Spin Dependent Fragmentation Functions for Heavy Flavor Baryons and Single Heavy Hyperon Polarization*

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Abstract

Spin dependent fragmentation functions for heavy flavor quarks to fragment into heavy baryons are calculated in a quark-diquark model. The production of intermediate spin 1/2 and 3/2 excited states is explicity included. Λ_b , Λ_c and Ξ_c production rate and polarization at LEP energies are calculated and, where possible, compared with experiment. A different approach, also relying on a heavy quark-diquark model, is proposed for the small momentum transfer inclusive production of polarized heavy flavor hyperons. The predicted Λ_c polarization is roughly in agreement with experiment.

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1 Introduction

The fragmentation of quarks into hadrons has been of considerable theoretical interest as a means of exploring both the perturbative and soft regime of QCD. The spin dependence of the fragmentation process is also of interest because of the light it can shed on the related longstanding, puzzling experimental results on how the nucleon spin is shared by its partons. An important theoretical step toward predicting fragmentation functions was made several years ago – it was realized that renormalization group improved QCD perturbation theory, along with the non-relativistic constituent quark model of the hadrons, could apply to the fragmentation of heavy flavor quarks into heavy flavor mesons [1, 2]. In a series of papers, Adamov and Goldstein [3, 4] have extended this model to the fragmentation of heavy flavor quarks into heavy flavor baryons via the quark-diquark structure of the baryons. (At the same time a Russian group [5] made a similar extension to baryons. More recently the light cone expansion of the fragmentation functions was used by the Amsterdam group [6] with an algebraic model to generate predictions similar to ours.) In the first section following, a summary of the recent extension of this model for heavy baryon fragmentation to include baryon excited states will be presented. It is through excited state production that *depolarization* of the heavy quark can occur, as will be shown.

Another longstanding puzzle in hadronic scattering has been the sizeable, nearly energy independent polarization of inclusively produced hyperons. Early theoretical expectations were that single polarization effects should be small [7], since (massless) QCD conserves chirality. Finite mass corrections allow some polarization, but the magnitude remains small [8]. However, it was expected that in the case of heavy flavor quarks and baryons the mass effects begin to produce sizeable polarizations [9]. This has been born out by recent data on inclusively produced Λ_c [10]. In the second section below I will summarize the model calculations [11] that confront this data.

2 Fragmentation functions for heavy baryons

The fragmentation of a heavy flavor quark (Q_1) into a doubly heavy flavor meson ($Q_1 + \bar{Q}_2$) is in the perturbative regime of QCD; the required momentum transfer to produce a heavy pair ($Q_2 + \bar{Q}_2$) is large compared to the

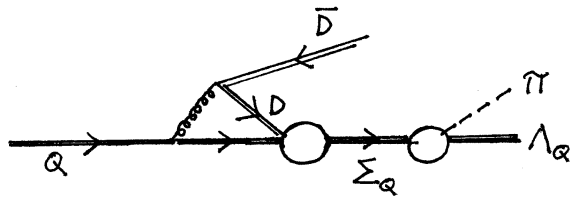

Figure 1: Diagram representing the combined perturbative and soft QCD calculation of fragmentation through excited baryons.

QCD scale λ_{QCD}. The final meson is formed via the non-relativistic potential between the comoving quark and antiquark. The corresponding picture for baryon production has the heavy quark pair replaced by a diquark pair, as shown in Fig. 1. The heavy quark emits a hard gluon that creates the diquark pair. Diquarks can be either scalar or vector, with the vector diquark having the larger mass (as suggested by the nucleon-Δ mass difference). If the diquark is scalar, the heavy quark forms a ground state spin $\frac{1}{2}$ baryon with it. Since the scalar diquark is spinless and the ground state has no orbital angular momentum, the heavy baryon has nearly the same helicity as the heavy quark. Vector diquarks have spin 1 and the resulting helicity of a baryon (composed of a heavy quark and a vector diquark) is not the same as that of the single heavy quark, but naturally depends on the helicity of the diquark. That does not produce any depolarization of the heavy quark by itself - the heavy quark inside of the baryon does not change its helicity. However, depolarization may occur during the "relaxation" of the baryon into the ground state. The depolarizing features of the spin excited baryon decay into the ground state were discussed by Falk and Peskin [12]. Two parameters are crucial in determining the amount of depolarization relaxation, ΔM, the mass splitting between the $\frac{3}{2}$ and $\frac{1}{2}'$ excited states of the heavy baryon (which is related to the spin-dependent part of the QCD interaction with scale determined by $\frac{\Lambda^2_{QCD}}{m_Q}$ and is responsible for flipping the quark spin) and the excited baryon lifetimes , $\frac{1}{\Gamma}$, which will be nearly the same. The longer this lifetime is, in comparison to the time required for heavy quark spin flipping, the more likely it is for the diquark and quark to mix together forming a randomized spin state. For the heavy mass limit ΔM disappears and there is no depolarization. For finite mass depolarization occurs. Recent data on Λ_b confirm this expectation; OPAL [13] finds an average longitudinal polarization of -0.56 ± 0.22 where the Standard Model prediction *without* excited state contributions would be -0.94. This and related results provide the impetus for studying the fragmentation via intermediate excited states.

