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Cosmological Solutions of Supergravity in Singular Spaces

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Abstract

We study brane-world solutions of five-dimensional supergravity in singular spaces. We exhibit a self-tuned four-dimensional cosmological constant when five-dimensional supergravity is broken by an arbitrary tension on the brane-world. The brane-world metric is of the FRW type corresponding to a cosmological constant $\Omega_\Lambda = \frac{5}{7}$ and an equation of state $\omega = -\frac{5}{7}$ which are consistent with experiment.

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1 Introduction

An outstanding problem in cosmology is understanding the origin of the cosmological constant[1, 2]. This has been compounded by the recent evidence of a small, non-zero value in type IA supernova data[3], suggesting that the dark energy density is of the same order of magnitude as the matter density. There has been some progress towards understanding this fundamental problem. On the one hand, there have been various proposals that it may arise as 'dark energy' due to the evolution of a scalar field[4]. These quintessence models have met with some success. However, the origin of the scalar field has yet to be addressed in such models. There has been an alternative proposal, that the cosmological constant may arise from a rather deeper understanding of space-time in higher dimensionsal theories [5]. In particular, [6] have suggested that the cosmological constant in our Universe could be induced by the properties of the 'bulk' in a five dimensional model. Ref [6] showed that it was possible to tune the bulk gravitational dynamics so that the contribution of the standard model vacuum energy density was carried off the brane-world into the bulk. Further progress was made in this direction by considering the approach of a 'self-tuning' domain wall[7] and developed in [8], which also pointed out short comings in the approach. Both[7] and[8] used five dimensional Einstein equations coupled to a scalar field to cancel the vacuum energy of the brane tension, and in both cases the scalar field became singular in the bulk at a finite distance from the brane world.

Considerable progress has been made in understanding the origin of brane-worlds by[9], who considered five-dimensional supergravity in singular spaces. Their formalism properly accounts for the boundary conditions on our brane-world. Indeed, the conditions arising for a BPS solution are just those arising from the junction conditions in[8]. As a consequence, the natural framework in which to realise the possibility of understanding the cosmological constant as arising from gravitational solutions in extra dimensions seem to be that of five dimensional supergravity in singular spaces.

Here we develop the idea of the cosmological constant in our brane world being induced by the behaviour of the bulk solution. We consider five dimensional supergravity in singular spaces, taking careful account of the boundary conditions on the brane. When supersymmetry is broken, a time dependent scale factor is induced on the brane world. This gives rise to an induced cosmological constant and equation of state on the brane world. The induced cosmological constant is independent of the supersymmetry breaking parameter. Using an exponential superpotential in the bulk and the simplest one-dimensional example of vector multiplet theory [10] there are no free parameters. For a flat universe we obtain a cosmological constant of $\Omega_\Lambda = \frac{5}{7}$ and corresponding matter density $\Omega_m = \frac{2}{7}$, consistent with experiment. In our equation of state we obtain $\omega = -\frac{5}{7}$. Moreover, we show that the form of our potential is the only one which gives consistent cosmology on the brane world. Our solution has an evolving singularity in the bulk. If the singularity hits the brane world, then the five dimensional supergravity approximation used here breaks down and the full string theory must be used.

In the next section we introduce supergravity in singular spaces, considering the structure of the bulk theory and deriving the special solutions which we use in the following section. In section 3 we consider cosmological solutions. Starting with static solutions, we show that time dependent solutions arise when supersymmetry is broken on the brane. The induced metric on the brane is of FRW type. We then study the luminosity distance and deduce the acceleration parameter. This allows us to infer the value of the effective cosmological constant on the brane. The FRW dynamics appears to be due to a perfect fluid whose equation of state always respects the dominant energy condition. In the conclusions we comment on the possibility of connecting our analysis with the usual matter dominated cosmological eras and the possibility of brane-world quintessence.

2 Supergravity in Singular Spaces

In this section we shall recall the main ingredients of supergravity in singular spaces[9]. The newly constructed supergravity theory differs from the usual five-dimensional supergravity theories since space-time boundaries are taken into account. These boundaries provide new terms in the Lagrangian and require new fields in order to close the supersymmetry algebra and ensure the invariance of the Lagrangian.

More precisely space-time is supposed to be non-compact four dimensional Minkowski space enlarged to five dimensions by the adjunction of a $Z2$ orbifold of a circle. The two boundaries of the interval are identified as branes, in particular the brane at the origin is identified with our brane-world. The bulk physics far from the boundaries is identical to five dimensional supergravity coupled to $n-1$ vector multiplets. Let us recall briefly the structure of the bulk theory.

