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USE OF Λ_b POLARIMETRY IN TOP QUARK SPIN-CORRELATION FUNCTIONS

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Abstract

Due to the absence of hadronization effects and the large m_t mass, top quark decay will be uniquely sensitive to fundamental electroweak physics at the Tevatron, at the LHC, and at a future linear collider. A "complete measurement" of the four helicity amplitudes in $t \rightarrow W^+ b$ decay is possible by the combined use of Λ_b and W polarimetry in stage-two spin-correlation functions (S2SC). In this paper, the most general Lorentz-invariant decay density matrix is obtained for the decay sequence $t \rightarrow W^+ b$ where $b \rightarrow l^- \bar{\nu} c$ and $W^+ \rightarrow l^+ \nu_l$ [or $W^+ \rightarrow j_{\bar{d}} j_u$], and likewise for $\bar{t} \rightarrow W^- \bar{b}$. These density matrices are expressed in terms of b-polarimetry helicity parameters which enable a unique determination of the relative phases among the $A(\lambda_{W^+}, \lambda_b)$ amplitudes. Thereby, S2SC distributions and single-sided b-W-interference distributions are expressed in terms of these parameters. The four b-polarimetry helicity parameters involving the $A(-1, -1/2)$ amplitude are considered in detail. Λ_b polarimetry signatures will not be suppressed in top quark analyses when final $\bar{\nu}$ angles-and-energy variables are used for $b \rightarrow l^- \bar{\nu} c$.

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1 Introduction:

Assuming only W-polarimetry information[1], in Ref.[2] we used the helicity formalism to derive stage-two spin-correlation functions for top quark decays. However, a complete determination of the decay amplitudes requires information from b-polarimetry, such as from Λ_b decays. In this paper we accordingly generalize the earlier results to enable measurement of the relative phase of b_L and b_R amplitudes by Λ_b polarimetry [3-6][7-10].

The significant point [11] is that by this technique a "complete measurement" of the four helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$ in $t \to W^+ b$ decay is possible: If only b_L coupling's existed, there would be only 2 amplitudes, so 3 quantities would determine $t \to W^+ b$: measurement of the magnitude's ratio $r_a^L \equiv \frac{|A(-1,-\frac{1}{2})|}{|A(0,-\frac{1}{2})|}$, of the L-handed relative phase β_a^L, and of the partial width Γ. But, since $m_b \neq 0$, there are 2 more amplitudes, so to achieve an "almost" complete measurement [2], 3 additional quantities must be determined: e.g. $r_a^R \equiv \frac{|A(1,\frac{1}{2})|}{|A(0,\frac{1}{2})|}$, the R-handed relative phase β_a^R, and a L-R magnitude's ratio $r_a^1 \equiv \frac{|A(1,\frac{1}{2})|}{|A(0,-\frac{1}{2})|}$. However, a further measurement is required to determine the relative phase of the b_L and b_R amplitudes, such as $\gamma_+^a \equiv \phi_1^a - \phi_o^a$ which is between the two $\lambda_b = \pm 1/2$ amplitudes with the largest moduli in the standard model (SM). See the upper sketch in Fig. 1 for definition of these relative phases [12]. By a combined use of Λ_b and W^+ polarimetry, it is possible to obtain this missing phase through a measurement of the interference between the b_L and b_R amplitudes in $t \to W^+ b$ with $b \to c l^- \bar{\nu}$ where the b-quark is required to occur in a bound state in the Λ_b mass region. For instance, the helicity parameters

$$\epsilon_+ \equiv \frac{1}{\Gamma} \left| A(1, \frac{1}{2}) \right| \left| A(0, -\frac{1}{2}) \right| \cos \gamma_+, \qquad {\epsilon_+}' \equiv \frac{1}{\Gamma} \left| A(1, \frac{1}{2}) \right| \left| A(0, -\frac{1}{2}) \right| \sin \gamma_+ \tag{1}$$

appear in the stage-two spin-correlation distributions (S2SC), such as (56,59) below, and in single-sided sequential-decay distributions, such as (62). Primed helicity parameters depend on "sine" functions of the relative phases and so can be used to test for $\tilde{T}_{FS}$-violation [2, 13].

At present, the available experimental and theoretical information regarding future application of Λ_b polarimetry is promising: The pre-measurement, heavy-quark-effective-theory predictions (HQET) [14-16] for Z^o decays were $< P_{\Lambda_b} > \approx -0.7 \pm 0.1$ with small QCD corrections. Initial LEP1 measurements in 1996-7 were significantly smaller but with large errors [ALEPH reported (-0.23 ± 0.25) [3] and DELPHI reported (-0.08 ± 0.39) [4]]. The later OPAL result [5] of $< P_{\Lambda_b} >= -0.56(+0.20/-0.13) \pm 0.09$ and the more recent DELPHI result [6] of $< P_{\Lambda_b} >= -0.49(+0.32/-0.30) \pm 0.17$ are both consistent with a large polarization as in the HQET. The OPAL measurement of the product of branching ratios is $B(b \rightarrow \Lambda_b X) B(\Lambda_b \rightarrow \Lambda X) = 2.67 \pm 0.38(+0.67/-0.60)\%$. There is no information on Λ_b decays from CESR or from the on-going B-factory CP-violation experiments because of their choice of operating at an upsilon resonance, the $\Upsilon(4S)$, at too low an E_{cm} energy so as to maximize B-meson production.

Throughout this S2SC analysis, we use the following notations: Per the $A(\lambda_{W^+}, \lambda_b)$ amplitudes, lowercase "a" or "1" subscripts denote quantities describing the **t-decay side**, whereas per the use of $B(\lambda_{W^-}, \lambda_{\bar{b}})$ amplitudes for the CP-conjugate $\bar{t} \rightarrow W^- \bar{b}$ decay, lowercase "b" or "2" subscripts similarly denote quantities for the **$\bar{t}$-decay side**. To be clear versus the notation for the fundamental $t \rightarrow W^+ b$ process, we will use "barred" accents for helicity parameters and other quantities describing the CP-conjugate sequential-decay $\bar{t} \rightarrow W^- \bar{b} \rightarrow (l^- \bar{\nu}_l)(l^+ \nu \bar{c})$, see Fig. 2 and (19-32). For the second-stage of $t \rightarrow W^+ b \rightarrow (l^+ \nu_l)(l^- \bar{\nu} c)$, we use **tilde accents** to denote angles for $W^+ \rightarrow l^+ \nu_l$ [or $W^+ \rightarrow j_{\bar{d}} j_u$] and for density matrices **depending only** on W-polarimetry. We use **hat accents** to denote angles for $b \rightarrow l^- \bar{\nu} c$ and for density matrices **depending** on b-polarimetry. For the second-stage of the CP-conjugate sequence we correspondingly use these "tilde" and "hat" accents. Invariance under any fundamental discrete symmetry such as CP, T, or CPT is not assumed in this analysis nor in the earlier papers because the framework is the helicity formalism. A brief introduction to this accent labeling can be obtained by inspection of the figures. We also use a **slash notation** for the "full" or double-sequential-decay density matrices which use both b-quark and W-polarimetry, e.g. (30).

Below, in Sec. 2, we construct the most general Lorentz-invariant decay density matrix for the decay sequence $t \rightarrow W^+ b$ where $b \rightarrow l^- \bar{\nu} c$ and also $W^+ \rightarrow l^+ \nu_l$ [or $W^+ \rightarrow j_{\bar{d}} j_u$]. The corresponding quantities are also obtained for the CP-conjugate sequential-decay. For it and associated with Fig. 2, we list explicitly the "barred" formulas for its helicity parameters and relative phases. Simple CP tests were treated in [2]. In Sec. 3, we then generalize the derivation of the earlier stage-two spin-correlation functions in the case of both W-polarimetry and b-polarimetry. We similarly generalize the single-sided sequential-decay distributions.

In Sec. 4, the b-polarimetry helicity parameters ϵ_-, κ_1 involving the L-handed b-quark amplitude $A(-1, -1/2)$ are considered in detail regarding tests for single-additional Lorentz-invariant couplings. We also consider the analogous "primed parameters" regarding signatures for $\tilde{T}_{FS}$-violation. Those involving $A(0, -1/2)$ were treated in [12, 13]. In the SM and at the $(S + P)$ and $(f_M + f_E)$ ambiguous moduli points, the values of the four b-polarimetry interference parameters $\epsilon_\pm, \kappa_{o,1}$ are small$\sim$ 1%. However at low-effective mass scales ($< 320 GeV$), the values of these parameters can be large, 0.1 to 0.4 versus their unitarity limit of 0.5, for single-additional Lorentz structures having sizable R-handed b-quark amplitudes, c.f. Figs. 9-10 below and Figs. 8-9 in [12].

In the near future, Λ_b polarimetry could be used in top quark spin-correlation analyses at the

Tevatron, at the LHC, and at a future linear collider [18]. If the heavy-quark-effective theory prediction is correct, then depending on the dynamics occurring in top quark decay and on good detector/accelerator polarimetry capabilities, we think that Λ_b polarimetry could be a very important technique in studying fundamental electroweak physics through top quark decay processes.

2 Use of Λ_b Polarimetry in Sequential-Decay Density Matrices

In order to include b-polarimetry, we generalize the derivation of state-two-spin-correlation functions given in [2].

In the t rest frame, the matrix element for $t \to W^+ b$ is

$$\langle \theta_1^t, \phi_1^t, \lambda_{W^+}, \lambda_b | \frac{1}{2}, \lambda_1 \rangle = D^{(1/2)*}_{\lambda_1, \mu}(\phi_1^t, \theta_1^t, 0) A\left(\lambda_{W^+}, \lambda_b\right) \tag{2}$$

where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the W^+ and b-quark helicities. The asterisk denotes complex conjugation. The final W^+ momentum is in the θ_1^t, ϕ_1^t direction and the b-quark momentum is in the opposite direction. λ_1 gives the t-quark's spin component quantized along the z_1^t axis in Fig. 3. So, upon a boost back to the $(t\bar{t})$ center-of-mass frame, or on to the $\bar{t}$ rest frame, λ_1 also specifies the helicity of the t-quark. For the CP-conjugate process, $\bar{t} \to W^- \bar{b}$, in the $\bar{t}$ rest frame

$$\langle \theta_2^t, \phi_2^t, \lambda_{W^-}, \lambda_{\bar{b}} | \frac{1}{2}, \lambda_2 \rangle = D^{(1/2)*}_{\lambda_2, \bar{\mu}}(\phi_2^t, \theta_2^t, 0) B\left(\lambda_{W^-}, \lambda_{\bar{b}}\right) \tag{3}$$

with $\bar{\mu} = \lambda_{W^-} - \lambda_{\bar{b}}$ and by the analogous argument λ_2 is the $\bar{t}$ helicity.

To use Λ_b-polarimetry in the S2SC functions, we consider the decay sequence $t \to W^+ b$ followed by $b \to l^- \bar{\nu} X$. In Figs. 3 and 4, the spherical angles $\widehat{\theta_1^t}$ and $\widehat{\phi_1^t}$ describe the b momentum in the "first stage" $t \to W^+ b$. Note that $\widehat{\theta^t_1} = \pi - \theta^t_1$. Note that $\hat{\phi}$ is the important opening-angle between the t-quark and $\bar{t}$-quark decay planes, and $\widehat{\theta^t_2} = \pi - \theta^t_2$.

In Fig. 4, the angles $\widehat{\theta_a}$ and $\widehat{\phi_a}$ describe the l^- momentum in the "second stage" $b \to l^- \bar{\nu} X$ when the boost is from the t_1 rest frame. In Fig. 5 the spherical angles $\widehat{\theta}_b$, $\widehat{\phi_b}$ similarly specify the l^+ momentum in the $\bar{b}$ rest frame when the boost is from the $\bar{t}_2$ rest frame.

As shown in Fig. 6, when the boost to these b and $\bar{b}$ rest frames is directly from the $(t\bar{t})_{cm}$ center-of-mass frame, we use respectively the subscripts "$1,2$" in place of the subscripts "a,b". Physically these angles, $\widehat{\theta}_a$, $\widehat{\phi}_a$ and $\widehat{\theta}_1$, $\widehat{\phi}_1$, are simply related by a Wigner-rotation, see below following (58). For the CP-conjugate mode, one only needs to change the subscripts $a \to b, 1 \to 2$.