The fragmentation functions (at leading twist, i.e. large Q^2) that I will discuss can be related to simple probability functions for quarks of fixed or mixed helicities to produce ground state (spin $\frac{1}{2}$) baryons with the same or different helicities. Symbolically,

$$\begin{aligned}
\hat{f}_1 &\sim \left| |\lambda_Q = +\frac{1}{2} > \rightarrow |\lambda_{B_Q} = +\frac{1}{2} > \right|^2 + \left| |\lambda_Q = +\frac{1}{2} > \rightarrow |\lambda_{B_Q} = -\frac{1}{2} > \right|^2 \\
\hat{g}_1 &\sim \left| |\lambda_Q = +\frac{1}{2} > \rightarrow |\lambda_{B_Q} = +\frac{1}{2} > \right|^2 - \left| |\lambda_Q = +\frac{1}{2} > \rightarrow |\lambda_{B_Q} = -\frac{1}{2} > \right|^2 \\
\hat{h}_1 &\sim \left| ||S_T^Q = +\frac{1}{2} > \rightarrow |S_T^{B_Q} = +\frac{1}{2} > \right|^2 - \left| |S_T^Q = +\frac{1}{2} > \rightarrow |S_T^{B_Q} = -\frac{1}{2} > \right|^2
\end{aligned} \quad (1)$$

where ${S_T}^Q \sim |+> \pm |->$ is the *transversity*, a variable introduced originally by Moravcsik and Goldstein [14] to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. The chiral odd fragmentation function $\hat{h}_1$ is of particular interest, since the analogous structure function can not be measured in deep inelastic scattering.

The perturbative calculation of the baryon B fragmentation functions proceeds from the partial width [2] for the inclusive decay process $Z^0 \rightarrow B+X$. The expression for that width will be simplified if we restrict ourselves to the fragmentation channel shown in Fig. 1. Then, while z is kept fixed, the limit of large mass of the Z^0 along with large energy of the heavy quark, q_0, and the baryon, l_0, yields

$$\lim_{l_0 \longrightarrow \infty} d\Gamma(Z^0 \rightarrow B(E) + X) = \lim_{q_0 \longrightarrow \infty} \int_0^1 dz\, d\hat{\Gamma}(Z^0 \rightarrow Q(E/z) + X, \mu)\, f_1(z, \mu). \quad (2)$$

for the spin averaged case, with

$$\int_0^1 dz\, f_1(z, \mu) = \frac{\Gamma_1}{\Gamma_0}, \quad (3)$$

where Γ_1 is the decay width of Z^0 into the ground state baryon and appropriate remnants - antiquark, spectator diquark and pion, while Γ_0 is the total decay width of the Z^0 into the heavy quark pair [3].

For the fragmentation into an excited baryon, followed by π (or γ) decay into the ground state the full Z^0 width has multiple phase space integrations.

$$\Gamma_1 = \frac{1}{2M_Z}\int [d\overline{q}][dl][dp'][d\overline{\pi}](2\pi)^4\delta^4(Z-\overline{q}-l-p'-\overline{\pi})\,|M_1|^2 \qquad (4)$$

where $\bar{q}$, l, p' and π are the 4-momenta of the $\bar{Q}$, B_Q,$\overline{D}$ and the pion (or photon), respectively, and the amplitude M_1 is summed and averaged over unobserved spins and colors.

