DISCUSSION

SALECKER: To what length have you tested quantum electrodynamics if you introduce in the hyperfine structure the conventional form factors?

HUGHES: To our present accuracy the conventional form factors will appear in the same way as they do for the spin magnetic moment. Our present accuracy is only 5 parts in 10^4 as compared to an accuracy of 5 parts in 10^6 in the (g-2) experiment. Hence we do not yet have anywhere nearly as precise a test of quantum electrodynamics as is provided by the (g-2) experiment. However, I feel quite convinced that we will eventually be able to do the hfs measurement with comparable accuracy and hence will test form factor effects of the same size as does the (g-2) experiment. I also believe that in principle somewhat different form factor effects may be tested with hfs as compared with the spin magnetic moment.

(g-2) AND ITS CONSEQUENCES

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(presented by A. Zichichi)

I am reporting on an experiment performed by Charpak, Garwin, Farley, Muller, Sens and myself at CERN, to measure the anomalous magnetic moment of the muon to an accuracy of $\pm 0.4\%$, i.e. ± 4 parts per million in the total g factor. As the relevance of this experiment with respect to the previous one [1,2] performed at a level of 2% lies in the factor of 5 improved accuracy, I will only discuss the main problems with which we have been faced, and whose solution has been vital to reach the desired accuracy. Unfortunately time obliges me to make a choice: either to try to be clear in the problems and only state their solution, or vice versa. I will adopt the first choice because once the problems are clear, if you want to understand their solution you can use the time devoted to the discussion to ask questions. All this, I hope, will convince you that the transition from 2% to 0.4% has not been a simple matter of collecting statistics. Finally, I will discuss some consequences of our experimental result.

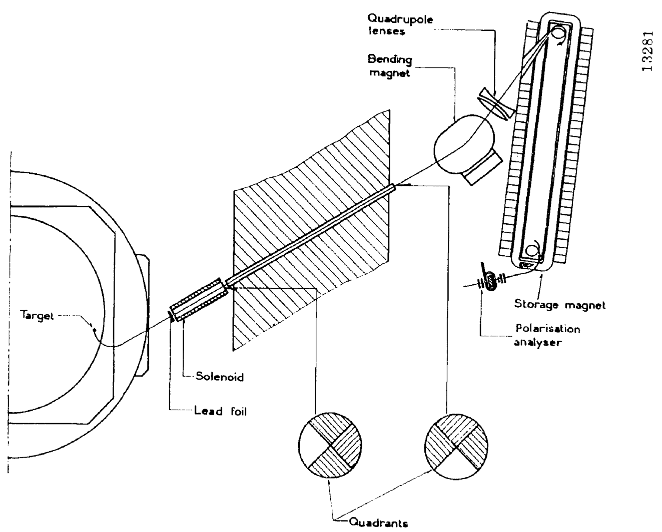

Fig. 1 General sketch of the (g-2) experiment, showing the CERN Synchro-cyclotron (left). The emerging muon beam passes through a lead scattering foil and a solenoid, then through a vacuum pipe across the shielding wall and into the 6-metre storage magnet, to be finally stopped in the polarization analyser.

I would like to remind you of the sketch of our apparatus (Fig. 1). We have a magnet able to store muons for as many as 1500 turns. In principle the experiment is very simple. We have to measure where the spin of the muon is pointing with respect to the momentum vector before and after storage, the change in this angle between spin and momentum being proportional to (g-2).

Now comes the first difficulty. The beam stored and analysed represents only one per cent of the injected beam. What tells us that the state of polarization of this stored beam is well represented by the average polarization of the total beam coming out of the pipe? In fact, we must think of a beam coming out from an accelerator as the superposition of a number of partial beams, each one having its own state of polarization which can, as we have experimentally measured, scatter from the value of the average polarization by as much as 15°, which corresponds to $\sim$4% in (g-2). There is therefore a risk that the magnet selects a partial beam whose properties are quite different from the average we measure before injection. To avoid this we make the beam uniform in its polarization properties

a) by using a solenoid to eliminate the momentum dependence, and

b) by a lead scattering foil to eliminate the phase space correlation, (see Fig. 1).

