A phenomenological description of quantum-gravity-induced space-time noise

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ABSTRACT

I propose a phenomenological description of space-time foam and discuss the experimental limits that are within reach of forthcoming experiments.

"Space-time foam" is a geometric picture of the smallest size scales of the Universe, which is characterized mainly by the presence of quantum uncertainties in the measurement of distances. All quantum-gravity theories should have some kind of foam [1, 2], but the description of foam varies according to the theory. Experimental observations establishing some of the properties of space-time foam would provide a crucial hint for the search of the correct quantum gravity. I previously showed [3] that foam-induced distance fluctuations would affect gravity-wave interferometers by introducing a new source of noise, but the present level of development of candidate theories of quantum gravity does not allow [4] to derive detailed distance-fluctuation predictions to guide the work of experimentalists. Here I propose a phenomenological approach that describes directly space-time foam and this new approach naturally leads to a picture of quantum distance fluctuations that is independent of the specific setup of a given interferometer. The only unknown in the model is the length scale that sets the overall magnitude of the effect. I find that recent data [5, 6] already rule out the possibility that this length scale be identified with the "string length" ($10^{-34}m < L_s < 10^{-33}m$). Experiments that will soon start operating will probe values of the length scale even smaller than the "Planck length" ($L_p \sim 10^{-35}m$).

One of the most robust [1, 2] expectations for quantum gravity, as the theory describing the interplay between gravity and quantum mechanics, is that space-time itself at the smallest scales should manifest quantum fluctuations of geometry, which could be described in terms of a highly non-trivial structure of (3+1-dimensional) space-time. This can be roughly visualized using an analogy with ordinary "foamy" or "spongy" materials, imagining however that physical processes are confined to the material of the "sponge". Another useful intuition-building analogy can be made with the Heisenberg uncertainty principle: while that principle assigns a minimum on uncertainties relevant for the combined measurement of the position and the momentum of a particle in a fixed (background) space-time, we now expect an uncertainty principle for space-time itself. This would set an absolute limit on the measurability of distances.

It is natural [3, 4, 7, 8, 9] to characterize operatively this space-time foam through its implications for an ideal interferometer. Quantum fluctuations of distances would be observed in an interferometer as a source of noise. Theoretical predictions for this noise could be tested by comparing them with the noise levels actually found experimentally. In particular, a given picture of foam-induced distance fluctuations is of course ruled out if it predicts more noise than the total noise seen experimentally.

The first studies on this subject [3, 4, 7, 8, 9] indicated in various ways that the sensitivity of modern interferometers could be sufficient for the detection of space-time fluctuations originating at Planckian distance scales. Of course, in order to provide guidance to the experimentalists involved in interferometric tests, it would be useful to have a detailed description of the fluctuations induced by space-time foam. Unfortunately, the scarcity of experimental information on the quantum-gravity realm has not yet allowed a proper "selection process", so there are a large number of quantum-gravity candidates. Moreover, even the two approaches whose mathematical/logical consistency has been already explored in some depth, the one based on "critical superstrings" [10, 11] and the one based on "canonical/loop quantum gravity" [12, 13, 14], have not yet matured a satisfactory understanding of their physical implications, such as the properties of space-time foam. In the few phenomenological programmes investigating other quantum properties of space-time [15, 16, 17, 18, 19] the difficulties deriving from the preliminary status of quantum-gravity theories have been circumvented by developing direct phenomenological descriptions of the relevant phenomena. I propose to apply the same strategy to the description of the noise induced in interferometers by quantum gravity.

My task is partly facilitated by the fact that in order to guide interferometric studies of foam it is only necessary to estimate a relatively simple (single-variable) function: the power spectrum $\rho_h(f)$ of the strain noise [20, 21]. [Strain here has the standard engineering definition $h \equiv \Delta L/L$ in terms of the displacement ΔL in a given distance L.] In fact, the strain noise power spectrum, through its dependence on the frequency f at which observations are performed, contains the most significant information on the distance fluctuations, such as the mean square deviation (which is given by the integral of the power spectrum over the bandwidth of operation of the detector), and is the quantity against which the observations are compared.

