Dynamics of Symmetry Breaking and Tachyonic Preheating

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(December 18, 2000)

We reconsider the old problem of the dynamics of spontaneous symmetry breaking using 3d lattice simulations, and develop a theory of tachyonic preheating, which occurs due to the spinodal instability of the scalar field. Tachyonic preheating is so efficient that symmetry breaking typically completes within a single oscillation of the field distribution as it rolls towards the minimum of its effective potential. As an application of this theory we consider preheating in the hybrid inflation scenario, including SUSY-motivated F-term and D-term inflationary models. We show that preheating in hybrid inflation is typically tachyonic and the stage of oscillations of a homogeneous component of the scalar fields driving inflation ends after a single oscillation. Our results may also be relevant for the theory of the formation of disoriented chiral condensates in heavy ion collisions.

PACS: 98.80.Cq, CITA-2000-60, SU-ITP-00-35, IFT- UAM/CSIC-00-40, FT-UAM-00/26, CERN-TH/2000-365 hep-ph/0012142

I. INTRODUCTION

Spontaneous symmetry breaking is a basic feature of all realistic theories of elementary particles. In the simplest models, this instability appears because of the presence of tachyonic mass terms such as $-m^2\phi^2/2$ in the effective potential. As a result, long wavelength quantum fluctuations ϕ_k of the field ϕ with momenta $k < m$ grow exponentially, $\phi_k \sim \exp(t\sqrt{m^2-k^2})$, which leads to spontaneous symmetry breaking.

This process may occur gradually, as in the theory of second order phase transitions, when the parameter m^2 slowly changes from positive to negative and the degree of symmetry breaking gradually increases in time [1]. Sometimes the symmetry breaking occurs discontinuously, due to a first order phase transition [2]. But there is also another possibility, which we will study in this paper: The tachyonic mass term may appear suddenly, on a time scale that is much shorter than the time required for symmetry breaking to occur. This may happen, for example, when the hot plasma created by heavy ion collisions in the 'little Big Bang' suddenly cools down [3]. A more important application from the point of view of cosmology is the process of preheating in the hybrid inflation scenario [4,5], where inflation ends in a 'waterfall' regime triggered by tachyonic instability.

During the last few years we have learned that particle production by an oscillating scalar field may occur within a dozen oscillations due to the nonperturbative process of particle production called preheating [6]. Usually preheating is associated with broad parametric resonance in the presence of a coherently oscillating inflaton field [6], but other mechanisms are also possible, see e.g. [7,8].

In this paper we will concentrate on what we call *tachyonic preheating*, which occurs due to tachyonic (spinodal) instabilities in the field responsible for the symmetry breaking. The process of symmetry breaking has been studied before by advanced methods of perturbation theory, see e.g. [9] and references therein. However, spontaneous symmetry breaking is a strongly nonlinear and nonperturbative effect. It usually leads to the production of particles with large occupation numbers inversely proportional to the coupling constants. As a result the perturbative description, including the Hartree and $1/N$ approximations, has limited applicability. It does not properly describe rescattering of created particles and other important features such as production of topological defects.

For further theoretical understanding of the issue one should go beyond perturbation theory. Fortunately, during the last few years new methods of lattice simulations have been developed. They are based on the observation that quantum states of bose fields with large occupation numbers can be interpreted as classical waves and their dynamics can be fully analyzed by solving relativistic wave equations on a lattice [10,11]. In our paper we will further develop these methods, including effects of renormalization, and apply them to the investigation of spontaneous symmetry breaking and tachyonic preheating. A significant advantage of these methods as compared to other lattice simulations of quantum processes is that the semi-classical nature of the effects under investigation allows us to have a clear visual picture of all the processes involved. That is why this paper is accompanied by several computer generated movies that show the development of symmetry breaking in various models.