For W-polarimetry, we proceed as in [2] and use the angles shown in Figs. 7 and 8. In Fig. 7, the angles θ^t_1, ϕ^t_1 and $\widetilde{\theta_a}$,$\widetilde{\phi_a}$ describe the respective stages in the sequential decay $t \to W^+ b$ followed by $W^+ \to j_{\bar{d}} j_u$ [or $W^+ \to l^+ \nu$]. For the hadronic W^+ decay mode, we use the notation that the momentum of the charge $\frac{1}{3}e$ jet is denoted by $j_{\bar{d}}$ and the momentum of the charge $\frac{2}{3}e$ jet by j_u. Similarly, in Fig. 8, $\widetilde{\theta}_b$, $\widetilde{\phi_b}$ specify the j_d jet (or the l^-) momentum in the W^-rest frame. When the boost to these $W^\pm$ rest frames is directly from the $(t\bar{t})_{cm}$ center-of-mass frame, we also use subscripts "$1,2$" in place of "a,b". This is shown explicitly in Fig. 4 of [2]; we omit that figure here for it is exactly analogous to Fig. 6 of the present paper. The angles in the W^+ rest frame, $\tilde{\theta}_a$, $\tilde{\phi}_a$ and $\tilde{\theta}_1$, $\tilde{\phi}_1$, are simply related by a Wigner-rotation [see below following (58)]. For the CP-conjugate mode's W^- rest frame, one again only needs to change the subscripts $a \to b, 1 \to 2$.

In the W^+ rest frame, the matrix element for $W^+ \to l^+ \nu$ [or for $W^+ \to j_{\bar{d}} j_u$] is [2]

$$\langle \tilde{\theta}_a, \tilde{\phi}_a, \lambda_{l^+}, \lambda_\nu | 1, \lambda_{W^+} \rangle = D^{1*}_{\lambda_{W^+},1}(\tilde{\phi}_a, \tilde{\theta}_a, 0) c \qquad (4)$$

since $\lambda_\nu = -\frac{1}{2}, \lambda_{l^+} = \frac{1}{2}$, neglecting $(\frac{m_l}{m_W})$ corrections [neglecting $(\frac{m_{jet}}{m_W})$ corrections]. Since the amplitude "c" in this matrix element is then independent of the helicities, we will usually suppress it in the following formulas since it only effects the overall normalization. We will use below

$$\rho_{\lambda_1\lambda_1';\lambda_W\lambda_W'}(t\to W^+b) = \sum_{\lambda_b=\mp 1/2} D^{(1/2)*}_{\lambda_1,\mu}(\phi_1^t,\theta_1^t,0)D^{(1/2)}_{\lambda_1',\mu'}(\phi_1^t,\theta_1^t,0)A(\lambda_W,\lambda_b)A(\lambda_W',\lambda_b)^*$$

$$\widetilde{\rho}_{\lambda_W\lambda_W'}(W^+\to l^+\nu) = D^{1*}_{\lambda_W,1}(\widetilde{\phi_a},\widetilde{\theta_a},0)D^{1}_{\lambda_W',1}(\widetilde{\phi_a},\widetilde{\theta_a},0)|c|^2$$

where $\mu = \lambda_{W^+} - \lambda_b$ and $\mu' = \lambda_{W^+} - \lambda_b'$.

2.1 Sequential-decay density matrices

Case: Only b-quark polarimetry:

The decay density matrix for the first stage of the decay sequence when the W helicities are summed over is

$$\hat{\rho}_{\lambda_1\lambda_1',\lambda_b\lambda_b'}(t\to W^+b) = \sum_{\lambda_{W^+}=\pm 1,0} D^{1/2*}_{\lambda_1\mu}(\phi_1^t,\theta_1^t,0)D^{1/2}_{\lambda_1'\mu'}(\phi_1^t,\theta_1^t,0)A(\lambda_{W^+},\lambda_b)A^*(\lambda_{W^+},\lambda_b') \quad (5)$$

where $\mu = \lambda_{W^+} - \lambda_b$ and $\mu' = \lambda_{W^+} - \lambda_b'$.

Similarly for the second stage of the decay sequence, the decay density matrix is [c.f. Eqs.(3-9) in [17]]

$$\begin{aligned}\hat{\rho}_{\lambda_b\lambda_b'}(b &\to l^-\overline{\nu}c) = \hat{\rho}_{\lambda_b\lambda_b'}(\widehat{\theta_a},\widehat{\phi_a},E_l)\\ &= \sum_{\Lambda=\pm\frac{1}{2}} D^{1/2*}_{\lambda_b\Lambda}(\widehat{\phi_a},\widehat{\theta_a},0)D^{1/2}_{\lambda_b'\Lambda}(\widehat{\phi_a},\widehat{\theta_a},0)\left|R^b_\Lambda(E_l)\right|^2 \quad (6)\end{aligned}$$

As discussed in the Appendix, the two $\left|R^b_\Lambda(E_l)\right|^2$ factors can be expressed in terms of formulas for the final lepton energy spectra [17] in the $b\to l^-\overline{\nu}c$ decay in the Λ_b mass region. Different methods to determine the Λ_b polarization have been investigated and used for Λ_b's arising from

Z^o decays. These include spectra measures such as $< E_l > / < E_{\bar{\nu}} >$ and $< E_l^n > / < E_{\bar{\nu}}^n >$ [3-6,8-10], and spin-momentum correlation measures [10].

When the final $\bar{\nu}$ angles-and-energy-distribution are used, the Λ_b polarimetry signatures in all the elements of the composite decay density matrix $\widehat{\mathbf{R}}$ of (13), and similarly for $\widehat{\not{\mathbf{R}}}$ of (32), will not be suppressed by the ratio $\frac{|R_+|^2-|R_-|^2}{|R_+|^2+|R_-|^2}$ since the $|R_-|^2$ term then vanishes, see the Appendix. Thereby, $|R_+|^2$ can be completely factored out of $\widehat{\mathbf{R}}$ and $\widehat{\not{\mathbf{R}}}$. For the CP-conjugate mode when the final ν angles-and-energy-distribution are used, the analogous suppression factor is absent in all the elements of $\overline{\widehat{\mathbf{R}}}$ of (14), and of $\overline{\widehat{\not{\mathbf{R}}}}$ of (45), and $\left|\overline{R_-}\right|^2$ completely factors out of $\overline{\widehat{\mathbf{R}}}$ and $\overline{\widehat{\not{\mathbf{R}}}}$. This same factorization occurs in the S2SC distributions, e.g. (60,61), and in the single-sided b-W-interference distributions, e.g. (63-67).

Using (4) and (6), we define the composite decay density matrix for $t \rightarrow W^+ b \rightarrow W^+(l^- \bar{\nu} c)$ by

$$\widehat{R}_{\lambda_1 \lambda_1'}(\theta_1^t, \phi_1^t; \widehat{\theta_a}, \widehat{\phi_a}) = \sum_{\lambda_b \lambda_b'} \hat{\rho}_{\lambda_1 \lambda_1', \lambda_b \lambda_b'}(t \rightarrow W^+ b) \hat{\rho}_{\lambda_b \lambda_b'}(b \rightarrow l^- \bar{\nu} c) \tag{7}$$

where the summation is understood to be over $\lambda_b = \pm\frac{1}{2}, \lambda_b' = \pm\frac{1}{2}$. This gives

$$\begin{aligned} \widehat{R}_{\lambda_1 \lambda_1'} = e^{i(\lambda_1 - \lambda_1')\phi_1^t} \textstyle\sum_{\lambda_{W^+} = \pm 1, 0} \sum_{\lambda_b \lambda_b'} \{ d^{\frac{1}{2}}_{\lambda_1, \lambda_{W^+} - \lambda_b}(\theta_1^t) d^{\frac{1}{2}}_{\lambda_1', \lambda_{W^+} - \lambda_b'}(\theta_1^t) A(\lambda_{W^+}, \lambda_b) A^*(\lambda_{W^+}, \lambda_b') \\ e^{i(\lambda_b - \lambda_b')\widehat{\phi_a}} \left[\textstyle\sum_{\Lambda = \pm\frac{1}{2}} d^{\frac{1}{2}}_{\lambda_b, \Lambda}(\widehat{\theta_a}) d^{\frac{1}{2}}_{\lambda_b', \Lambda}(\widehat{\theta_a}) \left| R^b_\Lambda(E_l) \right|^2 \right] \} \end{aligned} \tag{8}$$

In later equations, we will often use the simplifying notation that

$$|R_\pm|^2 \equiv \left| R^b_{\pm\frac{1}{2}}(E_l) \right|^2 \tag{9}$$

The two diagonal elements of the matrix $\widehat{R}_{\lambda_1 \lambda_1'}$, with $\lambda_1, \lambda_1' = \pm\frac{1}{2}, \pm\frac{1}{2}$, are purely real. They can be written in terms of the angles in Fig. 4 and in terms of the helicity parameters defined in

[2, 12, 13].

$$\begin{aligned}\widehat{\mathbf{R}}_{\pm\pm} = \tfrac{\Gamma}{4}\{ &\left[|R_+|^2 + |R_-|^2\right](1 \pm \zeta \cos\theta_1^t) - \cos\widehat{\theta_a}\left[|R_+|^2 - |R_-|^2\right](\xi \pm \sigma\cos\theta_1^t) \\ &\mp 2\sin\widehat{\theta_a}\sin\theta_1^t\left[|R_+|^2 - |R_-|^2\right]\left(\kappa_o\cos\widehat{\phi_a} - \kappa_o'\sin\widehat{\phi_a}\right)\}\end{aligned} \tag{10}$$

where $\Gamma = \Gamma_L^+ + \Gamma_T^+$ is the partial width for $t \to W^+ b$. The two off-diagonal elements are (read "upper"/"lower" lines)

$$\widehat{\mathbf{R}}_{\pm\mp} = e^{\pm i\phi_1^t}\hat{\mathbf{r}}_{\pm\mp} \tag{11}$$

with the complex-valued

$$\begin{aligned}\hat{\mathbf{r}}_{+-} = \tfrac{\Gamma}{4}\{ &\left[|R_+|^2 + |R_-|^2\right]\zeta\sin\theta_1^t - \cos\widehat{\theta_a}\left[|R_+|^2 - |R_-|^2\right]\sigma\sin\theta_1^t \\ &+2\sin\widehat{\theta_a}\left[|R_+|^2 - |R_-|^2\right](\cos\theta_1^t[\kappa_o\cos\widehat{\phi_a} - \kappa_o'\sin\widehat{\phi_a}] - i[\kappa_o\sin\widehat{\phi_a} + \kappa_o'\cos\widehat{\phi_a}])\}\end{aligned} \tag{12}$$

and $(\widehat{\mathbf{r}_{-+}})^* = \widehat{\mathbf{r}_{+-}}$. Throughout this paper the symbol $i \equiv \sqrt{-1}$. Thus we have

$$\widehat{\mathbf{R}} = \begin{pmatrix} \widehat{\mathbf{R}}_{++} & e^{i\phi_1^t}\widehat{\mathbf{r}_{+-}} \\ e^{-i\phi_1^t}\widehat{\mathbf{r}_{-+}} & \widehat{\mathbf{R}}_{--} \end{pmatrix} \tag{13}$$

***CP*-conjugate decay sequence: Only $\bar{b}$-quark polarimetry:**

For the *CP*-conjugate decay sequence $\bar{t} \to W^- \bar{b} \to W^-(l^+\nu\bar{c})$, we obtain

$$\overline{\widehat{\mathbf{R}}} = \begin{pmatrix} \overline{\widehat{\mathbf{R}}_{++}} & e^{i\phi_2^t}\overline{\widehat{\mathbf{r}_{+-}}} \\ e^{-i\phi_2^t}\overline{\widehat{\mathbf{r}_{-+}}} & \overline{\widehat{\mathbf{R}}_{--}} \end{pmatrix} \tag{14}$$

where $(\overline{\widehat{\mathbf{r}_{-+}}})^* = \overline{\widehat{\mathbf{r}_{+-}}}$ and

$$\begin{aligned}\overline{\widehat{\mathbf{R}}_{\pm\pm}} = \tfrac{\overline{\Gamma}}{4}\{ &\left[\left|\overline{R_+}\right|^2 + \left|\overline{R_-}\right|^2\right]\left(1 \mp \overline{\zeta}\cos\theta_2^t\right) - \cos\widehat{\theta}_b\left[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2\right]\left(-\overline{\xi} \pm \overline{\sigma}\cos\theta_2^t\right) \\ &\mp 2\sin\widehat{\theta}_b\sin\theta_2^t\left[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2\right]\left(\overline{\kappa_o}\cos\widehat{\phi_b} + \overline{\kappa_o'}\sin\widehat{\phi_b}\right)\}\end{aligned} \tag{15}$$

$$\begin{aligned}\overline{\widehat{\mathbf{r}}_{+-}} = \tfrac{\overline{\Gamma}}{4}\{ &-\left[\left|\overline{R_+}\right|^2 + \left|\overline{R_-}\right|^2\right]\overline{\zeta}\sin\theta_2^t - \cos\widehat{\theta}_b\left[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2\right]\overline{\sigma}\sin\theta_2^t \\ &+2\sin\widehat{\theta}_b\left[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2\right](\cos\theta_2^t[\overline{\kappa_o}\cos\widehat{\phi_b} + \overline{\kappa_o'}\sin\widehat{\phi_b}] - i[\overline{\kappa_o}\sin\widehat{\phi_b} - \overline{\kappa_o'}\cos\widehat{\phi_b}])\}\end{aligned} \tag{16}$$

Here we are using the above mentioned "bar" notation for the CP conjugate quantities such as the partial width $\overline{\Gamma} = \overline{\Gamma}_L^+ + \overline{\Gamma}_T^+$ for $\bar{t} \to W^- \bar{b}$. The fundamental CP relation is

$$B(\lambda_{W^-}, \lambda_{\bar{b}}) = A(-\lambda_{W^+}, -\lambda_b) \tag{17}$$

This relationship is useful for constructing "substitution rules" for transcribing to the CP conjugate quantities. The results, e.g. the composite decay-density matrices, do not themselves assume CP and so the S2SC functions, etc. can be used to test for whether CP holds or not.

