Noncommutative branes from M theory

Yolanda Lozano*
Theory Division, CERN, 1211 Genève 23, Switzerland
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The analysis of the worldvolume effective actions of the M-theory Kaluza-Klein monopole and 9-brane suggests that it should be possible to describe non-Abelian configurations of M2-branes or M5-branes if the M2-branes are transverse to the eleventh direction and the M5-branes are wrapped on it. This is determined by the fact that the Kaluza-Klein monopole and the M9-brane are constrained to move in particular isometric spacetimes. We show that the same kind of situation is implied by the analysis of the brane descent relations in M theory. We compute some of the noncommutative couplings of the worldvolume effective actions of these non-Abelian systems of M2 and M5 branes, which may be responsible for the existence of configurations corresponding to *N* branes expanding into a higher dimensional M-brane. The reduction to type II brings up new descriptions of coincident D-branes at strong coupling. We show that these systems have the right noncommutative charges to describe certain expanded configurations playing a role in the framework of the AdS/conformal field theory correspondence. Finally, we discuss the realization of noncommutative brane configurations as topological solitons in non-Abelian brane-antibrane systems.

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I. INTRODUCTION AND SUMMARY

Non-Abelian D-brane systems have recently attracted a lot of attention. In remarkable papers Myers and Taylor and Van Raamsdonk observed that a system of *N* coincident D*p*-branes can develop multipole moments under Ramond-Ramond fields that would normally couple to higher dimensional branes [1,2]. This is possible because the non-Abelian D*p*-brane embedding coordinates become noncommutative. An external field $C^{(q+1)}$, $q>p$ [p, q even (odd) in type IIA (IIB)] can polarize the *p*-branes to expand into a noncommutative configuration that can be interpreted as a D*q*, N D*p* bound state. Alternatively the original D*p*-branes can be represented as a single D*q*-brane with N units of worldvolume instanton-like density [3]. Both approaches agree in the large N limit [1,4].

The "dielectric" property has been shown to play an important role within the AdS conformal field theory (CFT) correspondence. Dielectric branes have been used to find non-singular string theory duals of gauge theories living on D-branes with reduced supersymmetry [5–7]. In [5] the supergravity dual of a four dimensional $\mathcal{N}=1$ confining gauge theory, obtained by perturbing the $\mathcal{N}=4$ gauge theory living on N D3-branes, was identified as a non-singular spacetime with an expanded brane source arising from Myers' dielectric effect. In particular, a mapping between the gauge theory vacua and states corresponding to D3-branes being polarized into D5-branes and Neveu-Schwarz 5-branes (NS5-branes), with worldvolume $R^4\times S^2$, in $AdS_5\times S^5$ was found, with the Higgs vacuum represented by a single D5-brane configuration and the confining vacuum by an NS5-brane. Similar issues have also been investigated in an M-theory framework. A perturbation of the $\mathcal{N}=8$ three dimensional gauge theory living on N M2-branes to $\mathcal{N}=2$ has been shown to be dual to M2-branes expanding into an M5-brane of geometry $R^3\times S^3$ in $AdS_4\times S^7$ [8]. The existence of polarized M2-branes is expected from duality. However, the system cannot be described as a collection of branes in an external field developing a dipole moment and expanding, given that the degrees of freedom of the worldvolume theory of coincident M2-branes are not known. The approach of Ref. [8] was to consider the alternative description as an M5-brane in $AdS_4\times S^7$ with a non-trivial flux on S^3, carrying M2-brane charge N. This was also the approach of [7], where the dual of the confining vacua of a perturbed three dimensional gauge theory living on D2-branes was identified as D2-branes polarized into an NS5-brane.

Expanding N M0, M$(n-2)$ systems have also been proposed to describe gravitons carrying angular momentum in $AdS_m\times S^n$, with the M$(n-2)$-brane moving on the S^n sphere [9]. The "giant" gravitons of [9] in $AdS_7\times S^4$ are identified with (N M0, M2) bound states in a constant four-form magnetic field strength, where the M2-brane expands on the S^4 of $AdS_7\times S^4$ [10]. Similarly (N M0, M2) bound states in an electric four-form field strength can be constructed such that the M2-branes expand into the AdS_4 component of $AdS_4\times S^7$. Configurations of (N M0, M$(m-2)$) branes in which the M$(m-2)$-brane expands out into the AdS_m component of the spacetime have been associated to "dual giant gravitons" (see [11]).

In this paper we give an indication that it may be possible to understand all these configurations in terms of non-Abelian branes expanding into a higher dimensional brane. We start in Sec. II by summarizing the main results in [1,4], putting special emphasis in the interpretation of the couplings in the Wess-Zumino action describing the non-Abelian system in terms of expanding configurations. In Sec. III we study non-Abelian brane configurations in M-theory. By analyzing the couplings present in the Kaluza-Klein monopole and M9-brane effective actions we give an indication of the special type of non-Abelian M2 and M5 configurations that can arise as topological solutions in M-theory. The M2-

*Email address: yolanda.lozano@cern.ch

branes appear delocalized in the eleventh direction, whereas the M5-branes are wrapped on it. This situation is consistent with the fact that in the non-Abelian case the M2 and M5 branes should be able to expand into the Kaluza-Klein monopole and the M9-brane, which see the eleventh direction as a special isometric direction. The configurations of M2 and M5 branes that we find were shown to play a role as supergravity duals of (S)$YM_{3,6}$ in certain regions of the parameter space [12].

We will propose a worldvolume Wess-Zumino effective action describing non-Abelian M2 and M5 branes. The requirement is that they should reproduce the effective actions of non-Abelian D2 and D4 branes upon reduction to type IIA along the special isometric direction. We will argue that the terms that couple in the effective action may be associated to non-commutative configurations corresponding to the branes expanding into higher dimensional branes. Among these we find the case in which N M2-branes expand into an M5-brane, considered in [8]. We will also discuss the worldvolume effective action describing coincident M0-branes, and we will identify the couplings that could be responsible for the configurations in which N M0-branes expand into M2 and M5 branes [9,11,10].

In Sec. IV we show that the non-Abelian configurations that we have constructed in M-theory give rise, upon reduction to type IIA, to strongly coupled Dp-brane systems with the right worldvolume couplings to explain some non-Abelian configurations that have been identified as supergravity duals of certain gauge theories living on D-branes. We will see for instance that a delocalized D2-brane system contains a coupling $i_\Phi i_\Phi i_k B^{(6)}$[1] in its effective action, that would be associated to the N D2, NS5 bound state considered in [7]. This system is related by T duality with the N D3, NS5 bound state of [5]. In Sec. V we show that the strongly coupled Dp-brane systems constructed in type IIA by reduction from M-theory are connected to strongly coupled Dp-brane systems in type IIB. Delocalized N D2-branes are mapped for instance onto N D3-branes at strong coupling. The reason a distinction must be made between weakly coupled and strongly coupled D3-branes is that in the non-Abelian case the corresponding actions cannot be shown to be related by an SL(2,Z) worldvolume duality transformation. This has the implication that non-Abelian D2-branes at strong coupling should only have manifest O(6) transverse rotational symmetry, which is consistent with the fact that the strongly coupled D2-branes that we can construct by reduction from M-theory are delocalized in one of the space directions, derived from the fact that our M2-brane system is $O(7)\subset O(8)$ (transverse) rotationally invariant. There are arguments however showing that the three dimensional theory describing the M2-brane system should be superconformal in the infra-red, with global symmetry enhanced from O(7) to O(8) [13,14]. We will discuss various of these strongly coupled configurations.

Finally, in Sec. VI we consider non-Abelian Dp brane-antibrane systems, and show that polarized branes may arise as worldvolume solitons from these systems after tachyon condensation. As we will see, the non-Abelian brane-antibrane system couples to the right terms to describe this situation. In Sec. VII we identify the corresponding M-theory brane-antibrane configurations and show in Sec. VIII that the reduction to type IIA predicts as well the appearance of non-Abelian expanding F1-branes, NS5-branes and Kaluza-Klein monopoles as topological solitons.

II. THE NON-ABELIAN ACTION

In this section we summarize the main results in [1] and [2]. We will be using the notation of [1]. The reader is referred to these references for more details.

The Born-Infeld part of the worldvolume effective action proposed in [1,2] to describe a system of N coincident Dp-branes is given by

$$S_{\rm BI}=-T_p\int \mathrm{Tr}\left(e^{-\phi}\sqrt{-\det\{P[E_{ab}+E_{ai}(Q^{-1}-\delta)^{ij}E_{jb}]+(2\pi\alpha')F_{ab}\}\det(Q^i_j)}\right). \tag{2.1}$$

Here $Q^i_j\equiv\delta^i_j+i(2\pi\alpha')[\Phi^i,\Phi^k]E_{kj}$, $E_{\mu\nu}\equiv g_{\mu\nu}+B^{(2)}_{\mu\nu}$, and $g_{\mu\nu}$, $B^{(2)}_{\mu\nu}$ are the ten dimensional spacetime metric and NS-NS 2-form. Static gauge is assumed, i.e. the worldvolume coordinates are taken as $\xi^a=X^a$ for $a=0,1,\dots,p$, whereas the remaining spacetime coordinates are rescaled as $X^i=2\pi\alpha'\Phi^i$, with $i=p+1,\dots,9$, in such a way that the adjoint scalars Φ^i have dimensions of length^{-1}, like the gauge fields. F_{ab} is the non-Abelian Born-Infeld field strength: $F_{ab}=\partial_aA_b-\partial_bA_a+i[A_a,A_b]$, with $A_a=A^{(n)}_aT_n$ and T_n the N^2 generators of U(N), normalized such that $\mathrm{Tr}(T_nT_m)=N\delta_{nm}$. Finally, P denotes the pull-back to the $p+1$ dimensional worldvolume, which in the non-Abelian case is defined with covariant derivatives [15] $\mathcal{D}_a\Phi^i=\partial_a\Phi^i+i[A_a,\Phi^i]$, as required by gauge invariance and implied by T-duality. In this action the symmetrized trace prescription of Tseytlin [16] is adopted (see however [17] and references therein).