The quantum-gravity-induced strain noise should depend only on the Planck length, the speed-of-light constant c ($c \simeq 3{\cdot}10^8 m/s$), and, perhaps, a length scale characterizing the properties of the apparatus with respect to quantum gravity. I observe that within this conceptual framework there is a unique compellingly-simple candidate for a foam-induced "white noise" (noise with constant, f-independent, power spectrum). White noise is to be expected whenever the relevant stochastic phenomena are such that there is no correlation between one fluctuation and the next, an hypothesis which appears rather plausible for the case of space-time fluctuations. The hypothesis that foam-induced noise be white is also consistent with the the intuition emerging from analogies [22] between thermal environments and the environment provided by foam as a (dynamical) arena for physical processes. According to these studies one can see foam-induced noise as essentially analogous to thermal noise in various physical contexts (such as electric circuits, where noise is generated by the thermal agitation of the electrons), which is indeed white whenever the bandwidth of interest is below some characteristic (resonant) frequency. In the case of foam-induced noise the characteristic frequency (which should be somewhere in the neighborhood of the quantum-gravity frequency scale c/L_p) would be much higher than the frequencies of operation of our interferometers, and foam noise would be white at those frequencies.

Within a white-noise model, by observing that the strain noise power spectrum carries dimensions of Hz^{-1}, one is naturally led to the estimate

$$\rho_h(f) = \text{constant} \sim \frac{L_p}{c} \sim 5{\cdot}10^{-44} Hz^{-1} \ . \tag{1}$$

I also observe that, since, as mentioned, the frequencies we can access experimentally are much smaller than c/L_p, white noise is actually the only admissable structure for foam-induced strain noise within the hypothesis that this noise be independent of the characteristics of the apparatus which is used as a space-time probe. In fact this hypothesis implies that ρ_h can only depend on its argument f, on the Planck length and on the speed-of-light constant, and therefore the most general low-frequency expansion is of the type

$$\rho_h(f) = a_0 \frac{L_p}{c} + a_1 \left(\frac{L_p}{c}\right)^2 f + a_2 \left(\frac{L_p}{c}\right)^3 f^2 + ... \qquad (2)$$

where the a_i are numerical coefficients and all monomials of the type $f^{-|n|}$ were not included in the expansion because they would require coefficients of the type $L_p^{-|n|+1}$ (which would be inconsistent with the fact that quantum-gravity effects must disappear in the limit $L_p \to 0$). For $f \ll c/L_p$ the expansion (2) is well approximated by its first term, which corresponds to the dimensional estimate (1). From the point of view of experimental tests it is also important to consider the value of the coefficient a_0, *i.e.* to take into account the inherent uncertainty associated with the dimensional estimate (1). In this type of studies based on dimensional analysis, the natural guess, which often turns out to be correct, is that a_0 is of order 1, but it is not uncommon to find a disagreement between the dimensional estimate and the experimental result of a few orders of magnitude. In testing (1) we shall therefore be looking for sensitivities extending a few orders of magnitude below the L_p/c level.

In the same sense that the estimate (1) provides a compelling candidate for foam-induced noise in quantum-gravity theories with ordinary point-like (particle) fundamental objects, in theories with extended (*e.g.* string-like) fundamental objects characterized by a length scale L_s it appears natural to consider the low-frequency estimate

$$\rho_h \sim \frac{L_s}{c} \; . \qquad (3)$$

In string theories L_s would be the string length, which is expected to be somewhere between a factor 10 and a factor 100 larger than the Planck length, and therefore for L_s/c there is a range of values $5 \cdot 10^{-43} Hz^{-1} < L_s/c < 5 \cdot 10^{-42} Hz^{-1}$.

Since they predict no dependence on the nature of the apparatus being used to probe space-time, these estimates (1) and (3) can be tested using any detector with sensitivity to distance strain, such as interferometers and resonant-bar detectors. Remarkably, in spite of the smallness of the effects predicted, these types of experiments are reaching such a high level of sensitivity that (1) and (3) are going to be completely tested (either discovered or ruled out) within a few years.

