How Neutrino and Charged Fermion Masses Are Connected Within Minimal Supersymmetric SO(10)

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Abstract: Massive neutrinos are a generic prediction of SO(10), and models of unification cry for supersymmetry. Since we have a rather detailed information on neutrino and charged fermion masses, the real question is: how/whether it is possible to build a SO(10) supersymmetric model, that correctly incorporates fermion masses. We show that a *simple construction* is possible in the context of a minimal theory. We concentrate on the two heaviest generations, discuss the predictions of the model, and briefly comment on open questions.

1. Yukawa Couplings at M_{GUT}

In order to avoid unacceptably big Dirac neutrino masses in SO(10) [1], one introduces $\underline{\mathbf{126}}$-plets scalars. These produce huge Majorana masses for ν^c [2], and decouple them from the light spectrum:

$$\mathcal{L} = -\mathbf{16}_i \left[Y^{(10)}_{ij} \underline{\mathbf{10}} + Y^{(126)}_{ij} \underline{\mathbf{126}} \right] \mathbf{16}_j + h.c. \tag{1.1}$$

The $\underline{\mathbf{10}}$-plet contains two Higgs doublets, that we call φ_u and φ_d, while the $\underline{\mathbf{126}}$-plet contains one singlet S (needed for ν^c), one triplet Δ (which may contribute to light neutrino

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masses [3]) and two doublets φ'_u and φ'_d (useful to make up for wrong SO(10) mass relations [4]). Indeed, with a self-explanatory notation for the Weyl fermions [5]:

$$\begin{cases} \mathbf{16}_i\ \underline{\mathbf{10}}\ \mathbf{16}_j \ni \varphi_u\ (u^c_i u_j + \nu^c_i \nu_j) + \varphi_d\ (d^c_i d_j + \mathrm{e}^c_i \mathrm{e}_j) +\ (i \leftrightarrow j); \\ \mathbf{16}_i\ \underline{\mathbf{126}}\ \mathbf{16}_j \ni \frac{1}{2}(S\ \nu^c_i \nu^c_j + \Delta\ \nu_i \nu_j) + \varphi'_u (u^c_i u_j - 3\nu^c_i \nu_j) + \varphi'_d (d^c_i d_j - 3\mathrm{e}^c_i \mathrm{e}_j) +\ (i \leftrightarrow j) \end{cases}$$

In this work, we propose a model of the Yukawa couplings, in which all the features of the minimal SO(10) theory are exploited.

2. Beyond the Great (Supersymmetric) Desert

The question of starting up model building is: what does the minimal supersymmetric standard model (MSSM) want from SO(10)? We get an answer by extrapolating the Yukawa couplings from $T = 0$ to $T = \log(M_{\mathsf{GUT}}/M_Z)/2\pi \approx 5.2$ (see appendix A for details). From figure 1, one sees that:

- For 3^{rd} family charged fermions masses: the *Hypothesis* of leading $\underline{\mathbf{10}}$−plet Yukawa coupling [6], that gives $y_t = y_b = y_\tau$ at M_{GUT} is OK.[1]
- For 2^{nd} family charged fermion masses: the *Hypothesis* of leading $\underline{\mathbf{126}}$−plet Yukawa coupling [7], that gives $y_\mu = -3 \times y_s$ at M_{GUT} is OK.

This could be an accidental fact, but is suggestive enough to take it seriously.

3. Determining Model and Parameters

Now that we defined the target, the question becomes: how to match MSSM and SO(10) Yukawa couplings? SO(10) can meet the MSSM needs (illustrated in previous figure) after the very simple identification of the MSSM Higgs fields: $H_u \approx \varphi_u$ and $H_d \approx \varphi_d + \varepsilon\ \varphi'_d$. (Of course, the orthogonal doublets should decouple from the MSSM spectrum, to maintain gauge coupling unification– namely, we need a "doublet-doublet" splitting).