The Wess-Zumino (WZ) part of the action reads [1,2]

$$S_{\rm WZ}=\mu_p\int \mathrm{Tr}\left\{P\left[e^{i(2\pi\alpha')i_\Phi i_\Phi}\left(\sum_n C^{(n)}e^{B^{(2)}}\right)\right]e^{2\pi\alpha' F}\right\}. \tag{2.2}$$

Here i_Φ denotes the interior product with Φ^i: $(i_\Phi C^{(r)})_{i_2\dots i_r}=\Phi^{i_1}C^{(r)}_{i_1\dots i_r}$. The T-duality analysis reveals

[1] Φ are the embedding scalars and $B^{(6)}$ the NS-NS 6-form. See the notation in the next section.

that it must act both on the NS-NS 2-form and the Ramond-Ramond (RR) potentials. The most striking aspect of this action is that it involves couplings to RR potentials with form degree larger than the dimensionality of the worldvolume. These fields can couple to the $p+1$ dimensional worldvolume by means of the interior products with the non-Abelian scalars. The presence of these couplings has been confirmed by direct examination of string scattering amplitudes in [18]. Some implications were already analyzed in [1] and [4], where it was shown that they gave charge to interesting non-commutative solutions in the worldvolume of the N Dp-branes.

Let us consider as an illustration the collection of N D0-branes in a constant RR 4-form field strength discussed in [1]. The coupling of the N D0-branes to this field can be read from Eq. (2.2), in particular from:

$$\int \mathrm{Tr}P[i_\Phi i_\Phi C^{(3)}]=\int dt\mathrm{Tr}\{\Phi^j\Phi^i[C^{(3)}_{ijt}(\Phi,t)+(2\pi\alpha')C^{(3)}_{ijk}(\Phi,t)\mathcal{D}_t\Phi^k]\}. \tag{2.3}$$

Since the background fields are functionals of the non-Abelian scalars, $C^{(3)}(\Phi,t)$ is defined in terms of the non-Abelian Taylor expansion [19]:

$$C^{(3)}(\Phi,t)=e^{(2\pi\alpha')\Phi^i\partial_{x^i}}C^{(3)}(t)=C^{(3)}(t)+(2\pi\alpha')\Phi^k\partial_k C^{(3)}(t)+\ \dots\ . \tag{2.4}$$

Contraction of the first term in Eq. (2.4) with $i_\Phi i_\Phi$ gives a vanishing trace, whereas the second term together with the first contribution to the expansion of the last term in Eq. (2.3) yield

$$\int dt\mathrm{Tr}(\Phi^i\Phi^j\Phi^k)F^{(4)}_{tijk}(t). \tag{2.5}$$

Combining this term with the leading order scalar potential from the Born-Infeld action, it is possible to construct an explicit non-commutative solution to the equations of motion with a non-vanishing dipole coupling [1]. One can also notice that going beyond leading order a whole series of higher order multipole couplings arises. This is the D-brane analog of the dielectric effect of electromagnetism. The nontrivial $F^{(4)}$ field has the effect of polarizing the D0-branes to expand into a non-commutative configuration which can be interpreted as a spherical D2-brane with N D0-branes bound to it. Reference [1] also investigated to what extent this configuration could be described as a solution in the Abelian worldvolume theory of the D2-brane, with the remarkable result that the two approaches agree up to $1/\mathrm{N}^2$ corrections.

The previous analysis can be generalized to Dp-branes in a background of constant $F^{(p+4)}$ field strength. Starting with a flat Dp-brane it is energetically favorable for the brane to expand into a non-commutative configuration with spatial geometry $R^p\times S^2$ and non-vanishing dipole charge, as implied by the term

$$\int d\xi^0 d\xi^1\dots d\xi^p\mathrm{Tr}(\Phi^i\Phi^j\Phi^k)F^{(p+4)}_{01\dots pijk}(\xi). \tag{2.6}$$

The contribution of this term to the scalar potential for N coincident D-strings was discussed in [4], and it was shown that there exists a non-commutative solution describing polarized D-strings, which is interpreted as a spherical (N D1, D3) bound state. Reference [20] considered a more general $(p+4)$-form field strength than the one considered in [1] and obtained other non-commutative configurations corresponding to certain fuzzy cosets.

As another interesting application Ref. [4] considered a similar type of noncommutative solution in the worldvolume of N coincident D-strings in the absence of any nontrivial background fields. This solution was interpreted as a funnel describing the expansion of the D-strings into an orthogonal D3-brane. This configuration acts as a source for the RR 4-form potential thanks to the non-trivial expectation values of the non-Abelian scalars, which contribute through the term: $\int_{R^{1+1}}\mathrm{Tr}(i_\Phi i_\Phi C^{(4)})$. The dual description in the worldvolume of the D3-brane is in terms of a spike solution corresponding to D-strings attached to the D3-brane, a configuration that has been extensively studied in the literature [21]. In [4] it is shown that the two descriptions have complementary ranges of validity, so that the non-commutative D-string theory point of view is reliable at the center of the spike, where the D3-brane description is expected to break down. Again both descriptions turn out to be in agreement in the large N limit. As pointed out in [4] the extension to Dp-branes opening up into orthogonal D$(p+2)$-branes is straightforward. It is governed by the term $\int_{R^{p+1}}\mathrm{Tr}(i_\Phi i_\Phi C^{(p+3)})$.

In the next section we will discuss similar non-commutative couplings in M-theory.

III. NON-ABELIAN CONFIGURATIONS IN M-THEORY

Let us start by analyzing the kind of non-Abelian configurations of branes that one can construct in M-theory. Our approach will be to represent these non-Abelian systems as Kaluza-Klein monopoles (or M9-branes) with N units of worldvolume instanton-like density.

The worldvolume field content of the M-theory Kaluza-Klein monopole is that of the seven dimensional U(1) vector multiplet, involving 3 scalars and 1 vector.[2] A system of N coincident Kaluza-Klein monopoles is then described by a seven dimensional U(N) vector multiplet. Therefore the Wess-Zumino action contains the couplings (see [23])[3]

[2]Recall that the embedding coordinates contribute with 3 degrees of freedom because one of the scalars is eliminated through the gauging of the Taub-NUT (Newman-Unti-Tamburine) isometry of the background [22].

[3]We ignore all numerical prefactors and the contribution of the A-roof genus. Hats indicate eleven dimensional fields. l_p denotes the eleven dimensional Planck length. Our conventions are that $(i_{\hat{k}}\hat{L})_{\hat{\mu}_2\dots\hat{\mu}_r}=\hat{k}^{\hat{\mu}_1}\hat{L}_{\hat{\mu}_1\dots\hat{\mu}_r}$, and $i_{\hat{k}}\hat{N}^{(8)}=C^{(7)}$, $i_{\hat{k}}\hat{\tilde{C}}=C^{(5)}+\dots$ upon reduction along the Killing direction.

$$S_{WZ}^{NKK}=\mu_6\int_{R^{6+1}}\mathrm{Tr}(i_{\hat{k}}\hat{N}^{(8)}+l_p^2 i_{\hat{k}}\hat{\tilde{C}}\wedge\hat{\mathcal{F}}+l_p^4\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}D\hat{X}^{\hat{\mu}}D\hat{X}^{\hat{\nu}}D\hat{X}^{\hat{\rho}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}+\ \dots).\quad(3.1)$$

$\hat{k}^{\hat{\mu}}$ denotes the Killing vector along the Taub-NUT direction, and $\hat{N}^{(8)}$ is its Poincaré dual when considered as a 1-form $\hat{k}_{\hat{\mu}}$. $\hat{C}$ ($\hat{\tilde{C}}$) denote the 3-form (6-form) of eleven dimensional supergravity. The D-derivatives are defined as $D\hat{X}^{\hat{\mu}}=\mathcal{D}\hat{X}^{\hat{\mu}}-\hat{k}^{-2}\hat{k}_{\hat{\nu}}\mathcal{D}\hat{X}^{\hat{\nu}}\hat{k}^{\hat{\mu}}$, with $\hat{k}^2=\hat{g}_{\hat{\mu}\hat{\nu}}\hat{k}^{\hat{\mu}}\hat{k}^{\hat{\nu}}$. Note that covariant derivatives substitute partial derivatives in the non-Abelian case, in such a way that $\hat{C}D\hat{X}D\hat{X}D\hat{X}=(\hat{C}-3\hat{k}^{-2}\hat{k}^{(1)}i_{\hat{k}}\hat{C})\mathcal{D}\hat{X}\mathcal{D}\hat{X}\mathcal{D}\hat{X}$, and the contribution of the Taub-NUT direction cancels out.[4] $\hat{\mathcal{F}}$ is the field strength of the U(N) vector field describing M2-branes, wrapped in the Killing direction,[5] ending on the monopoles $\hat{\mathcal{F}}=2\partial\hat{A}+i[\hat{A},\hat{A}]+l_p^{-2}i_{\hat{k}}\hat{C}\equiv\hat{F}+l_p^{-2}i_{\hat{k}}\hat{C}$. The spacetime fields are understood to be pulled-back onto the worldvolume as explained in Sec. II.

The second term in Eq. (3.1) shows that wrapped M5-branes can arise as solitonic solutions when there is a non-trivial magnetic flux in R^2. Similarly, the third term shows that an instanton-like configuration $\int_{R^4}\mathrm{Tr}\hat{F}\wedge\hat{F}=Z$ induces M2-brane charge, but with the M2-branes delocalized along the Killing direction, since the resulting coupling contains $D\hat{X}$ derivatives: $\int_{R^{2+1}}\hat{C}D\hat{X}D\hat{X}D\hat{X}$.

The same situation in terms of wrapped M5-branes and delocalized M2-branes arises from the analysis of the Wess-Zumino action of a system of M9-branes. The field content of the M9-brane is that of the nine dimensional U(1) vector multiplet, containing 1 scalar and 1 vector. The vector contributes however with 7 degrees of freedom when one of the worldvolume directions is gauged away[6] (see [24]). Therefore, a system of coincident N M9-branes is described by a nine dimensional U(N) vector multiplet. The Wess-Zumino action contains the terms [25]

$$S_{WZ}^{NM9}=\mu_8\int_{R^{8+1}}\mathrm{Tr}(i_{\hat{k}}\hat{B}^{(10)}+l_p^2 i_{\hat{k}}\hat{N}^{(8)}\wedge\hat{\mathcal{F}}+l_p^4 i_{\hat{k}}\hat{\tilde{C}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}+l_p^6\hat{C}D\hat{X}D\hat{X}D\hat{X}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}+\ \dots).\quad(3.2)$$

From here we see that Kaluza-Klein monopole charge is induced when $\int_{R^2}\mathrm{Tr}\hat{F}=Z$, (wrapped) M5-brane charge when $\int_{R^4}\mathrm{Tr}(\hat{F}\wedge\hat{F})=Z$, and (delocalized) M2-brane charge when $\int_{R^6}\mathrm{Tr}(\hat{F}\wedge\hat{F}\wedge\hat{F})=Z$.