We will show that tachyonic preheating can be extremely efficient. In many models it leads to the transfer of the initial potential energy density $V(0)$ into the energy of scalar particles within a single oscillation. Contrary to standard expectations, the first stage of preheating in hybrid inflation is typically tachyonic, which means that the stage of oscillations of a homogeneous component of the scalar fields driving inflation either does not exist at all or ends after a single oscillation. A detailed description of our results will be given elsewhere [12].

II. TACHYONIC INSTABILITY AND SYMMETRY BREAKING

Symmetry breaking occurs due to tachyonic instability and may be accompanied by the formation of topological defects. Here we will consider two toy models that are prototypes for many interesting applications, including symmetry breaking in hybrid inflation.

A. Quadratic potential

The simplest model of spontaneous symmetry breaking is based on the theory with the effective potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 \equiv \frac{m^4}{4\lambda} - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \,, \qquad (1)$$

where $\lambda \ll 1$. $V(\phi)$ has a minimum at $\phi = \pm v$ and a maximum at $\phi = 0$ with curvature $V'' = -m^2$.

The development of tachyonic instability in this model depends on the initial conditions. We will assume that initially the symmetry is completely restored so that the field ϕ does not have any homogeneous component, i.e. $\langle\phi\rangle = 0$. But then $\langle\phi\rangle$ remains zero at all later stages, and for the investigation of spontaneous symmetry breaking one needs to find the spatial distribution of the field $\phi(x,t)$. To avoid this complication, many authors assume that there is a small but finite initial homogeneous background field $\phi(t)$, and even smaller quantum fluctuations $\delta\phi(x,t)$ that grow on top of it. This approximation may provide some interesting information, but quite often it is inadequate. In particular, it does not describe the creation of topological defects, which, as we will see, is not a small nonperturbative correction but an important part of the problem.

For definiteness, we suppose that in the symmetric phase $\phi = 0$ there are usual quantum fluctuations of the massless field with the mode functions $\frac{1}{\sqrt{2k}} e^{-ikt + i\vec{k}\vec{x}}$ and then at $t = 0$ we 'turn on' the term $-m^2\phi^2/2$ corresponding to the negative mass squared $-m^2$. The modes with $k = |\vec{k}| < m$ grow exponentially so the dispersion of these fluctuations can be estimated as

$$\langle\delta\phi^2\rangle = \frac{1}{4\pi^2}\int_0^m dk\, k\, e^{2t\sqrt{m^2-k^2}} \,. \qquad (2)$$

To get a qualitative understanding of the process of spontaneous symmetry breaking, instead of many growing waves with momenta $k < m$ in (2) let us consider first a single sinusoidal wave $\delta\phi = \Delta(t)\cos kx$ with $k \sim m$ and with initial amplitude $\sim \frac{m}{2\pi}$ in one-dimensional space. The amplitude of this wave grows exponentially until its becomes $\mathcal{O}(v) \sim m/\sqrt{\lambda}$. This leads to the division of the universe into domains of size $\mathcal{O}(m^{-1})$ in which the field changes from $\mathcal{O}(v)$ to $\mathcal{O}(-v)$. The gradient energy density of domain walls separating areas with positive and negative ϕ will be $\sim k^2\delta\phi^2 = O(m^4/\lambda)$. This energy is of the same order as the total initial potential energy of the field $V(0) = m^4/4\lambda$. This is one of the reasons why any approximation based on perturbation theory and ignoring topological defect production cannot give a correct description of the process of spontaneous symmetry breaking.

Thus a substantial part of the false vacuum energy $V(0)$ is transferred to the gradient energy of the field ϕ when it rolls down to the minimum of $V(\phi)$. Because the initial state contains many quantum fluctuations with different phases growing at a different rate, the resulting field distribution is very complicated, so it cannot give all of its gradient energy back and return to its initial state $\phi = 0$. This is one of the reasons why spontaneous symmetry breaking and the main stage of preheating in this model may occur within a single oscillation of the field ϕ.