This position leads us to identify the MSSM Yukawa couplings in the following manner:

$$\begin{cases} Y_u \approx Y^{(10)} \quad \textit{diagonal by definition} \\ Y_d \approx Y^{(10)} + \epsilon\ Y^{(126)} \\ Y_e \approx Y^{(10)} - 3\ \epsilon\ Y^{(126)} \end{cases} \tag{3.1}$$

Since we know the Yukawa couplings (after extrapolation at M_{GUT}), we can deduce the size of several elements of the SO(10) Yukawa matrices. The chain of deduction we follow and the numerical values we obtain at M_{GUT} are shown in this table:

y_t, y_b, y_τ	$\Rightarrow$	$Y^{(10)}_{33} \simeq 0.94 \gg \epsilon\, Y^{(126)}_{33}$
y_μ, y_s	$\Rightarrow$	$\epsilon\, Y^{(126)}_{22} \simeq 1.4 \times 10^{-2} > Y^{(10)}_{22}$
y_c	$\Rightarrow$	$Y^{(10)}_{22} \simeq 1.8 \times 10^{-3}$
V_{cb}	$\Rightarrow$	$\epsilon\, Y^{(126)}_{23} \simeq 2.7 \times 10^{-2}$

[1] We tuned the *vev* ratio $\tan\beta = \langle H_u \rangle / \langle H_d \rangle \sim 55.4$ to get this. We use 1 loop "running" and $\alpha_3 = 0.118$.

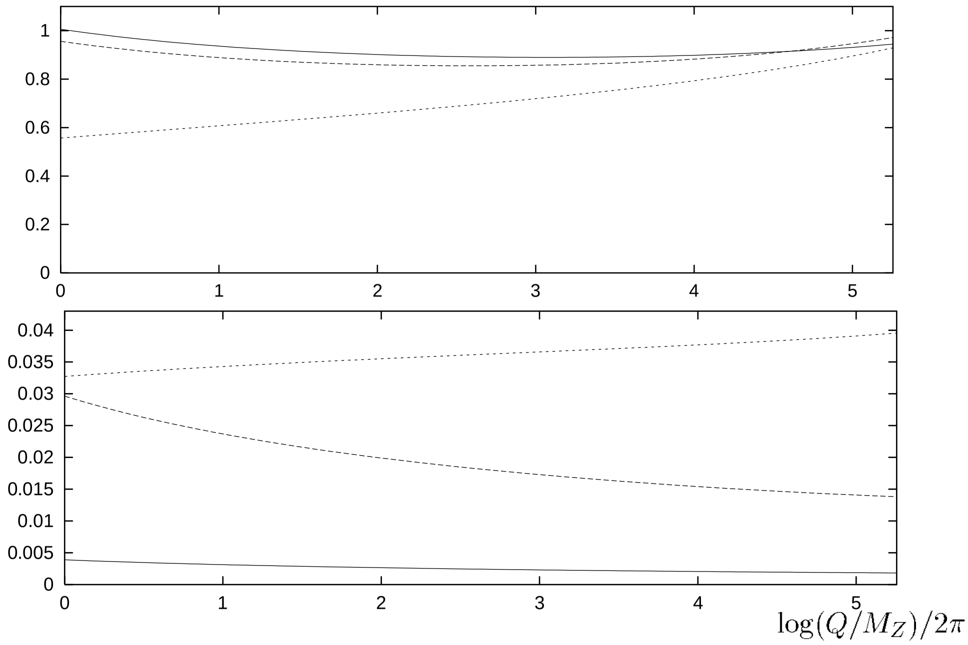

Figure 1: Upper panel: Running of MSSM Yukawa couplings of third generation from M_Z till M_{GUT} (y_t is the largest at M_Z, y_τ the smallest). Lower panel: same for second generation (y_μ is the largest, y_c is the smallest). (We denote by y_x the Yukawa coupling of the particle x, e.g.: y_t for top, y_c for charm, y_μ for muon. For a given $\tan\beta$, y_x is computed from the mass of x at $T=0$.)

Two remarks are in order:

- We kept the deduction as simple as possible *e.g.* we did not perform detailed diagonalizations to get these numbers, which saves us from considering their phases. (However, we feel that it is fair to say that higher order effects, threshold and non-log corrections *etc.* could make a much more accurate treatment meaningless.)
- The only unknown element of the $2^{nd}-3^{rd}$ family blocks is $\epsilon\, Y_{33}^{(126)}$ (though one may reasonably guess that it is not too far from $\epsilon\, Y_{22}^{(126)}$ or $\epsilon\, Y_{23}^{(126)}$).

Till here, we showed that the model is not contradicting known things...