The analysis of the brane descent relations of M-theory points towards the same situation. It is possible to construct brane descent relations in M-theory from non-Abelian configurations of non-Bogomol'nyi-Prasad-Sommerfield (BPS) M10-branes. The M10-brane is constructed in such a way that it gives rise to the non-BPS D9-brane of type IIA [26] after reduction along a Killing direction, and to the BPS M9-brane after condensation of its tachyonic mode. This connection with the M9-brane determines that it should contain a Killing direction in its worldvolume. The analysis of the WZ couplings reveals the following pattern of brane descent relations: M9=M10, M6=2 M10, M5=4 M10, M2=8 M10, M0=16 M10, where M6 denotes the Kaluza-Klein monopole and M0 the M-wave. This analysis was made in [27]. There it was pointed out that the M5-brane should again be wrapped around the Killing direction and the M2-brane delocalized in this direction. This is implied by the following terms in the M10-brane effective action:

$$S_{\mathrm{WZ}}^{NM10}=\mu_9\int_{R^{9+1}}\mathrm{Tr}([l_p^4 i_{\hat{k}}\hat{\tilde{C}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}+l_p^6\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}D\hat{X}^{\hat{\mu}}D\hat{X}^{\hat{\nu}}D\hat{X}^{\hat{\rho}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}+\ \dots]\wedge\mathcal{D}\hat{T}),\quad(3.3)$$

where $\hat{T}$ stands for the real adjoint tachyon induced in the worldvolume by wrapped M2-branes, and $\mathcal{D}\hat{T}=d\hat{T}+i[\hat{A},\hat{T}]$.[7]

The analysis of the brane descent relations points out that arbitrary M2 and M5 branes cannot be connected to the other branes of M-theory through a hierarchy of embeddings. The fact that the higher dimensional M-branes live in spacetimes with special Killing directions implies that the branes that can be constructed from them as bound states should also see the special direction. The analysis of the Wess-Zumino terms of the higher dimensional branes shows in particular that the M5-brane should be wrapped and the M2-brane should not move along the Killing direction. In this situation one can construct non-Abelian configurations of M5 and M2 branes as bound states of 4N M10-branes and 8N M10-branes respectively. We can conclude that the M5-brane that arises as a bound state in a system of higher dimensional branes behaves effectively as a 4-brane propagating in a ten dimensional spacetime. Thus, its field content must be that of

[4] Here $\hat{k}^{(1)}$ denotes the Killing vector considered as a 1-form, with components $\hat{k}_{\hat{\mu}}$.

[5] This is implied by the fact that $\hat{C}$ is contracted with the Killing vector [23].

[6] The Killing vector points at a worldvolume direction, in such a way that the D8-brane is obtained upon reduction.

[7] The full Wess-Zumino action for coincident non-BPS D-branes has been constructed recently in [28], extending previous results in [29,30]. In this reference it is shown that this action contains an infinite sum of terms with different powers of the tachyon and its covariant derivatives. Similar kinds of terms will also couple in the worldvolume effective action describing non-Abelian M10-branes. For our purposes it will be sufficient to just consider the contribution of the previous term in this expansion, from where the desired couplings can already be read. In Eq. (3.3) the condensation of the tachyon through a kink-like configuration, which in the limit of zero size can be written as: $d\hat{T}=\hat{T}_0\delta(x-x_0)dx$ [30], gives rise to the Wess-Zumino term of a system of N M9-branes located at $x=x_0$.

the five dimensional vector multiplet, whose non-Abelian extension is well-known. Similarly, the M2-branes behave like 2-branes in ten dimensions, and therefore their field content should be that of the three dimensional U(M) vector multiplet.

Moreover, the existence of the expanded configurations discussed in the previous section in the type IIA theory implies that it should be possible to construct configurations in M-theory corresponding to M2-branes or M5-branes expanding into higher dimensional branes. However, the fact that the higher dimensional branes are coupled to spacetime fields that are constrained by the presence of the Killing direction implies that the non-Abelian worldvolume theory that would describe the M2 and M5 brane configurations should also be constrained by the existence of the Killing direction, which is consistent with our discussion above. This fact does not exclude however the possibility of constructing non-Abelian configurations of arbitrary M2-branes opening up into unwrapped M5-branes.[8] The existence of this kind of configuration is in fact predicted by the coupling $\int_{R^{5+1}}\hat{C}\wedge d\hat{a}^{(2)}$ present in the worldvolume effective action of a single M5-brane. The difficulty stands however in the construction of the non-Abelian worldvolume theory that describes the set of coinciding M2-branes. This problem is related to the problem of implementing duality transformations in non-Abelian gauge theories, as we will further discuss in the paper.

Let us now discuss which could then be the M-theory description of a system of N coincident D2-branes. The Wess-Zumino action of this type IIA system, up to linear terms in the non-Abelian Born-Infeld field strength, includes [1]:

$$
\begin{aligned}
S_{WZ}^{ND2}=\mu_2\int_{R^{2+1}}\mathrm{Tr}\Big[&C^{(3)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(5)}\\
&-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(7)}+\dots+(2\pi\alpha')\\
&\times\Big(C^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(3)}\\
&-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(5)}+\dots\Big)\wedge\mathcal{F}+\dots\Big],
\end{aligned}
\tag{3.4}
$$

where $\mathcal{F}=F+1/(2\pi\alpha')B^{(2)}$. From our discussion in the previous section the interpretation of the couplings in the first line of Eq. (3.4) should be clear. The contraction of the embedding scalars with the RR 5-form indicates the existence of a non-commutative configuration corresponding to the D2-branes expanding into a D4-brane, which acts as a source for this RR potential, and the next term, $(i_\Phi i_\Phi)^2C^{(7)}$, should represent the N D2-branes expanding into a D6-brane.[9]

Uplifting these couplings to M-theory one finds[10]

$$
\begin{aligned}
S_{WZ}^{NM2_t}=\mu_2\int_{R^{2+1}}\mathrm{Tr}\Big[&\hat{C}D\hat{X}D\hat{X}D\hat{X}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat{\hat{C}}\\
&-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat{N}^{(8)}+\dots\\
&+l_p^2\Big(\hat{k}^{-2}\hat{k}^{(1)}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}(\hat{C}-\hat{k}^{-2}\hat{k}^{(1)}\wedge i_{\hat k}\hat{C})\\
&-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat{\hat{C}}+\dots\Big)\wedge\hat{\mathcal{F}}+\dots\Big],
\end{aligned}
\tag{3.5}
$$

with $\hat{\mathcal{F}}=2\partial\hat{A}+i[\hat{A},\hat{A}]+l_p^{-2}i_{\hat k}\hat{C}$. This action describes a non-Abelian configuration of M2-branes delocalized along the eleventh direction, which appears as a special Killing direction mainly for two reasons. First, in the non-Abelian case it is not possible to dualize the Born-Infeld field of the D2-branes into the scalar associated with the eleventh coordinate, a necessary step in order to connect the fully eleven dimensional M2-brane and the D2-brane. Therefore, we are constrained to introduce the eleventh direction as a special isometric direction. Second, the spacetime fields contracted with the embedding scalars cannot be uplifted into any, unwrapped, eleven dimensional field. Thus, Eq. (3.5) is describing the same type of M2-branes that can arise as worldvolume solitons in a single M6 or M9 brane. The second coupling in Eq. (3.5) is likely to describe a non-commutative configuration corresponding to the N M2-branes expanding into a wrapped M5-brane. This configuration has been considered in [8] in the context of the AdS/CFT correspondence. Here we find that it should be possible to describe it from the point of view of the non-Abelian system of branes if the M2-branes are delocalized and the M5-branes are wrapped around the eleventh direction. With respect to the interpretation of the third coupling, we have seen that $i_{\hat k}\hat{N}^{(8)}$ is the field to which the M-theory Kaluza-Klein monopole couples minimally.[11] Therefore its contraction with the four non-commutative scalars would give charge to a configuration corresponding to the N M2-branes expanding into a Kaluza-Klein monopole. The existence of this configuration would also justify why the M2-branes contain a special Killing direction. We will give an interpretation of the terms in the second line of Eq. (3.5) in Sec. VII, where we analyze solitonic configurations in non-Abelian brane-antibrane systems.

[8]These branes will however not be able to expand into other higher dimensional branes such as the monopole and the M9-brane, since they cannot see their special isometric directions.

[9]See the discussions in [4] for the similar configuration of D-strings opening up into a D5-brane, and [31] for a solution associated to D-instantons in a RR 5-form field strength. Reference [20] also considers different four dimensional non-commutative configurations.

[10]We have denoted these branes as $M2_t$ to specify that they are transverse to the Killing direction.

[11]It appears in the N M2-branes effective action because it reduces to the term $(i_\Phi i_\Phi)^2C^{(7)}$ in Eq. (3.4) (see [23]).

Similarly, the Wess-Zumino part of the worldvolume effective action describing a system of N M5-branes is constructed from that of N D4-branes. This includes the terms:

$$S_{\rm WZ}^{\rm N\,D4}=\mu_4\int_{R^{4+1}}{\rm Tr}\Big[C^{(5)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(7)}-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(9)}+(2\pi\alpha')\times\Big(C^{(3)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(5)}-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(7)}+\dots\Big)\wedge\mathcal{F}+\dots\Big]. \tag{3.6}$$

The contraction of the embedding scalars with the RR 7-form indicates the existence of a configuration corresponding to the D4-branes expanding into a D6-brane, whereas the term $(i_\Phi i_\Phi)^2C^{(9)}$, would be associated to the N D4-branes expanding into a D8-brane. Uplifting this non-Abelian system onto M-theory we find a non-Abelian configuration of wrapped M5-branes containing the couplings:

$$S_{\rm WZ}^{NM5_w}=\mu_4\int_{R^{4+1}}{\rm Tr}\Big[i_{\hat k}\hat{\tilde{C}}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat N^{(8)}-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat B^{(10)}+l_p^2\Big(\hat C D\hat X D\hat X D\hat X+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat{\tilde{C}}-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat N^{(8)}+\dots\Big)\wedge\hat{\mathcal{F}}+\dots\Big]. \tag{3.7}$$

The Killing direction emerges, on the one hand, because the vector field of the D4-branes cannot be (worldvolume) dualized into the worldvolume 2-form of an unwrapped M5-brane and, on the other hand, because it is not possible to uplift the fields contracted with the embedding scalars into any eleven dimensional fields not involving the Killing vector. The contraction of $i_{\hat k}\hat N^{(8)}$ with the two non-commutative scalars would give charge to a configuration corresponding to the N M5-branes opening up into a Kaluza-Klein monopole. The third coupling would give charge in turn to a configuration corresponding to the N M5-branes opening up into an M9-brane, since as we have seen the field $i_{\hat k}\hat B^{(10)}$ couples minimally to this brane.[12] Similarly to the M2-brane case, the M5-branes must be wrapped on the Killing direction, consistently with the fact that they should be able to expand into the monopole or the M9-brane.