Meanwhile if one were to make the usual assumption that initially there exists a small homogeneous background field $\phi \ll v$ with an amplitude greater than the amplitude of the growing quantum fluctuations $\delta\phi$, so that $m/2\pi \ll \phi < m/\sqrt{\lambda}$, one would find out that when ϕ falls to the minimum of the effective potential the gradient energy of the fluctuations remains relatively small. One would thus come to the standard conclusion that the field should experience many fluctuations before it relaxes near the minimum of $V(\phi)$. To avoid this error, we need to perform a complete study of the growth of all tachyonic modes and their subsequent interaction without making this simplifying assumption about the existence of the homogeneous field ϕ.

Consider the tachyonic growth of all fluctuations with $k < m$, i.e. those that contribute to $\langle\delta\phi^2\rangle$ in Eq. (2). This growth continues until $\sqrt{\langle\delta\phi^2\rangle}$ reaches the value $\sim v/2$, since at $\phi \sim v/\sqrt{3}$ the curvature of the effective potential vanishes and instead of tachyonic growth one has the usual oscillations of all the modes. This happens within the time $t_* \sim \frac{1}{2m}\ln\frac{\pi^2}{\lambda}$. The exponential growth of fluctuations up to that moment can be interpreted as the growth of the occupation number of particles with $k \ll m$. These occupation numbers at the time t_* grow up to

$$n_k \sim \exp(2mt_*) \sim \exp\left(\ln\frac{\pi^2}{\lambda}\right) = \frac{\pi^2}{\lambda} \gg 1 \,. \qquad (3)$$

One can easily verify that t_* depends only logarithmically on the choice of the initial distribution of quantum fluctuations. For small λ the fluctuations with $k \ll m$ have very large occupation numbers, and therefore they can be interpreted as classical waves of the field ϕ.

The dominant contribution to $\langle \delta\phi^2 \rangle$ in Eq. (2) at the moment t_* is given by the modes with wavelength $l_* \sim 2\pi k_*^{-1} \sim \sqrt{2}\pi m^{-1} \ln^{1/2}(C\pi^2/\lambda) > m^{-1}$, where $C = \mathcal{O}(1)$. As a result, at the moment when the fluctuations of the field ϕ reach the minimum of the effective potential, $\sqrt{\langle \phi^2 \rangle} \sim v$, the field distribution looks rather homogeneous on a scale $l \lesssim l_*$. On average, one still has $\langle \phi \rangle = 0$. This implies that the universe becomes divided into domains with two different types of spontaneous symmetry breaking, $\phi \sim \pm v$. The typical size of each domain is $l_*/2 \sim \frac{\pi}{\sqrt{2}}\, m^{-1} \ln^{1/2} \frac{C\pi^2}{\lambda}$, which differs only logarithmically from our previous estimate m^{-1}. At later stages the domains grow in size and percolate (eat each other up), and spontaneous symmetry breaking becomes established on a macroscopic scale.

Of course, these are just simple estimates which should be followed by a detailed quantitative investigation. When the field rolls down to the minimum of its effective potential, its fluctuations scatter off each other as classical waves. It is difficult to study this process analytically, but fortunately one can do it numerically using the method of lattice simulations developed in [10,11]. We performed our simulations on lattices with either 128^3 and 256^3 gridpoints. A detailed description of our calculations will be given in [12]; here we will only present our main results.

Figure 1 illustrates the dynamics of symmetry breaking in the model (1). It shows the probability distribution $P(\phi, t)$, which is the fraction of the volume containing the field ϕ at a time t if at $t = 0$ one begins with the probability distribution concentrated near $\phi = 0$, with the quantum mechanical dispersion (2).

As we see from this figure, after the first oscillation the probability distribution $P(\phi, t)$ becomes narrowly concentrated near the two minima of the effective potential corresponding to $\phi = \pm v$. In this sense one can say that symmetry breaking completes within one oscillation. To demonstrate that this is not a strong coupling effect, we show the results for the model (1) with $\lambda = 10^{-4}$. Note that only when the distribution stabilizes and the domains become large can one use the standard language of perturbation theory describing scalar particles as excitations on a (locally) homogeneous background. That is why the use of the nonperturbative approach based on lattice simulations was so important for our investigation.