4. Neutrino Features

Now we come to the fermion of the day: the neutrino. In order to formulate our proposal, we will base our discussion on this provocative question: what do these neutrinos want? We recapitulate the experimental situation by means of the following table:

Δm^2_{31}	$[1.5, 5]\times 10^{-3}$ eV^2	atmospheric neutrinos
Δm^2_{21}	$[2, 50]\times 10^{-5}$ eV^2	solar LMA (or $< 2\times 10^{-7}$ eV^2)
θ_{23}	$[35°, 55°]$	atmospheric neutrinos
θ_{13}	$< 10°$	CHOOZ+atm.+K2K (depends on Δm^2_{31})
θ_{12}	$[25°, 43°]$	solar neutrinos (99 % CL)

We will be mostly concerned with the first three items. As remarked by several people (see *e.g.* [8]) a neutrino mass matrix with a "dominant block" is strongly suggested:

$$\frac{\mathbf{M}_\nu}{\sqrt{\Delta m^2_{31}}} = \frac{1}{2} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathcal{O}\left(\theta_{13},\ \theta_{23} - \frac{\pi}{4},\ \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\right)$$

But, due to hierarchical Yukawa couplings, the seesaw does not yield this pattern *generically* (however, see also [10]). Often, small values of θ_{23} are found, as pointed out in [4, 9] and as illustrated here:

$$M_D M_R^{-1} M_D = \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ b & c \end{pmatrix} \cdot \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix}$$

Thus, we are lead to try another mass mechanism, and we welcome the fact that we have the triplet Δ at our disposal [3] (by the way, we arrived at a common sense answer to the question on "neutrino wishes": neutrinos want to be different from the other fermions).

5. The Triplet Option

We are assuming that neutrinos take mass *mostly* from the triplet Δ : $\mathbf{M}_\nu \propto Y^{(126)}$. Running back to M_Z the MSSM Yukawa couplings, we get a simple expression for the $\nu_\mu - \nu_\tau$ block of the neutrino mass matrix:

$$\mathbf{M}_\nu \propto \begin{pmatrix} 1 & 1.7 \\ 1.7 & x \end{pmatrix} \tag{5.1}$$

(We have "x", for $Y^{(126)}_{33}$ is unknown, and also because seesaw *might* contribute to 33-entry—see *e.g.* [11]). Clearly, eq. (5.1) can underlie a "dominant block", thus:

θ_{23} can be large

we expect a weak mass hierarchy (not $m_3 \gg m_2$)

These two properties correlate, as can be seen in figure 2. To further illustrate this result (assuming $m_3^2 \simeq \Delta m^2_{31} = 3 \times 10^{-3}$ eV^2 and $m_2^2 \simeq \Delta m^2_{21}$) we note that:

- If $\theta_{23} = 45°$, then $\Delta m^2_{21} > 2 \times 10^{-4}$ eV^2;
- If $\Delta m^2_{21} = 5 \times 10^{-5}$ eV^2, then[2] $\theta_{23} < 40°$.

We conclude that the minimal SO(10) model for Yukawa coupling we propose *is* predictive, despite (thanks to?) its simplicity.

[2] Quite tough to test experimentally, since it is equivalent to $\sin^2 2\theta_{23} < 0.97...$

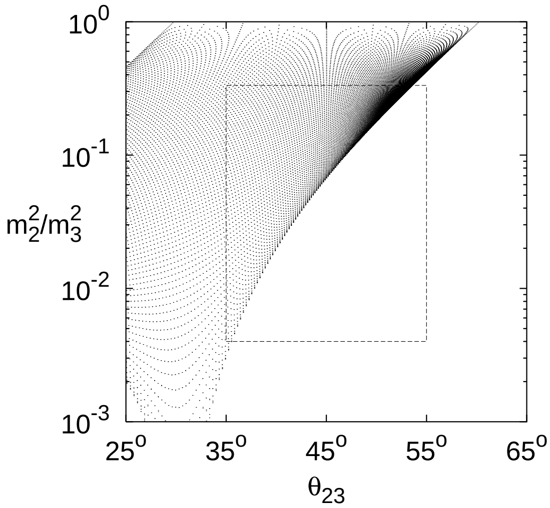

Figure 2: Possible values of the mass hierarchy parameter m_2^2/m_3^2 and of the atmospheric mixing angle θ_{23}, obtained varying the complex input parameter x (eq. 5.1). A rectangle encloses the range of permitted values, estimated assuming that the lightest neutrino mass m_1 is negligible.