The helicity parameters for the CP conjugate mode, $\bar{t} \to W^- \bar{b}$ include, c.f. lower part of Fig. 2,

$$\bar{\xi} \equiv \frac{1}{\overline{\Gamma}}(\overline{\Gamma}_L^- + \overline{\Gamma}_T^-), \qquad \bar{\zeta} \equiv \frac{1}{\overline{\Gamma}}(\overline{\Gamma}_L^- - \overline{\Gamma}_T^-) \tag{18}$$

$$\overline{\Gamma} = \overline{\Gamma}_L^+ + \overline{\Gamma}_T^+, \qquad \bar{\sigma} \equiv \frac{1}{\overline{\Gamma}}(\overline{\Gamma}_L^+ - \overline{\Gamma}_T^+) \tag{19}$$

where

$$\overline{\Gamma}_L^\pm \equiv \left|B(0, \frac{1}{2})\right|^2 \pm \left|B(0, -\frac{1}{2})\right|^2, \qquad \overline{\Gamma}_T^\pm \equiv \left|B(1, \frac{1}{2})\right|^2 \pm \left|B(-1, -\frac{1}{2})\right|^2 \tag{20}$$

This means that

$\bar{\xi} =$(Prob $\bar{b}$ is R-handed) $-$ (Prob $\bar{b}$ is L-handed)

$\bar{\sigma} =$(Prob W^- is Longitudinally-polarized) $-$ (Prob W^- is Transversely-polarized)

The W^- polarimetry interference parameters are given by

$$\bar{\omega} \equiv \frac{\overline{I_R^-}}{\overline{\Gamma}}, \qquad \bar{\eta} \equiv \frac{\overline{I_R^+}}{\overline{\Gamma}} \tag{21}$$

$$\bar{\omega}' \equiv \frac{\overline{I_I^-}}{\overline{\Gamma}}, \qquad \bar{\eta}' \equiv \frac{\overline{I_I^+}}{\overline{\Gamma}} \tag{22}$$

where

$$I_R^\pm \equiv \left|B(0, \frac{1}{2})\right| \left|B(1, \frac{1}{2})\right| \cos \overline{\beta_b^R} \pm \left|B(0, -\frac{1}{2})\right| \left|B(-1, -\frac{1}{2})\right| \cos \overline{\beta_b^L} \tag{23}$$

$$I_I^{\pm} \equiv \left|B(0,\frac{1}{2})\right| \left|B(1,\frac{1}{2})\right| \sin \overline{\beta_b^R} \pm \left|B(0,-\frac{1}{2})\right| \left|B(-1,-\frac{1}{2})\right| \sin \overline{\beta_b^L} \tag{24}$$

For $B(\lambda_{W^-}, \lambda_{\overline{b}}) = |B(\lambda_{W^-}, \lambda_{\overline{b}})| \exp i\phi^{bR/L}_{\lambda_{W^-}}$, the relative phases are

$$\begin{aligned}
\overline{\alpha_o} &= \phi_0^{bL} - \phi_0^{bR} & \overline{\alpha_1} &= \phi_{-1}^{b} - \phi_1^{b} \\
\overline{\beta_b^L} &= \phi_{-1}^{b} - \phi_0^{bL} & \overline{\beta_b^R} &= \phi_1^{b} - \phi_0^{bR} \\
\overline{\gamma_-} &= \phi_{-1}^{b} - \phi_0^{bR} & \overline{\gamma_+} &= \phi_1^{b} - \phi_0^{bL}
\end{aligned} \tag{25}$$

We suppress the "R/L" superscript for $\lambda_b = \pm\frac{1}{2}$ when it is not needed.

The $\overline{b}$-polarimetry helicity parameters for the CP conjugate decay then are

$$\overline{\epsilon_-} \equiv \frac{1}{\overline{\Gamma}} \left|B(-1,-\frac{1}{2})\right| \left|B(0,\frac{1}{2})\right| \cos \overline{\gamma_-}, \qquad \overline{\epsilon_-}' \equiv \frac{1}{\overline{\Gamma}} \left|B(-1,-\frac{1}{2})\right| \left|B(0,\frac{1}{2})\right| \sin \overline{\gamma_-} \tag{26}$$

$$\overline{\epsilon_+} \equiv \frac{1}{\overline{\Gamma}} \left|B(1,\frac{1}{2})\right| \left|B(0,-\frac{1}{2})\right| \cos \overline{\gamma_+}, \qquad \overline{\epsilon_+}' \equiv \frac{1}{\overline{\Gamma}} \left|B(1,\frac{1}{2})\right| \left|B(0,-\frac{1}{2})\right| \sin \overline{\gamma_+} \tag{27}$$

$$\overline{\kappa_o} \equiv \frac{1}{\overline{\Gamma}} \left|B(0,-\frac{1}{2})\right| \left|B(0,\frac{1}{2})\right| \cos \overline{\alpha_o}, \qquad \overline{\kappa_o}' \equiv \frac{1}{\overline{\Gamma}} \left|B(0,-\frac{1}{2})\right| \left|B(0,\frac{1}{2})\right| \sin \overline{\alpha_o} \tag{28}$$

$$\overline{\kappa_1} \equiv \frac{1}{\overline{\Gamma}} \left|B(-1,-\frac{1}{2})\right| \left|B(1,\frac{1}{2})\right| \cos \overline{\alpha_1}, \qquad \overline{\kappa_1}' \equiv \frac{1}{\overline{\Gamma}} \left|B(-1,-\frac{1}{2})\right| \left|B(1,\frac{1}{2})\right| \sin \overline{\alpha_1} \tag{29}$$

In the S2SC distributions and in the single-sided sequential-decay distributions, sometimes their linear-combinations

$$\overline{\delta} \equiv \overline{\epsilon_+} + \overline{\epsilon_-}, \quad \overline{\epsilon} \equiv \overline{\epsilon_+} - \overline{\epsilon_-}, \quad \overline{\lambda} \equiv \overline{\kappa_o} + \overline{\kappa_1}, \quad \overline{\kappa} \equiv \overline{\kappa_o} - \overline{\kappa_1}$$

and the corresponding primed linear-combinations occur, see (59-67). Simple CP tests were treated in [2].

Case: Both b-quark polarimetry and W-polarimetry:

We now consider both branches in the decay sequence $t \to W^+ b$ so $b \to l^- \bar{\nu} c$ and also $W^+ \to l^+ \nu_l$ [or $W^+ \to j_{\overline{d}} j_u$].

We define the "full" or double-sequential-decay density matrix for $t \rightarrow W^+ b \rightarrow (l^+ \nu_l)(l^- \bar{\nu} c)$ by

$$
\begin{gathered}
\not{R}_{\lambda_1 \lambda_1'}(\theta_1^t, \phi_1^t; \widetilde{\theta_a}, \widetilde{\phi_a}; \widehat{\theta_a}, \widehat{\phi_a}) = \sum_{\lambda_b \lambda_b'} \sum_{\mu_1 \mu_1'} D^{1/2*}_{\lambda_1 \mu_1 - \lambda_b}(\phi_1^t, \theta_1^t, 0) D^{1/2}_{\lambda_1' \mu_1' - \lambda_b'}(\phi_1^t, \theta_1^t, 0) \\
A(\mu_1, \lambda_b) A^*(\mu_1', \lambda_b') \widetilde{\rho}_{\mu_1 \mu_1'}(W^+ \rightarrow l^+ \nu_l) \hat{\rho}_{\lambda_b \lambda_b'}(b \rightarrow l^- \bar{\nu} c)
\end{gathered}
\tag{30}
$$

where the summations are over $\lambda_b = \pm\frac{1}{2}$, $\lambda_b' = \pm\frac{1}{2}$ and the W^+ helicities $\mu_1, \mu_1' = \pm 1, 0$. For the "full" quantities we use a "slash" notation. Note that all possible $A(\mu_1, \lambda_b) A^*(\mu_1', \lambda_b')$ interference terms occur in (30). $\widetilde{\rho}_{\mu_1 \mu_1'}$ is given after (4) and $\hat{\rho}_{\lambda_b \lambda_b'}$ is (6).

This gives the "full master-equation"

$$
\begin{gathered}
\not{R}_{\lambda_1 \lambda_1'} = e^{i(\lambda_1 - \lambda_1')\phi_1^t} \sum_{\lambda_b \lambda_b'} \sum_{\mu_1 \mu_1'} \{ d^{\frac{1}{2}}_{\lambda_1, \mu_1 - \lambda_b}(\theta_1^t) d^{\frac{1}{2}}_{\lambda_1', \mu_1' - \lambda_b'}(\theta_1^t) A(\mu_1, \lambda_b) A^*(\mu_1', \lambda_b') \\
e^{i(\mu_1 - \mu_1')\widetilde{\phi_a}} d^1_{\mu_1 1}(\widetilde{\theta_a}) d^1_{\mu_1' 1}(\widetilde{\theta_a}) e^{i(\lambda_b - \lambda_b')\widehat{\phi_a}} \sum_{\Lambda = \pm\frac{1}{2}} d^{\frac{1}{2}}_{\lambda_b, \Lambda}(\widehat{\theta_a}) d^{\frac{1}{2}}_{\lambda_b', \Lambda}(\widehat{\theta_a}) \left| R^b_\Lambda(E_l) \right|^2 \}
\end{gathered}
\tag{31}
$$

It can be expressed in terms of the previously defined helicity parameters. In matrix form the result is

$$
\widehat{\not{\mathbf{R}}} = \begin{pmatrix} \widehat{\not{\mathbf{R}}}_{++} & e^{i\phi_1^t} \, \widehat{\not{r}_{+-}} \\ e^{-i\phi_1^t} \, \widehat{\not{r}_{-+}} & \widehat{\not{\mathbf{R}}}_{--} \end{pmatrix}
\tag{32}
$$

where $(\widehat{\not{r}_{-+}})^* = \widehat{\not{r}_{+-}}$. The elements of this matrix are each conveniently written as the sum of three contributions:

$$
\widehat{\not{\mathbf{R}}} = \widehat{\not{\mathbf{R}}}^W + \widehat{\not{\mathbf{R}}}^c + \widehat{\not{\mathbf{R}}}^s
\tag{33}
$$

where the first $\widehat{\not{\mathbf{R}}}^W$ is proportional to Eq.(12) in [2] which only makes use of W-polarimetry information, the second $\widehat{\not{\mathbf{R}}}^c$ is proportional to the b-polarimetry "$\cos\widehat{\theta_a}$", and the third $\widehat{\not{\mathbf{R}}}^s$ is proportional to the b-polarimetry "$\sin\widehat{\theta_a}$".