In order to factor the fictitious decay width Γ_0 out of Eq. 4 we transform the phase space variables to production independent variables $x_1 = \frac{p_0+p_L}{q_0+q_L}$ and $x_2 = \frac{l_0+l_L}{p_0+p_L}$ that can be loosely thought of as Feynman scaling variables for each subprocess, i.e. excited baryon production and decay. We also introduce a further simplification - the ratio of the narrow decay width to the mass of the excited baryons allows the narrow width approximation to be used. The resulting phase space integral can be written as:

$$\begin{aligned}\Gamma_1 &= \frac{1}{2M_Z}\frac{1}{256\pi^4}\int [d\overline{q}][dq](2\pi)^4\delta^4(Z-q-\overline{q}) \\ &\cdot\int ds_q\theta\left(s_q-\frac{M_\Sigma^2}{z}-\frac{m_d^2}{1-z}\right)\int d\phi d\varphi dx_1 dx_2\,|A_1|^2 \qquad (5)\end{aligned}$$

Here $|A_1|^2\,\delta(p^2-M^2) = |M_1|^2$. The two angles ϕ and φ are associated with the position of the transverse momentum vector in two frames of reference. The first is the frame determined by the three-momentum of the heavy quark and a fixed vector perpendicular to it, which is arbitrary unless it is the heavy quark's transverse spin vector (which enters in h_1 only). The angle ϕ is the azimuthal angle between this plane and the transverse momentum vector of the excited baryon. The second plane is constructed out of the three-momentum of the excited baryon and the spin vector perpendicular to that three-momentum but having no transverse component relative to the first frame. The second angle φ is defined as the azimuthal angle between this latter plane and the transverse momentum of the baryon.

After factoring out the production decay width we are left with the somewhat simpler expression for f_1:

$$f_1(z,\mu) = \frac{1}{16\pi^2}\lim_{q_0\to\infty}\int_{s_{th}}^{\infty} ds\frac{dx_2}{x_2}\frac{|A_1|^2}{|M_0|^2} \qquad (6)$$

The spin dependent fragmentation functions can be obtained directly, using the modified Eq.6:

$$g_1(Q,z) = \frac{1}{256\pi^4} \lim_{q_0\to\infty} \int_{s_{th}}^{\infty} ds \frac{dx_2}{x_2} d\phi d\varphi \frac{|A_{1+}|^2 - |A_{1-}|^2}{|M_0|^2} \quad (7)$$

$$h_1(Q,z) = \frac{1}{256\pi^4} \lim_{q_0\to\infty} \int_{s_{th}}^{\infty} ds \frac{dx_2}{x_2} d\phi d\varphi \frac{|A_{1y+}|^2 - |A_{1y-}|^2}{|M_0|^2} \quad (8)$$

with new indices specifying the spin alignment ($|y+ >\sim |+> +i|->$). Angular integration is especially complicated for h_1, because matrix elements involve spin projections that make them no longer azimuthally symmetric.

The expressions above are general for the four body final state. The model dependence is hidden inside of the $|A_1|^2$ with the delta function obtained in the narrow width approximation integrated out. The final ground state baryon can be produced via one of the two intermediate states. The amplitudes for both $\frac{1}{2}$ and $\frac{3}{2}$ have been obtained (details can be found in [4]).

The amplitudes for $\frac{1}{2}$ and $\frac{3}{2}$ depend on the gluon–vector diquark pair production amplitude, which involves chromodynamic diquark currents [15] and form factors. The important contributions yield the chromoelectric part of the matrix element contributing to the spin $\frac{1}{2}'$ baryon

$$A_{E\,1/2} = -\frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2)\gamma_5\gamma^\mu \frac{1+\mathbf{v}}{2} g_s \bar{\epsilon}^*_\mu [k_\lambda - 2m_d v_\lambda] P^\lambda. \quad (9)$$

and

$$A^\nu_{E\,3/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2) g_s \bar{\epsilon}^{*\nu} [k_\lambda - 2m_d v_\lambda] P^\lambda, \quad (10)$$

where

$$P^\lambda = \triangle^{\lambda\nu} g_s \gamma_\nu \frac{m_Q(1+\mathbf{v}) + \mathbf{k}}{(s - m_Q^2)} \Gamma. \quad (11)$$

for the spin $\frac{3}{2}$. The wavefunctions for the diquark–quark baryon formation are obtained from the power law potential of Eichten and Quigg [16].