The growth of fluctuations in this model is shown in Fig. 2. It shows how fluctuations grow in a two-dimensional slice of 3D space. Maxima correspond to domains with $\phi > 0$, minima correspond to domains with $\phi < 0$.

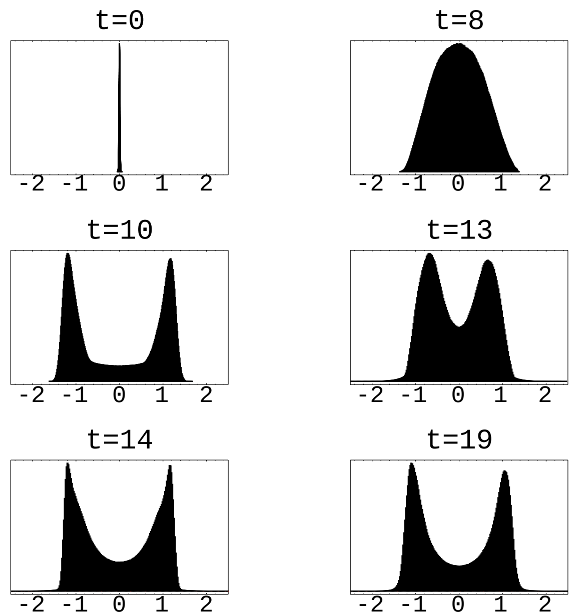

FIG. 1. The process of symmetry breaking in the model (1) for $\lambda = 10^{-4}$. In the beginning the distribution is very narrow. Then it spreads out and shows two maxima which oscillate about $\phi = \pm v$ with an amplitude much smaller than v. These maxima never come close to the initial point $\phi = 0$. The values of the field are shown in units of v.

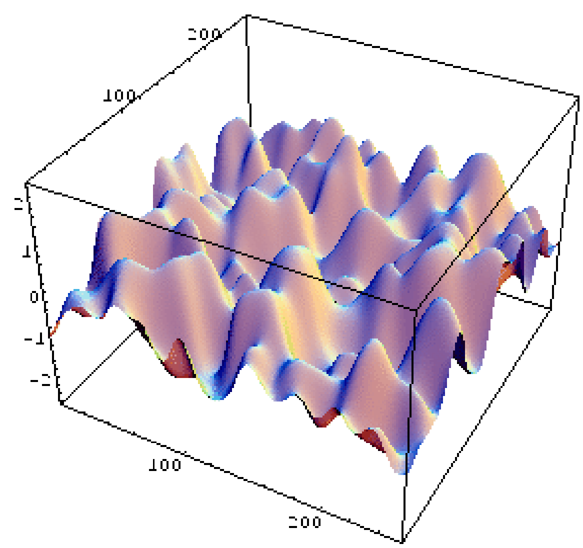

FIG. 2. Growth of quantum fluctuations in the process of symmetry breaking in the quadratic model (1).

The dynamics of spontaneous symmetry breaking in this model is even better illustrated by the computer generated movie is included in this submission as a file 1.gif, which can be found also at http://physics.stanford.edu/gfelder/hybrid/1.gif. It consists of an animated sequence of images similar to the one shown in Fig. 2. These images show the whole process of spontaneous symmetry breaking from the growth of small gaussian fluctuations of the field ϕ to the creation of domains with $\phi = \pm v$.

Similar results are valid for the theory of a multi-component scalar field ϕ_i with the potential (1). For example, the behavior of the probability distribution $P(\phi_1, \phi_2, t)$ in the theory of a complex scalar field $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ is shown in Fig. 3. As we see, after a single oscillation this probability distribution has stabilized at $|\phi| \sim v$. A computer generated movie illustrating this process is included in this submission as a file 2.gif, which can be found also at http://physics.stanford.edu/gfelder/hybrid/2.gif.

