As previously discussed, many classes of SUSY breaking models [29] lead to off-diagonal LR entries of the down-squark sector that are naturally suppressed compared with those of the up-squark sector:

$$(\delta_{ij}^{LR})^d \approx \frac{\max(m_i, m_j)}{m_t} (\delta_{ij}^{LR})^u \quad (34)$$

in which  $m_{i,j}$  are down-quark masses. In particular, the  $(\delta_{23}^{LR})^d$  entries, which are relevant for the  $b \rightarrow s\gamma$  process, receive a  $O(m_b/m_t)$  suppression as can be derived from eqs.(5-8). For  $(\delta_{23}^{LR})^u \approx \mathcal{O}(1)$ , a natural value  $(\delta_{23}^{LR})^d \approx \mathcal{O}(m_b/m_t) \approx 10^{-2}$  is obtained. With this mechanism at work, off-diagonal chargino and gluino contributions to flavor changing processes are naturally of the same order. The  $\alpha_s/\alpha_w$  enhancement of the gluino contribution with respect to the chargino one is compensated by the  $m_b/m_t$  suppression of the LR off-diagonal entries. Clearly a complete analysis of the regions of uMSSM parameter space where the  $C_{7,8}$  cancellation takes place is an important task, necessary for studying the details of this scenario. However, a detailed analysis is beyond the scope of this paper will be discussed elsewhere [39]. It is worth mentioning that in preliminary scans we checked that it is not difficult to find a candidate set of parameters where  $C_{7,8}$  numerically yield small contributions to the  $b \rightarrow s\gamma$  branching ratio. Of course, this set is not obviously not expected to be unique, and further checking that any such parameter sets are consistent with all the other existing measurements of FCNC and CP-violating observables will impose further strong constraints.

Finally, we stress that in the following analysis we do not make any specific assumptions as to the size of the off-diagonal entries of the down-squark mass matrix. In particular, we are not using any of the relations described in eqs.(4-8). The previous arguments have been intended as a theoretical framework for the following model independent analysis. A general discussion of the CP-violating sector, using the factorization ansatz of eq.(4), will be the subject of a forthcoming paper [39].

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<sup>7</sup>The main constraint on this scenario is the requirements of the  $C_7$  cancellation. The  $C_8$  contribution enters in the  $b \rightarrow s\gamma$  branching ratio at  $\mathcal{O}(\alpha_s)$  and usually cannot account for more than 10% of the measured branching ratio.

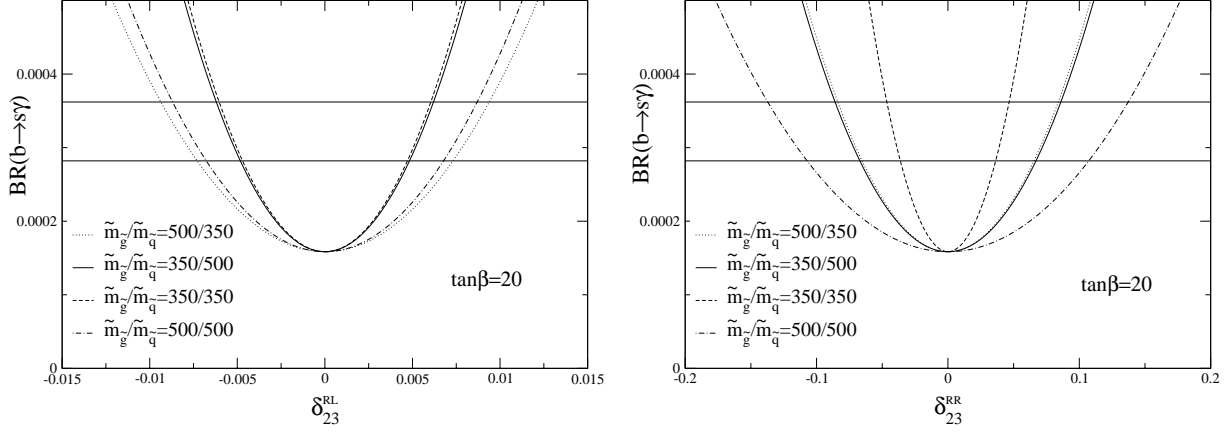


Figure 1: The dependence of  $b \rightarrow s\gamma$  branching ratio on  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  for different values of  $\tilde{m}_{\tilde{g}}/\tilde{m}_D$ , for  $\tan\beta = 20$  and  $\mu = 350$  GeV. All of the other off-diagonal entries except the one displayed in the axes, are assumed to vanish.  $C_{7,8}(\mu_W) = 0$  is assumed. The horizontal lines represent the  $1\sigma$  experimental allowed region.