This reduces the non-uniformity to $\sim\pm 2°$, but we still measure the polarization structure in detail by dividing the beam into 16 independent cells in phase space. Finally, we measure the contribution of each cell to the over-all stored intensity in order to calculate the correct average initial polarization.

Now for the second difficulty. We have to moderate the beam in order to capture it, and this inevitably involves scattering. If the beam is scattered through an angle ψ, the polarization angle changes by ψ/γ ($\gamma = E/m$). We keep this scattering small ($\sim\pm 3°$) by injecting into a steep magnetic gradient and then making an adiabatic transition to the storage region. This allows us to fill all the available phase space of the storage region with the smallest perturbation in the polarization properties of the injected beam. But the residual $\pm 3°$ scattering ($\sim\pm 1\%$ in $g-2$) must still be measured, and in fact we measure the scattering as a function of particle momentum and as a function of storage time. We can understand from Fig. 2 that particles scattered to the left are in a steep gradient, walk fast, and arrive at early times, while particles scattered to the right sit in a weak gradient and arrive late. As you see, the scattering angle is strictly related to the storage time. Because scattering means change of polarization, we have a change of polarization which is time-dependent, and we have to measure it to $\pm 1°$. How can we measure such a change of polarization? Remember the muons are

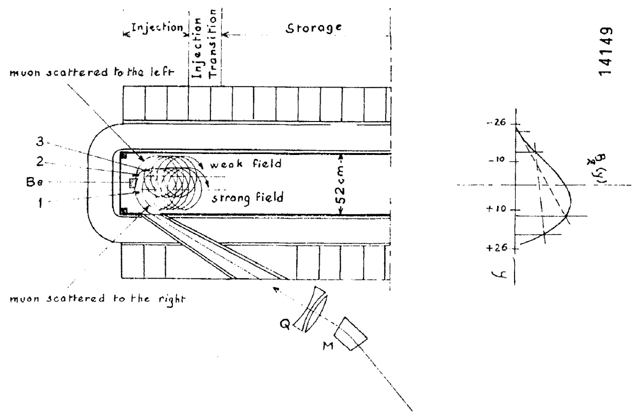

Fig. 2 Correlation between scattering angle and storage time. Muons scattered to the left will have orbits centered in the upper part of the figure and will consequently have a stronger gradient than muons scattered to the right. This is illustrated by the diagram on the right of the figure which shows the shape of the storage field (full line) and the gradients (dotted line) relative to the above-mentioned orbits.

already in the magnet when they suffer this change of polarization.

The solution consists in measuring the position of the stored beam and of the unscattered beam in the injection region after one quarter turn by scanning along the line F with an anticoincidence counter (see Fig. 3).

The difference Δ between the position of the unscattered beam and of the stored beam is clearly related to the scattering angle ψ and therefore to the

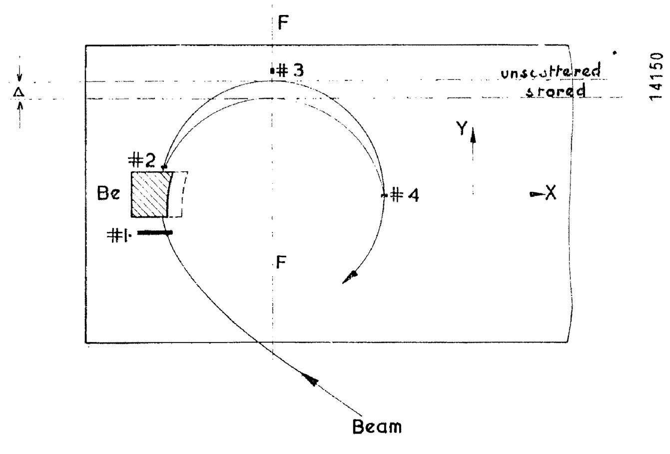

Fig. 3 Counter assembly used for measuring scattering at injection. The distribution of injected particles along the line FF is studied by varying the position of No. 3 and recording 1234 coincidences. No. 4 defines the orbit radius. The distribution of stored particles along the line FF is obtained in a separate experiment by probing with an anticoincidence counter (not shown).

change of polarization. In Fig. 4 you can see the results of our measurements. All these were the main problems to be solved in order to know the initial polarization.