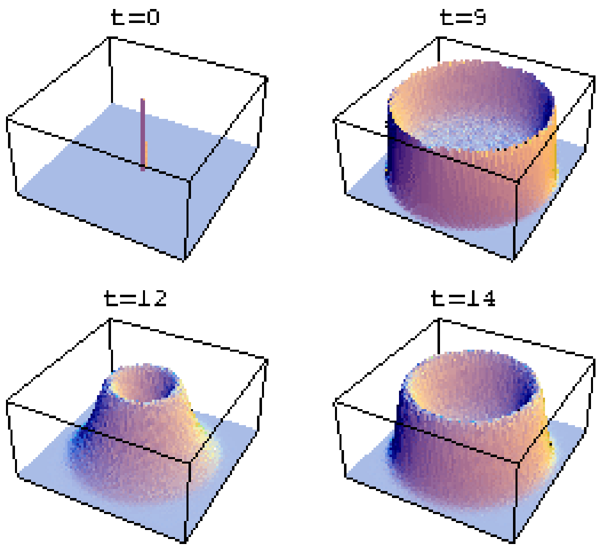

FIG. 3. The process of symmetry breaking in the model (1) for a complex field ϕ. The field distribution falls down to the minimum of the effective potential at $|\phi| = v$ and experiences only small oscillations with rapidly decreasing amplitude $|\Delta\phi| \ll v$.

B. Cubic potential

Another important example of tachyonic preheating is provided by the theory

$$V = -\frac{\lambda}{3} v\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{\lambda}{12} v^4 \ . \qquad (4)$$

This potential is a prototype of the potential that appears in descriptions of symmetry breaking in F-term hybrid inflation [13,14].

The first question to address concerns the initial amplitude of the tachyonic modes in this model. This is nontrivial because $m^2(\phi) = -2\lambda v\phi + 3\lambda\phi^2$ vanishes at $\phi = 0$. However, eq. (2) implies that scalar field fluctuations with momentum $\sim k$ have initial amplitude $\langle\delta\phi^2\rangle \sim \frac{k^2}{8\pi^2}$. They enter a self-sustained tachyonic regime if $k^2 < |m^2_{\rm eff}| = 2\lambda v\sqrt{\langle\delta\phi^2\rangle} \sim \frac{\lambda v k}{2\pi}$, i.e. if $k < \frac{\lambda v}{2\pi}$. The average initial amplitude of the growing tachyonic fluctuations with momenta smaller than $\frac{\lambda v}{2\pi}$ is

$$\delta\phi_{\rm rms} \sim \frac{\lambda v}{4\pi^2}. \qquad (5)$$

These fluctuations grow until the amplitude of $\delta\phi$ becomes comparable to $2v/3$, and the effective tachyonic mass vanishes. At that moment the field can be represented as a collection of waves with dispersion $\sqrt{\langle\delta\phi^2\rangle} \sim v$, corresponding to coherent states of scalar particles with occupation numbers $n_k \sim \left(\frac{4\pi^2}{\lambda}\right)^2 \gg 1$.

Because of the nonlinear dependence of the tachyonic mass on ϕ, a detailed description of this process is more involved than in the theory (1). Indeed, even though the typical amplitude of the growing fluctuations is given by (5), the speed of the growth of the fluctuations increases considerably if the initial amplitude is somewhat bigger than (5). Thus even though the fluctuations with amplitude a few times greater than (5) are exponentially suppressed, they grow faster and may therefore have greater impact on the process than the fluctuations with amplitude (5). Low probability fluctuations with $\delta\phi \gg \delta\phi_{\rm rms}$ correspond to peaks of the initial Gaussian distribution of the fluctuations of the field ϕ. Such peaks tend to be spherically symmetric [15]. As a result, the whole process looks not like a uniform growth of all modes, but more like bubble production (even though there are no instantons in this model). The results of our lattice simulations for this model are shown in Fig. 4. The bubbles (high peaks of the field distribution) grow, change shape, and interact with each other, rapidly dissipating the vacuum energy $V(0)$. A computer generated movie illustrating this process is included in this submission as a file 3.gif. It can be found also at http://physics.stanford.edu/gfelder/hybrid/3.gif.