The three contributions to the diagonal elements are:

$$
\widehat{\not{\mathbf{R}}}^W_{\pm\pm} = \frac{1}{8}[|R_+|^2 + |R_-|^2] \mathbf{R}_{\pm\pm}
\tag{34}
$$

where $\mathbf{R}_{\pm\pm}$ is Eq.(13) in [2]. The second term in (33) is

$$\widehat{\not{\mathbf{R}}}^{c}_{\pm\pm} = \tfrac{1}{8}[|R_+|^2 - |R_-|^2] \cos\widehat{\theta}_a (\mathbf{n}_{\mathbf{a}}^{(-)}[1 \pm \mathbf{f}_{\mathbf{a}}^{(-)} \cos\theta_1^t] \mp \tfrac{1}{\sqrt{2}} \sin\theta_1^t \{ \sin 2\widetilde{\theta}_a [\eta \cos\widetilde{\phi}_a + \omega' \sin\widetilde{\phi}_a] - 2 \sin\widetilde{\theta}_a [\omega \cos\widetilde{\phi}_a + \eta' \sin\widetilde{\phi}_a]\}) \tag{35}$$

where

$$\begin{pmatrix} \mathbf{n}_{\mathbf{a}}^{(-)} \\ \\ \mathbf{n}_{\mathbf{a}}^{(-)} \mathbf{f}_{\mathbf{a}}^{(-)} \end{pmatrix} = -\sin^2\widetilde{\theta}_a \frac{\Gamma_L^{\mp}}{\Gamma} \mp \frac{1}{4}(3 + \cos 2\widetilde{\theta}_a) \frac{\Gamma_T^{\mp}}{\Gamma} \pm \cos\widetilde{\theta}_a \frac{\Gamma_T^{\pm}}{\Gamma} \tag{36}$$

with a superscript-tagging per the l^- tag of the decaying b-quark. Equivalently,

$$\mathbf{n}_{\mathbf{a}}^{(-)} = \frac{1}{8}(4[1-\sigma]\cos\widetilde{\theta}_a - \xi[5 - \cos 2\widetilde{\theta}_a] + \zeta[1 + 3\cos 2\widetilde{\theta}_a]) \tag{37}$$

$$\mathbf{n}_{\mathbf{a}}^{(-)} \mathbf{f}_{\mathbf{a}}^{(-)} = \frac{1}{8}(1 + 3\cos 2\widetilde{\theta}_a - \sigma[5 - \cos 2\widetilde{\theta}_a] - 4[\xi - \zeta]\cos\widetilde{\theta}_a) \tag{38}$$

The third term in (33) is

$$\begin{aligned}\widehat{\not{\mathbf{R}}}^{s}_{\pm\pm} = \tfrac{1}{8}[|R_+|^2 - |R_-|^2] \sin\widehat{\theta}_a \{ &\mp \sin^2\widetilde{\theta}_a \sin\theta_1^t [\cos(2\widetilde{\phi}_a + \widehat{\phi}_a)\kappa_1 + 2\cos(\widehat{\phi}_a)\kappa_o \\ &- \sin(2\widetilde{\phi}_a + \widehat{\phi}_a)\kappa_1' - 2\sin(\widehat{\phi}_a)\kappa_o'] \\ &+ \sqrt{2}\sin\widetilde{\theta}_a (\cos(\widetilde{\phi}_a + \widehat{\phi}_a)[\delta(1 \pm \cos\widetilde{\theta}_a \cos\theta_1^t) + \epsilon(\cos\widetilde{\theta}_a \pm \cos\theta_1^t) \\ &- \sin(\widetilde{\phi}_a + \widehat{\phi}_a)[\delta'(1 \pm \cos\widetilde{\theta}_a \cos\theta_1^t) + \epsilon'(\cos\widetilde{\theta}_a \pm \cos\theta_1^t)])\}\end{aligned} \tag{39}$$

where $\delta \equiv \epsilon_+ + \epsilon_-, \epsilon \equiv \epsilon_+ - \epsilon_-$ and analogously for δ' and ϵ'. Note from Fig. 1 that δ and ϵ are b-W-interference parameters. They only appear in the third term, i.e. in $\widehat{\not{\mathbf{R}}}^{s}_{\pm\pm}$. The b-polarimetry helicity parameters $\kappa_{o,1}$ also appear only in this term. Note above in the case of only b-polarimetry the parameters $\kappa_{o,1}$ do appear, but δ and ϵ do not. This is expected since the latter two helicity parameters only occur due to b-W-interference.

The three contributions to the off-diagonal elements are:

$$\widehat{\not{\mathbf{r}}}^{W}_{+-} = \frac{1}{8}[|R_+|^2 + |R_-|^2]\mathbf{r}_{+-} \tag{40}$$

where $\mathbf{r}_{+-}$ is Eq.(14) in [2]. The second contribution is

$$\begin{aligned} \widehat{\not{r}}^{c}_{+-} &= \tfrac{1}{8}[|R_+|^2 - |R_-|^2] \cos\widehat{\theta_a}\{(\mathbf{n}^{(-)}_{\mathbf{a}}\mathbf{f}^{(-)}_{\mathbf{a}} \sin\theta^t_1 \\ &-\sqrt{2}\sin\widetilde{\theta_a}(\cos\theta^t_1[\omega\cos\widetilde{\phi_a} + \eta^{'}\sin\widetilde{\phi_a}] + i[\omega\sin\widetilde{\phi_a} - \eta^{'}\cos\widetilde{\phi_a}]) \\ &+\tfrac{1}{\sqrt{2}}\sin 2\widetilde{\theta_a}(\cos\theta^t_1[\eta\cos\widetilde{\phi_a} + \omega^{'}\sin\widetilde{\phi_a}] + i[\eta\sin\widetilde{\phi_a} - \omega^{'}\cos\widetilde{\phi_a}])\} \end{aligned} \tag{41}$$

and the third contribution is

$$\begin{aligned} \widehat{\not{r}}^{s}_{+-} &= \tfrac{1}{8}[|R_+|^2 - |R_-|^2] \sin\widehat{\theta_a}\{\sqrt{2}\sin\theta^t_1 \sin\widetilde{\theta_a}(\cos(\widetilde{\phi_a}+\widehat{\phi_a})\epsilon - \sin(\widetilde{\phi_a}+\widehat{\phi_a})\epsilon^{'} \\ &+\cos\widetilde{\theta_a}[\cos(\widetilde{\phi_a}+\widehat{\phi_a})\delta - \sin(\widetilde{\phi_a}+\widehat{\phi_a})\delta^{'}]) \\ &+\sin^2\widetilde{\theta_a}(\cos\theta^t_1[2\cos\widehat{\phi_a}\kappa_o - 2\sin\widehat{\phi_a}\kappa^{'}_o + \cos(2\widetilde{\phi_a}+\widehat{\phi_a})\kappa_1 - \sin(2\widetilde{\phi_a}+\widehat{\phi_a})\kappa^{'}_1] \\ &+i[-2\sin\widehat{\phi_a}\kappa_o - 2\cos\widehat{\phi_a}\kappa^{'}_o + \sin(2\widetilde{\phi_a}+\widehat{\phi_a})\kappa_1 + \cos(2\widetilde{\phi_a}+\widehat{\phi_a})\kappa^{'}_1])\} \end{aligned} \tag{42}$$

Here also, $\delta, \epsilon, \kappa_{o,1}$ and $\delta^{'}, \epsilon^{'}, \kappa^{'}_{o,1}$ only appear in $\widehat{\not{r}}^{s}_{+-}$ and not in $\widehat{\not{r}}^{c}_{+-}$.

In summary, the $\sin\widehat{\theta_a}$ dependence of the "s" superscript terms in $\widehat{\not{\mathbf{R}}}$ of (32) must be used in order to measure the eight b-polarimetry helicity parameters.

CP-conjugate process: Both $\bar{b}$-quark polarimetry and W-polarimetry:

For both branches in the CP-conjugate decay sequence $\overline{t} \rightarrow W^-\overline{b}$ so $\overline{b} \rightarrow l^+\nu\overline{c}$ and also $W^- \rightarrow l^-\overline{\nu_l}$ [or $W^- \rightarrow j_d j_{\overline{u}}$], we define the "full" sequential-decay density matrix by

$$\begin{aligned} \overline{\not{R}}_{\lambda_2\lambda^{'}_2}(\theta^t_2,\phi^t_2;\widetilde{\theta}_b,\widetilde{\phi}_b;\widehat{\theta}_b,\widehat{\phi}_b) &= \textstyle\sum_{\lambda_{\bar{b}}\lambda^{'}_{\bar{b}}}\sum_{\mu_2\mu^{'}_2} D^{1/2*}_{\lambda_2\mu_2-\lambda_{\bar{b}}}(\phi^t_2,\theta^t_2,0) D^{1/2}_{\lambda^{'}_2\mu^{'}_2-\lambda^{'}_{\bar{b}}}(\phi^t_2,\theta^t_2,0) \\ & B(\mu_2,\lambda_{\bar{b}})B^*(\mu^{'}_2,\lambda^{'}_{\bar{b}})\overline{\rho}_{\mu_2\mu^{'}_2}(W^-\rightarrow l^-\bar{\nu}_l)\overline{\rho}_{\lambda_b\lambda^{'}_b}(\bar{b}\rightarrow l^+\nu\bar{c}) \end{aligned} \tag{43}$$

where the summations are over $\lambda_{\overline{b}} = \pm\frac{1}{2}, \lambda^{'}_{\overline{b}} = \pm\frac{1}{2}$ and the W^- helicities $\mu_2, \mu^{'}_2 = \pm 1, 0$. This gives the "full master-equation"

$$\begin{aligned} \overline{\not{R}}_{\lambda_2\lambda^{'}_2} &= e^{i(\lambda_2-\lambda^{'}_2)\phi^t_2}\textstyle\sum_{\lambda_{\bar{b}}\lambda^{'}_{\bar{b}}}\sum_{\mu_2\mu^{'}_2}\{d^{\frac{1}{2}}_{\lambda_2,\mu_2-\lambda_{\bar{b}}}(\theta^t_2) d^{\frac{1}{2}}_{\lambda^{'}_2,\mu^{'}_2-\lambda^{'}_{\bar{b}}}(\theta^t_2) B(\mu_2,\lambda_{\bar{b}})B^*(\mu^{'}_2,\lambda^{'}_{\bar{b}}) \\ & e^{i(\mu_2-\mu^{'}_2)\widetilde{\phi}_b} d^1_{\mu_2,-1}(\widetilde{\theta}_b) d^1_{\mu^{'}_2,-1}(\widetilde{\theta}_b) e^{i(\lambda_{\bar{b}}-\lambda^{'}_{\bar{b}})\widehat{\phi}_b} \textstyle\sum_{\overline{\Lambda}=\pm\frac{1}{2}} d^{\frac{1}{2}}_{\lambda_{\bar{b}},\overline{\Lambda}}(\widehat{\theta}_b) d^{\frac{1}{2}}_{\lambda^{'}_{\bar{b}},\overline{\Lambda}}(\widehat{\theta}_b)\left|R^{\overline{b}}_{\overline{\Lambda}}(E_{\overline{l}})\right|^2\} \end{aligned} \tag{44}$$

It can be expressed in terms of the previously defined "barred" helicity parameters. The result is

$$\overline{\widehat{\not{R}}} = \begin{pmatrix} \overline{\widehat{\not{R}}_{++}} & e^{i\phi_2^t}\,\overline{\widehat{\not{r}}_{+-}} \\ e^{-i\phi_2^t}\,\overline{\widehat{\not{r}}_{-+}} & \overline{\widehat{\not{R}}_{--}} \end{pmatrix} \tag{45}$$

where $(\overline{\widehat{\not{r}}_{-+}})^* = \overline{\widehat{\not{r}}_{+-}}$, and

$$\overline{\widehat{\not{R}}} = \overline{\widehat{\not{R}}^W} + \overline{\widehat{\not{R}}^c} + \overline{\widehat{\not{R}}^s} \tag{46}$$

where $\overline{\widehat{\not{R}}^W}$ is proportional to Eq.(17) in [2] , $\overline{\widehat{\not{R}}^c}$ is proportional to the $\overline{b}$-polarimetry "$\cos\widehat{\theta}_b$", and $\overline{\widehat{\not{R}}^s}$ is proportional to the $\overline{b}$-polarimetry "$\sin\widehat{\theta}_b$".