The Wess-Zumino action describing a non-Abelian configuration of Kaluza-Klein monopoles is obtained from that of a system of N D6-branes, following the same procedure described in [23]. One obtains the action (3.1) supplemented with typically non-commutative couplings corresponding to the contraction of the spacetime fields with the embedding scalars:

$$S_{WZ}^{NKK}=\mu_6\int_{R^{6+1}}{\rm Tr}\Big[i_{\hat k}\hat N^{(8)}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat B^{(10)}+l_p^2\Big(i_{\hat k}\hat{\tilde{C}}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat N^{(8)}-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat B^{(10)}\Big)\wedge\hat{\mathcal{F}}+\dots\Big]. \tag{3.8}$$

The second term in Eq. (3.8) would be responsible for a configuration of N monopoles opening up into an M9-brane.

Finally, the non-Abelian action associated to coincident M-waves is obtained by uplifting the Wess-Zumino action of a system of D0-branes. One finds the couplings:

$$S_{WZ}^{NM0}=\mu_0\int_R{\rm Tr}\Big(\hat k^{-2}\hat k^{(1)}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}(\hat C-\hat k^{-2}\hat k^{(1)}\wedge i_{\hat k}\hat C)-\frac{1}{2}l_p^4(i_{\hat\Phi}i_{\hat\Phi})^2 i_{\hat k}\hat{\tilde{C}}+\dots\Big). \tag{3.9}$$

The second term would be responsible for a configuration of M-waves expanding into an M2-brane transverse to the direction of propagation, and therefore, to the "dual giant graviton" of $AdS_4\times S^7$ [11]. In turn the third term would be associated to the N M0-branes expanding into an M5-brane wrapped in the direction of the propagation, and therefore to the dual giant graviton of $AdS_7\times S^4$. These configurations can also be studied from the point of view of the M2 and M5 branes. The first term in the second line of Eq. (3.5) indicates that the transverse M2-brane carries momentum in a direction orthogonal to the electric 3-form. In the case of the wrapped M5-brane the term

$$\int_{R^{4+1}}{\rm Tr}[\hat k^{-2}\hat k^{(1)}\wedge\hat{\mathcal{F}}\wedge\hat{\mathcal{F}}], \tag{3.10}$$

that we have omitted in Eq. (3.7), shows that it carries momentum along the compact direction and that this arises as a non-trivial instanton charge.

Let us finish this section with some comments on the M2 and M5 brane effective actions that we have presented. In the Abelian case the action of the $M2_t$-brane would be related to that of an ordinary M2-brane by means of a worldvolume duality transformation. This is easily seen by considering the Abelian version of the action (3.5) and adding to it a Lagrange multiplier term $\int_{R^{2+1}}d\hat A\wedge d\hat y=\int_{R^{2+1}}(\hat{\mathcal{F}}-l_p^{-2}i_{\hat k}\hat C)\wedge d\hat y$. The integration on $\hat y$ would impose the constraint that $\hat{\mathcal{F}}-l_p^{-2}i_{\hat k}\hat C$ is derived from a vector potential and the original action would be recovered. On the other hand, the integration of $\hat{\mathcal{F}}$ would give rise to a coupling

[12] It gives the 9-form RR-potential of type IIA after reduction along the Killing direction.

$\int_{R^{2+1}}[\hat{C}+i_{\hat{k}}\hat{C}\wedge d\hat{y}]$ in the dual action. This term describes an M2-brane, with $\hat{y}$ playing the role of the eleventh direction. This connection between $M2_t$ and M2 does not exist however in the non-Abelian case and only systems involving the first type of branes can be constructed explicitly. The $M5_w$-brane that we have constructed can also be related to an ordinary M5-brane in the Abelian case. The worldvolume duality transformation would be performed by adding the Lagrange multiplier term $\int_{R^{4+1}}d\hat{A}\wedge d\hat{a}^{(2)}=\int_{R^{4+1}}(\hat{\mathcal{F}}-l_p^{-2}i_{\hat{k}}\hat{C})\wedge d\hat{a}^{(2)}$ to the action (3.7). Now a coupling $\int_{R^{4+1}}i_{\hat{k}}\hat{C}\wedge d\hat{a}^{(2)}$ would remain after integrating out $\hat{\mathcal{F}}$, which would arise in the double dimensional reduction of an M5-brane.

IV. OTHER NON-ABELIAN TYPE IIA CONFIGURATIONS

We have seen that it is possible that a system of N M2-branes can open up into a wrapped M5-brane when they are delocalized along the direction on which the M5-brane is wrapped. This configuration gave rise upon reduction along the special direction to N D2-branes opening up into a D4-brane. One could however consider other possibilities. Consider for instance reducing this system along a transverse direction different from the Killing direction. This would give rise to a configuration of N D2-branes expanding into an NS5-brane, with the NS5-brane wrapped and the D2-branes constrained to move in the transverse space. This is in fact described by the following couplings in the D2-branes effective action:

$$S_{WZ}^{ND2_t}=\mu_2\int_{R^{2+1}}\mathrm{Tr}\Big[C^{(3)}DXDXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k B^{(6)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(8)}+\dots+(2\pi\alpha') \times\Big(k^{-2}k^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi(C^{(3)}DXDXDX) -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(6)}+\dots\Big)\wedge\mathcal{H}^{(2)}+\dots\Big], \tag{4.1}$$

derived from Eq. (3.5) upon reduction.[13] $\mathcal{H}^{(2)}$ is defined as: $\mathcal{H}^{(2)}=2\partial b^{(1)}+i[b^{(1)},b^{(1)}]+1/(2\pi\alpha')i_kC^{(3)}+2(i_kB^{(2)})\mathcal{D}c^{(0)}$. Here $c^{(0)}$ is the scalar field arising from the eleventh direction $c^{(0)}=y/(2\pi\alpha')$ and $b^{(1)}$ comes from the reduction of the vector field, which we have denoted as $b^{(1)}$ to specify that its Abelian part has a different gauge transformation rule than the vector field A that couples in ordinary Dp-brane effective actions. We have also taken $C^{(1)}=0$ for simplicity.[14] $\mathcal{H}^{(2)}$ is associated to wrapped D2-branes ending on the non-Abelian system of branes. Before we discuss the interpretation of the different couplings in this effective action, let us consider other possibilities.

[13]We denote these branes as $D2_t$ to specify that they are transverse to the Killing direction.

[14]We will be doing the same in the coming actions.

We could also reduce the N M2-branes along a wordvolume direction. This would give rise to N fundamental strings, constrained to move in a nine dimensional spacetime, which could expand into a wrapped D4-brane. The corresponding couplings that one finds after the reduction are

$$S_{WZ}^{NF1_t}=\mu_1\int_{R^{1+1}}\mathrm{Tr}\Big[B^{(2)}DXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k C^{(5)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(7)}+\dots+(2\pi\alpha') \times\Big(k^{-2}k^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi(C^{(3)}DXDXDX) -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(6)}+\dots\Big)\wedge\mathcal{K}^{(1)}+(2\pi\alpha') \times\Big(i(2\pi\alpha')i_\Phi i_\Phi(B^{(2)}DXDX) -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k C^{(5)}+\dots\Big)\wedge\mathcal{K}^{(2)}+\dots\Big], \tag{4.2}$$

where

$$\mathcal{K}^{(1)}=\mathcal{D}\omega^{(0)}+1/(2\pi\alpha')(i_kB^{(2)})+i(2\pi\alpha')(i_kC^{(3)})\times\mathcal{D}\Phi[\omega^{(0)},\Phi]$$

and

$$\mathcal{K}^{(2)}=2\partial\omega^{(1)}+i[\omega^{(1)},\omega^{(1)}]+1/(2\pi\alpha')i_kC^{(3)}.$$

$\omega(0)$ and $\omega^{(1)}$ arise as the components of $\hat{A}$ along the direction in which we reduce and along a different direction, and are associated to wrapped F1-branes and wrapped D2-branes ending on the F1-branes. Note that the coupling to the wrapped D2-branes only occurs in the non-Abelian case, since the spacetime fields must be contracted with the embedding scalars. Non-Abelian configurations of type IIA fundamental strings have been studied in [32].

Similarly, the case in which N wrapped M5-branes opened up into a Kaluza-Klein monopole gave rise to N D4-branes opening up into a D6-brane when reducing along the Killing direction. Reduction along a transverse direction would give rise instead to N wrapped NS5-branes, containing the couplings:

$$S_{WZ}^{N(NS5)_w}=\mu_4\int_{R^{4+1}}\mathrm{Tr}\Big[i_kB^{(6)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(8)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(10)}+(2\pi\alpha') \times\Big(C^{(3)}DXDXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k B^{(6)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(8)}+\dots\Big) \wedge\mathcal{H}^{(2)}+\dots\Big]. \tag{4.3}$$

The second term may be associated to a configuration corresponding to the N wrapped NS5-branes expanding into a KK6-brane.[15] Reduction along a worldvolume direction of the M5's different from that on which they are wrapped would give rise to N wrapped D4-branes, containing the couplings:

$$S_{WZ}^{ND4_w}=\mu_3\int_{R^{3+1}}\mathrm{Tr}\bigg[i_kC^{(5)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(7)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(9)}+(2\pi\alpha') \times\bigg(C^{(3)}DXDXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k B^{(6)} -\frac{1}{2}(2\pi\alpha')^2 i_k N^{(8)}+\dots\bigg)\wedge\mathcal{K}^{(1)}+(2\pi\alpha') \times\bigg(B^{(2)}DXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k C^{(5)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(7)}+\dots\bigg)\wedge\mathcal{K}^{(2)}+\dots\bigg]. \tag{4.4}$$

The second term would be associated to a configuration in which the N D4-branes open up into a Kaluza-Klein monopole, given that $i_kN^{(7)}$ is the field to which the type IIA monopole couples minimally.[16]