#### 4.1 Single MI dominance analysis

From eqs.(25-32), one can read (in MI language) the off-diagonal entries that are relevant for the gluino contribution to the  $C_{7,8}$  and  $C'_{7,8}$  Wilson coefficients<sup>8</sup>. Note that limits on  $\delta_{23}^{LR} \approx \mathcal{O}(10^{-2})$  have previously been obtained in [13]. No stringent bound has been derived there for  $\delta_{23}^{LL}$ , as this term at lowest order does not come with the  $\tilde{m}_{\tilde{g}}/m_b$  enhancement (see eqs.(25)). No limits were derived on  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  because in the specific scenario used in [13], the “opposite chirality” MIs are suppressed by a factor  $m_s/m_b$  and so negligible. An analysis of the  $\delta_{23}^{RL}$  dependence has been performed in [28], in which the  $W$  contribution to  $C_{7,8}$  was not set to zero (sometimes also Higgs and MFV chargino contributions to  $C_{7,8}$  were included). Consequently their bounds on the down-squark off-diagonal MIs contributing to  $C'_{7,8}$  are more stringent than the bounds we derive in our scenario, for which the total contribution to  $C_{7,8}$  is assumed to be negligible. It is clearly only in the scenario we study that an *absolute* constraint on these MIs be derived. Moreover no analysis on  $\delta_{23}^{LL}$  and  $\delta_{23}^{RR}$  was performed in [28] as these contributions are not relevant in the small  $\tan\beta$  region, as can be seen from eqs.(29-32).

In Fig. 1 we show the dependence of the  $b \rightarrow s\gamma$  branching ratio on the MI terms  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  for different values of  $x_D^g = \tilde{m}_{\tilde{g}}^2/\tilde{m}_D^2$  and for  $\tan\beta = 20$  and  $\mu = 350$  GeV.

<sup>8</sup>From now on, for the sake of simplicity the symbol  $\delta_{ij}^{AB}$  will be used instead of  $(\delta_{ij}^{AB})^d$  for referring to the down-squark MIs.

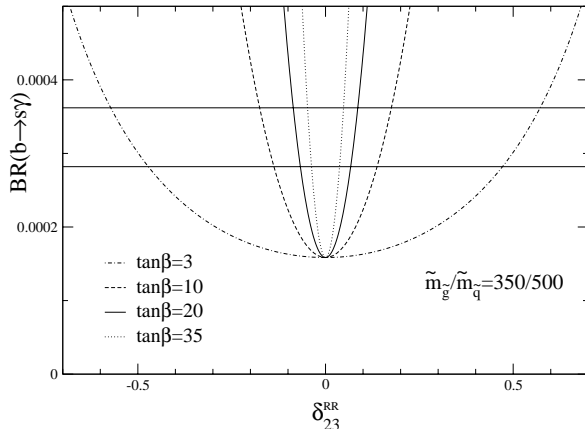


Figure 2: *Dependence of  $b \rightarrow s\gamma$  branching ratio on  $\delta_{23}^{RR}$  for different three values of  $\tan\beta$ , with the other parameters fixed to  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$  and  $\mu = 350$  GeV. All of the other off-diagonal entries, except the one displayed in the axes, are assumed to vanish. The horizontal lines represent the  $1\sigma$  experimental allowed region.*

All the other off-diagonal entries in the down-squark mass matrix are assumed to vanish for simplicity. “Individual” limits  $\delta_{23}^{RL} < 10^{-2}$  and  $\delta_{23}^{RR} < 1.5 \times 10^{-1}$  can be obtained respectively from the left and right side plot of Fig. 1. Horizontal full lines represent  $1\sigma$  deviations from the experimental results reported in eq.(1). Of course, the required cancellation of the total  $C_{7,8}$  contribution may in general need nonvanishing off-diagonal entries of the up and down squark mass matrices. However, the specific values of these entries do not significantly affect the absolute limits on  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  MIs shown in Fig. 1.