The remaining calculations of the amplitudes are straightforward, but quite complex. The squares of the matrix elements involve the trace of various amplitudes and spin projection operators ($\frac{1+\gamma_5 \not{S}_\alpha}{2}$) with α being the spin projection index and S_α being the spin four-vector corresponding to that projection. The two cases for this problem are the longitudinal or helicity

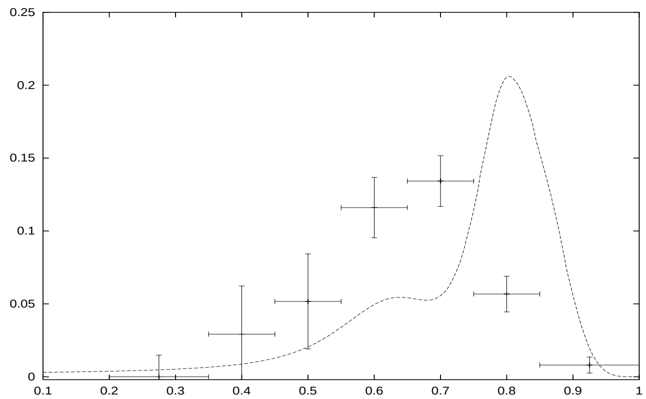

Figure 2: Spin independent function $f_1(z)$ for Λ_c evolved to μ= 5.5 GeV. The data are from CLEO [18].

and transverse spin vectors. After simplifying the squares of matrix elements and leaving only the leading terms in the high momentum limit, scalar products of all the involved four-momenta and spin vectors remain. The angular dependences can then be integrated over.

The integrations produce spin-dependent fragmentation functions that are defined at the scale $\mu_0 = m_Q + m_{diquark}$. To evolve them to higher values of the defining scale (or the typical Q^2) we utilize the appropriate spin-dependent Altarelli-Parisi integro-differential equations as determined by Artru and Mekhfi [17].

The resulting Λ_c $f_1(z)$ evolved to 5.5 GeV is shown in Fig. 2. The data are probably at the low edge of reliable energy for our evolution. Predictions for 45 GeV are in ref. [4]. The corresponding Λ_b is given in Fig. 3. To compare with yield measurements at LEP f_1 is integrated over z. The results are in Table 1 and agree with data.

Table 1: Total Fragmentation Probabilities

Particle	Experiment	Prediction
Λ_c	5.6± 2.6% [OPAL [19]]	3.9%
Ξ_c		0.59%
Λ_b	7.6± 4.2% [ALEPH [20]]	6.7%

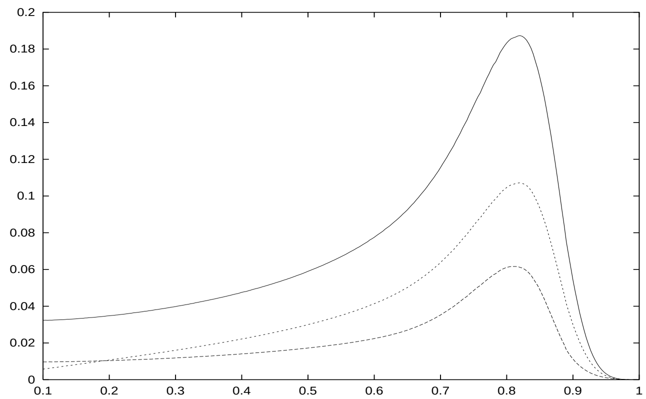

Figure 3: Fragmentation functions for Λ_b evolved to 45 GeV. At the peak f_1 is largest, followed by h_1 and g_1.