The first coupling in Eq. (4.4) seems to imply that the 4-branes that we have obtained after the reduction from M-theory are just wrapped D4-branes. However, one notices that a system of wrapped D4-branes would contain in its effective action a coupling $i_\Phi i_\Phi i_k C^{(7)}$, corresponding to the D4-branes expanding into a wrapped D6-brane, and not the coupling $i_\Phi i_\Phi i_k N^{(7)}$ that we have found after the reduction from M-theory. The same thing happens if we try to give a meaning to the effective action (4.1) as associated to a delocalized D2-brane in the type IIA theory. Moreover, the reduction of the field strength $\hat{\mathcal{F}}$ shows that the dynamics of these objects is governed by wrapped D2-branes ending on them.[17] Thus, the reduction from M-theory provides a description of the D2 and D4 branes which is fully nonperturbative, and therefore valid in the strong coupling regime. As we have mentioned already the origin of this different description at strong coupling can be traced to the fact that the worldvolume duality transformations that are needed in order to prove the equivalence between the transverse or wrapped brane and the fully ten dimensional one cannot be performed in the non-Abelian case. The worldvolume effective actions that we have derived in this section must be used however to describe those situations which are typically non-perturbative and cannot be predicted by the weakly coupled effective actions. Indeed, the expanding brane configurations that we have discussed, predicted by the contraction of the spacetime fields with the embedding scalars, cannot be explained from the weakly coupled effective actions, like the one associated to N D2 branes expanding into an NS5-brane, considered in [7]. The analysis presented in this section shows that this configuration can be studied from the point of view of the non-Abelian D2-branes if they are delocalized in one spatial direction. The NS5-brane into which they would expand would then be wrapped on this direction. The second coupling in Eq. (4.1) shows that this system can carry $B^{(6)}$ charge, which is however not the case for weakly coupled D2-branes. This special situation arises naturally from the T-duality of a configuration of N D3-branes expanding into an NS5-brane, which we will discuss in the next section, and is in agreement with the type IIA NS5-brane solution with non-trivial $C^{(3)}$ charge that was constructed in [35], and considered further in [7]. The same configuration can also be represented as a single (wrapped) NS5 with N units of (wrapped) D2-brane magnetic flux. This is predicted by the first coupling in the second line of Eq. (4.3).

Other non-commutative terms show that the delocalized D2-branes and the wrapped D4-branes could expand into higher dimensional branes defined in isometric spacetimes, namely D2 into KK6, D4 into a monopole, etc. which is possible because these strongly coupled configurations can also see the special Killing direction.

We have found as well the Wess-Zumino part of the worldvolume effective actions describing coincident systems of F1-branes and NS5-branes. The F1-branes cannot move along a special Killing direction and the NS5-branes are wrapped on it. The dynamics of these objects is governed by wrapped D2-branes ending on them.[18] Both effective actions can be shown to reduce to the effective action of ordinary, localized, F1-branes and unwrapped NS5-branes in the Abelian case, by means of a worldvolume duality transformation.

Finally, there are two cases in which the reduction from M-theory provides the only possible description of the brane. This happens when reducing a system of N coincident Kaluza-Klein monopoles along a worldvolume direction, which gives rise to coinciding type IIA monopoles. We find the couplings

[15]The KK6-brane arises when reducing the M-theory Kaluza-Klein monopole along a transverse direction different from the Taub-NUT direction [33,34], and it couples minimally to $i_kN^{(8)}$, obtained from the reduction of $i_{\hat{k}}\hat{N}^{(8)}$. This brane is however not predicted by the spacetime supersymmetry algebra, and in this sense is referred to as an exotic brane. See however the recent work [49] for a possible extension of the SUSY algebra including this, and other exotic, charges.

[16]It arises in the reduction of $i_{\hat{k}}\hat{N}^{(8)}$ along a worldvolume direction (see [23]).

[17]Also wrapped F1-branes in the case of the D4-brane.

[18]Also wrapped F1-branes in the case of the F1's.

$$S_{WZ}^{NKK}=\mu_5\int_{R^{5+1}}\mathrm{Tr}\bigg[i_kN^{(7)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(9)}+\dots+(2\pi\alpha')\bigg(i_kC^{(5)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(7)}-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(9)}\bigg)\wedge\mathcal{K}^{(2)}+(2\pi\alpha')\times\bigg(i_kB^{(6)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(8)}-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(10)}\bigg)\wedge\mathcal{K}^{(1)}+\dots\bigg].\tag{4.5}$$

The second term would be associated to a configuration of N monopoles opening up into a so-called KK8-brane, exotic brane that appears after double dimensionally reducing the M9-brane [34,36]. Equation (4.5) generalizes the action for a Kaluza-Klein monopole [23] to the non-Abelian case[19] and shows that it also contains couplings to higher dimensional spacetime fields, which could be interpreted in terms of noncommutative brane configurations. The reduction of Eq. (3.8) along a transverse direction different from the Taub-NUT direction produces the effective action of a system of KK6-branes[20] containing as well couplings to higher dimensional spacetime fields, one of which could be interpreted as the KK6-branes expanding into an NS9-brane.

The reduction of the effective action describing a system of M0-branes gives rise to the effective action of non-Abelian pp-waves in type IIA. One finds:

$$S_{WZ}^{Nwaves}=\mu_0\int_R\mathrm{Tr}\bigg(k^{-2}k^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi(C^{(3)}-k^{-2}k^{(1)}\wedge i_kC^{(3)})-\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(6)}+\dots\bigg).\tag{4.6}$$

Here the second term could be responsible for a configuration corresponding to the waves expanding into a D2-brane transverse to the direction in which they propagate, and the third term could be associated to pp-waves expanding into a wrapped NS5-brane.

In the next section we will see that the non-Abelian brane systems that we have derived in this section by reduction from M-theory are connected by T-duality to non-Abelian type IIB systems, predicted by the S-duality symmetry of the theory, consisting of strongly coupled p-branes. Finally in Sec. VIII we will give an interpretation of the terms that couple to these actions through the Born-Infeld field strengths in terms of topological solitons in brane-antibrane systems.

[19]Up to linear terms in the Born-Infeld field strengths.

[20]See [36] for the Abelian case.

V. NON-ABELIAN TYPE IIB BRANES AND S-DUALITY

In this section we discuss the interplay between S-duality and the non-Abelian effective actions for Dp-branes derived in [1] (see [2] for a related discussion for D3-branes). We will consider non-Abelian systems of Dp-branes, with $p=1,3,5$, F1-branes and NS5-branes.

A. Non-Abelian D3-branes

The worldvolume effective action describing a single D3-brane is S-duality invariant. Although an S-duality transformation maps the action into a different one, in which NS-NS and RR 2-forms are interchanged and the Abelian Born-Infeld field describing open strings ending on the 3-brane is mapped into a dual vector field associated to open D1-branes ending on it, one can see that this action is equivalent to the original one under a worldvolume duality transformation that interchanges the two vector fields. In the non-Abelian case, however, the explicit worldvolume duality transformation that connects weakly coupled and strongly coupled D3-branes is not known. As a consequence one seems to have independent worldvolume effective actions to describe the system in the weak and strong coupling regimes.

Let us consider the following couplings in the Wess-Zumino action of a U(N) system of D3-branes:

$$S_{WZ}^{ND3}=\mu_3\int_{R^{3+1}}\mathrm{Tr}\bigg(C^{(4)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(6)}-\frac{1}{2}C^{(2)}\wedge B^{(2)}+(2\pi\alpha')C^{(2)}\wedge\mathcal{F}+\dots\bigg),\tag{5.1}$$

where we have chosen the basis in which $C^{(4)}$ is S-duality invariant: $C^{(4)}\rightarrow C^{(4)}-\frac{1}{2}C^{(2)}\wedge B^{(2)}$. These terms are mapped under S-duality into

$$S_{WZ}^{ND3}=\mu_3\int_{R^{3+1}}\mathrm{Tr}\bigg(C^{(4)}+i(2\pi\alpha')i_\Phi i_\Phi B^{(6)}+\frac{1}{2}C^{(2)}\wedge B^{(2)}-(2\pi\alpha')B^{(2)}\wedge\tilde{\mathcal{F}}+\dots\bigg),\tag{5.2}$$

where $\tilde{\mathcal{F}}=2\partial\tilde{A}+i[\tilde{A},\tilde{A}]+1/(2\pi\alpha')C^{(2)}$.

Let us recall how the worldvolume duality transformation works in the Abelian case (see for instance [37]). It proceeds in two steps. First one substitutes $dA+1/(2\pi\alpha')B^{(2)}$ by a gauge invariant field strength $\mathcal{F}$, and adds a Lagrange multiplier term:

$$\mu_3(2\pi\alpha')^2\int_{R^{3+1}}\bigg(\mathcal{F}-\frac{1}{2\pi\alpha'}B^{(2)}\bigg)\wedge d\tilde{A}\tag{5.3}$$

which imposes the constraint that $\mathcal{F}-1/(2\pi\alpha')B^{(2)}$ derives from a vector potential upon integration over $\tilde{A}$. This way one recovers the original action. On the other hand the dual action is obtained through the equation of motion for $\mathcal{F}$, which is given by a non-linear expression in terms of $\tilde{\mathcal{F}}$ and the spacetime fields due to the contribution of the Born-

Infeld Lagrangian. One readily sees however from Eq. (5.3) that a term $\int_{R^{3+1}} B^{(2)} \wedge d\tilde{A}$ will couple in the dual action. Therefore the worldvolume duality transformation substitutes the term $\int_{R^{3+1}} C^{(2)} \wedge \mathcal{F}$ in the worldvolume effective action of the D3-brane by its S-dual: $\int_{R^{3+1}} B^{(2)} \wedge \tilde{\mathcal{F}}$. The presence of the couplings $i_\Phi i_\Phi C^{(6)}$ and $i_\Phi i_\Phi B^{(6)}$ in the non-Abelian case makes clear that the embedding scalars should transform as well, and very non-trivially, under worldvolume duality. This is in agreement with the observations in [2], where the duality transformation properties of certain operators containing the embedding scalars were derived. However the explicit worldvolume duality transformation that connects the two actions is not known. Therefore, perturbative processes governed by open strings should be described by Eq. (5.1) and non-perturbative ones, governed by open D-strings, should be described by the strongly coupled action (5.2). The second term in Eq. (5.1) indicates the existence of a configuration corresponding to N D3-branes expanding into a D5-brane, which represents the Higgs vacuum of [5]. The confining vacuum is in turn represented by D3-branes polarized into an NS5-brane [5], and would be described by the second term in Eq. (5.2). Note that $B^{(6)}$ does not couple in the weakly coupled action (5.1), and therefore the configuration corresponding to the N D3-branes expanding into an NS5-brane cannot be described at weak coupling. Consistently with this picture the N D3-branes can also be represented at weak coupling as a single D5-brane with N units of magnetic flux, associated to the coupling $S^{D5} \sim \int_{R^{5+1}} C^{(4)} \wedge F$ in the D5-brane effective action, or as a single NS5-brane with N units of D1 flux, associated to the coupling in the NS5-brane effective action $S^{NS5} \sim \int_{R^{5+1}} C^{(4)} \wedge \tilde{F}$, at strong coupling.