The diagonal elements are:

$$\overline{\widehat{\not{R}}^W_{\pm\pm}} = \frac{1}{8}[\left|\overline{R_+}\right|^2 + \left|\overline{R_-}\right|^2]\overline{\mathbf{R}_{\pm\pm}} \tag{47}$$

where $\overline{\mathbf{R}_{\pm\pm}}$ is Eq.(18) in [2],

$$\begin{aligned}\overline{\widehat{\not{R}}^c_{\pm\pm}} = \tfrac{1}{8}[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2]\cos\widehat{\theta}_b(-\mathbf{n_b}^{(+)}[1 \mp \mathbf{f_b}^{(+)}\cos\theta_2^t] \mp \tfrac{1}{\sqrt{2}}\sin\theta_2^t\{\sin 2\widetilde{\theta}_b[\bar{\eta}\cos\widetilde{\phi_b} \\ -\bar{\omega}'\sin\widetilde{\phi_b}] - 2\sin\widetilde{\theta}_b[\bar{\omega}\cos\widetilde{\phi_b} - \bar{\eta}'\sin\widetilde{\phi_b}]\})\end{aligned} \tag{48}$$

where

$$\begin{pmatrix} \mathbf{n_b}^{(+)} \\ \mathbf{n_b}^{(+)}\mathbf{f_b}^{(+)} \end{pmatrix} = -\sin^2\widetilde{\theta}_b\frac{\overline{\Gamma_L^{\mp}}}{\overline{\Gamma}} \mp \frac{1}{4}(3+\cos 2\widetilde{\theta}_b)\frac{\overline{\Gamma_T^{\mp}}}{\overline{\Gamma}} \pm \cos\widetilde{\theta}_b\frac{\overline{\Gamma_T^{\pm}}}{\overline{\Gamma}} \tag{49}$$

with a superscript-tagging per the l^+ tag of the decaying $\overline{b}$-quark, or

$$\mathbf{n_b}^{(+)} = \frac{1}{8}(4[1-\overline{\sigma}]\cos\widetilde{\theta}_b - \overline{\xi}[5-\cos 2\widetilde{\theta}_b] + \overline{\zeta}[1+3\cos 2\widetilde{\theta}_b]) \tag{50}$$

$$\mathbf{n_b}^{(+)}\mathbf{f_b}^{(+)} = \frac{1}{8}(1+3\cos 2\widetilde{\theta}_b - \overline{\sigma}[5-\cos 2\widetilde{\theta}_b] - 4[\overline{\xi}-\overline{\zeta}]\cos\widetilde{\theta}_b) \tag{51}$$

and

$$\overline{\widehat{\not{\mathbf{R}}}^{s}_{\pm\pm}} = \tfrac{1}{8}[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2]\sin\widehat{\theta}_b\{\mp\sin^2\widetilde{\theta}_b\sin\theta_2^t[\cos(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1} + 2\cos(\widehat{\phi_b})\bar{\kappa_o}$$
$$+\sin(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1}' + 2\sin(\widehat{\phi_b})\bar{\kappa_o}']$$
$$-\sqrt{2}\sin\widetilde{\theta}_b(\cos(\widetilde{\phi_b}+\widehat{\phi_b})[\bar{\delta}(1\mp\cos\widetilde{\theta}_b\cos\theta_2^t) + \bar{\epsilon}(\cos\widetilde{\theta}_b\mp\cos\theta_2^t)$$
$$+\sin(\widetilde{\phi_b}+\widehat{\phi_b})[\bar{\delta}'(1\mp\cos\widetilde{\theta}_b\cos\theta_2^t) + \bar{\epsilon}'(\cos\widetilde{\theta}_b\mp\cos\theta_2^t)])\} \tag{52}$$

The off-diagonal elements are:

$$\overline{\widehat{\not{\mathbf{r}}}^{W}_{+-}} = \frac{1}{8}[\left|\overline{R_+}\right|^2 + \left|\overline{R_-}\right|^2]\overline{\mathbf{r}_{+-}} \tag{53}$$

where $\overline{\mathbf{r}_{+-}}$ is Eq.(19) in [2],

$$\overline{\widehat{\not{\mathbf{r}}}^{c}_{+-}} = \tfrac{1}{8}[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2]\cos\widehat{\theta}_b\{(\mathbf{n_b}^{(+)}\mathbf{f_b}^{(+)}\sin\theta_2^t$$
$$-\sqrt{2}\sin\widetilde{\theta}_b(\cos\theta_2^t[\bar{\omega}\cos\widetilde{\phi_b} - \bar{\eta}'\sin\widetilde{\phi_b}] + i[\bar{\omega}\sin\widetilde{\phi_b} + \bar{\eta}'\cos\widetilde{\phi_b}])$$
$$+\tfrac{1}{\sqrt{2}}\sin 2\widetilde{\theta}_b(\cos\theta_2^t[\bar{\eta}\cos\widetilde{\phi_b} - \bar{\omega}'\sin\widetilde{\phi_b}] + i[\bar{\eta}\sin\widetilde{\phi_b} + \bar{\omega}'\cos\widetilde{\phi_b}])\} \tag{54}$$

and

$$\overline{\widehat{\not{\mathbf{r}}}^{s}_{+-}} = \tfrac{1}{8}[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2]\sin\widehat{\theta}_b\{\sqrt{2}\sin\theta_2^t\sin\widetilde{\theta}_b(\cos(\widetilde{\phi_b}+\widehat{\phi_b})\bar{\epsilon} + \sin(\widetilde{\phi_b}+\widehat{\phi_b})\bar{\epsilon}'$$
$$+\cos\widetilde{\theta}_b[\cos(\widetilde{\phi_b}+\widehat{\phi_b})\bar{\delta} + \sin(\widetilde{\phi_b}+\widehat{\phi_b})\bar{\delta}'])$$
$$+\sin^2\widetilde{\theta}_b(\cos\theta_2^t[2\cos\widehat{\phi_b}\bar{\kappa_o} + 2\sin\widehat{\phi_b}\bar{\kappa_o}' + \cos(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1} + \sin(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1}']$$
$$+i[-2\sin\widehat{\phi_b}\bar{\kappa_o} + 2\cos\widehat{\phi_b}\bar{\kappa_o}' + \sin(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1} - \cos(2\widetilde{\phi_b}+\widehat{\phi_b})\bar{\kappa_1}'])\} \tag{55}$$

The $\sin\widehat{\theta}_b$ dependence of the "s" superscript terms in $\overline{\widehat{\not{\mathbf{R}}}}$ of (45) must be used in order to measure the eight $\bar{b}$-polarimetry helicity parameters.

3 Stage-Two Spin-Correlation Functions Including b-Polarimetry

We include both branches in the decay sequence $t \to W^+ b$ so $b \to l^- \bar{\nu} c$ and also $W^+ \to l^+ \nu_l$ [or $W^+ \to j_{\overline{d}} j_u$]. We also include both branches for the CP-conjugate sequence.

3.1 The full S2SC function:

The complete S2SC function is relatively simple in structure even though it depends on "5+4+4" variables [c.f. Eq.(66) of [2]]. Each of the last 4 variables $\widetilde{\theta_a}, \widetilde{\phi_a}; \widehat{\theta_a}, \widehat{\phi_a}$ and $\widetilde{\theta_b}, \widetilde{\phi_b}; \widehat{\theta_b}, \widehat{\phi_b}$ describe the two second-stage branches:

$$
\begin{aligned}
\not{I}(\Theta_B, \Phi_B; \phi; \theta_1^t, \widetilde{\theta_a}, \widetilde{\phi_a}; \widehat{\theta_a}, \widehat{\phi_a}; \theta_2^t, \widetilde{\theta_b}, \widetilde{\phi_b}; \widehat{\theta_b}, \widehat{\phi_b}) = \textstyle\sum_{h_1 h_2} \{ \rho^{prod}_{h_1 h_2, h_1 h_2} \; \not{R}_{h_1 h_1} \overline{\not{R}_{h_2 h_2}} \\
+ (\rho^{prod}_{++,--} \; \not{r}_{+-} \overline{\not{r}_{+-}} + \rho^{prod}_{--,++} \; \not{r}_{-+} \overline{\not{r}_{-+}}) \cos\phi \\
+ i (\rho^{prod}_{++,--} \; \not{r}_{+-} \overline{\not{r}_{+-}} - \rho^{prod}_{--,++} \; \not{r}_{-+} \overline{\not{r}_{-+}}) \sin\phi \}
\end{aligned}
\tag{56}
$$

The "slashed" composite density matrix elements have been discussed above. The production density matrix elements are given in [2, 21].

The production density matrices ρ^{prod} in (56) depend on the angles Θ_B, Φ_B which give [19, 20, 2] the direction of the incident parton beam, i.e. the quark's momentum or the gluon's momentum, arising from the incident p in the $p\bar{p}$, or pp, $\to t\bar{t}X$ production process:

$$
q\overline{q}, \quad or \quad gg, \to t\overline{t} \to (W^+ b)(W^- \overline{b}) \tag{57}
$$

The important angle between the t and $\bar{t}$ decay planes is

$$
\phi = \phi_1^t + \phi_2^t = \widehat{\phi} = \widehat{\phi}_1^t + \widehat{\phi}_2^t
$$

The other angles have been discussed previously in consideration of the earlier figures in this paper. The θ_1^t angular dependence can be replaced by the W^+ energy in the $(t\bar{t})_{cm}$ and similarly θ_2^t by the W^- energy [2].

In the $(t\bar{t})_{cm}$ system where θ_t is the angle between the t-quark momentum and the incident parton-beam this simplifies to

$$\not{I}(\theta_t; \theta_1^t; \widetilde{\theta_a}, \widetilde{\phi_a}; \widehat{\theta_a}, \widehat{\phi_a}; \theta_2^t; \widetilde{\theta_b}, \widetilde{\phi_b}; \widehat{\theta_b}, \widehat{\phi_b}) = \sum_{h_1 h_2} \rho^{prod}_{h_1 h_2, h_1 h_2}(\theta_t) \ \not{R}_{h_1 h_1} \overline{\not{R}_{h_2 h_2}} \tag{58}$$

which depends on only the diagonal elements of the "full" sequential-decay density matrices $\widehat{\mathbf{R}}_{\pm\pm}$ and $\overline{\widehat{\mathbf{R}}_{\pm\pm}}$ given respectively in (32) and (45). Note that $\widehat{\mathbf{R}}_{\pm\pm}$ depends on (i) all 8 of the W-polarimetry helicity parameters [the partial-width Γ, the probablility that the emitted W is longitudinally polarized $P(W_L) = \frac{1}{2}(1+\sigma)$, the probability that the emitted b-quark is L-handed $P(b_L) = \frac{1}{2}(1+\xi)$, ζ, $\eta_{L,R}$, and ${\eta_{L,R}}'$] see [2], and it also depends on (ii) all 8 of the new b-polarimetry parameters [$\epsilon_\pm, \kappa_{o,1}$ and ${\epsilon_\pm}'$, ${\kappa_{o,1}}'$].

Remark: Use of Alternative-Angles: The alternative-angle-labeling of the final $l^\mp$ as shown in Fig. 6 can be a significant issue in some circumstances, see [2, 19] and references therein. These angles occur when the boosts to the b and $\bar{b}$ rest frames are directly from the $(t\bar{t})_{cm}$ frame. Recall that this same choice arises for the labeling of the $W^\pm$ decays, see Fig. 4 in [2]. At a hadron collider, this alternative-angle-labeling would be useful when both $W^\pm$ decay into hadrons. The necessary Wigner-rotation for Fig. 6 is exactly analogous to that given in [2] in (74,75) with respect to Fig.4 therein. For the t-decay-side, the explicit "transformation-equations" (involving the Wigner-rotations) are a simple relabeling of Eqs. (3.22a,b,c) in [19], see Fig. 1 therein, and for the $\bar{t}$-decay-side there is an exactly analogous relabeling of the transformation-Eqs.(3.31a,b,c). At a future linear collider, use of these alternative-angles for specifying the final stage-two momenta for some or all of the $b, \bar{b}, W^{\pm}$ decays might also be preferable when W^+ and/or W^- decay leptonically.

In the derivation of S2SC distributions and of single-sided distributions, some care is needed as to at what step to use the explicit "transformation-equations" to the alternative-angles, see [19]. Second, the sensitivity in regard to the measurement of a specific helicity parameter can vary significantly depending on which minimum variable choice is made. For instance at an e^+e^- collider, integrations over some of the $\theta_{1,2}, \phi_{1,2}$-type variables (which follow after using the "transformation-equations") can yield minimum-variable-distributions which are significantly more sensitive to some parameters than are the analogous same-number-of-variable distributions in which the integrations are performed on the analogous $\theta_{a,b}, \phi_{a,b}$-type variables, see [2, 19].

3.2 A simple two-sided b-W spin-correlation function:

After integrating out the polar angles describing the second-stage branches, $\widetilde{\theta_a}, \widehat{\theta_a}; \widetilde{\theta_b}, \widehat{\theta_b}$, we obtain

$$\begin{aligned} I\!\!/(\theta_t; \theta_1^t; \widetilde{\phi_a}; \widehat{\phi_a}; \theta_2^t; \widetilde{\phi_b}; \widehat{\phi_b}) = \textstyle\sum_{q_i's, g_i's} \{ \rho_{+-}^{prod}(\theta_t)[\hat{\rho}_{++}\overline{\hat{\rho}_{--}} + \hat{\rho}_{--}\overline{\hat{\rho}_{++}}] \\ + \rho_{++}^{prod}(\theta_t)[\hat{\rho}_{++}\overline{\hat{\rho}_{++}} + \hat{\rho}_{--}\overline{\hat{\rho}_{--}}] \} \end{aligned} \tag{59}$$

The production density matrix elements are given in [2].