As expected from eqs.(31,32), the bounds obtained for  $\delta_{23}^{RR}$  are strongly dependent on the product  $\mu \tan\beta$ . In Fig. 2 we show the  $\tan\beta$  dependence of this limit, for fixed  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$  and  $\mu = 350$  GeV. More stringent bounds on  $\delta_{23}^{RR}$  can be obtained for larger  $\tan\beta$ . For  $\tan\beta > 35$  the bounds on  $\delta_{23}^{RR}$  can become as stringent as the  $\delta_{23}^{RL}$  bounds. Similar considerations and bounds obviously hold also for the  $\delta_{23}^{LL}$  MI. As we are only interested here in the gluino contributions to  $C'_{7,8}$ , we do not discuss this sector in detail. Clearly this term must be taken into consideration if a similar analysis was performed for the  $C_{7,8}$  coefficient in the large  $\tan\beta$  region.

In Fig. 1 and Fig. 2, we set  $C_7 = C_8 = 0$  so that the only contribution to the  $b \rightarrow s\gamma$  branching ratio is due to the gluino contribution to  $C'_7$  and  $C'_8$ . Thus one should think that for vanishing  $\delta_{23}^{RL}$  and/or  $\delta_{23}^{RR}$  the branching ratio in our scenario should vanish. The reason for the finite, nonzero contribution is the fact that we are using a NLO formula for the  $b \rightarrow s\gamma$  branching ratio [9]. At NLO, imposing the condition  $C_{7,8}(\mu_W) = 0$  still leaves constant terms that arise from the mixing of the SM operators (specifically,

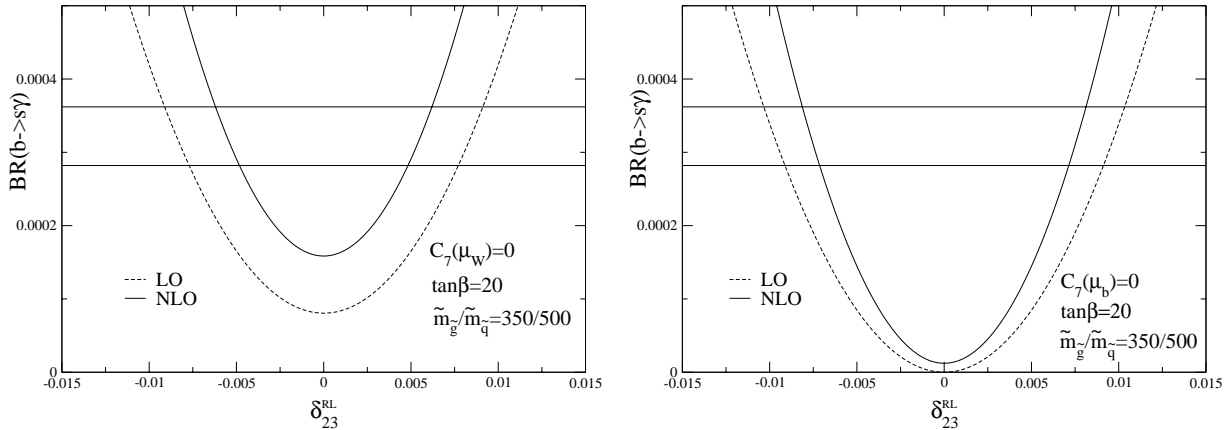


Figure 3: *Dependence of  $b \rightarrow s\gamma$  branching ratio on  $\delta_{23}^{RL}$  for  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$ ,  $\tan\beta = 20$  and  $\mu = 350$  GeV. All the other off-diagonal entries, except the one displayed in the axes, are assumed to vanish. In the plots we show the result obtained using LO (dashed line) and NLO (full line) formula for the  $b \rightarrow s\gamma$  branching ratio, setting respectively  $C_{7,8}(\mu_W) = 0$  (left plot) and  $C_{7,8}(\mu_b) = 0$  (right plot). The horizontal lines represent the  $1\sigma$  experimental allowed region.*