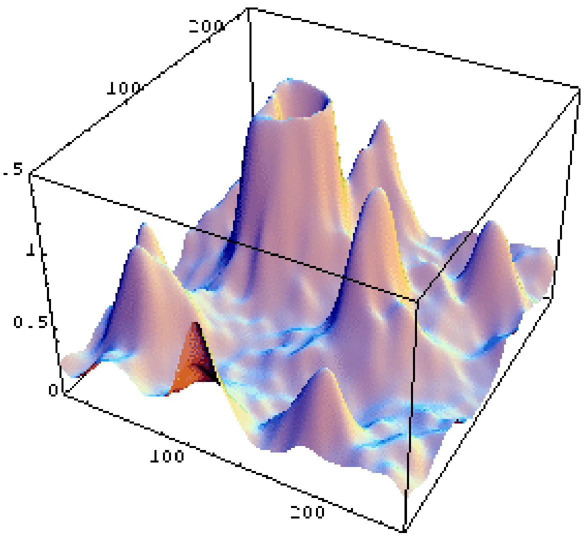

FIG. 4. Fast growth of the peaks of the distribution of the field ϕ in the cubic model (4). It should be compared with Fig. 2 for the quadratic model (1).

Fig. 5 shows the probability distribution $P(\phi, t)$ in the model (4). As we see, in this model the field also relaxes near the minimum of the effective potential after a single oscillation.

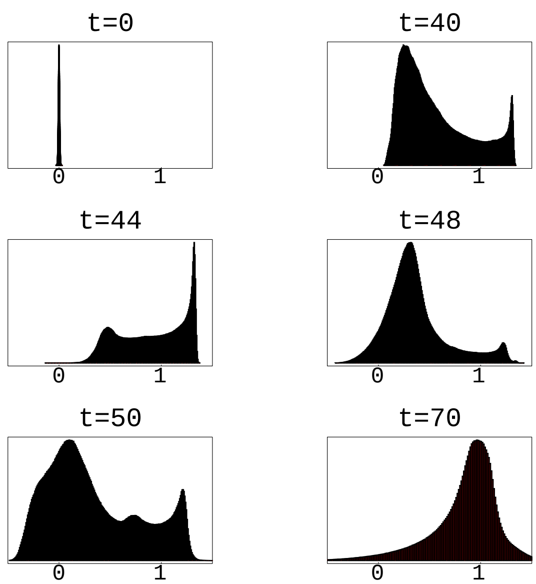

FIG. 5. Histograms describing the process of symmetry breaking in the model (4) for $\lambda = 10^{-2}$. After a single oscillation the distribution acquires the form shown in the last frame and after that it practically does not oscillate.

One should note that numerical investigation of this model involved specific complications due to the necessity of performing renormalization. Lattice simulations involve the study of modes with large momenta that are limited by the inverse lattice spacing. These modes give an additional contribution to the effective parameters of the model. In the simple model (1) these corrections were relatively small, but in the cubic model they induce an additional linear term $\lambda v \phi \langle \phi^2 \rangle$. This term should be subtracted by the proper renormalization procedure, which brings the effective potential back to its form (4).

For completeness we would like to mention also that in the theory with the quartic potential $V = V(0) - \frac{1}{4}\lambda\phi^4$ the decay of the symmetric phase occurs via tunneling and the formation of bubbles, even though there is no barrier between $\phi = 0$ and $\phi \neq 0$ [16]. In this case quantum tunneling can be heuristically interpreted as a building up of stochastic fluctuations $\delta\phi$ [17]. In this respect the character of tachyonic instability for the cubic potential is intermediate between the quadratic and quartic potentials.