The integrated composite decay density matrix elements are found to be: For the $t \to W^+ b \to (l^+\nu_l)(l^-\bar{\nu}c)$ decay sequence, we obtain $\widehat{\rho}_{\pm\pm}(\theta_1^t, \widetilde{\phi_a}, \widehat{\phi_a})$ which depends on the t-quark polar angle and the two second-stage azimuthal angles

$$\begin{aligned} \widehat{\rho}_{\pm\pm} = \tfrac{1}{8}[|R_+|^2 + |R_-|^2]\{\tfrac{2}{3}[1 \pm \zeta \cos\theta_1^t] \mp \tfrac{\pi}{2\sqrt{2}} \sin\theta_1^t [\eta \cos\widetilde{\phi_a} + \omega' \sin\widetilde{\phi_a}]\} \\ + \tfrac{\pi}{32}[|R_+|^2 - |R_-|^2]\{\tfrac{\pi}{2\sqrt{2}}(\cos(\widetilde{\phi_a} + \widehat{\phi_a})[\delta \pm \epsilon \cos\theta_1^t] - \sin(\widetilde{\phi_a} + \widehat{\phi_a})[\delta' \pm \epsilon' \cos\theta_1^t]) \\ \mp \tfrac{2}{3} \sin\theta_1^t [\cos(2\widetilde{\phi_a} + \widehat{\phi_a})\kappa_1 + 2\cos(\widehat{\phi_a})\kappa_o - \sin(2\widetilde{\phi_a} + \widehat{\phi_a})\kappa_1' - 2\sin(\widehat{\phi_a})\kappa_o']\} \end{aligned} \tag{60}$$

For the $\bar{t} \to W^- \bar{b} \to (l^- \bar{\nu}_l)(l^+ \nu \bar{c})$ decay sequence, we obtain $\overline{\widehat{\rho}_{\pm\pm}}(\theta_2^t, \widetilde{\phi_b}, \widehat{\phi_b})$ which depends on the $\bar{t}$-quark polar angle and the two second-stage azimuthal angles

$$\begin{aligned} \overline{\widehat{\rho}_{\pm\pm}} = & \tfrac{1}{8}\left[\left|\overline{R_+}\right|^2 + \left|\overline{R_-}\right|^2\right]\{\tfrac{2}{3}[1 \mp \overline{\zeta}\cos\theta_2^t] \pm \tfrac{\pi}{2\sqrt{2}}\sin\theta_2^t[\overline{\eta}\cos\widetilde{\phi_b} - \overline{\omega}'\sin\widetilde{\phi_b}]\} \\ & + \tfrac{\pi}{32}\left[\left|\overline{R_+}\right|^2 - \left|\overline{R_-}\right|^2\right]\{-\tfrac{\pi}{2\sqrt{2}}(\cos(\widetilde{\phi_b} + \widehat{\phi_b})[\overline{\delta} \mp \overline{\epsilon}\cos\theta_2^t] + \sin(\widetilde{\phi_b} + \widehat{\phi_b})[\overline{\delta}' \mp \overline{\epsilon}'\cos\theta_2^t]) \\ & \mp \tfrac{2}{3}\sin\theta_2^t[\cos(2\widetilde{\phi_b} + \widehat{\phi_b})\overline{\kappa}_1 + 2\cos(\widehat{\phi_b})\overline{\kappa}_o + \sin(2\widetilde{\phi_b} + \widehat{\phi_b})\overline{\kappa}_1' + 2\sin(\widehat{\phi_b})\overline{\kappa}_o']\} \end{aligned} \tag{61}$$

It is important to note that the helicity parameters discussed in the following section of this paper appear in the above two density matrices. The density matrix $\widehat{\rho}_{\pm\pm}(\theta_1^t, \widetilde{\phi_a}, \widehat{\phi_a})$ depends on (i) 3 of the helicity parameters measurable [2] with only W -polarimetry: ζ, η and the $\tilde{T}_{FS}$-violating ω' parameter which is zero in the SM, and on (ii) 8 b-polarimetry helicity parameters: $\delta \equiv \epsilon_+ + \epsilon_-, \epsilon \equiv \epsilon_+ - \epsilon_-, \kappa_o, \kappa_1$ and the corresponding primed quantities which are non-zero if there is $\tilde{T}_{FS}$-violation in $t \to W^+ b$.

3.3 Simple single-sided b-W distributions:

Three simple single-sided distributions for $t \to W^+ b \to (l^+ \nu_l)(l^- \bar{\nu} c)$ (or $W^+ \to j_{\bar{d}} j_u$), are the following:

$$I\!\!/(\theta_t; \widetilde{\theta_a}, \widehat{\theta_a}, \widetilde{\phi_a} + \widehat{\phi_a}) = \sum_{q_i, g_i} (\rho_{++}^{prod} + \rho_{--}^{prod})\{\widehat{R\!\!/}_{++} + \widehat{R\!\!/}_{--}\} \tag{62}$$

where

$$\begin{aligned} \{\widehat{R\!\!/}_{++} + \widehat{R\!\!/}_{--}\} = & \tfrac{1}{4}[|R_+|^2 + |R_-|^2]\mathbf{n_a} + \tfrac{1}{4}[|R_+|^2 - |R_-|^2]\{\cos\widehat{\theta_a}\mathbf{n_a}^{(-)} \\ & + \tfrac{1}{\sqrt{2}}\sin\widehat{\theta_a}(\cos(\widetilde{\phi_a} + \widehat{\phi_a})[2\delta\sin\widetilde{\theta_a} + \epsilon\sin 2\widetilde{\theta_a}] \\ & - \sin(\widetilde{\phi_a} + \widehat{\phi_a})[2\delta'\sin\widetilde{\theta_a} + \epsilon'\sin 2\widetilde{\theta_a}])\} \end{aligned} \tag{63}$$

Note that this distribution depends on (i) the W-polarimetry parameters σ, ξ, ζ of [2] and on (ii) the b-polarimetry parameters $\delta \equiv \epsilon_+ + \epsilon_-, \epsilon \equiv \epsilon_+ - \epsilon_-$ and on δ', ϵ', see Fig. 1.

By integrating out " $\cos\widehat{\theta_a}$", we obtain

$$
\begin{aligned}
\not{I}(\theta_t;\widetilde{\theta_a},\widetilde{\phi_a}+\widehat{\phi_a}) &= \tfrac{1}{2}\int d(\cos\widehat{\theta_a}) I(\theta_t;\widetilde{\theta_a},\widehat{\theta_a},\widetilde{\phi_a}+\widehat{\phi_a}) \\
&= \textstyle\sum_{q_i,g_i}(\rho_{++}^{prod}+\rho_{--}^{prod}) \\
&\{\tfrac{1}{4}[|R_+|^2+|R_-|^2]\mathbf{n_a}+\tfrac{\pi}{8\sqrt{2}}[|R_+|^2-|R_-|^2]\sin\widetilde{\theta_a}(\cos(\widetilde{\phi_a}+\widehat{\phi_a})[\delta+\epsilon\cos\widetilde{\theta_a}] \\
&-\sin(\widetilde{\phi_a}+\widehat{\phi_a})[\delta^{'}+\epsilon^{'}\cos\widetilde{\theta_a}])\}
\end{aligned}
\tag{64}
$$

which displays the difference between $\epsilon,\epsilon^{'}$ and $\delta,\delta^{'}$ in the $\cos\widetilde{\theta_a}$ dependence. Next, by also integrating out " $\cos\widetilde{\theta_a}$",

$$
\begin{aligned}
\not{I}(\theta_t;\widetilde{\phi_a}+\widehat{\phi_a}) &= \tfrac{1}{2}\int d(\cos\widetilde{\theta_a})\tfrac{1}{2}\int d(\cos\widehat{\theta_a}) I(\theta_t;\widetilde{\theta_a},\widehat{\theta_a},\widetilde{\phi_a}+\widehat{\phi_a}) \\
&= \textstyle\sum_{q_i,g_i}(\rho_{++}^{prod}+\rho_{--}^{prod}) \\
&\{\tfrac{1}{6}[|R_+|^2+|R_-|^2]\mathbf{n_a}+\tfrac{\pi^2}{32\sqrt{2}}[|R_+|^2-|R_-|^2](\delta\cos(\widetilde{\phi_a}+\widehat{\phi_a}) \\
&-\delta^{'}\sin(\widetilde{\phi_a}+\widehat{\phi_a}))\}
\end{aligned}
\tag{65}
$$

which only depends on δ and $\delta^{'}$.

For the CP-conjugate sequential-decay $\bar{t}\rightarrow W^-\bar{b}\rightarrow(l^-\bar{\nu}_l)(l^+\nu\bar{c})$, or $W^-\rightarrow j_d j_{\bar{u}}$, the distribution analogous to (62) is:

$$
\overline{\not{I}}(\theta_t;\widetilde{\theta}_b,\widehat{\theta}_b,\widetilde{\phi_b}+\widehat{\phi_b}) = \sum_{q_i,g_i}(\rho_{++}^{prod}+\rho_{--}^{prod})\{\overline{\not{R}}_{++}+\overline{\not{R}}_{--}\}
\tag{66}
$$

where

$$
\begin{aligned}
\{\overline{\not{R}}_{++}+\overline{\not{R}}_{--}\} &= \tfrac{1}{4}[\left|\overline{R}_+\right|^2+\left|\overline{R}_-\right|^2]\mathbf{n_b}+\tfrac{1}{4}[\left|\overline{R}_+\right|^2-\left|\overline{R}_-\right|^2]\{\cos\widehat{\theta}_b\mathbf{n_b^{(+)}} \\
&+\tfrac{1}{\sqrt{2}}\sin\widehat{\theta}_b(\cos(\widetilde{\phi_b}+\widehat{\phi_b})[2\bar{\delta}\sin\widetilde{\theta}_b+\bar{\epsilon}\sin 2\widetilde{\theta}_b] \\
&-\sin(\widetilde{\phi_b}+\widehat{\phi_b})[2\bar{\delta}^{'}\sin\widetilde{\theta}_b+\bar{\epsilon}^{'}\sin 2\widetilde{\theta}_b])\}
\end{aligned}
\tag{67}
$$

where $\mathbf{n_b}$ is Eq.(20-21) in [2], $\mathbf{n_b^{(+)}}$ is (49-51) above, and $\bar{\delta}\equiv\bar{\epsilon}_+ + \bar{\epsilon}_-, \bar{\epsilon}\equiv\bar{\epsilon}_+ - \bar{\epsilon}_-$.

Remark: Use of Single-Sided b-W Sequential-Decay Distributions: In the context of b-polarimetry and of joint b-W-polarimetry, especially at a linear collider, it is important to note what information can be garnered from simpler single-sided sequential-decay distributions versus S2SC distributions: Both the "full" 11-angle-variable S2SC distribution (58) and the 7-angle-variable S2SC distribution (59) depend on all four b-polarimetry interference parameters, $\epsilon_\pm, \kappa_{o,1}$ and on the analogous primed parameters. The 4 and 3-angle-variable single-sided distributions, (62) and (64), both depend on all of $\epsilon_\pm, {\epsilon_\pm}'$ but not on any of $\kappa_{o,1}, {\kappa_{o,1}}'$. However, the 2-angle-variable single-sided distribution (65) only depends on the sum $\delta = \epsilon_+ + \epsilon_-$ and on δ'.

These differences can be considered versus single-additional Lorentz structures where the presence or absence of in signatures occur similarly for ϵ_- and κ_o (and for ${\epsilon_-}'$ and ${\kappa_o}'$), and for ϵ_+ and κ_1 (and for ${\epsilon_+}'$ and ${\kappa_1}'$). With respect to measurement of only δ and/or δ' by the simplest 2-angle-variable distribution (65), there is almost complete cancellations in δ and δ' in the case of single-additional $V + A, V$ or A couplings, and some cancellation in δ' for $f_M - f_E, f_M$ or f_E couplings. This means that the 2-angle-variable single-sided distribution (65) should not be solely used because the other linear-combinations $\epsilon = \epsilon_+ - \epsilon_-$ and ϵ' need also to be measured. Also, if ϵ and ϵ' are not measured, then the simple " $S - P, S$, or P coupling's signature" of a presence/absence of effects in (ϵ_- and κ_o)/(ϵ_+ and κ_1) and likewise for the primed quantities would not be available.

Obviously, a sensitivity analysis of the "ideal statistical errors" and of the systematic errors in regard to the various helicity parameters in the case of b-polarimetry and of joint b-W-polarimetry distributions would be useful in the context of different Λ_b polarimetry methods, the expected number of events, the details of specific experiments/detectors, and available experimental information on the dynamics occurring/not-occurring in top quark decay.