3 Polarization of inclusively produced heavy flavor baryons

Several years ago, with W.G.D. Dharmaratna, I developed a hybrid model for hyperon polarization in inclusive reactions [8]. The model involved the order α_s^2 QCD perturbative calculation of strange quark polarization due to the hard QCD subprocesses. The interference between tree level and one loop diagrams gives rise to significant, albeit small polarization. Of particular importance for describing then existing data on $p + p \to \Lambda + X$ were one loop diagrams leading to strange quark pair production from gluons or quark-antiquark pairs, with the gluon fusion being more significant. For strange quarks, their low current or constituent quark mass (compared to Λ_{QCD}) made the application of PQCD marginal. Nevertheless, it was realized that the hadronization process, by which the polarized strange quark would recombine with a (ud) diquark system to form a Λ, was crucial for understanding the subsequent hadron polarization. A simple prescription was introduced to "pull" or accelerate the negatively polarized, relatively slow s-quark along with a fast moving diquark resulting from a pp collision to form the hadron with particular x_F and p_T. This recombination prescription is similar to the "Thomas precession" model [21], which posits that the s-quark needs to be accelerated by a confining potential or via a "flux tube" [22] at an angle to its initial momentum in order to join with the diquark to form

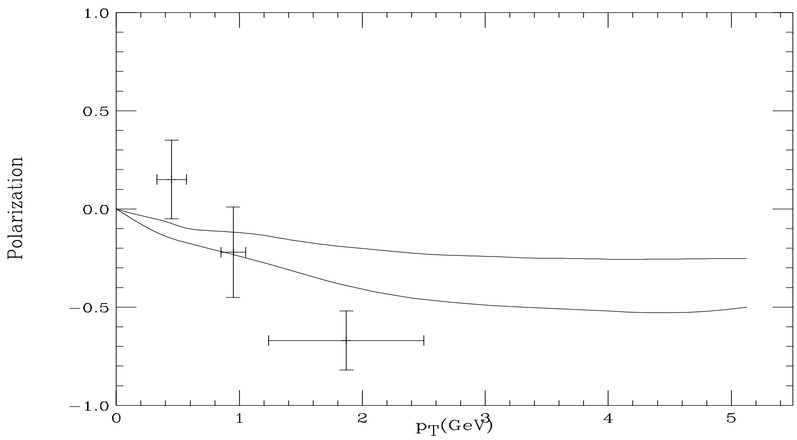

Figure 4: *Lambda*$_c$ polarization from $\pi^- p \rightarrow \Lambda_c + X$. The larger polarization includes heavy mass enhancements. The data [10] are from E791.

the hyperon. The skewed acceleration gives rise to a spin precession for the s-quark. That latter enhances the s-quark's polarization by the Thomas precession. The hybrid model combines hard perturbative QCD with a simple model for non-perturbative recombination. It accounted for the contemporaneous polarization data which were determined for a range of x_F and p_T values and the predicted kinematic dependence was confirmed, along with the nearly negligible overall energy dependence of the polarization. However, the PQCD based hybrid model is best tested in the production of heavy flavor hadrons, wherein the heavy quark needs to be produced at large energies compared to the Λ_{QCD} scale. The polarization in the QCD subprocesses was calculated [9] for all flavors and indeed, the polarization increases substantially with constituent mass; the peak polarization goes roughly as the mass.

I applied this reasoning to $\pi + p \rightarrow \Lambda_c + X$. It is known that π induced inclusive Λ production also produces significant polarization of $-28 \pm 10\%$ for p_T values near and above 1 GeV/c [23]. In this process the ud-diquark system that combines with the strange or heavy flavor polarized quark must be pulled from the target proton or the pion sea. Various possibilities were considered in determining the polarization for Λ_c via π production, where new data exist [10]. I adopted the same recombination prescription used for the $p + p \rightarrow \Lambda + X$.

The basic equation for production is given by

$$d^2\sigma(\uparrow \text{ and } \downarrow)/dx_Q dp_T = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i^{p,\pi}(x_1) f_j^p(x_2) d^2\sigma(\uparrow \text{ and } \downarrow)/dx_Q dp_T. \tag{12}$$

Next the algebraic recombination formula is applied to obtain the corresponding Λ_Q polarized cross section at $x_F (= a + bx_Q)$ and p_T.

The full dependence on both variables x_F, p_T is similar to the Λ data. The overall polarization of Λ_c in fig. 4 is consistent with the new data. It will be of considerable interest for the hybrid model to see how well this detailed behavior will be confirmed when more data are available.