Let us now discuss the behavior under T-duality of the action (5.2) representing a set of D3-branes at strong coupling. T-duality along a transverse direction gives rise to the following couplings:

$$S_{WZ}^{ND4_w} = \mu_3 \int_{R^{3+1}} \mathrm{Tr}[i_k C^{(5)} + i(2\pi\alpha') i_\Phi i_\Phi i_k N^{(7)} + \dots], \tag{5.4}$$

corresponding to the wrapped D4-branes that we obtained in the previous section from the reduction of the non-Abelian M5-branes. A basic difference with the same operation in the weakly coupled action (5.1), giving rise to D4-branes, is that Eq. (5.4) cannot be unwrapped. At the level of the terms that we have included this is a consequence of the T-duality transformation:

$$B^{(6)}_{\mu_1 \dots \mu_6} \to (i_k N^{(7)})_{\mu_1 \dots \mu_6} + \dots \tag{5.5}$$

for the NS-NS 6-form.[21] Therefore, we encounter again the situation in which the same brane, in this case a D4-brane, is described by different effective actions at weak and strong couplings. As we mentioned already in the previous section, Eq. (5.4) can indicate the existence of a configuration in which (wrapped) D4-branes open up into a monopole, which cannot be explained however through the usual couplings present in the weakly coupled D4-brane effective action.

Let us now consider a T-duality transformation along a direction in the worldvolume of the N D3-branes at strong coupling. We find N D2-branes that cannot move in the direction of the duality transformation. The relative minus sign of the $C^{(2)} \wedge B^{(2)}$ term in Eqs. (5.1) and (5.2) plays a key role in this derivation. While in Eq. (5.1) the second term in the T-duality rule for $C^{(4)}$:

$$C^{(4)}_{\mu\nu\rho z} \to C^{(3)}_{\mu\nu\rho} - \frac{3}{2} C^{(3)}_{[\mu\nu z} \frac{g_{\rho]z}}{g_{zz}}, \tag{5.6}$$

where z denotes the direction of the T-duality transformation, is cancelled with the T-dual of $C^{(2)} \wedge B^{(2)}$, so that only the $C^{(3)}$ coupling of the D2-brane effective action remains, the relative minus sign of this term in Eq. (5.2) gives an overall coupling:

$$C^{(3)}_{abc} - 3 C^{(3)}_{[abz} \frac{g_{c]z}}{g_{zz}} = C^{(3)} D_a X D_b X D_c X.$$

This, together with the T-duality tranformation rule of $B^{(6)}$ (see [38]):

$$B^{(6)}_{\mu_1 \dots \mu_5 z} \to B^{(6)}_{\mu_1 \dots \mu_5 z} + \dots \tag{5.7}$$

gives

$$S_{WZ}^{ND2_t} = \mu_2 \int_{R^{2+1}} \mathrm{Tr}[C^{(3)} DXDXDX + i(2\pi\alpha') i_\Phi i_\Phi i_k B^{(6)} + \dots], \tag{5.8}$$

which is the expression (4.1) that we derived from M-theory representing transverse D2-branes. Again, this strongly coupled description of the D2-branes can be responsible for the existence of configurations that cannot be explained by looking at the weakly coupled action. This is the case for the situation in which N D2-branes expand into an NS5-brane, discussed in [7].

That these configurations may exist is deduced both from the analysis of the non-Abelian systems that we can construct in M-theory and the combined action of S- and T-duality transformations for coinciding D3-branes. We find new possibilities corresponding to N D2-branes expanding into a wrapped NS5-brane [7] and N D4-branes opening up into a type IIA Kaluza-Klein monopole. The description in terms of the effective actions that we have constructed would be valid at strong coupling, because the dynamics of both the D2-branes and the wrapped D4-branes is governed by wrapped D2-branes ending on them, as can be inferred both from the reduction from M-theory and from the T-duality transformation of the dual Born-Infeld field strength in Eq. (5.2).

[21]This T-duality rule was derived in [38] as a key ingredient in order to prove the connection between the type IIB NS5-brane and the type IIA Kaluza-Klein monopole under T-duality.

B. Non-Abelian D1-branes

Let us consider now a system of N coincident D1-branes. The corresponding effective action, according to Myers prescription, contains the terms:

$$S_{WZ}^{ND1}=\mu_1\int_{R^{1+1}}\mathrm{Tr}\bigg(C^{(2)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(4)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(6)}+\dots\bigg). \tag{5.9}$$

An S-duality transformation gives rise to the effective action describing a system of coincident fundamental strings:

$$S_{WZ}^{NF1}=\mu_1\int_{R^{1+1}}\mathrm{Tr}\bigg(B^{(2)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(4)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2B^{(6)}+\dots\bigg). \tag{5.10}$$

Here the second term indicates the existence of a configuration corresponding to the N F1's opening up into a D3-brane, a configuration that has been studied in [4], and in [39] from the point of view of the Abelian D3-brane theory.

A T-duality transformation along a transverse direction to the N F1's gives rise to the following action in type IIA:

$$S_{WZ}^{NF1}=\mu_1\int_{R^{1+1}}\mathrm{Tr}\bigg(B^{(2)}DXDX+i(2\pi\alpha')i_\Phi i_\Phi i_k C^{(5)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k N^{(7)}+\dots\bigg), \tag{5.11}$$

where the first term arises from the T-duality transformation of $B^{(2)}$:

$$B_{ab}^{(2)}\rightarrow B_{ab}^{(2)}-2B_{[az}^{(2)}\frac{g_{b]z}}{g_{zz}}=B^{(2)}D_aXD_bX$$

and $B^{(6)}\rightarrow i_kN^{(7)}$ as in Eq. (5.5). Note that these are the same couplings present in Eq. (4.2), which was derived by reduction from M-theory. This action describes a delocalized fundamental string. Again, in the Abelian case a worldvolume duality transformation would map this action into the action of an ordinary fundamental string.

Finally, T-duality along a worldvolume direction gives rise to N pp-waves in type IIA. We find the same couplings that we obtained in the reduction from M-theory:

$$S_{WZ}^{Nwaves}=\mu_0\int_R\mathrm{Tr}\bigg(k^{-2}k^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi \times(C^{(3)}-k^{-2}k^{(1)}\wedge i_kC^{(3)}) -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2 i_k B^{(6)}+\dots\bigg). \tag{5.12}$$

C. Non-Abelian D5-branes

Let us now perform the same analysis of the previous subsection but with a non-Abelian system of D5-branes. The Wess-Zumino action contains the terms

$$S_{WZ}^{ND5}=\mu_5\int_{R^{5+1}}\mathrm{Tr}\{C^{(6)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(8)}+\dots +(2\pi\alpha')[C^{(4)}+i(2\pi\alpha')C^{(6)}+\dots]\wedge\mathcal{F}+\dots\}, \tag{5.13}$$

from where S-duality gives

$$S_{WZ}^{NNS5}=\mu_5\int_{R^{5+1}}\mathrm{Tr}\{B^{(6)}+i(2\pi\alpha')i_\Phi i_\Phi \tilde{C}^{(8)}+\dots +(2\pi\alpha')[C^{(4)}+i(2\pi\alpha')B^{(6)}+\dots]\wedge\tilde{\mathcal{F}}+\dots\}. \tag{5.14}$$

Here $\tilde{C}^{(8)}$ is the field to which the D7-brane couples minimally at strong coupling (see [36]). Therefore the second term represents the N NS5-branes opening up into a D7-brane. Making now a T-duality transformation along a transverse direction, we find:

$$S_{WZ}^{NKK}=\mu_5\int_{R^{5+1}}\mathrm{Tr}[i_kN^{(7)}+i(2\pi\alpha')(i_\Phi i_\Phi i_k N^{(9)})+\dots], \tag{5.15}$$

where we have used Eq. (5.5) and $i_kN^{(9)}$ arises from the dualization of $\tilde{C}^{(8)}$ (see [36]):

$$\tilde{C}^{(8)}_{\mu_1\dots\mu_8}\rightarrow(i_kN^{(9)})_{\mu_1\dots\mu_8}+\dots. \tag{5.16}$$

This action describes N coincident Kaluza-Klein monopoles, and we encountered it already in our reduction from M-theory.

T-dualizing Eq. (5.14) along a worldvolume direction we find:

$$S_{WZ}^{N(NS5)_w}=\mu_4\int_{R^{4+1}}\mathrm{Tr}[i_kB^{(6)}+i(2\pi\alpha')i_\Phi i_\Phi i_k N^{(8)}+\dots], \tag{5.17}$$

where $i_kN^{(8)}$ arises from the dualization of $\tilde{C}^{(8)}$ [see Eq. (3.3) in [36]]:

$$\tilde{C}^{(8)}_{\mu_1\dots\mu_7 z}\rightarrow(i_kN^{(8)})_{\mu_1\dots\mu_7}+\dots. \tag{5.18}$$

The KK6-brane of type IIA couples minimally to this field. Therefore $i_\Phi i_\Phi i_k N^{(8)}$ could be associated to a configuration of N wrapped NS5-branes opening up into a KK6-brane, as we discussed in the previous section.

VI. BPS SOLITONS FROM NON-ABELIAN BRANE-ANTIBRANE CONFIGURATIONS

The analysis of the couplings in the Wess-Zumino action describing a non-Abelian brane-antibrane system reveals the emergence of new solitonic solutions after the tachyonic mode of the open strings stretched between branes and antibranes condenses.[22] This study reveals that typically noncommutative configurations can also be described as topological configurations in non-Abelian brane-antibrane systems.

To see how these solutions arise let us start by recalling the Abelian case.

The Wess-Zumino part of the effective action corresponding to a Dp brane-antibrane system contains couplings to the two U(1) fields of the brane and the antibrane as well as to the complex tachyon field [41]. It includes in particular the very simple term:

$$\int_{R^{p+1}} C^{(p-1)} \wedge (dA^{(1)} - dA^{(1)'}) \tag{6.1}$$

where $dA^{(1)}$ and $dA^{(1)'}$ denote the Born-Infeld field strengths of the brane and the antibrane.[23] It is by now well understood that the tachyon condenses through a Higgs-like mechanism in which the relative U(1) field on the brane-antibrane system gets a mass and there is a localized magnetic flux on R^2 that acts as a source for the RR $(p-1)$-form field, as can be deduced from the coupling above. This signals the emergence of a D$(p-2)$-brane as the associated topological defect [42], a qualitative conclusion that is supported by the CFT analysis of the system [43,44].