in our chosen basis,  $C_2$ ) that do not contribute to the branching ratio at LO. In Fig. 3 (left side), we compare the results obtained using the LO and NLO expression for the  $b \rightarrow s\gamma$  branching ratio imposing the condition  $C_{7,8}(\mu_W) = 0$ . As can be seen explicitly, the difference in using the LO or NLO is sizeable. In Fig. 3 (right side), we compare the results obtained using the LO and NLO expression for the  $b \rightarrow s\gamma$  branching ratio imposing the condition  $C_{7,8}(\mu_b) = 0$ . As one can see now, the LO contribution to the  $b \rightarrow s\gamma$  branching ratio vanishes for vanishing MIs. This does not happen for the LO contribution of the left plot, as a finite contribution to the branching ratio appears from the running  $\mu_W \rightarrow \mu_b$  when the condition  $C_{7,8}(\mu_W) = 0$  is taken. In all the plots, except Fig. 3 (right side), we use  $C_{7,8}(\mu_W) = 0$ , as this is the natural scale where cancellations could be explained in terms of the underlying fundamental theory, while the choice  $C_{7,8}(\mu_b) = 0$  seems highly accidental. Finally, it should be noted that the strongest restriction comes from imposing the condition  $C_7 = 0$ . The same requirement on  $C_8$  could easily be relaxed, and our results would remain almost unchanged. The  $C_8$  contribution to the  $b \rightarrow s\gamma$  branching ratio represents in fact only a 10% effect of the total.

It is important to notice at this point that a consistent analysis of  $b \rightarrow s\gamma$  at NLO would require the calculation of the two-loop (QCD and SQCD) contribution to the  $C'_7$  coefficient. In the general uMSSM the calculation of the  $\mathcal{O}(\alpha_s^2)$  contribution to  $C'_7$  (and obviously  $C_7$ )

is extremely complicated. In [37], the contribution to  $C_7$  from the two-loop diagrams with one gluino and one gluon internal line has been calculated. This represents the dominant MSSM two-loop contribution only in the limit of very heavy gluino mass of  $\mathcal{O}(1\text{TeV})$  and small  $\tan\beta$  ( $\approx 1$ ). Thus it cannot be applied to our analysis, in which SUSY masses (and the gluino mass in particular) below 500 GeV and large  $\tan\beta$  are assumed. In fact, if the gluino mass is light the two-loop diagrams with two gluino internal lines should also be taken into account. Moreover, if  $\tan\beta$  is large, diagrams with internal Higgsino lines cannot be neglected anymore as Yukawa couplings can become of  $\mathcal{O}(1)$ . Using the results of [37], one obtains an effect of a few percent in the  $b \rightarrow s\gamma$  branching ratio. It should be remembered, however, that in our analysis this provides only a very crude estimation. It seems reasonable to expect a possible 10% modification of the  $b \rightarrow s\gamma$  branching ratio results from the inclusion of the complete NLO calculation of the  $C_7'$  coefficient. Moreover, while the two-loop diagrams with gluino/gluon internal lines have the same MI structure and as such are proportional to the one-loop gluino contribution to  $C_7'$ , this is not the case for the diagrams with gluino/Higgsino internal lines, for which the CKM flavor changing structure also enters.

## 4.2 General MI analysis

A general analysis of the gluino contribution to  $C_{7,8}'$  depends simultaneously on both the  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  MIs. For a complete specification of our scenario the only other free parameters that need to be fixed are the ratio between the gluino mass and the common down-squark mass,  $\tilde{m}_{\tilde{g}}/\tilde{m}_D$ , the product  $\mu \tan\beta$ , and the relative phase between  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$ . The influence in of all the other down-sector squark matrix off-diagonal entries and MSSM parameters in the  $C_{7,8}'$  sector can safely be neglected<sup>9</sup>. Thus, we can have a complete description in terms of only five free parameters of the  $b \rightarrow s\gamma$  phenomenology in our MSSM “ $C_7'$  dominated” scenario.