We also need to measure the polarization after storage as function of storage time.

Fig. 5 gives the final curve of $(g-2)$ precession; the experimental points scatter according to statistics around the fitted curve with $\chi^2 = 60$ (58 expected). The final result is

$$\tfrac{1}{2}(g-2) = 0.001162 \pm 0.000005$$

to be compared with the theoretical value

$$\tfrac{1}{2}(g-2) = 0.001165\,.$$

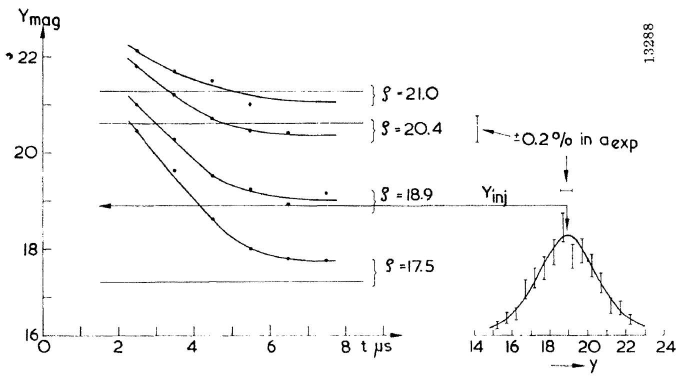

Fig. 4 Injection scattering measurements. The position of the stored particles, y_{mag}, as a function of storage time for particles of various radii, ϱ. Inset at right: typical distribution curve of injected particles used to find the value y_{inj} corresponding to zero scattering.

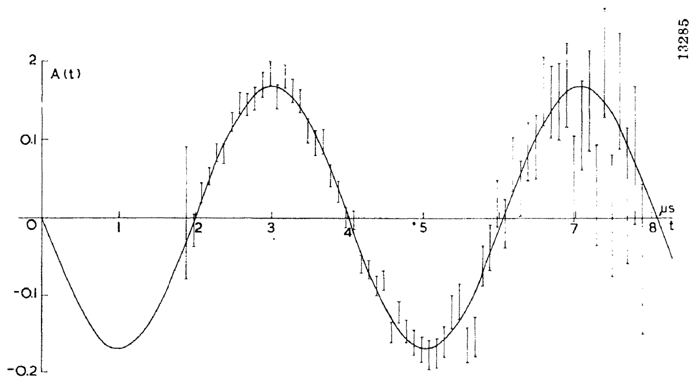

Fig. 5 Experimental data of the (*g*-2) experiment. Observed electron decay asymmetry *A*(*t*) as a function of storage time. The curve represents the best fit of the data.

As you see, within a few parts in a million, experimental result and theoretical prediction agree very well.

In Table I you will find the summary of the consequences of our experimental result.

I would like to add a few comments to these results. We say that to one standard deviation our result checks the validity of *QED* down to distances of $\sim 0.2 \times 10^{-13}$ cm, or equivalently up to momentum transfers of ~ 1 GeV/c. This result is obtained by assuming that *QED* breakdown is really described with the replacement of the photon propagator $1/K^2$ with

$$1/K^2 - 1/(K^2 + \Lambda^2)\,. \tag{1}$$

As you know, to break *QED* is a delicate matter for the specialists, essentially because nobody knows how to break *QED* in a self-consistent way. The choice (1)