4 b-Polarimetry Interference Parameters involving $A(-1,-\frac{1}{2})$

Case: In Standard Model and at Ambiguous Moduli Points

The two b-polarimetry interference parameters ϵ_+ and κ_o involving the standard model's largest amplitude, $A(0,-\frac{1}{2})$ were considered in [12]. Plots for these parameters were given therein for the case of a single-additional, real coupling g_i. For the other L-handed b-quark amplitude, $A(-1,-\frac{1}{2})$, the two analogous helicity parameters are

$$\begin{aligned}
\epsilon_- &\equiv \tfrac{1}{\Gamma}|A(-1,-\tfrac{1}{2})||A(0,\tfrac{1}{2})|\cos\gamma_- \\
&= \tfrac{1}{\Gamma} Re\left\{A(-1,-\tfrac{1}{2})A^*(0,\tfrac{1}{2})\right\} \\
\kappa_1 &\equiv \tfrac{1}{\Gamma}|A(1,\tfrac{1}{2})||A(-1,-\tfrac{1}{2})|\cos\alpha_1 \\
&= \tfrac{1}{\Gamma} Re\left\{A(1,\tfrac{1}{2})A^*(-1,-\tfrac{1}{2})\right\}
\end{aligned} \tag{68}$$

where $\gamma_- = \phi^L_{-1} - \phi^R_0$ and $\alpha_1 = \phi^R_1 - \phi^L_{-1}$, see Fig. 1.

In the SM, the two $\mathcal{O}(LR)$ helicity parameters which are between the amplitudes with the largest moduli product are ϵ_+ and κ_1, which respective depend on $A(0,-\frac{1}{2}) \sim 338$ and $A(-1,-\frac{1}{2}) \sim 220$ in $g_L = 1$ units. The R-handed amplitudes are $A(1,\frac{1}{2}) \sim -7.16$ and $A(0,\frac{1}{2}) \sim -2.33$. Unfortunately, as shown in Table 1, the tree-level values of the four b-interference parameters are only about 1% in the SM, and also at the $(S+P)$ and $(f_M + f_E)$ ambiguous moduli points [12].

Case: "(V-A) + Single Additional Lorentz Structure"

Some single-additional Lorentz structures can produce sizable effects in these four b-interference parameters: In [12], this was shown to occur in ϵ_+ for additional non-chiral V, f_M or A, f_E couplings, and for the chiral combinations $V + A, f_M - f_E$ in Figs. 9, and in κ_o for additional V, S, f_M or A, P, f_E couplings, and for $V + A, S - P, f_M - f_E$ in Figs. 10.

In the present paper, analogous plots are given for the b-interference parameters involving the $A(-1, -\frac{1}{2})$ amplitude. In Fig. 9 are plots of the b-polarimetry interference parameter ϵ_- versus η_L for the case of a single-additional, real coupling. By W-polarimetry, the η_L parameter can be measured since

$$\eta_L = \frac{1}{2}(\eta + \omega) \equiv \frac{1}{\Gamma}|A(-1, -\tfrac{1}{2})||A(0, -\tfrac{1}{2})| \cos \beta_L \tag{69}$$

where $\beta_L = \phi_{-1}^L - \phi_0^L$ is the relative phase difference the two helicity amplitudes in (69). Similarly, in Fig. 10 are plots of the b-polarimetry interference parameter κ_1 versus η_L. In both sets of plots, Figs. 9 and 10, the upper(lower) figures are for the case of an additional chiral (non-chiral) coupling.

Note that an additional $(V - A)$ coupling only effects the overall magnitude or phase of the $A(\lambda_{W^+}, \lambda_b)$ amplitudes and so an additional $(V - A)$ coupling will only effect the overall normalization of the spin-correlation functions. Note also that due to their L-handed b-quark structure, an additional $f_M + f_E$ or $S + P$ coupling does not significantly effect any of the four $\epsilon_\pm, \kappa_{o,1}$ interference parameters. For the same reason, these couplings do not significantly effect the four $\tilde{T}_{FS}$-violation ${\epsilon_\pm}', {\kappa_{o,1}}'$ parameters, see Figs. 11 and 12 below and see Figs. 1 and 2 of [13].

An additional $S - P$ coupling effects significantly only κ_o and ϵ_-. In the non-chiral case, an additional S or P coupling effects significantly only κ_o and ϵ_-. As in [12], the "oval shapes" of the curves in Figs. 9 and 10 as the coupling strength varies is due to the non-zero value of the m_b mass, $m_b = 4.5 GeV$.

Case: Explicit $\tilde{T}_{FS}$ Violation from a Single-Additional Lorentz Structure

By "explicit $\tilde{T}_{FS}$ violation", we mean an additional complex-coupling, $g_i/2\Lambda_i$ or g_i, associated with a specific single-additional Lorentz structure, $i = S, P, S \pm P, \ldots$. For a single-additional gauge-type coupling V, A, or $V+A$, there is not a significant signature in ${\eta_L}'$ due to the T-violation "masking mechanism" associated with gauge-type couplings [2]. For example: for an additional pure imaginary g_R coupling plus a purely real g_L, ${\eta_L}' \sim m_b/m_t$. In [13], we considered the effects on the ${\epsilon_+}'$ and ${\kappa_o}'$ helicity parameters of pure-imaginary couplings. These two parameters involve the $A(0, -\frac{1}{2})$ amplitude. Here we consider the analogous effects on ${\epsilon_-}'$ and ${\kappa_1}'$ which involve the $A(-1, -\frac{1}{2})$ amplitude.

However, as in [13], there are large indirect effects on other helicity parameters in the case of explicit $\tilde{T}_{FS}$-violation due to a single-additional pure-imaginary coupling. Therefore, while the following plots do show that sizable $\tilde{T}_{FS}$-violation signatures can occur due to pure-imaginary additional couplings, such additional couplings can usually be more simply excluded by 10% precision measurement of the probabilities $P(W_L)$ and $P(b_L)$, and of the W-polarimetry interference parameters η and ω.

Figs. 11 and 12 display plots of the b-polarimetry interference parameters ${\epsilon_-}'$ and ${\kappa_1}'$ versus the coupling strength for the case of a single-additional, pure-imaginary coupling. These helicity parameters are defined by

$$\begin{aligned} \epsilon_-' &\equiv \frac{1}{\Gamma}|A(-1,-\tfrac{1}{2})||A(0,\tfrac{1}{2})| \sin\gamma_- \\ \kappa_1' &\equiv \frac{1}{\Gamma}|A(1,\tfrac{1}{2})||A(-1,-\tfrac{1}{2})| \sin\alpha_1 \end{aligned} \tag{70}$$

The "cosine's" of γ_- and α_1 occurred above in (68).

In Figs. 11 and 12 the upper plots (lower plots) are respectively for the case of an additional non-gauge (gauge) type coupling. The SM limits are correspondingly at the "wings" where $|\Lambda_i| \rightarrow$

∞ ("origin" where $g_i \to 0$). As in the case of a purely real additional coupling, an additional $S-P, S$ or P coupling effects significantly only κ_o' and ϵ_-'. In Figs. 11 and 12, the peaks in the curves usually do correspond to where $|\sin\alpha_1| \sim 1$ and $|\sin\gamma_-| \sim 1$. The exceptions are: in Fig. 11 for ϵ_-' for f_M, f_E where respectively $|\sin\gamma_-| \sim 0.52, 0.48$ at $|\Lambda_i| = 70GeV$ and for V, A where $|\sin\gamma_-| \sim 0.81$ at $|g_i| = 0.75$; and in Fig. 12 for κ_1' for f_M, f_E where respectively $|\sin\alpha_1| \sim 0.52, 0.48$ at $|\Lambda_i| = 70GeV$ and for V, A where $|\sin\alpha_1| \sim 0.86$ at $|g_i| = 0.75$. The drops in the curves for small $|\Lambda_i|$'s is due to the vanishing of the "sine" of the corresponding relative phase.

5 Concluding Remarks

The implications and directions for further development of many of the results in this paper will have to deduced as data accumulates from model-independent top quark spin-correlation analyses. However, even if the standard model predictions are correct to better than the 10% level, top quark decay will still be uniquely sensitive to "new" short-distance physics at nearby-but-not-yet-explored distance scales because of the absence of hadronization effects and the large m_t mass.

(1) From analyses only using W-polarimetry, it will important to ascertain the magnitude of the two R-handed b-quark amplitudes for $t \to W^+ b$. If these are indeed 30 to 100 times smaller than the L-handed b-quark amplitudes as occurs in the SM and occurs at the $(S+P)$ and $(f_M + f_E)$ ambiguous moduli points, then the data should show that $[P(b_L) = \frac{1}{2}(1+\sigma)] \simeq 1$, that $[2P(W_L) - P(b_L) = \frac{1}{2}(1 + 2\sigma - \xi)] \simeq \zeta$, and that $\omega \simeq \eta$. Consequently, the b-polarimetry interference parameters $\epsilon_\pm, \kappa_{o,1}$, and their primed analogues, must be small versus their unitarity

limit of 0.5. In the SM and for the very interesting $(f_M + f_E)$ dynamical ambiguity $\frac{\epsilon_+}{0.5} \sim 3\%$, $\frac{\kappa_1}{0.5} \sim 2\%$, $\frac{\kappa_o}{0.5} \sim 1\%$ and $\frac{\epsilon_-}{0.5} \sim 0.6\%$. In this case, **both** a large and clean sample of t and $\bar{t}$ decays **and** mature Λ_b polarimetry methods will be required for a "complete measurement" of the $A(\lambda_{W^+}, \lambda_b)$ and the $B(\lambda_{W^-}, \lambda_{\bar{b}})$ amplitudes.

However, as shown by the last 4 figures in this paper and by the analogous ones in [12, 13], the situation is very different it there are sizable R-handed b-quark amplitudes. Such amplitudes can occur in the case of single-additional Lorentz structures. In this case, the use of Λ_b polarimetry in top quark spin-correlation analyses could be uniquely sensitive and useful in disentangling new physics at the Tevatron and LHC.

(2) Being able to neglect R-handed amplitudes is also important with respect to searching for $\tilde{T}_{FS}$-violation signatures. If R-handed amplitudes are negligible, then from analyses only using W-polarimetry, the data can show that $0 \neq [\omega' \simeq \eta']$ or that $0 \neq [({\eta_L}')^2 \simeq \frac{1}{4}[P(b_L) + \zeta][P(b_L) - \zeta] - (\eta_L)^2]$ as signatures for $\tilde{T}_{FS}$-violation. With respect to b-polarimetry information the situation is as discussed in remark (1) above: If the R-handed amplitudes are $\sim 1\%$ then the four "primed" $\tilde{T}_{FS}$-violation b-polarimetry parameters will have magnitudes at most of $\sim 1\%$, c.f. the lower parts of Figs. 1 and 2. However, if R-handed b-quark amplitudes are sizable, then $\tilde{T}_{FS}$-violation signatures can be much larger as shown by the "primed" helicity-parameter figures in this paper and in [13].

(3) When final $\bar{\nu}$ angles-and-energy variables are used in top quark analyses for $b \to l^- \bar{\nu} c$ decay in the Λ_b mass region, then Λ_b polarimetry signatures will not be suppressed. This is shown above by a simple argument in the paragraph following (6), and see Appendix. Similarly, ν angles-and-energy variables will not suppress signatures from $\bar{b} \to l^+ \nu \bar{c}$ decay.

(4) At the time of a future linear collider, it will be important to reconsider what information about top quark decays can be better obtained from (i) the use of the alternative-angles of Fig.6, and from (ii) the simpler single-sided b-W-interference distributions versus S2SC distributions. These issues are briefly discussed in the "remarks" following (58) and (67). The single-sided distributions considered above (62-67) depend on $\epsilon_\pm, {\epsilon_\pm}'$ but not on $\kappa_{o,1}, {\kappa_{o,1}}'$.

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A Appendix: Formulas for $\left|R^b_\Lambda(E_l)\right|^2$ type factors when $b \to l^- \overline{\nu} c$, and $\overline{b} \to l^+ \nu \overline{c}$:

In this appendix are listed explicit formulas for the $\left|R^b_\Lambda(E_l)\right|^2$ and $\left|\overline{R^b_{\overline{\Lambda}}}(E_{\bar{l}})\right|^2$ factors, assuming the SM's pure $V-A$ ($V+A$) couplings respectively for b ($\bar{b}$) decay. These formulas follow by use of [17], the second paper in [8] and its references.