In Fig. 4 we show the  $1\sigma$  experimentally allowed region in the  $(\delta_{23}^{RL}, \delta_{23}^{RR})$  parameter space for a specific choice of  $\tilde{m}_{\tilde{g}}/\tilde{m}_D = 350/500$ ,  $\mu = 350$  GeV, and for three different values of  $\tan\beta = 3, 20$  and  $35$ . For  $\delta_{23}^{RL}$  or  $\delta_{23}^{RR}$  vanishing, one obtains the regions depicted in Figs. 1 and 2. Larger regions in the  $(\delta_{23}^{RL}, \delta_{23}^{RR})$  parameter space are obtained when both the MIs take nonvanishing values. It is clear no absolute limit can be derived for the two MIs simultaneously. The values  $(\delta_{23}^{RR}, \delta_{23}^{RR}) \approx (1, 0.1)$  are, for example, possible<sup>10</sup> for  $\tan\beta = 35$ . In fact, as can be seen in Fig. 4, there is always a “flat direction” where

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<sup>9</sup>Of course all of the other off-diagonal entries of the down-squark and up-squark mass matrices as well all the other flavor conserving MSSM parameters enter in our analysis, as we assume to choose them in such a way that the condition  $C_{7,8} = 0$  is satisfied. However, as previously mentioned the detailed analysis of this condition will be discussed in a following paper [39].

<sup>10</sup>One should check if, for such large MI values, charge and color breaking minima appear. Anyway as these are usually rather model depend assumptions we don't introduce here the constraints discussed, for

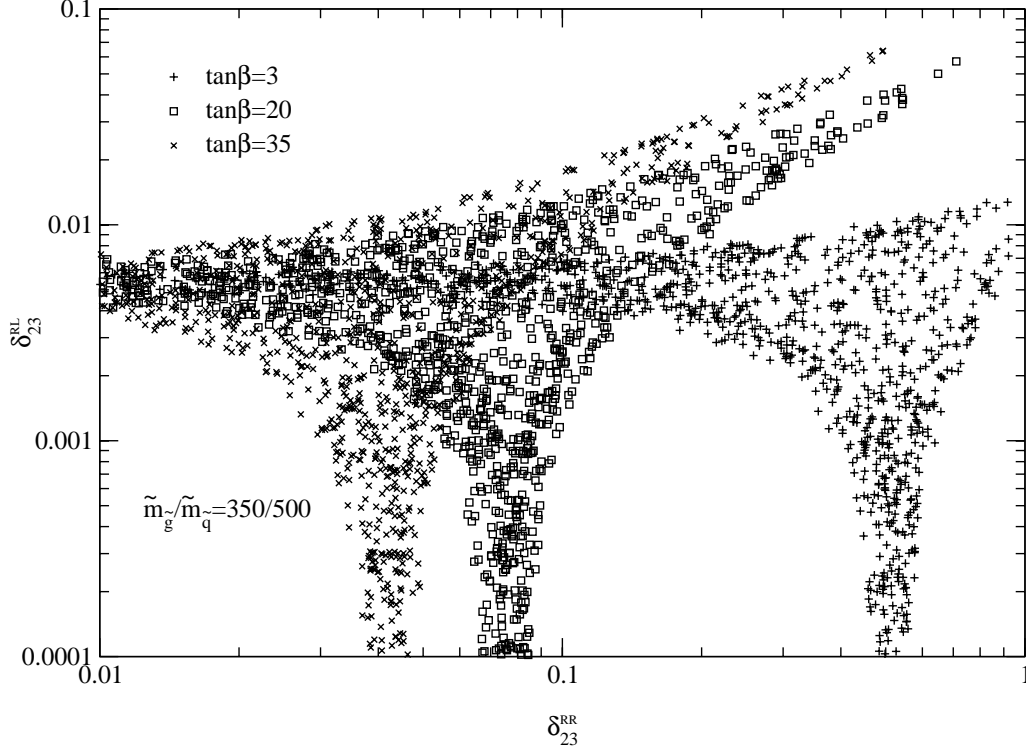


Figure 4:  $1\sigma$ -allowed region in the  $(\delta_{23}^{RR}, \delta_{23}^{RL})$  parameter space for three different values of  $\tan\beta$ , with the other parameters fixed to  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$ , and  $\mu = 350$  GeV. All the other off-diagonal entries, except the one displayed in the axes, are assumed to vanish.

large values of  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  can be tuned in such a way that the gluino contribution to  $C'_{7,8}$  is consistent with the experimental bound. This flat direction clearly depends on the chosen values for  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}}$  and  $\mu\tan\beta$ . The presence of this particular direction is explained by the fact that we are allowing complex off-diagonal entries. Hence the relative phase between  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  can be fixed in such a way that the needed amount of cancellation can be obtained between the first and second order MI contribution. In the notation used in eqs.(27,31) the line of maximal cancellation is obtained for  $\varphi = \arg[\delta_{23}^{RL} \delta_{23}^{RR}] = \pm\pi$ .

example, in [40].