The vector multiplets comprise scalar fields ϕ^i parameterizing the manifold

$$C_{IJK}h^I(\phi)h^J(\phi)h^K(\phi) = 1 \tag{1}$$

with the functions $h^I(\phi), I = 1\dots n$ playing the role of auxiliary variables. In string theory the symmetric tensor C_{IJK} has the meaning of an intersection tensor. Defining the metric

$$G_{IJ} = -2C_{IJK}h^K + 3h_I h_J \tag{2}$$

where $h_I = C_{IJK}h^J h^K$, the bosonic part of the Lagrangian reads

$$S_{bulk} = \frac{1}{2\kappa_5^2}\int \sqrt{-g_5}(R - \frac{3}{4}(g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + V)) \tag{3}$$

where the induced metric is

$$g_{ij} = 2G_{IJ}\frac{\partial h^I}{\partial\phi^i}\frac{\partial h^J}{\partial\phi^j} \tag{4}$$

and the potential is given by

$$V = W_i W^i - W^2. \tag{5}$$

The superpotential W defines the dynamics of the theory. It is given by

$$W = 4\sqrt{\frac{2}{3}}gh^I V_I \tag{6}$$

where g is a gauge coupling constant and the V_I's are real number such that the $U(1)$ gauge field is $A^I_\mu V_I$. The vectors A^I_μ belong to the vector multiplets.

The $Z2$ orbifold implies that a boundary action has to be incorporated. The boundary action depends on two new fields. There is a supersymmetry singlet G and a four form $A_{\mu\nu\rho\sigma}$. One also introduces a modification of the bulk action by replacing $g \to G$ and adding a direct coupling

$$S_A = \frac{2}{4!\kappa_5^2}\int d^5x\epsilon^{\mu\nu\rho\sigma\tau}A_{\mu\nu\rho\sigma}\partial_\tau G. \tag{7}$$

The boundary action is taken as

$$S_{bound} = -\frac{1}{\kappa_5^2}\int d^5x(\delta_{x_5} - \delta_{x_5-R})(\sqrt{-g_4}\frac{3}{2}W + \frac{2g}{4!}\epsilon^{\mu\nu\rho\sigma}A_{\mu\nu\rho\sigma}). \tag{8}$$

The supersymmetry algebra closes on shell where

$$G(x) = g\epsilon(x_5), \tag{9}$$

and $\epsilon(x_5)$ jumps from -1 to 1 at the origin of the fifth dimension. On shell the bosonic Lagrangian reduces to (3) coupled to the boundaries as,

$$S_{bound} = -\frac{3}{4\kappa_5^2}\int d^5x(\delta_{x_5} - \delta_{x_5-R})(\sqrt{-g_4}f), \tag{10}$$

where we have defined

$$f = 2W. \tag{11}$$

The equations of motion of such a Lagrangian show a BPS property and can be rewritten in a first order form.

Let us give the simplest example of models based on supergravity in singular spaces, i.e. one vector multiplet scalar such that the only component of the symmetric tensor C_{IJK} is $C_{122} = 1$. The moduli space of vector multiplets is then defined by the algebraic relation

$$h^1(h^2)^2 = 1. \tag{12}$$

This allows to parameterize this manifold using the coordinate ϕ such that h^1 is proportional to $e^{\sqrt{\frac{1}{3}}\phi}$ and h^2 to $e^{-\phi/2\sqrt{3}}$. The induced metric $g_{\phi\phi}$ can be seen to be one. The most general superpotential is a linear combination of the two exponentials $W = ae^{\sqrt{\frac{1}{3}}\phi} + be^{-\phi/2\sqrt{3}}$. We will analyse this example in the next section.

3 Cosmological Solutions

In the following we shall restrict ourselves to a single scalar ϕ and normalize $g_{\phi\phi} = 1$. Let us look for static solutions with the metric

$$ds^2 = e^{-A/2}dx_i^2 + dx_5^2. \tag{13}$$

The equations of motions read

$$\begin{aligned} A'' &= \phi'^2 + f(\phi)\delta_{x_5} \\ A'^2 &= \phi'^2 - V(\phi) \\ \phi'' - A'\phi' &= \frac{1}{2}\frac{\partial V}{\partial \phi} + \frac{\partial f}{\partial \phi}\delta_{x_5} \end{aligned} \tag{14}$$

leading to the junction conditions

$$\begin{aligned} \Delta(\frac{\partial W}{\partial \phi}) &= \frac{\partial f}{\partial \phi}|_{x_5=0} \\ \Delta W &= f|_{x_5=0} \end{aligned} \tag{15}$$

where the left-hand sides represent the discontinuity across the origin. Notice that these conditions are automatically satisfied due to the assignment (11). The equations of motion can be written in BPS form

$$\begin{aligned} \phi' &= \frac{\partial W}{\partial \phi} \\ A' &= W. \end{aligned} \tag{16}$$

These equations depend explicitly on the choice of the superpotential W.