Denoting with ρ_h^{TOT} the total strain noise power spectrum observed by the experiments, the present level of interferometric data is best characterized by the results obtained by the *40-meter interferometer* [5] at Caltech and the *TAMA interferometer* [6] at the Mitaka campus of the Japanese National Astronomical Observatory, both reaching ρ_h^{TOT} of order $10^{-40} Hz^{-1}$ (the lowest level has been achieved by *TAMA* around $1kHz$: $\rho_h^{TOT} \sim 3 \cdot 10^{-41} Hz^{-1}$). Even more remarkable is the present sensitivity $\rho_h^{TOT} \simeq 5 \cdot 10^{-43} Hz^{-1}$ of resonant-bar detectors such as NAUTILUS [23] (which achieved it near $924Hz$). This is already quite close to the natural quantum-gravity estimate L_p/c of (1), and is already at the level L_s/c. We are already probing a potentially interesting region and in order to complete a satisfactory test of the estimates (1) and (3) we only need to improve the sensitivity by a few orders of magnitude (in order to exclude also the possibility that the coefficient a_0 be somewhat smaller than 1).

This will be accomplished in the near future. Planned upgrades of the NAUTILUS resonant-bar detector are expected [23, 24] to reach sensitivity at the level $7 \cdot 10^{-45} Hz^{-1}$. The LIGO/VIRGO generation of interferometers [25, 26] should achieve sensitivity of the order of $10^{-44} Hz^{-1}$ within a year or two, during its first phase of operation. A few years later, with the space interferometer LISA [27] and especially with the "advanced phase" [24, 25] of the LIGO/VIRGO interferometers, another significant sensitivity improvement should be achieved: according to recent estimates [25] it should be possible to reach sensitivity levels in the neighborhood of $10^{-48} Hz^{-1}$, more than four orders of magnitude below the natural L_p/c estimate here considered!

This expected experimental progress is described in the figure together with the L_p/c white-noise level and the analogous noise-level predictions that can be obtained by assuming instead that the foam-induced noise be of "random-walk" type (*i.e.* with f^{-2} frequency dependence of the power spectrum [21]). Through the example of random-walk noise the figure shows that the sensitivity of modern interferometers is significant also with respect to non-white models of foam-induced noise. This is an important consideration in assessing the overall significance of the interferometric studies here considered; in fact, the quantum-gravity realm is very far from the experimental contexts that formed our intuition, and, while the simple L_p-linear white-noise model may appear natural at present, it is reassuring that this experimental programme can explore a rather wide class of noise models.

The example of random-walk noise can also be used to illustrate what would be the implications of having noise that, unlike L_p-linear white noise, necessarily depends on some experiment-characteristic length scale Λ. A model with random-walk strain noise linearly suppressed by the Planck length would have to predict a power spectrum of the form $\rho_h \sim c L_p f^{-2} \Lambda^{-2}$. Our capability to test such a model is to be described with the range of values of Λ which we can exclude. As shown in the figure, for the L_p-linear random-walk-noise model the excluded range of values of Λ extends all the way up to values of Λ of the order of the optical length of the arms of the interferometer. In the random-walk case we will soon even reach some sensitivity to models with effects quadratically suppressed by the Planck length; in fact, as shown in the figure, the LISA interferometer [27] will be able to test the possibility of noise levels of the type $\rho_h \sim c L_p^2 f^{-2} \Lambda^{-3}$ for plausible values of the experiment-characteristic length scale Λ. Since other quantum-gravity-motivated experimental programmes can only achieve sensitivity to effects linear in the Planck length [9, 15, 16, 17, 18, 19], LISA's capability to reach "L_p^2 sensitivity" will mark the beginning of another significant phase in the search of quantum properties of space-time.

In order to render more powerful the phenomenological approach here proposed the most urgent challenge to theory concerns the understanding of the role of energy considerations in quantum gravity. If one applied a similar phenomenological approach to the analysis of other interferometric noise sources, it would be easy to find ways to discriminate between different noise models by evaluating the amount of energy required by the corresponding fluctuation schemes [24]. Unfortunately, while energy considerations are rather elementary when the analysis is supported by a fixed background space-time, the fact that quantum gravity cannot rely on a background space-time renders energy considerations much more subtle [28]. The role that this issue could play in the development of the approach here proposed will be discussed in detail elsewhere [29].