TABLE I

Cut-off in the photon propagator	$>$ 1 GeV/c	($<$0.2 $\times 10^{-13}$ cm)
Cut-off in the muon vertex	$>$ 1.3 GeV/c	(0.15$\times 10^{-13}$ cm)
Cut-off in the muon propagator	$>$ 2.7 GeV/c	(0.07$\times 10^{-13}$ cm)
Universal length	$>$ 3.3 GeV/c	(0.06$\times 10^{-13}$ cm)

Coupling of the muon to a hypothetical neutral field (scalar, pseudoscalar or vector) of nucleonic mass must be less than the electromagnetic coupling, i.e. $\sim 10^{-3}$.

$m_{\mu^+} = (206.768 \pm 0.003) m_e$	These data are obtained by combining four experiments:
$e_\mu = (1.00000 \pm 0.00005) e_e$	1) $g-2$; 2) total precession frequency; 3) μ-mesic phosphorous x-ray energy; 4) limit on the neutron charge.
$e_{\nu_\mu} = (0.00000 \pm 0.00005) e_e$	

is undoubtedly the most popular and widely accepted; however, the negative sign in (1) is not a result of a theory, but merely a good guess. The physical significance of this (—) sign is that we subtract all contributions coming from distances smaller than Λ^{-1} from the effect we want to calculate. Nobody forbids us to imagine a breakdown such as to increase at distances below Λ^{-1} what would be the normal *QED* contributions coming from these distances. If this were the case the magnetic moment of the muon would increase. Experimentally we are sensitive to both signs of the effect.

Let me now go to other experiments which verify *QED* to a comparable accuracy (*), i.e. to the $(e-p)$ scattering experiments of Hofstadter and collaborators, and of Wilson and collaborators. Their limit on the validity of *QED* is 0.35×10^{-13} cm or 560 MeV/c momentum transfer [4]. But the (ep) scattering experiments can be used to fix a limit on *QED* only on the hypothesis that the sign in (1) is negative. In fact in this experiment the increase in the cross-section, due to *QED* breakdown with (+) sign in (1), would be masked by the decrease due to the proton structure. So the $(g-2)$ of the muon is an experiment sensitive to both ways in which *QED* could break.

Finally, I would like to make a remark concerning vacuum polarization effects in muon physics. As you know, the contribution of the electron-vacuum polarization to the muon $(g-2)$ is only $\sim5\times10^{-6}$, that is just our error. So we can say that we measure this effect with 100% error. But there is a way which allows us to use $(g-2)$ in order to establish the presence of electron-vacuum polarization effect in muon physics to 4% accuracy. This result is obtained by combining our $(g-2)$ result with the total precession frequency [5] and the x-ray energy in the transition $(3D-2P)$ of the μ-mesic phosphorous [6, 7]. As you know, the energy of the μ-mesic x-ray is proportional to $m_\mu e_\mu C_\mu$ where m_μ is the mass of the muon, e_μ its electric charge, and C_μ a constant which contains first and second order vacuum polarization effects, reduced mass, and α [8]. If we combine $(g-2)$ and the total precession frequency experiment we get the mass of the muon, indicated in Table I. The μ-mesic x-ray experiment can then be used to measure C_μ instead of the mass. The most relevant quantity in C_μ is the vacuum polarization effects which amount to 331.42 eV. The experimental uncertainty amounts to $(^{+15}_{-10})$ eV; we conclude that this experiment can be used to give the best check of vacuum polarization in μ-mesic atoms, the check being good to $\sim\pm4\%$.

As you know, for electron-atoms the vacuum polarization effects are checked by the Lamb shift to an accuracy of $\sim\pm1\%$. If we calculate the contribution to vacuum polarization coming from hypothetical leptons of mass "m", using the μ-mesic atom data, we get $\sim10\ m_e$ as the lower limit for this mass value. The corresponding limit dictated by the more precise determination of the vacuum polarization in the Lamb shift is also $\sim10m_e$. I am indebted to Dr. J. S. Bell for the calculations on the hypothetical lepton mass limits.