Here we will use the following model. We assume that there exists a field ϕ with $g_{\phi\phi} = 1$ such that one can choose the Fayet-Iliopoulos scalars V_I in such a way that

$$W(\phi) = \xi e^{\alpha\phi} \tag{17}$$

where ξ is a characteristic scale. Here the parameter α will play a crucial role. In the previous section we have shown that $\alpha = \sqrt{\frac{1}{3}},\ -\sqrt{\frac{1}{12}}$ correspond to a one dimensional moduli space.

It is easy to show that the solutions to the BPS conditions are

$$\begin{aligned} \phi &= -\frac{1}{\alpha}\ln(1-\alpha^2\xi|x_5|) \\ A &= -\frac{1}{\alpha^2}\ln(1-\alpha^2\xi|x_5|). \end{aligned} \tag{18}$$

Notice that we have chosen the scale factor

$$a^2 = e^{-A/2} \tag{19}$$

to be normalized to one on the brane-world at $y = 0$. Another feature of this solution is the existence of a singularity at

$$x_{5*} = \frac{1}{\alpha^2\xi}. \tag{20}$$

This implies that the supergravity description breaks down in the vicinity of x_{5*}, and one has to use the full string theory underlying this approach.

Since nature is not supersymmetric, we deform our model to introduce supersymmetry breaking on the brane world in a phenomenological way. A viable theory should incorporate the standard model of particle physics at low energy and describe the coupling between the standard model fields and the bulk fields living in five dimensions. We can modify the previous study in a minimal way in order to take into account the matter fields living on the brane. Let us assume that these matter fields couple universally to the superpotential $W(\phi)$

$$-\frac{3}{2\kappa_5^2}\int d^4x\sqrt{-g_4}W(\phi)V(\Phi) \tag{21}$$

and that to a good approximation the matter fields are fixed to their vev's -in particular we exclude time-dependent phenomena such as inflation- in such a way that the effective coupling becomes

$$f(\phi) = 2TW(\phi) \tag{22}$$

where

$$T \equiv V(< \Phi >) \tag{23}$$

encapsulates supersymmetry breaking for $T \neq 1$ which occurs only on the brane world. We assume that the bulk is still supersymmetric. In particular the tension T is subject to radiative corrections and phase transitions. We will now discuss the deformations of the previous static solutions when the supersymmetry breaking parameter T is turned on.

We can generate time-dependent conformally flat solutions from the static solutions. This is most easily achieved by using a boost along the x_5 direction. To do so we shall first introduce conformal coordinates so that the metric becomes

$$ds_5^2 = a^2(u)(dx^2 + du^2). \tag{24}$$

Under a boost the new solutions of the bulk equations of motions are

$$\begin{aligned} a(u,\eta) &= \sqrt{1-h^2}a(u+h\eta,\xi) \\ \phi(u,\eta) &= \phi(u+h\eta,\xi) \end{aligned} \tag{25}$$

where $x_1 \equiv \eta$ is the conformal time. The junction conditions for conformally flat metrics read[4]

$$\begin{aligned} \Delta(\partial_n A) &= f|_{\partial M} \\ \Delta(\partial_n \phi) &= \frac{\partial f}{\partial \phi}|_{\partial M} \end{aligned} \tag{26}$$

where the normal vector is $\partial_n \equiv a^{-1}\partial_u$ and ∂M is the brane world. In order to verify these conditions we will use a transformation which rescales the conformal factor and the potential, i.e. the scale ξ

$$a \longrightarrow \lambda a,\ V \longrightarrow \frac{V}{\lambda^2}. \tag{27}$$

The new solution of the bulk equations of motion is given by

$$\begin{aligned} \tilde{a}(u,\eta) &= \frac{\sqrt{1-h^2}}{\lambda}a(u+h\eta,\frac{\xi}{\lambda}) \\ \tilde{\phi}(u,\eta) &= \phi(u+h\eta,\frac{\xi}{\lambda}). \end{aligned} \tag{28}$$

One can now use the BPS equations satisfied by (a,ϕ) to deduce that

$$\begin{aligned} \partial_{\tilde{n}}\tilde{\phi} &= \frac{1}{\sqrt{1-h^2}}\frac{\partial W}{\partial \tilde{\phi}}(\tilde{\phi}) \\ \partial_{\tilde{n}}\tilde{A} &= \frac{1}{\sqrt{1-h^2}}W(\tilde{\phi}) \end{aligned} \tag{29}$$