### 4.3 CP asymmetry and branching ratio

In addition to the  $b \rightarrow s\gamma$  branching ratio, the experimental collaborations will provide in the following years more precise measurements of the  $b \rightarrow s\gamma$  CP asymmetry:

$$\mathcal{A}_{CP}(b \rightarrow s\gamma) = \frac{BR(b \rightarrow s\gamma) - BR(\bar{b} \rightarrow \bar{s}\gamma)}{BR(b \rightarrow s\gamma) + BR(\bar{b} \rightarrow \bar{s}\gamma)}. \quad (35)$$

The present best experimental value available [41] gives at 90% CL level the following range:

$$-0.27 < \mathcal{A}_{CP}(b \rightarrow s\gamma) < 0.10, \quad (36)$$

which is still too imprecise for to provide useful tests for NP, although the measurement is expected to be upgraded soon.

The only flavor-violating and CP-violating source in the SM (and MFV scenarios) is given by the CKM matrix, which results in a very small prediction for the CP asymmetry. In the SM an asymmetry approximatively of 0.5% is expected [9]. If other sources of CP violation are present, a much bigger CP asymmetry could be produced (see references [9] and [42]).

In our  $C'_7$  dominated scenario, one can derive the following approximate relation for the CP asymmetry [9], in terms of the  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  MIs:

$$\mathcal{A}_{CP}(b \rightarrow s\gamma) = -\frac{4}{9}\alpha_s(\mu_b)\frac{\text{Im}[C'_7 C'_8]^*}{|C'_7|^2} \approx k(x_D^g) \left( \frac{m_b \mu \tan \beta}{\tilde{m}_D^2} \right) |\delta_{23}^{RL} \delta_{23}^{RR}| \sin \varphi, \quad (37)$$

in which  $\varphi$  is the relative phase between  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  as previously defined. The constant of proportionality  $k(x_D^g)$  depends only on the ratio  $\tilde{m}_{\tilde{g}}/\tilde{m}_D$  through the integrals  $F_i$  and can be easily obtained from eqs.(27,31). One can immediately note from eq.(37) that if only one MI is considered, the CP asymmetry is automatically zero. Moreover, a nonvanishing phase in the off-diagonal down-squark mass matrix is necessary<sup>11</sup>. No sensitive bounds on this phase can be extracted from EDM's in a general flavor violating scenario.

In Fig. 5, we show the results obtained for the branching ratio and CP asymmetry in which  $\delta_{23}^{RL}$ ,  $\delta_{23}^{RR}$  and the relative phase  $\varphi$  are varied arbitrarily for a fixed value  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$  and  $\tan \beta = 35$ . The full vertical lines represents the  $1\sigma$  region experimentally allowed by the  $b \rightarrow s\gamma$  branching ratio measurements. It is possible, using  $C'_{7,8}$  alone, to saturate the  $b \rightarrow s\gamma$  measured branching ratio and at the same time have a CP asymmetry even larger than  $\pm 10\%$ , the sign of the asymmetry being determined by the sign of  $\sin \varphi$ . As Fig. 5 shows, in the relevant branching ratio range the CP asymmetry range is constant. No strong dependence from  $\tan \beta$ , in the large  $\tan \beta$  region, is present. The points with

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<sup>11</sup>Recall that for avoiding EDM constraints reparameterization invariant combinations of flavor-independent phases (such as the phase of  $\mu$  in a particular basis) are taken to be zero.