III. TACHYONIC PREHEATING IN HYBRID INFLATION

The results obtained in the previous section have important implications for the theory of reheating in the hybrid inflation scenario. The basic form of the effective potential in this scenario is [4]

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2 + \frac{1}{2}m^2\phi^2 \,. \qquad (6)$$

The point where $\phi = \phi_c = M/g$ and $\sigma = 0$ is a bifurcation point. Here $M \equiv \sqrt{\lambda} v$. The global minimum is located at $\phi = 0$ and $|\sigma| = v$. However, for $\phi > \phi_c$ the squares of the effective masses of both fields $m_\sigma^2 = g^2\phi^2 - \lambda v^2 + 3\lambda\sigma^2$ and $m_\phi^2 = m^2 + g^2\sigma^2$ are positive and the potential has a valley at $\sigma = 0$. Inflation in this model occurs while the ϕ field rolls slowly in this valley towards the bifurcation point. When ϕ reaches ϕ_c, inflation ends and the fields rapidly roll towards the global minimum at $\phi = 0$, $|\sigma| = v$. If σ is a real one-component scalar, this may lead to the formation of domain walls. To avoid this problem, we assume that σ is a complex field. In this case symmetry breaking after inflation produces cosmic strings instead of domain walls [4].

In realistic versions of this model the mass m is extremely small, as well as its initial velocity $\dot{\phi}$. The fields fall down along a certain trajectory $\phi(t), \sigma(t)$ in such a way that initially this trajectory is absolutely flat, then it rapidly falls down, and then it becomes flat again near the minimum of $V(\phi, \sigma)$. This implies that the curvature of the effective potential along this curve is initially negative. Therefore the fields should experience tachyonic instability along the way.

The decay of the homogeneous inflaton field and preheating in hybrid inflation were considered in two papers: for the simplest non-supersymmetric scenario with a variety of parameters [5] and for a SUSY F-term model [14]. Both papers were focused on the possibility of parametric resonance. However, in [5] it was also pointed out that for $g^2 \gg \lambda$ the field σ falls down only when the field ϕ reaches some point $\phi \ll \phi_c$. As a result, the motion of the field σ occurs just like the motion of the field ϕ in the theory (1). In this case one has a tachyonic instability and the fields relax near the minimum of $V(\phi, \sigma)$ within a single oscillation [5]. For all other relations between g^2 and λ the fields follow more complicated trajectories. One might expect that the fields would in general experience many oscillations, which might or might not lead to parametric resonance [5,14].

We performed an investigation of preheating in hybrid inflation in the model (6) with two scalar fields (one real and one complex) and in SUSY-motivated F-term and D-term inflation models with three complex fields. We used methods similar to those that we applied in the previous section to the investigation of spontaneous symmetry breaking, including 3D lattice simulations. We have found that efficient tachyonic preheating is a generic feature of the hybrid inflation scenario, which means that the stage of oscillations of the quasi-homogeneous components of the scalar fields driving inflation is typically terminated by the backreaction of fluctuations. Here we report the qualitative results of our findings, leaving the technical details for the longer paper [12].

In particular, Fig. 6 shows the process of spontaneous symmetry breaking in the theory (6) for $g^2 = 10^{-4}$, $\lambda = 10^{-2}$, $M = 10^{15}$ GeV. The probability distribution oscillates along the ellipse $g^2\phi^2 + \lambda\sigma^2 = g^2\phi_c^2$. As before, it relaxes near the minimum of the effective potential within a single oscillation. A computer generated movie illustrating this process is included in this submission as a file 4.gif. It can be found also at http://physics.stanford.edu/gfelder/hybrid/4.gif.