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(*) In spite of being two times more accurate than the muon $(g-2)$, the electron $(g-2)$ experiment is $\sim$10 thousand times less sensitive to *QED* breakdown. The Lamb shift is also quite insensitive; a cut-off $\Lambda^{-1}\sim10^{-13}$ cm would alter the Lamb shift by 0.01 Mc, which is a quantity 10 times smaller than the experimental error [see Drell [3]].

DISCUSSION

MICHEL: Can you give us the limit on the electric dipole moment of the muon that your experiment must have measured too?

ZICHICHI: This limit has been measured independently in a separate experiment. The result is

$$\text{EDM of muon} = e\,(0.6 \pm 1.1) \times 10^{-17}\ \text{cm}$$

where " e " is the electron charge.

SALECKER: If you take another form factor instead of this here, how much does it change the result?

ZICHICHI: Which one do you want to change? This one:

$$\frac{1}{K^2} - \frac{1}{K^2 + \Lambda^2}\ ?$$

SALECKER: Yes.

ZICHICHI: Well, we do not know. All we can say is that if you assume that quantum electrodynamics breaks in such a way that the break is described by this change in the photon propagator, then it is obviously this. But the point is that nobody knows how to break quantum electrodynamics, so there is no point to produce other changes; I mean you can conceive an enormous number of ways in which QED could break down. But I think that only when the theoreticians come with a consistent way of breaking quantum electrodynamics will the answer be unique. The experimental result, of course, is there and any theoretical model can just be taken and fitted in. If you give another theoretical model, we shall give you the answer but at the moment this is what we have.

HUGHES: Do you expect to improve the accuracy of your experiment further or have you completed now?

ZICHICHI: No, we do not expect to improve our accuracy further. If we did, it would have to be an improvement by a factor 10 or better.

FEINBERG: On this question of introducing a different photon propagator I would think the best thing to do would be to use the Lehmann-Källén expression for the photon propagator in terms of a spectral function and then express the deviation from electrodynamics in terms of an integral of the spectral function

$$\left[\int_0^\infty dm^2 \frac{\sigma(m^2)}{m^2}\right]$$

because even if one has a theory different from electrodynamics, it is likely that the general expression for the propagator which is summarized in the Lehmann-Källén representation would still be right, since that depends on fairly general properties. So instead of summarizing a result in terms of a length which is as you say ambiguous, one might do better to express it in terms of a limit on the size of the integral, which also would restrict possible models of the deviation from electrodynamics.

ZICHICHI: Well, I have nothing more to say there. The problem is, as I said before, always the same: one has too many ways of breaking QED in so far as nobody produces a self consistent theory of QED breakdown. If you want to use our result to get another limit, please do.

KÄLLÉN: I would like to comment on the point made by Feinberg. The magnetic moment is not a 2 point function, but really a 3 point function, so you have no reason to believe in the absolute correctness of this representation for the magnetic moment. It may be right but we do not know. However, if you accept it, I believe that the cut-off length generally comes out to be smaller than the limit you get with this Feynman model.

FEINBERG: I was not suggesting writing the vertex as a spectral function and saying something about that, but rather that if you just take a model for deviation from quantum electrodynamics in which the photon propagator is different from $1/K^2$, you can use the Lehmann-Källén representation for that.

BERMAN: I would like to remark in answer to Feinberg's question that the work has already been done by Berestetskii, and it appears I think in the translated JETP. The method is essentially what he has suggested, so one can say what is the change in $(g-2)$ in terms of a cut-off integral on the spectral function. I do not know the number, but it is something like ¾.

ZICHICHI: In this case we get a 3.3 GeV, three times better result. If I understand well the question, what Feinberg says is equivalent to our fundamental length.

MICHEL: It is the same cut-off. You can also give a limit on the change of the photon propagator due to the existence of light charged particles or $\pi - \pi$ resonances.