[4] The first of these conditions stems from the Israel conditions relating the extrinsic curvature tensor $K_{ij} = \partial_n h_{ij}/2$, where h_{ij} is the metric on the hypersurface orthogonal to the normal vector ∂_n, and the energy momentum tensor on the brane world. In our case this reads $\Delta(K_{ij} - Kh_{ij}) = 3fh_{ij}/4|_{\partial M}$ which leads to the first condition.

where $\tilde{a} = e^{-\tilde{A}/4}$ and the normal vector is

$$\partial_{\tilde{n}} = \tilde{a}^{-1}\partial_u. \tag{30}$$

Under this rescaling the junction conditions become

$$\begin{aligned} \Delta(\frac{\partial W}{\partial \tilde{\phi}}) &= T\sqrt{1-h^2}\frac{\partial f}{\partial \tilde{\phi}}|_{\partial M} \\ \Delta(W) &= T\sqrt{1-h^2} f|_{\partial M} \end{aligned} \tag{31}$$

which are automatically satisfied thanks to the identity $f = 2W$ provided that

$$h = \pm\frac{\sqrt{T^2-1}}{T}. \tag{32}$$

Notice that this requires $T > 1$. For smaller values of the breaking parameter the solutions are not defined and we thus need to go beyond the approximations used here.

In the following we choose $\lambda = \sqrt{1-h^2}$ for clarity. The cosmological solutions are then

$$a(u,\eta) = (\frac{u + h\eta}{u_0(h)})^{1/(4\alpha^2-1)}. \tag{33}$$

The singularity is now located at

$$u_* = -h\eta \tag{34}$$

and the brane world at

$$u_0(h) \equiv \sqrt{1-h^2}u_0 = \frac{\sqrt{1-h^2}}{(\frac{1}{4}-\alpha^2)\xi}. \tag{35}$$

Depending on the sign of h the singularity either converges towards the brane-world or recedes towards the other end of the fifth dimension. In either cases the supergravity approximation breaks down in finite time. In particular the collision between the singularity and the brane-world occurs at

$$\eta_0 = -\frac{u_0(h)}{h}. \tag{36}$$

Let us now focus on the brane-world. The induced metric is of the FRW type

$$ds^2_{BW} = a^2(\eta)(-d\eta^2 + dx^i dx_i) \tag{37}$$

where the scale factor is

$$a(\eta) = (1 - \frac{\eta}{\eta_0})^{1/(4\alpha^2-1)}. \tag{38}$$

The type of geometry on the brane-world depends on α and h. Notice that there is always a singularity, either in the past or in the future.

The four dimensional Planck constant appears to be time-dependent

$$M_p^2 = 2M_5^3 \int_{-h\eta}^{u_0(h)} \frac{a^3(u,\eta)}{a^2(\eta)} du. \tag{39}$$

This leads to

$$M_p^2 = \frac{4T}{2\alpha^2+1}\frac{M_5^3}{\xi}a(\eta)^{4\alpha^2} \tag{40}$$

Now one can perform a Weyl rescaling $ds_4^2 \to Ta(\eta)^{4\alpha^2} ds_4^2$ of the induced metric on the brane-world. This makes the Planck mass constant

$$M_p^2 = \frac{4}{2\alpha^2+1}\frac{M_5^3}{\xi}. \tag{41}$$

The new scale factor becomes

$$a(\eta) = T(1-\frac{\eta}{\eta_0})^{(2\alpha^2+1)/(4\alpha^2-1)}. \tag{42}$$

Notice that the exponent is now $(2\alpha^2+1)/(4\alpha^2-1)$ instead of $1/(4\alpha^2-1)$. In particular if one uses cosmic time defined by $dt = a(\eta)d\eta$ there is a singularity at $t_0 = (1/3 + 1/6\alpha^2)T\eta_0$. More precisely the scale factor becomes

$$a(t) = T(1-\frac{t}{t_0})^{1/3+1/6\alpha^2} \tag{43}$$

leading to the Hubble parameter

$$H = \frac{2\alpha^2+1}{6\alpha^2(t-t_0)}. \tag{44}$$

The universe is decelerating when $t_0 > 0$, i.e. when the singularity converges to the brane-world. It is accelerating when $t_0 < 0$ and the singularity recedes away from the brane-world. Notice that the supersymmetry breaking parameter appears in the overall normalization of the scale factor and in the time scale t_0. A variation of the parameter T implies an adaptation of the scale factor. However the characteristic exponent is independent of T.