On the experiment side, my analysis of quantum-gravity noise provides additional motivation for the studies planned by LISA and by the "advanced phase" of the LIGO/VIRGO interferometers. The original classical-gravity objective of modern interferometers, the discovery of Einstein's gravity waves, might well be achieved already by the "first phase" of LIGO/VIRGO, but even in that case it appears to be necessary to maintain the present ambitious sensitivity objectives for LISA and the LIGO/VIRGO "advanced phase". The payoff could be the first experimental evidence of a quantum property of space-time.

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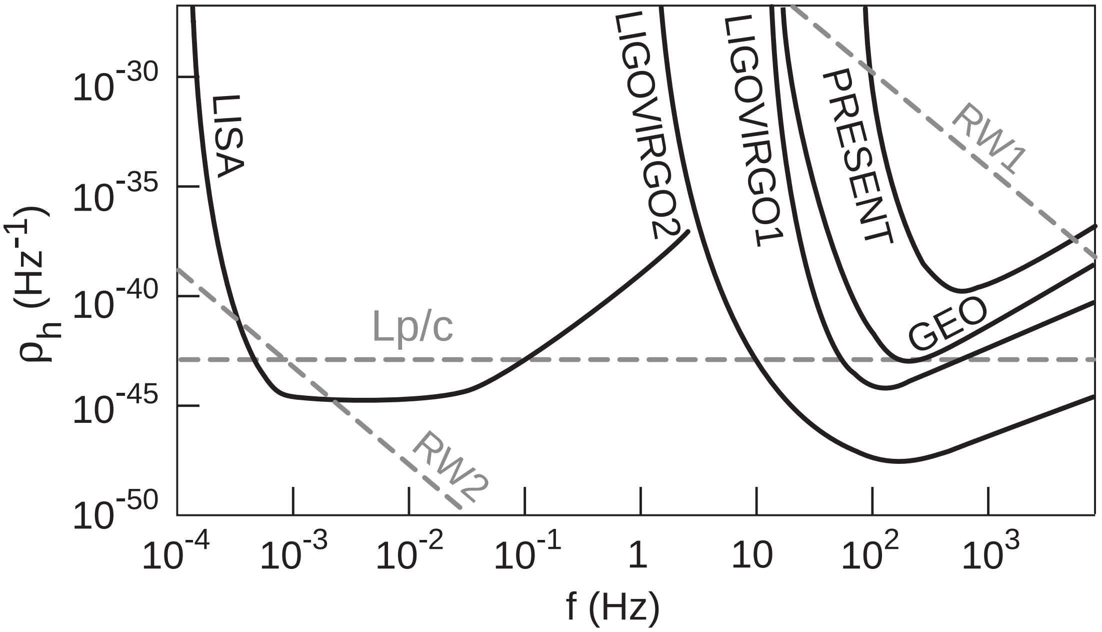

Figure 1: A qualitative (at best semi-quantitative) comparison between the sensitivity of certain interferometers and the types of strain noise power spectra here considered. The evolution from the level of sensitivity ("PRESENT") of interferometers already in operation, to GEO ("GEO") and the first phase of the LIGO and VIRGO interferometers ("LIGOVIRGO1"), and finally to LISA ("LISA") and the second phase of LIGO and VIRGO ("LIGOVIRGO2"), will take us through some significant phenomenological milestones among candidate foam-induced noise levels. The white-noise line "L_p/c" will be crossed already by the first phase of LIGO and VIRGO. The line "RW1" is representative of the random-walk scenario with magnitude suppressed linearly by the Planck length, and, as mentioned, is ruled out by "PRESENT" data. The figure also shows that with LISA we will even start probing a small range of values of the overall coefficient c/Λ^3 (where Λ should be a scale characteristic of the experimental setup) of the scenario with random-walk noise levels suppressed by the square of the Planck length. In fact, the line "RW2" corresponds to $\rho_h \sim c L_p^2/(\lambda^3 f^2)$, where λ is the wavelength of the LISA beam.