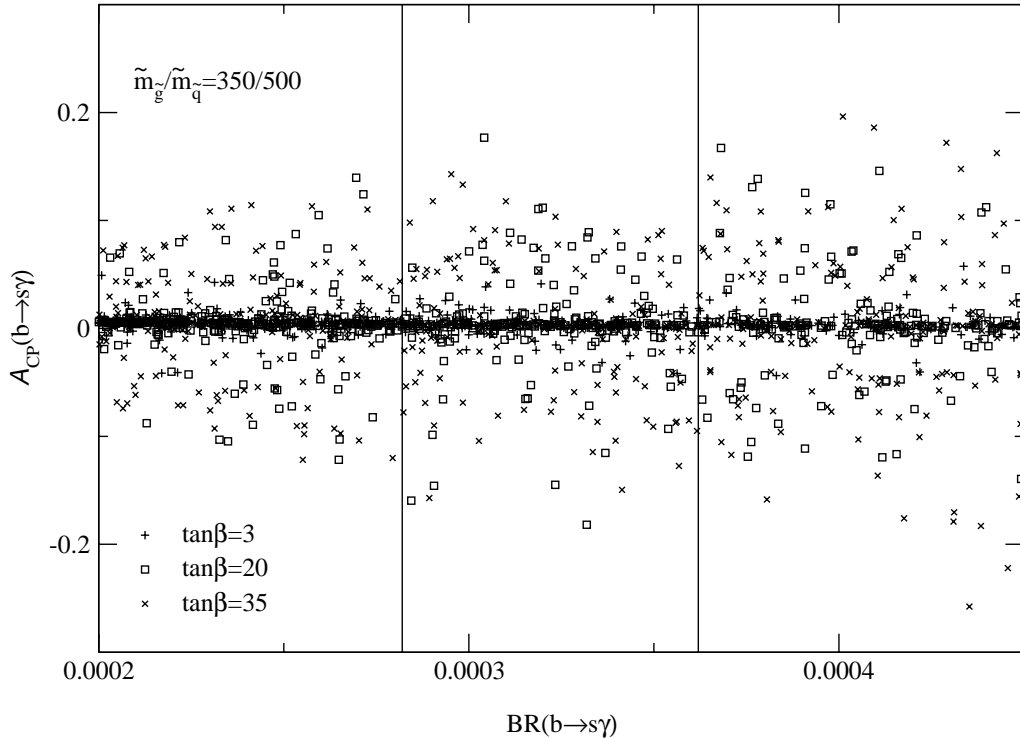


Figure 5: *Asymmetry vs branching ratio for three different values of  $\tan\beta$ , with  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$ , and  $\mu = 350$  GeV. All the off-diagonal entries except  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$  are assumed to vanish. The vertical lines represent the  $1\sigma$  experimental allowed region.*

large asymmetry ( $> 5\%$ ) lie in the “flat direction” observed in Fig.4 and they have almost  $\varphi \approx \pm\pi$  (obviously for  $\varphi = \pm\pi$  the CP asymmetry vanishes). The explanation of this fact is the following. The numerator is proportional to  $\sin\varphi$  and so goes to 0 as  $\varphi$  approaches  $\pm\pi$ . However, at the same time it is enhanced for large MI values. This happens when the flat direction condition is (almost) satisfied. Here, in fact, a cancellation between the two (large) MI terms takes place, providing the enhancement of the CP asymmetry as the denominator remains practically constant, fixed by the allowed experimental measurement on the branching ratio. Note also that for parameter values outside the flat direction condition a CP asymmetry of a few % can still be observed, about ten times bigger than the SM prediction. The same order of magnitude can be observed in MFV when large  $\tan\beta$  effects are taken into account [24]. In our scenario even smaller values of the CP asymmetry can be obtained, e.g. if one of the two off-diagonal entries is negligible, or the

two MIs are “aligned”.

#### 4.4 Distinguishing the “ $C'_7$ dominated” scenario from the “ $C_7$ dominated”

A possible method for disentangling the relative contributions to the  $b \rightarrow s\gamma$  branching ratio from the  $Q_7$  and  $Q'_7$  operators utilizes an analysis of the photon polarization. A detailed analysis of how it is possible to extract information from the photon polarization in radiative B decays is given in [43]. For simplicity, let us define the following “theoretical” LR asymmetry at LO:

$$\mathcal{A}_{LR}(b \rightarrow s\gamma) = \frac{BR(b \rightarrow s\gamma_L) - BR(b \rightarrow s\gamma_R)}{BR(b \rightarrow s\gamma_L) + BR(b \rightarrow s\gamma_R)} = \frac{|C_7(\mu_b)|^2 - |C'_7(\mu_b)|^2}{|C_7(\mu_b)|^2 + |C'_7(\mu_b)|^2}, \quad (38)$$