Let us now analyse the cosmological implications of these FRW models. In particular it is phenomenologically highly relevant to study the luminosity distance defined by

$$d_L = (1+z)\int_0^z \frac{dz'}{H(z')} \tag{45}$$

where z is the red-shift factor

$$\frac{a(t)}{a(0)} = \frac{1}{1+z}. \tag{46}$$

The small z expansion reads in general

$$d_L \sim \frac{1}{H_0}(z + \frac{1-q_0}{2}z^2 + o(z^3)) \tag{47}$$

and the acceleration parameter is directly related to the matter energy density Ω_m and the effective cosmological constant density Ω_Λ

$$\Omega_\Lambda = \frac{\Omega_m}{2} - q_0. \tag{48}$$

Notice that from the observer's point of view a finite and small cosmological constant, i.e. of the order of the critical density, appears as a consequence of the interpretation of the four dimensional FRW geometry as resulting from the dynamics in five dimensions. In our models the effective cosmological constant becomes

$$\Omega_\Lambda = \frac{\Omega_m}{2} + 1 - \frac{6\alpha^2}{1+2\alpha^2}. \tag{49}$$

This can accommodate a small and positive cosmological constant, which is independent of the supersymmetry breaking parameter.

Another relevant observable from the four dimensional point of view is the equation of state of the brane-world $p = \omega\rho$ relating the pressure to the energy density. It is given by

$$\omega = -1 + \frac{4\alpha^2}{1+2\alpha^2}. \tag{50}$$

It can take any value between -1 and 1, i.e. it never violates the dominant energy condition. The observer will therefore be driven to conclude that the four dimensional dynamics of the universe is due to some matter with the previous equation of state.

The previous model has been obtained for a particular choice of superpotential. However it is easy to calculate the acceleration parameter before the Weyl rescaling. It happens to be $-1 + 4(\frac{d\ln W}{d\phi})^2|_{\phi_0}$ where ϕ_0 is the value of ϕ on the brane. Hence only an exponential superpotential yields a boundary value independent result for the acceleration parameter.

In the case of the one-dimensional moduli space discussed earlier with $\alpha = \sqrt{\frac{1}{3}}$, the induced cosmological constant is negative. In the other case of $\alpha = -\sqrt{\frac{1}{12}}$, the cosmological constant and the equation of state are given by

$$\Omega_\Lambda = \frac{\Omega_m}{2} + \frac{4}{7},\ \omega = -\frac{5}{7}. \tag{51}$$

Imposing a flat universe, $\Omega_0 \equiv \Omega_m + \Omega_\Lambda = 1$, yields

$$\Omega_\Lambda = \frac{5}{7},\ \Omega_m = \frac{2}{7},\ \omega = -\frac{5}{7}. \tag{52}$$

These numbers are compatible with the current experimental results from type Ia supernovae and CMB anisotropies.

In brief we have exhibited a mechanism where the effective cosmological constant is naturally of the right order of magnitude. This springs from the five dimensional origin of the FRW dynamics of the universe. Moreover we have shown that including the non-supersymmetric effects of the matter fields on the brane world does not affect the FRW features of the universe, in particular the cosmological constant is not modified by variations of the potential energy of the matter fields.

4 Conclusions

In this letter we have realised the original suggestion of [5] and [6] that the observed cosmological constant is induced on our brane world by the dynamics of fields in extra dimensions. In our case we exhibited a self-tuned cosmological constant induced by five dimensional supergravity once supersymmetry was broken on the brane world. Further, our mechanism gave a relationship between the matter density and the dark energy density. Imposing a flat universe resulted in values consistent with observation. Our induced cosmological constant is independent of the value of the supersymmetry breaking parameter.

Our mechanism relies on supersymmetry breaking on the brane world inducing time dependent solutions, which are of the FRW type. The FRW dynamics are of the perfect fluid type, with an equation of state that respects the dominant energy condition and is consistent with experiment.

At present we have considered static solutions in the bulk with cosmological solutions being induced on the brane world once supersymmetry is broken. This has given us an accelerating universe. In order to explore the transition between an accelerating universe and matter domination we need to consider the bulk solution in a perfect fluid. For example, [11] has considered a self-tuned domain wall in five dimensional gravity coupled to a scalar field with bulk fluid, obtaining matter dominated FRW dynamics induced on the brane. Coupling this to our mechanism for an accelerating universe should result in a rich cosmology, such as a modified Friedmann equation on the brane-world. This is in progress.

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