6. Summary and Discussion

$\star$ We discussed an "economical embedding" of MSSM into SO(10), in a sense that all features of **126**-plet have been exploited, namely: we use singlet, doublets *and* triplet *vev*'s.

$\star$ The most important step in the construction: how the masses of the charged fermions of the 2^{nd} and 3^{rd} generations are explained (Sects. 2 and 3). 3^{rd} family unification suggests the large $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ regime; this is not an appealing case, but perhaps it is still viable (incidentally, it permits us to accommodate a "heavy" Higgs field, $m_h < 135$ GeV).

$\star$ The *triplet* mechanism for neutrino mass generation is at least likely (discussion in Sect. 4). The correlations among $(\mathbf{M}_\nu)_{22} \leftrightarrow m_\mu, m_s$, and $(\mathbf{M}_\nu)_{23} \leftrightarrow V_{cb}$ imply (eq. (5.1)):

$$\theta_{23} \in [\ 35^\circ\ ,\ 55^\circ\ ] \Leftrightarrow \frac{m_2^2}{m_3^2} \in \left[\ \frac{1}{250}\ ,\ \frac{1}{3}\ \right]$$

Solar ν solutions with big hierarchy are disfavored, while LMA fits well the scheme. After the Δm^2_{21} measurement–at KamLAND?–we will get an upper bound on θ_{23} (fig. 2 and Sect. 5).

$\star$ A pending question is: masses of 1^{st} family fermions (also m_1); proton decay rate; feasibility of baryogenesis-through-leptogenesis mechanism. These features are strictly tied among them, and require further study.

To conclude, we stress the main goals achieved: We showed that it is possible to build a simple model for fermion masses based on supersymmetric SO(10), with renormalizable couplings only. This model accounts for the masses of second and third generation fermions. It has large θ_{23}, and prefers the solar neutrino solutions with weak mass hierarchy.

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A. 1 loop renormalization group equations

We assume supersymmetry, in order to comply with one-step unification of gauge couplings. The renormalization group equations relevant to our analysis are:

$$\begin{cases} \alpha_t' = \alpha_t[6\alpha_t + \alpha_b - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1] \\ \alpha_b' = \alpha_b[6\alpha_b + \alpha_t + \alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1] \\ \alpha_\tau' = \alpha_\tau[4\alpha_\tau + 3\alpha_b - 3\alpha_2 - 3\alpha_1] \\ \\ \alpha_c' = \alpha_c[3\alpha_t - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1] \\ \alpha_s' = \alpha_s[3\alpha_b + \alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1] \\ \alpha_\mu' = \alpha_\mu[3\alpha_b + \alpha_\tau - 3\alpha_2 - 3\alpha_1] \\ \\ A' = -A[\alpha_t + \alpha_b]/2 \\ \lambda' = 0 \\ \rho' = 0 \\ \eta' = 0 \\ \\ M_{ij}' = M_{ij}[\alpha_\tau(k_i + k_j)/2 + 3\alpha_t - 3\alpha_2 - \alpha_1] \end{cases}$$

The symbol ' (=prime) denotes derivative with respect to $T = \log(Q/M_Z)/2\pi$. We define $\alpha_x = y_x^2/4\pi$ for $x = t, b, \tau, c, s, \mu$, analogously to gauge α_i's. A, λ, η, ρ are the Wolfenstein parameters. M_{ij} are the entries of neutrino mass matrix; $k_3 = 1$, and $k_2 = 0$. α_1 is normalized in standard model fashion–not SU(5)'s.

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[10] There is *not* a "no-go theorem" for seesaw mechanism in SO(10). See K.T. Mahanthappa *et al.,* `hep-ph/0110037`, talk given in the session on CP violation of HEP2001, and K. Matsuda *et al.,* `hep-ph/0108202,` appeared after the conference. However, note that they find solutions with pronounced hierarchy as LOW and QVO, that are not expected in our model.

[11] As formalized in eq. (17) of A.S. Joshipura and E.A. Paschos, `hep-ph/9906498`.