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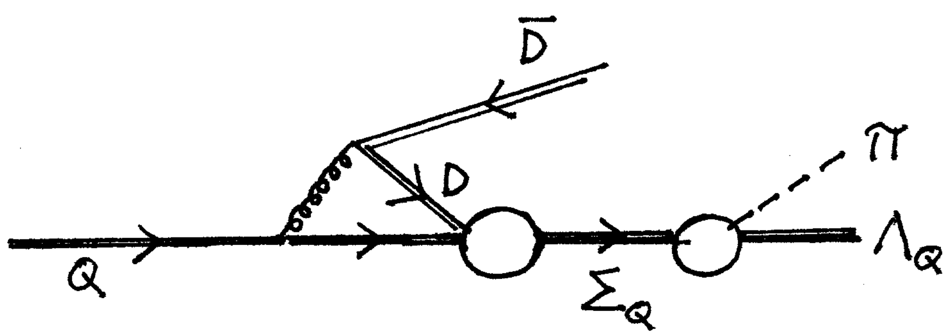

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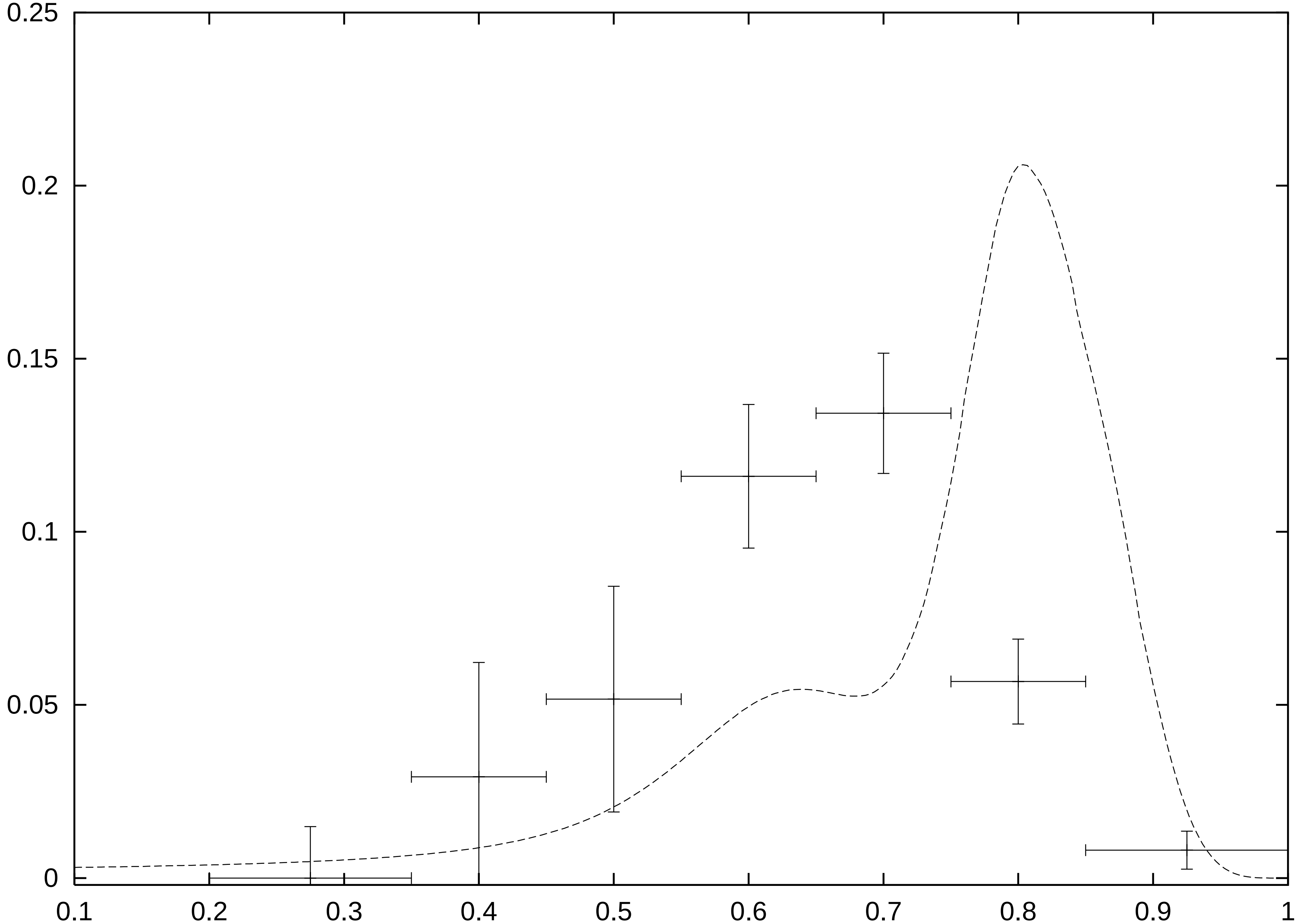
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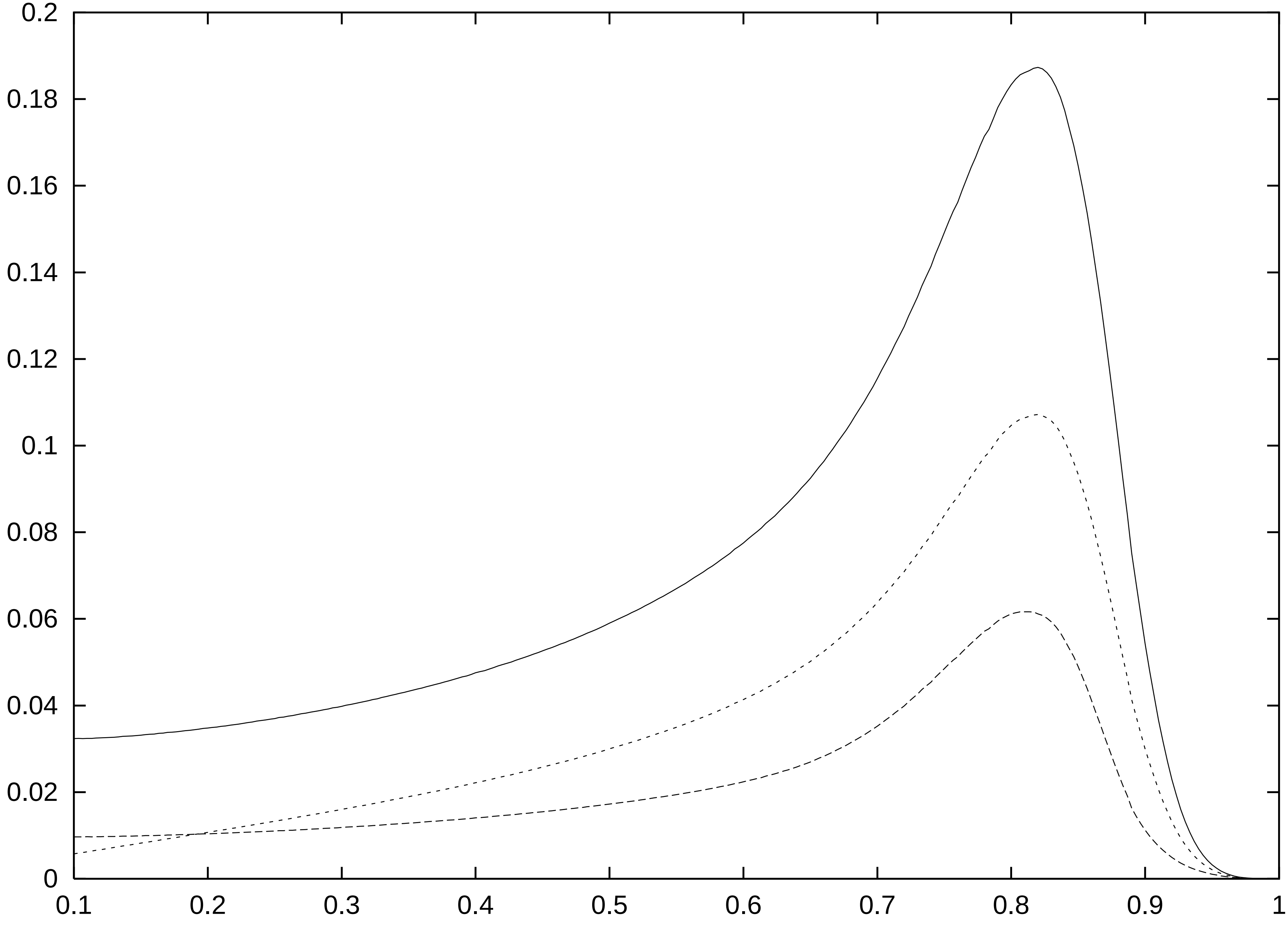