which could in principle distinguish between  $C_7$  or  $C'_7$  dominated scenarios. Here L,R is the polarization of the external photon. This quantity is related to the quark chiralities of the  $Q_7, Q'_7$  operators. Note that the photon polarization is the best possibility to gain information on the operator chirality, which gets almost lost in  $b$  and  $s$  quark hadronization into spin zero mesons (in principle if hadronization into spin one states could be isolated, perhaps some information could be obtained). Such a measurement is not yet available as only the average quantity  $BR(b \rightarrow s\gamma_L) + BR(b \rightarrow s\gamma_R)$  is reported experimentally.

In the SM case, and in general in all the MFV and mSUGRA scenarios, only the  $C_7$  coefficient gives a nonnegligible contribution to the  $b \rightarrow s\gamma$  branching ratio. Only the right-handed bottom quark (in the center of mass reference frame) can decay, producing a photon with Left polarization and  $\mathcal{A}_{LR}(b \rightarrow s\gamma) = 1$ . Small deviations from unity are possible due to subleading  $m_s/m_b$  terms and hadronization effects. In our scenario, where the  $C_7$  contribution is negligible, only left-handed bottom quarks can decay, emitting a photon with Right polarization, which in turn predict  $\mathcal{A}_{LR}(b \rightarrow s\gamma) = -1$ . In any other MSSM scenario, with nonminimal flavor violation, any LR asymmetry between 1 and  $-1$  is allowed. Consequently, a measurement of  $\mathcal{A}_{LR}(b \rightarrow s\gamma)$  different from 1 will be a clear indication of physics beyond the SM with a nonminimal flavor structure. It will be very interesting to know if (and how precisely) CLEO, BABAR, and BELLE can measure the LR asymmetry of eq.(38).

## 5 Conclusions

In this letter, we have discussed an alternative explanation of the  $b \rightarrow s\gamma$  branching ratio in the MSSM with a nonminimal flavor structure. We analyzed in particular the gluino contribution to the Wilson coefficient  $C'_7$  associated with the “wrong” chirality operator

$Q'_7$ . We show that this coefficient arises mainly from two off-diagonal entries:  $\delta_{23}^{RL}$  and  $\delta_{23}^{RR}$ . For scenarios in which where the  $C_{7,8}$  contributions to  $b \rightarrow s\gamma$  are small, (i.e. for regions in the MSSM parameter space where W, Higgs, chargino and gluino contributions to  $C_{7,8}$  tend to cancel each other)  $C'_{7,8}$  provides the dominant effect. We derived absolute bounds separately on each of these coefficients. We then described the allowed region of  $(\delta_{23}^{RL}, \delta_{23}^{RR})$  parameter space, as a function of  $\tan\beta$ . We observed that (for a fixed ratio  $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}}$  and for each chosen value of  $\mu \tan\beta$ ), there exists a “flat direction” where large (even  $\mathcal{O}(1)$ ) off-diagonal entries are allowed. Along this direction the relative phase between the two MI elements is  $\varphi \approx \pi$ . For the majority of parameter space in this scenario the CP asymmetry is less than 5%. Asymmetries as big as 20% can be obtained along the “flat directions”. Finally, we suggested a possible quantity (a LR asymmetry) that (if measured) can help to disentangle the  $C_7$  from the  $C'_7$  contribution to the  $b \rightarrow s\gamma$  branching ratio. Any  $\mathcal{A}_{LR}(b \rightarrow s\gamma) \neq 1$  would be an irrefutable proof of physics beyond the SM. In addition, in the framework of the general MSSM, it would indicate the existence of nonminimal flavor violation produced by off-diagonal entries in the down-squark mass matrix, generally related to a nonzero gluino contribution. In our “ $C'_7$  dominated” scenario, where the gluino contribution produce the only “visible” effect, we obtain in particular the extreme value  $\mathcal{A}_{LR}(b \rightarrow s\gamma) = -1$ . It would be very interesting if such a quantity could be measured. One implication of our analysis is that previous results on MSSM parameters, including constraints on the “sign of  $\mu$ ” (i.e. its phase relative to  $A_t$ ), are more model dependent than have been generally assumed.

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