The analysis of the couplings in the Wess-Zumino action provides a hint of the possible solitonic objects that can emerge after tachyonic condensation in situations in which string perturbation theory cannot be applied. One can explain for instance the emergence of the fundamental string as a solitonic solution in Dp, anti-Dp systems [45,46], configuration that should exist as a consequence of various duality arguments. A duality transformation in the $p+1$ dimensional worldvolume of the Dp, anti-Dp system maps the Born-Infeld vector into a $(p-2)$-form. Therefore, in this dual description the tachyonic field would be associated to a D$(p-2)$-brane stretched between the brane and the antibrane, and it would be charged under the relative $(p-2)$-form. The fundamental string arises after the condensation of this tachyon field, as can be seen qualitatively from the analysis of the WZ term of the dualized brane-antibrane effective action [45,46]. In this action one finds a coupling (see [46]):

$$\int_{R^{p+1}} B^{(2)} \wedge (dA^{(p-2)} - dA^{(p-2)'}). \tag{6.2}$$

This indicates that the fundamental string will arise as a topological soliton when the dual tachyon condenses through a Higgs-like mechanism, in which there is a localized magnetic flux in the transverse R^{p-1}.

The dual Higgs mechanism explains the decoupling of the overall $U(1)$ gauge group of the system at strong coupling.[24] At weak coupling it is interpreted in terms of confinement of the overall $U(1)$, with the fundamental string emerging as the confined electric flux string at the end of the annihilation process [47]. This mechanism has been used to explain the fate of the unbroken $U(1)$ on the worldvolume of the annihilating system [45].

Let us now consider N coincident Dp brane-antibrane pairs. The Wess-Zumino action describing this system has been constructed in [41], though this reference does not include the non-commutative couplings to the embedding scalars introduced by Myers. Our aim in this section is to discuss the interpretation of precisely these terms for the creation of branes as solitonic solutions. For our purposes we will just need to consider the sum of the Wess-Zumino terms for branes and antibranes, i.e. we will ignore the contribution from the tachyon field.

Let us consider for instance the case of N coincident (D4, anti-D4) brane pairs. The WZ effective action up to linear terms in the non-Abelian Born-Infeld field strength includes:

$$S_{\rm WZ}^{({\rm D4},\bar{\rm D4})} = \mu_4 \int_{R^{4+1}} (2\pi\alpha') {\rm Tr}\Bigg(\Bigg[C^{(3)} + i(2\pi\alpha') i_\Phi i_\Phi C^{(5)} - \frac{1}{2}(2\pi\alpha')^2 (i_\Phi i_\Phi)^2 C^{(7)} \Bigg] \wedge (F^{(2)} - F^{(2)'}) \Bigg), \tag{6.3}$$

where $F^{(2)}$, $F^{(2)'}$ are the Born-Infeld field strengths for branes and antibranes. Similar non-Abelian configurations to the ones discussed in the previous sections are possible when one considers non-Abelian (Dp, anti-Dp) systems. In this case the N pairs could open up into a (D6, anti-D6) system or into a (D8, anti-D8). In turn, annihilation of these brane configurations will produce solitonic solutions which have as well an interpretation in terms of expanding branes. Qualitatively one can see the emergence of these configurations from the analysis of the couplings in the WZ action describing the N (D4, anti-D4) pairs of branes. The first term in Eq. (6.3) describes the usual realization of the D2-brane as a vortex solution in a (D4, anti-D4) system, in this case N D2-branes from N (D4, anti-D4) pairs.[25] The next term:

$$\int_{R^{4+1}} {\rm Tr}[i_\Phi i_\Phi C^{(5)} \wedge (F^{(2)} - F^{(2)'})] \tag{6.4}$$

signals the emergence of a configuration representing the N D2-branes opening up into a D4-brane, which would act as a source for the RR 5-form. This configuration arises naturally from the annihilation via a Higgs-like mechanism of N (D4, anti-D4) pairs opening up into a (D6, anti-D6) system. Similarly, the next term in Eq. (6.3) would be associated to a non-commutative configuration of N D2-branes expanding

[22]See [40] for reviews on the role of tachyonic excitations in unstable brane systems.

[23]Since we will be dealing with worldvolume forms of different degree we indicate it explicitly for each field.

[24]Due to the opposite orientation of brane and antibrane one has $dA^{(p-2)} - dA^{(p-2)'} = *(dA^{(1)} + dA^{(1)'})$.

[25]See the discussion below Eq. (6.5).

into a D6-brane, a by-product of the annihilation of N (D4, anti-D4) pairs expanding into a (D8, anti-D8) system.

By analogy with the Abelian case one would expect that non-commutative configurations involving fundamental strings would arise after condensation of the dual tachyon, charged with respect to the worldvolume dual of the Born-Infeld field. However, the general mechanism for the dualization of the non-Abelian vector field is not known, and not even a qualitative description of non-Abelian configurations of fundamental strings can be made in these terms. We will see however in the next section that certain configurations can be described from non-Abelian $(p,\ \overline{p})$ systems in M-theory.

Going back to Eq. (6.3), including higher order terms in the Born-Infeld field strengths gives rise to new solitonic configurations. Bearing in mind that the topologically non-trivial character of the soliton can be carried by just one of the two field strengths, say $F^{(2)}$ (see for instance [43]), the next contribution can be read from the action associated to N D4-branes, namely from:

$$S^{\rm D4}_{\rm WZ,quad}=\mu_4\int_{R^{4+1}}\frac{1}{2}(2\pi\alpha')^2{\rm Tr}\bigg(\bigg[C^{(1)}+i(2\pi\alpha')i_\Phi i_\Phi C^{(3)} -\frac{1}{2}(2\pi\alpha')^2(i_\Phi i_\Phi)^2C^{(5)}+\ \dots\bigg]\wedge F^{(2)}\wedge F^{(2)}\bigg). \tag{6.5}$$

The first term describes the realization of a D0-brane as an instanton-like configuration in the N (D4, anti-D4) system. $\int_{R^4}{\rm Tr}F^{(2)}\wedge F^{(2)}$ gives an integer if the homotopy group $\Pi_3(U(N))=Z$, which happens generically for $N>2$. In the particular case $N=2^{k-1}$, where $2k$ denotes the codimension of the topological defect (in this case $k=2$) it is possible to give a representation of the tachyon vortex configuration [the generator of $\Pi_{2k-1}(U(N))$] such that all higher and lower dimensional charges vanish [48]. In this case a given Dp-brane can be realized as a bound state of N D$(p+2k)$ brane-antibrane pairs stepwise, i.e. through a hierarchy of embeddings onto higher dimensional brane-antibrane systems. Generically, for N D$(p+2k)$ brane-antibrane pairs this representation of the tachyon should give rise to N/2^{k-1} Dp-branes as instanton-like configurations. Therefore, in our case the N (D4, anti-D4) pairs would give rise to N/2 D0-branes.[26] The non-commutative configuration of N/2 D0-branes expanding into a D2-brane would arise as an instanton-like solution in the worldvolume of N (D4, anti-D4) pairs expanding into 2 (D6, anti-D6) pairs, as implied by the second term in Eq. (6.5). Similarly, the third term in this expression would describe the N/2 D0-branes expanding into a D4-brane as a bound state of N (D4, anti-D4) pairs expanding into 2 (D8, anti-D8) pairs.

This discussion can be generalized to arbitrary N Dp brane-antibrane systems. One finds that a non-commutative configuration of N/2^{k-1} D$(p-2k)$-branes expanding into a D$(p-2k+2r)$-brane[27] would be realized as a bound state of N (Dp, anti-Dp) branes opening up into 2^{k-1} [D$(p+2r)$, anti-D$(p+2r)$] branes.

A similar analysis can be performed in the type IIB theory. In this case it is easy to include as well non-Abelian systems of NS-NS branes and antibranes. For instance, one easily sees from Eq. (5.14) that a non-Abelian system of (NS5, anti-NS5) could support N D3-branes expanding into an NS5-brane as a topological configuration, arising after the condensation of the tachyon associated to open D1-branes stretched between the NS5 and the anti-NS5 branes. One notices as well that a system of coincident fundamental strings arises as a topological soliton from a non-Abelian (D3, anti-D3) brane configuration at strong coupling, with open D1-branes stretched between the branes and the anti-branes. Including higher order terms in Eq. (5.2) one finds that the system could also support a non-Abelian configuration associated to the N fundamental strings opening up into a D3-brane. Finally, although we did not consider D7-branes, one can easily see that coincident (D7, anti-D7) branes may give rise to N NS5-branes expanding into a D7-brane at strong coupling, i.e. when the tachyonic mode associated to open D1-branes stretched between branes and antibranes condenses.

VII. M-THEORY INTERPRETATION

For a single brane-antibrane system one possible way of inferring the emergence of the fundamental string as a topological soliton comes from M-theory [45,46]. Let us consider for instance a (D4, anti-D4) pair in type IIA. This system corresponds in M-theory to a coincident (M5, anti-M5) pair. The tachyonic mode in the open string stretched between the D4 and the anti-D4 branes occurs in M-theory in the form of a tachyonic mode in an M2-brane stretched between the M5 and the anti-M5 branes. The condensation of this tachyon through the previously discussed Higgs mechanism gives rise to an M2-brane as the resulting topological defect, as can be inferred both from the duality with type IIA and from the term:

$$\int_{R^{5+1}}\hat{C}\wedge d\hat{a}^{(2)} \tag{7.1}$$

in the worldvolume effective action of the (M5, anti-M5) system [45]. Here $\hat{a}^{(2)}$ is the six dimensional worldvolume antisymmetric tensor, which for the brane antibrane pair is not constrained by self or anti-self duality (see [45]). As Yi pointed out the reduction along a direction in the worldvolume of the (M5, anti-M5) pair transverse to the stretched M2-brane gives rise to a situation in which the tachyonic mode of an open D2-brane stretched between a D4 and an anti-D4 brane condenses to give rise to a fundamental string as the topological defect. This is predicted by the coupling above, where $\hat{a}^{(2)}$ reduces to a 2-form worldvolume field coupling to an open D2-brane in type IIA, and $B^{(2)}_{\mu\nu}=\hat{C}_{\mu\nu 11}$

[26]For this to make sense we need an even N.