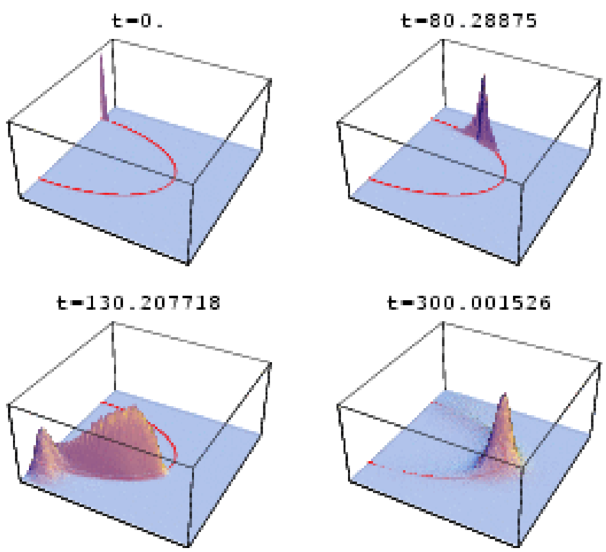

FIG. 6. The process of symmetry breaking in the hybrid inflation model (6) for $g^2 \ll \lambda$. The field distribution moves along the ellipse $g^2\phi^2+\lambda\sigma^2 = g^2\phi_c^2$ from the bifurcation point $\phi = \phi_c$, $\sigma = 0$.

The theory of preheating in D-term inflation for various relations between g^2 and λ [18] is very similar to the theory discussed above. Meanwhile, in the case $g^2 = 2\lambda$ the effective potential (6) has the same features as the effective potential of SUSY-inspired F-term inflation [13]. In this scenario the fields ϕ and σ fall down along a simple linear trajectory [14], so that instead of following each of these fields one may consider a linear combination of them and find the effective potential in this direction. This effective potential has exactly the same shape as our cubic potential (4). Thus all of the results which we obtained for tachyonic preheating in the theory (4) should be valid for F-term inflation as well, with minor modifications due to the presence of additional degrees of freedom that can be excited during preheating. Indeed, we were able to confirm these conclusions by lattice simulations of the F-term and D-term models.

Thus we see that tachyonic preheating is a typical feature of hybrid inflation. The production of bosons in this regime is nonperturbative, very fast, and efficient, but it is usually not related to parametric resonance. Instead it is related to the production and scattering of classical waves of the scalar fields. Of course, one should keep in mind that there may exist some particular versions of hybrid inflation in which tachyonic preheating is inefficient, e.g. because of fast motion of the field ϕ near the bifurcation point.

So far we have discussed tachyonic preheating in the inflaton sector of hybrid models, which leads to the decay of the homogeneous fields and the excitation of their fluctuations. Preheating in the non-inflaton sector and the subsequent development of equilibrium in hybrid models were considered in [19]. Light bosonic fields interacting with scalars from the inflaton sector are dragged into the process of preheating. Excitations of these fields rapidly acquire large occupation numbers and further evolve into equilibrium together with the scalars from the inflaton sector.

The tachyonic nature of preheating in hybrid inflation implies, in particular, that instead of the production of gravitinos by a coherently oscillating field [20,21], in hybrid inflation one should study gravitino production due to the scattering of classical waves of the scalar fields produced by tachyonic preheating. Our results may also be important for the theory of the generation of the baryon asymmetry of the universe at the electroweak scale [22].

From a more general point of view, however, the most important application of our results is to the general theory of spontaneous symmetry breaking. This theory constitutes the basis of all models of weak, strong and electromagnetic interactions. The new methods developed during the last few years in application to the theory of reheating after inflation have been applied in this paper to the theory of spontaneous symmetry breaking. These methods have for the first time allowed us not only to calculate correlation functions and spectra of produced particles, but to actually *see* the process of spontaneous symmetry breaking and to reveal some of its rather unexpected features. We will return to the discussion of this issue in the coming publication [12].

We thank NATO Linkage Grant 975389 for support. J.G.B. and G.F. thank CITA for its hospitality. L.K. and P.G. were supported by NSERC; L.K. was supported by CIAR; G.F. and A.L. were supported by NSF grant PHY-9870115 and by the Templeton Foundation; J.G.B. was supported by Spanish CICYT grant AEN97-1678, and also thanks the TH-Division at CERN for its hospitality.

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