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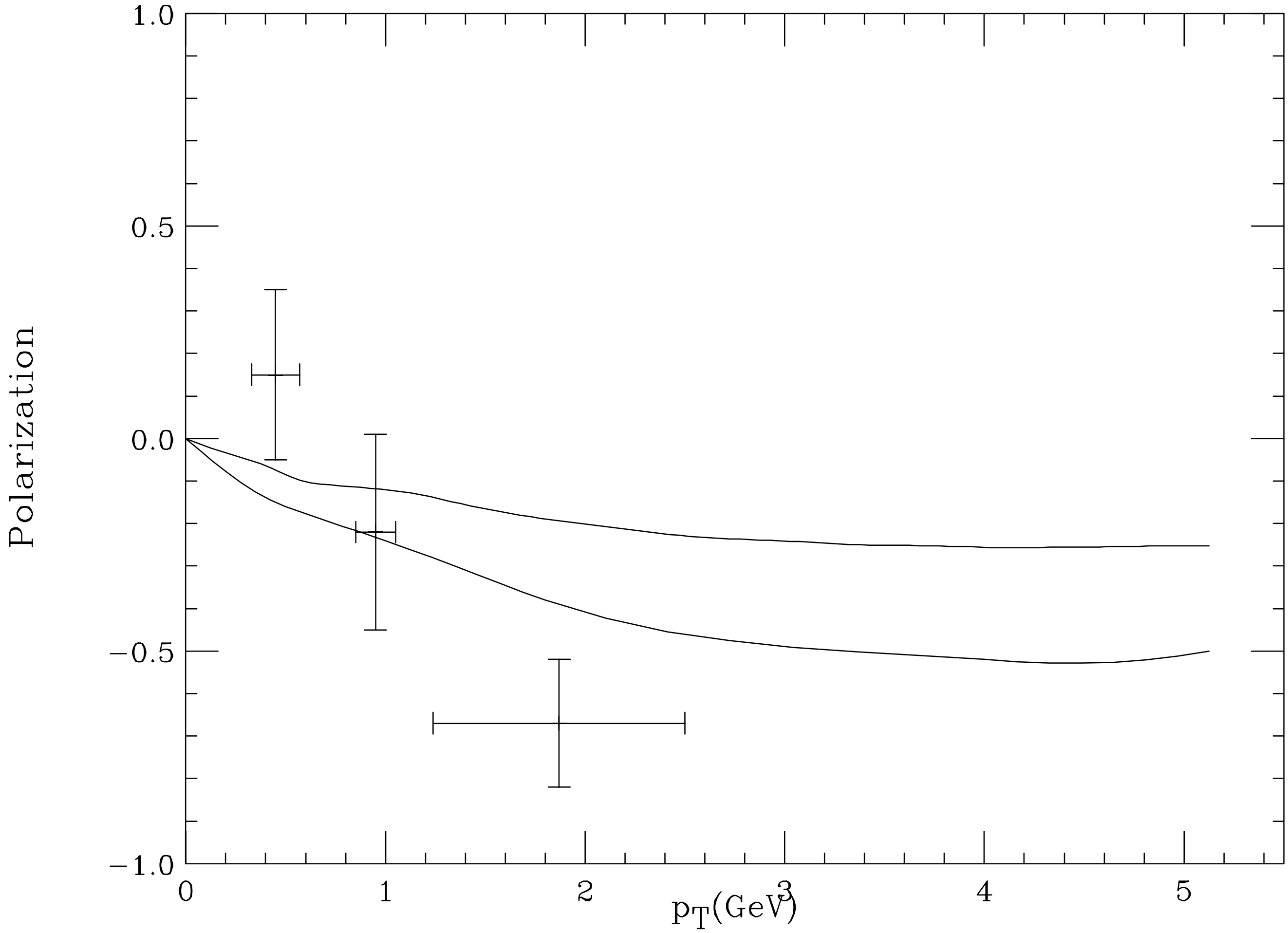

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