[27]With N a multiple of 2^{k-1}.

remains as the resulting field. A systematic study of the possible brane-antibrane configurations in M-theory and type II theories can be found in [46], where it is seen that Kaluza-Klein reduction of the different possible M-theory configurations predicts that generically fundamental strings arise as topological defects in (Dp, anti-Dp) systems after the tachyonic mode of a D$(p-2)$-brane stretched between the pair condenses, and that the NS5-brane, the wave, the Kaluza-Klein monopole and the so-called exotic branes may also arise as topological solitons from various types of configurations.

We can now consider non-Abelian brane-antibrane systems and try to provide a similar description. Let us start by considering N coincident (D4, anti-D4) pairs. The situation in which the N pairs expand into a (D6, anti-D6) pair, which supports as solitonic configuration N D2-branes opening up into a single D4-brane, can be described in M-theory as N pairs of wrapped (M5, anti-M5) branes opening up into a Kaluza-Klein monopole-antimonopole pair. The Wess-Zumino action describing N (M5, anti-M5) pairs contains the couplings:

$$S_{\rm WZ}^{N(M5_w,\bar{M}5_w)}=\mu_4\int_{R^{4+1}}{\rm Tr}\{l_p^2[\hat{C}D\hat{X}D\hat{X}D\hat{X}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}i_{\hat k}\hat{\hat{C}}+\ \dots]\wedge(\hat{F}^{(2)}-\hat{F}^{(2)\prime})\}, \tag{7.2}$$

as implied from Eq. (3.7). These couplings may describe a solitonic configuration representing N M2-branes opening up into a wrapped M5-brane, with the M2-branes delocalized in the direction on which the M5-brane is wrapped. This configuration would occur when the tachyonic mode of the wrapped M2-branes stretching between the M5 and the anti-M5 branes condenses, and would be consistent with the situation in which a wrapped M5-brane would arise as a soliton after tachyonic condensation of a single (M6, anti-M6) pair, into which the system of N ($M5_w$, anti-$M5_w$) branes would have expanded. It is again consistent that the possible M2-branes that can end on both the wrapped M5-branes and the M6-branes are wrapped M2-branes. Had the M5-branes been unwrapped we would have found an inconsistency in this description derived from the fact that only unwrapped M2-branes could have ended on them.

Let us now discuss the case in which N (D2, anti-D2) branes in type IIA annihilate to give rise to N D0-branes. This system may support as well a solitonic configuration corresponding to the N D0 branes opening up into a single D2-brane, which would be described by the coupling

$$\int_{R^{2+1}}{\rm Tr}[i_\Phi i_\Phi C^{(3)}\wedge(F^{(2)}-F^{(2)\prime})], \tag{7.3}$$

in the Wess-Zumino action of the (D2, anti-D2) branes [see Eq. (3.4)]. The M-theory description of a single (D2, anti-D2) pair consists on an (M2, anti-M2) system in which the tachyonic mode of a wrapped M2-brane stretched between the brane and the antibrane condenses, giving rise to an M-wave as the topological defect, moving in the same direction on which the stretched M2-brane is wrapped [46]. This is described by the coupling

$$\int_{R^{2+1}}\hat{k}^{-2}\hat{k}^{(1)}\wedge(\hat{F}^{(2)}-\hat{F}^{(2)\prime}) \tag{7.4}$$

in the (M2, anti-M2) effective action. In the non-Abelian case the Wess-Zumino action of the N (M2, anti-M2) system contains the couplings:

$$S_{WZ}^{N(M2_t,\bar{M}2_t)}=\mu_2\int_{R^{2+1}}{\rm Tr}\{l_p^2[\hat{k}^{-2}\hat{k}^{(1)}+il_p^2 i_{\hat\Phi}i_{\hat\Phi}\times(\hat{C}-\hat{k}^{-2}\hat{k}^{(1)}\wedge i_{\hat k}\hat{C})+\ \dots]\wedge(\hat{F}^{(2)}-\hat{F}^{(2)\prime})+\ \dots\}. \tag{7.5}$$

The second term could describe a situation in which a nontrivial localized magnetic flux on R^2 gave rise to a configuration corresponding to M0-branes opening up into a transverse M2-brane as a topological solution. As we discussed before, this is precisely the kind of M2-brane that can occur as a topological defect in an (M5, anti-M5) system in which the M5-branes are wrapped on the special direction transverse to the M2-brane. This is then consistent with the picture in which the N (M2, anti-M2) pairs would open up into a single (M5, anti-M5) pair giving N M0-branes expanding into an M2-brane as the resulting topological defect.

The analysis of the effective action describing N coincident (KK, anti-KK) pairs may predict the existence of a noncommutative configuration corresponding to N M5-branes expanding into a monopole as a topological defect, with the M5-branes wrapped in the Taub-NUT direction of the monopole. This would be consistent with the situation in which the N (KK, anti-KK) pairs opened up into an (M9, anti-M9) pair, that supports a Kaluza-Klein monopole as a topological defect (see [46]). Reducing this configuration along the Killing direction one obtains a configuration of N D4-branes opening up into a D6-brane, which we discussed in the previous section.

VIII. OTHER BRANE-ANTIBRANE CONFIGURATIONS IN TYPE IIA

The M-theory configurations that we have considered in the previous section were constructed in such a way that they reproduced non-Abelian (Dp, anti-Dp) systems when reduced along their isometric direction. We are now going to see that it is possible to obtain other interesting configurations in type IIA after reduction along a different direction. The solitonic configurations that we find in this section are connected via T-duality with the strongly coupled configurations of type IIB solitonic branes that we considered in Sec. VI.

Let us start by considering a system of coincident (M5, anti-M5) branes. Reducing along a transverse direction gives rise to coincident (NS5, anti-NS5) pairs, wrapped in some special direction. The leading term of the corresponding effective action is given by the second line in Eq. (4.3), with

$\mathcal{H}^{(2)}$ replaced by the relative field strength. The second term shows that a configuration corresponding to N (localized) D2-branes expanding into a (wrapped) NS5-brane could arise as a topological solution. This is consistent with the situation in which the N (NS5, anti-NS5) pairs expanded into a pair of (KK6, anti-KK6) branes, since this single pair supports an NS5-brane as a topological solution, with the brane wrapped in the special Killing direction of the KK6-brane (see [46]). On the other hand, double dimensional reduction of the N (M5, anti-M5) system gives rise to N (D4, anti-D4) pairs. The leading terms of the worldvolume effective action are given by the second and third lines in Eq. (4.4). The coupling to the two different field strengths $\mathcal{K}^{(2)}$ and $\mathcal{K}^{(1)}$ shows that the system can support two different types of solitonic configurations depending on which type of stretched brane has its tachyonic mode condensing. If the tachyonic mode of stretched wrapped D2-branes condenses one could end up with a configuration corresponding to N fundamental strings expanding into a D4-brane. This would be described by the second coupling in the third line of Eq. (4.4). On the other hand if the stretched branes are fundamental strings then N D2-branes expanding into an NS5-brane may arise as the topological defect. This would be described by the second coupling in the second line of Eq. (4.4). In both cases the N branes are transverse to the special direction in which the expanded brane is wrapped. As before, these configurations would be consistent with the situation in which N (D4, anti-D4) pairs expanded into a (KK, anti-KK) pair, since the latter can support D4 and NS5 branes, wrapped in the Taub-NUT direction, as topological solutions [46].

A similar analysis starting with a delocalized (M2, anti-M2) non-Abelian system would give rise to the following configurations. Direct dimensional reduction gives N coincident delocalized (D2, anti-D2) branes, which could support N pp-waves expanding into a transverse D2-brane as a topological solution. The coupling $\int_{R^{2+1}}\mathrm{Tr}\hat{k}^{-2}\hat{k}^{(1)}\wedge(db^{(1)}-db^{(1)\prime})$, present in the worldvolume effective action of the system [see Eq. (4.1)], shows that N pp-waves may arise as a topological defect for non-vanishing magnetic flux. Recall that this magnetic flux is associated to open, wrapped, D2-branes stretched between branes and antibranes. For a localized (D2, anti-D2) system one can only see the emergence of a pp-wave as a solitonic solution for a single pair. The reason is that one needs to perform a worldvolume duality transformation in order to identify the coupling responsible for this process. Recalling the results in [46], one first needs to select one of the transverse directions to the D2-brane system, and write the coupling to the 3-form RR-potential as $\int_{R^{2+1}}C^{(3)}=\int_{R^{2+1}}(C^{(3)}+i_kC^{(3)}\wedge dy)$. Then y is dualized into a worldvolume vector field by adding a Lagrange multiplier term, $\int_{R^{2+1}}dy\wedge db^{(1)}$, and the final dual action contains the term responsible for the creation of the pp-wave: $\int_{R^{2+1}}k^{-2}k^{(1)}\wedge db^{(1)}$ (see [46]). For a non-Abelian configuration of D2-branes one cannot however see the emergence of this coupling, given that the explicit worldvolume duality transformation cannot be made. One needs to start with a configuration in which the direction of propagation of the wave is already singled out, as is the case for a configuration of delocalized D2-branes. The emergence of this noncommutative configuration would be consistent with the situation in which the transverse D2-branes opened up into an NS5-brane, which can support a transverse D2-brane as a topological soliton, as we have discussed above.

Reducing the (M2, anti-M2) system along a worldvolume direction one obtains N coincident (F1, anti-F1) branes delocalized in one direction. This system may support as well a topological defect corresponding to N pp-waves opening up into a D2-brane, now arising after the tachyon of the stretched, wrapped, fundamental strings condenses. This can be seen from the second line in Eq. (4.2). This configuration would be consistent with the fact that the transverse F1-branes could open up into a ($D4_w$, anti-$D4_w$) pair, and this system admits a delocalized D2-brane as a topological soliton, as we have described above.

Finally, the reduction of a non-Abelian system of coincident Kaluza-Klein anti Kaluza-Klein monopoles gives, when reducing along a worldvolume direction, a system of coincident type IIA (KK, anti-KK) pairs, which could support two types of solitonic configurations: N D4-branes expanding into a monopole, and N NS5-branes expanding into a KK6-brane, depending on whether the condensing tachyon is associated to open, wrapped, D2-branes or F1-branes. The corresponding couplings can be read from Eq. (4.5). This would be consistent with the situation in which the Kaluza-Klein monopoles expanded into a (KK8, anti-KK8) pair, since this system can support both a Kaluza-Klein monopole and a KK6-brane as solitonic solutions (see [46]). The reduction along a transverse direction gives a system of coincident (KK6, anti-KK6) branes, which could support as a topological defect a configuration corresponding to N NS5-branes opening up into a KK6-brane. This would be consistent with the situation in which N (KK6, anti-KK6) pairs expanded into an (NS9, anti-NS9) pair, which can support a KK6-brane as a topological defect [46].

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