For the decay $b \to l^- \overline{\nu} c$, in (6)

$$\left|R^b_\pm\right|^2 = R(x_l) \mp S(x_l) \tag{71}$$

where

$$R(x_l) = \frac{1}{f(\varepsilon_c)} \frac{x_l^2(1-\varepsilon_c - x_l)^2}{(1-x_l)^2} \left[3 - 2x_l + \varepsilon_c \left(\frac{3-x_l}{1-x_l}\right)\right] \tag{72}$$

$$S(x_l) = \frac{1}{f(\varepsilon_c)} \frac{x_l^2(1-\varepsilon_c - x_l)^2}{(1-x_l)^2} \left[-1 + 2x_l + \varepsilon_c \left(\frac{1+x_l}{1-x_l}\right)\right] \tag{73}$$

with $x_l = 2E_l/m_b$, $\varepsilon_c = m_c^2/m_b^2$, and $f(\varepsilon_c) = 1-8\,\varepsilon_c+8\varepsilon_c^2-\varepsilon_c^4-12\varepsilon_c^2\log\varepsilon_c$. For the CP-conjugate mode $\overline{b} \rightarrow l^+\nu\overline{c}$, in (15-16, 48-55)

$$\left|\overline{R_{\pm}^b}\right|^2 = R(\overline{x_l}) \pm S(\overline{x_l}) \tag{74}$$

with

$$\begin{aligned} \overline{\hat{\rho}_{\lambda_{\bar{b}}\lambda'_{\bar{b}}}}(\overline{b} &\rightarrow l^+\nu\overline{c}) = \overline{\hat{\rho}_{\lambda_{\overline{b}}\lambda'_{\overline{b}}}}(\widehat{\phi_b},\widehat{\theta}_b,E_{\overline{l}}) \\ &= \sum_{\overline{\Lambda}=\pm\frac{1}{2}} D^{1/2*}_{\lambda_{\overline{b}}\overline{\Lambda}}(\widehat{\phi_b},\widehat{\theta}_b,0)D^{1/2}_{\lambda'_{\overline{b}}\overline{\Lambda}}(\widehat{\phi_b},\widehat{\theta}_b,0)\left|\overline{R^b_{\overline{\Lambda}}(E_{\overline{l}})}\right|^2 \end{aligned} \tag{75}$$

The $\bar{\nu}$ (or ν) angle-energy-spectra is very useful in Λ_b polarimetry methods, see [3-6, 8-10] and remarks above in paragraph after (6). Only simple changes are needed in the present formalism to use $\bar{\nu}$ (or ν) angles-and-energy variables: For describing $b \rightarrow l^-\overline{\nu}c$, the angles $\widehat{\varphi}'_a$, $\widehat{\theta}'_a$ can be used to label the anti-neutrino momentum direction in place of the l^- angles $\widehat{\varphi}_a$, $\widehat{\theta}_a$. This is only a matter of adding "primes" to these angles in the various expressions. In place of (6) one has

$$\begin{aligned} \hat{\rho}_{\lambda_b\lambda'_b}(b &\rightarrow l^-\bar{\nu}c) = \hat{\rho}_{\lambda_b\lambda'_b}(\widehat{\phi}'_a,\widehat{\theta}'_a,E_{\overline{\nu}}) \\ &= \sum_{\Lambda=\pm\frac{1}{2}} D^{1/2*}_{\lambda_b\Lambda}(\widehat{\phi}'_a,\widehat{\theta}'_a,0)D^{1/2}_{\lambda'_b\Lambda}(\widehat{\phi}'_a,\widehat{\theta}'_a,0)\left|N^b_{\Lambda}(E_{\overline{\nu}})\right|^2 \end{aligned} \tag{76}$$

where

$$\left|N^b_{\pm}\right|^2 = U(y_\nu) \mp V(y_\nu) \tag{77}$$

where $V(y_\nu) = -U(y_\nu)$ and

$$U(x_l) = \frac{1}{f(\varepsilon_c)}\frac{6y_\nu^2(1-\varepsilon_c-y_\nu)^2}{(1-y_\nu)} \tag{78}$$

with $y_\nu = 2E_{\overline{\nu}}/m_b$. Therefore, in the expressions in the text, when the $\bar{\nu}$ variables are used, $|R_+|^2$ of (9) is replaced by $2U(y_\nu)$ and $|R_-|^2$ is set equal to zero. This vanishing is occurring because in the b-quark rest frame, Λ in $\left|N^b_\Lambda(E_{\overline{\nu}})\right|^2$ is the eigenvalue of $\mathbf{J}\cdot\widehat{\mathbf{p}}_{\overline{\nu}}$ where $\mathbf{J}$ is the angular momentum operator. Therefore, for $m_\nu = 0$, the final $\overline{\nu}$ is purely R-handed so $\left|N_-^b\right|^2 = 0$. See paragraph after (5) in [17].

Similarly for the neutrino in $\overline{b} \rightarrow l^+\nu\overline{c}$: In place of the angles $\widehat{\varphi}_b$, $\widehat{\theta}_b$, the angles $\widehat{\varphi}_b'$, $\widehat{\theta}_b'$ can be used to label the neutrino momentum direction. Then in place of (75) one has

$$\begin{aligned} \overline{\hat{\rho}_{\lambda_{\bar{b}}\lambda'_{\bar{b}}}}(\overline{b} &\rightarrow l^+\nu\overline{c}) = \overline{\hat{\rho}_{\lambda_{\bar{b}}\lambda'_{\bar{b}}}}(\widehat{\phi}_b', \widehat{\theta}_b', E_\nu) \\ &= \sum_{\overline{\Lambda}=\pm\frac{1}{2}} D^{1/2*}_{\lambda_{\bar{b}}\overline{\Lambda}}(\widehat{\phi}_b', \widehat{\theta}_b', 0) D^{1/2}_{\lambda'_{\bar{b}}\overline{\Lambda}}(\widehat{\phi}_b', \widehat{\theta}_b', 0) \left|\overline{N_{\overline{\Lambda}}^{b}}(E_\nu)\right|^2 \end{aligned} \tag{79}$$

where

$$\left|\overline{N_\pm^b}\right|^2 = U(\overline{y_\nu}) \pm V(\overline{y_\nu}) \tag{80}$$

When the ν variables are used, $\left|\overline{R_-}\right|^2$ is replaced by $2U(\overline{y_\nu})$ and $\left|\overline{R_+}\right|^2 = 0$ because the final $\bar{\nu}$ is purely L-handed.

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Table Captions

Table 1: For the standard model and at the $(S+P)$ and (f_M+f_E) ambiguous moduli points, numerical values of η_L and of the four b-polarimetry interference parameters, $\epsilon_\pm, \kappa_{o,1}$ which are defined by the lower sketch in Fig. 1. $[m_t = 175 GeV,\ m_W = 80.35 GeV,\ m_b = 4.5 GeV\ ]$

Figure Captions

FIG. 1: For $t \rightarrow W^+ b$ decay, a display of the four helicity amplitudes $A(\lambda_{W^+}, \lambda_b) = |A| \exp(i\phi^{L/R}_{\lambda_{W^+}})$ relative to the W^+ and b helicities. The upper sketch defines the measurable

relative phases and the lower sketch defines their corresponding real-part and imaginary-part (primed) helicity parameters. Throughout this paper the symbol $i \equiv \sqrt{-1}$. For a pure $V - A$ coupling, the β's vanish and all the α's and γ's equal $+\pi$ (or $-\pi$) to give the overall minus sign in each of the standard model's R-handed b-quark amplitudes, see [12, 13].

FIG. 2: For $\bar{t} \rightarrow W^- \bar{b}$ decay (the CP-conjugate process), the relative phases and associated helicity parameters are defined as in Fig. 1 but now with "barred" accents. Compare (18-29). Now the R-handed $\bar{b}$ amplitudes, $B(\lambda_{W^-}, \frac{1}{2})$ reference the $\bar{\alpha}_0, \bar{\alpha}_1$ relative-phase-directions, $B(\lambda_{W^-}, \lambda_{\bar{b}}) = |B| \exp i\phi^{bR/L}_{\lambda_{W^-}}$.

FIG. 3: The three angles θ^t_1, θ^t_2 and ϕ describe the first stage in the sequential-decays of the $(t\bar{t})$ system in which $t \rightarrow W^+ b$ and $\bar{t} \rightarrow W^- \bar{b}$.

FIG. 4: For the sequential decay $t \rightarrow W^+ b$ followed by $b \rightarrow l^- \bar{\nu} X$, the two pairs of spherical angles $\widehat{\theta^t_1}$, $\widehat{\phi^t_1}$ and $\widehat{\theta_a}$,$\widehat{\phi_a}$ describe respectively the b momentum in the "first stage" $t \rightarrow W^+ b$ and the l^- momentum in the "second stage" $b \rightarrow l^- \bar{\nu} X$. Angles associated with the b, or Λ_b, branching's momenta directions have "hat" accents whereas those associated with the W^+ branching have "tilde" accents, as in Fig. 7 below. The spherical angles $\widehat{\theta_a}$, $\widehat{\phi_a}$ specify the l^- momentum in the b rest frame when the boost is from the t_1 rest frame. In this figure, $\widehat{\phi^t_1}$ is shown equal to zero for simplicity of illustration. The positive $\widehat{x_a}$ direction is specified by the $\bar{t}$ momentum direction.

FIG. 5: This figure is symmetric versus Fig. 4. The spherical angles $\widehat{\theta_b}$, $\widehat{\phi_b}$ specify the l^+ momentum in the $\bar{b}$ rest frame when the boost is from the $\bar{t}_2$ rest frame.

FIG. 6: The spherical angles $\widehat{\theta_1}$,$\widehat{\phi_1}$ specify the l^- momentum in the b rest frame when the boost is directly from the $(t\bar{t})_{cm}$ frame. Similarly, $\widehat{\theta_2}$, $\widehat{\phi_2}$ specify the l^+ momentum in the $\bar{b}$ rest frame. The $b\bar{b}$ production half-plane specifies the positive $\widehat{x_1}$ and $\widehat{x_2}$ axes.

FIG. 7: This figure labels the W^+ branch analogous to the labels in Fig. 4 for the b branch. The two pairs of spherical angles θ_1^t, ϕ_1^t and $\widetilde{\theta_a}$,$\widetilde{\phi_a}$ describe the respective stages in the sequential decay $t \rightarrow W^+ b$ followed by $W^+ \rightarrow j_{\bar{d}} j_u$ [or $W^+ \rightarrow l^+ \nu$]. The spherical angles $\widetilde{\theta_a}$, $\widetilde{\phi_a}$ specify the $j_{\bar{d}}$ jet [or the l^+] momentum in the W^+rest frame.

FIG. 8: The labels are as in Fig. 7 but here for the W^- branch. The spherical angles $\widetilde{\theta}_b$, $\widetilde{\phi_b}$ specify the j_d jet [or the l^-] momentum in the W^-rest frame.

FIG. 9: Plots of the b-polarimetry interference parameter ϵ_- versus η_L for the case of a single-additional, real coupling. The SM prediction is shown by the solid rectangle. The **upper plot** is for a single-additional, real chiral coupling. The value of $\epsilon_- \sim 0$ for the other couplings $S+P, f_M+f_E$. The **lower plot** is for a single-additional, real non-chiral coupling. The omitted curves for A, P, f_E are respectively almost mirror images about the η_L axis. Coupling strengths at representative points are given in the table associated with the respective plot. The unitarity limit is a circle of radius 0.5, centered at the origin, in each of these plots and in those in Fig. 10.

FIG. 10: Plots of the b-polarimetry interference parameter κ_1 versus η_L for the case of a single-additional, real coupling. The **upper plot** is for a single-additional, real chiral coupling; the value of $\kappa_1 \sim 0$ for the omitted couplings $S \pm P, f_M + f_E$. The **lower plot** is for a single-additional, real non-chiral coupling; the omitted curves for A, f_E are respectively almost mirror images about the η_L axis. The value of $\kappa_1 \sim 0$ for the omitted couplings S, P.

FIG. 11: Plots of the b-polarimetry, $\tilde{T}_{FS}$-violation, interference parameter ${\epsilon_-}'$ versus coupling strength for the case of a single-additional **pure-imaginary coupling**. Curves are for non-gauge-type couplings (**upper plot**), gauge-type couplings (**lower plot**), versus respectively the effective-mass scale Λ_i, or coupling strength g_i in $g_L = 1$ units. Curves are omitted in these plots

and in the following Fig. 12 when the couplings produce approximately zero deviations in the helicity parameter of interest.

FIG. 12: Plots of the b-polarimetry, $\tilde{T}_{FS}$-violation, interference parameter ${\kappa_1}'$ versus coupling strength for the case of a single-additional **pure-imaginary coupling**. Curves are for non-gauge-type coupling (**upper plot**), gauge-type coupling (**lower plot**), versus respectively the effective-mass scale Λ_i, or coupling strength g_i in $g_L = 1$ units.